1. Let $a \cdot b = ab - 4(a + b) + 20$. Evaluate
\[1 \cdot (2 \cdot (3 \cdot (\cdots (99 \cdot 100) \cdots))].\]

2. Suppose $z^3 = 2 + 2i$, where $i = \sqrt{-1}$. The product of all possible values of the real part of $z$ can be written in the form $\frac{p}{q}$ where $p$ and $q$ are relatively prime positive integers. Find $p + q$.

3. Let $\Gamma$ be the maximum possible value of $a + 3b + 9c$ among all triples $(a, b, c)$ of positive real numbers such that
\[\log_{30}(a + b + c) = \log_8(3a) = \log_{27}(3b) = \log_{125}(3c).\]
If $\Gamma = \frac{p}{q}$ where $p$ and $q$ are relatively prime positive integers, then find $p + q$.

4. Let $a_1, a_2, \ldots$ be a sequence of positive real numbers such that $a_n = 11a_{n-1} - n$ for all $n > 1$. The smallest possible value of $a_1$ can be written as $\frac{p}{q}$, where $p$ and $q$ are relatively prime positive integers. Find $p + q$.

5. Let $f_0(x) = x$, and for each $n \geq 0$, let $f_{n+1}(x) = f_n(x^2(3-2x))$. Find the smallest real number that is at least as large as
\[\sum_{n=0}^{2017} f_n(a) + \sum_{n=0}^{2017} f_n(1-a)\]
for all $a \in [0, 1]$.

6. Together, Kenneth and Ellen pick a real number $a$. Kenneth subtracts $a$ from every thousandth root of unity (that is, the thousand complex numbers $\omega$ for which $\omega^{1000} = 1$) then inverts each, then sums the results. Ellen inverts every thousandth root of unity, then subtracts $a$ from each, and then sums the results. They are surprised to find that they actually got the same answer! How many possible values of $a$ are there?

7. The sum
\[\sum_{k=0}^{\infty} \frac{2^k}{5^{2^k} + 1}\]
can be written in the form $\frac{p}{q}$ where $p$ and $q$ are relatively prime positive integers. Find $p + q$.

8. Let
\[\frac{p}{q} = \frac{2017}{2 - \frac{1}{3 - \frac{1}{2 - \frac{1}{2 - \frac{1}{3 - \frac{1}{2 - \frac{1}{2 - \cdots}}}}}}}\]
where $p$ and $q$ are relatively prime positive integers. Find $p + q$. 

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