Geometry A

1. Frist Campus Center is located 1 mile north and 1 mile west of Fine Hall. The area within 5 miles of Fine Hall that is located north and east of Frist can be expressed in the form $\frac{2}{5} \pi - c$, where $a, b, c$ are positive integers and $a$ and $b$ are relatively prime. Find $a + b + c$.

2. Let $AD$ be a diameter of a circle. Let point $B$ be on the circle, point $C$ be on $AD$ such that $A, B, C$ form a right triangle with right angle at $C$. The value of the hypotenuse of the triangle is 4 times the square root of its area. If $BC$ has length 30, what is the length of the radius of the circle?

3. Let $\triangle ABC$ satisfy $AB = 17$, $AC = \frac{70}{3}$ and $BC = 19$. Let $I$ be the incenter of $\triangle ABC$ and $E$ be the excenter of $\triangle ABC$ opposite $A$. (Note: this means that the circle tangent to ray $AB$ beyond $B$, ray $AC$ beyond $C$, and side $BC$ is centered at $E$.) Suppose the circle with diameter $IE$ intersects $AB$ beyond $B$ at $D$. If $BD = \frac{a}{b}$ where $a, b$ are coprime positive integers, find $a + b$.

4. Triangle $ABC$ has $\angle A = 90^\circ$, $\angle C = 30^\circ$, and $AC = 12$. Let the circumcircle of this triangle be $W$. Define $D$ to be the point on arc $BC$ not containing $A$ so that $\angle CAD = 60^\circ$. Define points $E$ and $F$ to be the feet of the perpendiculars from $D$ to lines $AB$ and $AC$, respectively. Let $J$ be the intersection of line $EF$ with $W$, where $J$ is on the minor arc $AC$. The line $DF$ intersects $W$ at $H$ other than at $D$. The area of the triangle $FHJ$ is in the form $\frac{a}{b}(\sqrt{c} - \sqrt{d})$ for positive integers $a, b, c, d$, where $a, b$ are relatively prime, and the sum of $a, b, c, d$ is minimal. Find $a + b + c + d$.

5. Let $\triangle ABC$ be triangle with side lengths $AB = 9$, $BC = 10$, $CA = 11$. Let $O$ be the circumcenter of $\triangle ABC$. Denote $D = AO \cap BC, E = BO \cap CA, F = CO \cap AB$. If $1/AD + 1/BE + 1/FC$ can be written in simplest form as $\frac{a\sqrt{b}}{c}$, find $a + b + c$.

6. Let triangle $ABC$ have $\angle BAC = 45^\circ$ and circumcircle $\Gamma$ and let $M$ be the intersection of the angle bisector of $\angle BAC$ with $\Gamma$. Let $\Omega$ be the circle tangent to segments $AB$ and $AC$ and internally tangent to $\Gamma$ at point $T$. Given that $\angle TMA = 45^\circ$ and that $TM = \sqrt{100 - 50\sqrt{2}}$, the length of $BC$ can be written as $a\sqrt{b}$, where $b$ is not divisible by the square of any prime. Find $a + b$.

7. Let $ABCD$ be a parallelogram such that $AB = 35$ and $BC = 28$. Suppose that $BD \perp BC$. Let $\ell_1$ be the reflection of $AC$ across the angle bisector of $\angle BAD$, and let $\ell_2$ be the line through $B$ perpendicular to $CD$. $\ell_1$ and $\ell_2$ intersect at a point $P$. If $PD$ can be expressed in simplest form as $\frac{m}{n}$, find $m + n$.

8. Let $\omega$ be a circle. Let $E$ be on $\omega$ and $S$ be outside $\omega$ such that line segment $SE$ is tangent to $\omega$. Let $R$ be on $\omega$. Let line $SR$ intersect $\omega$ at $B$ other than $R$, such that $R$ is between $S$ and $B$. Let $I$ be the intersection of the bisector of $\angle ESR$ with the line tangent to $\omega$ at $R$; let $A$ be the intersection of the bisector of $\angle ESR$ with $ER$. If the radius of the circumscribed circle of $\angle EIA$ is 10, the radius of the circumscribed circle of $\angle SAB$ is 14, and $SA = 18$, then $IA$ can be expressed in simplest form as $\frac{m}{n}$. Find $m + n$. 

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