1. Suppose we have a convex quadrilateral $ABCD$ such that $\angle B = 110^\circ$ and the circumcircle of $\triangle ABC$ has a center at $D$. Find the measure, in degrees, of $\angle D$.

   *Note:* The circumcircle of a $\triangle ABC$ is the unique circle containing $A$, $B$ and $C$.

2. A right cone in $xyz$-space has its apex at $(0, 0, 0)$, and the endpoints of a diameter on its base are $(12, 13, -9)$ and $(12, -5, 15)$. The volume of the cone can be expressed as $a\pi$. What is $a$?

3. Let $\triangle ABC$ be a triangle with circumcenter $O$ and orthocenter $H$. Let $D$ be a point on the circumcircle of $ABC$ such that $AD \perp BC$. Suppose that $AB = 6$, $DB = 2$, and the ratio $\frac{\text{area}(\triangle ABC)}{\text{area}(\triangle HBC)} = 5$. Then, if $OA$ is the length of the circumradius, then $OA^2$ can be written in the form $\frac{m}{n}$, where $m, n$ are relatively prime nonnegative integers. Compute $m + n$.

   *Note:* The circumradius is the radius of the circumcircle.

4. Suppose we choose two real numbers $x, y \in [0, 1]$ uniformly at random. Let $p$ be the probability that the circle with center $(x, y)$ and radius $|x - y|$ lies entirely within the unit square $[0, 1] \times [0, 1]$. Then $p$ can be written in the form $\frac{m}{n}$, where $m$ and $n$ are relatively prime nonnegative integers. Compute $m^2 + n^2$.

5. Let $BC = 6, BX = 3, CX = 5$, and let $F$ be the midpoint of $BC$. Let $AX \perp BC$ and $AF = \sqrt{247}$. If $AC$ is of the form $\sqrt{b}$ and $AB$ is of the form $\sqrt{c}$ where $b$ and $c$ are nonnegative integers, find $2c + 3b$.

6. Let $\Gamma$ be a circle with center $A$, radius 1 and diameter $BX$. Let $\Omega$ be a circle with center $C$, radius 1 and diameter $DY$, where $X$ and $Y$ are on the same side of $AC$. $\Gamma$ meets $\Omega$ at two points, one of which is $Z$. The lines tangent to $\Gamma$ and $\Omega$ that pass through $Z$ cut out a sector of the plane containing no part of either circle and with angle $60^\circ$. If $\angle YXC = \angle CAB$ and $\angle XCD = 90^\circ$, then the length of $XY$ can be written in the form $\frac{\sqrt{a} + \sqrt{b}}{c}$ for integers $a, b, c$ where $\gcd(a, b, c) = 1$. Find $a + b + c$.

7. Let two ants stand on the perimeter of a regular 2019-gon of unit side length. One of them stands on a vertex and the other one is on the midpoint of the opposite side. They start walking along the perimeter at the same speed counterclockwise. The locus of their midpoints traces out a figure $P$ in the plane with $N$ corners. Let the area enclosed by convex hull of $P$ be $\frac{A}{B} \sin^m \left( \frac{\pi m}{N} \right)$, where $A$ and $B$ are coprime positive integers, and $m$ is the smallest possible positive integer such that this formula holds. Find $A + B + m + N$.

   *Note:* The convex hull of a figure $P$ is the convex polygon of smallest area which contains $P$.

8. Let $ABCD$ be a trapezoid such that $AB \parallel CD$ and let $P = AC \cap BD$, $AB = 21$, $CD = 7$, $AD = 13$, $[ABCD] = 168$. Let the line parallel to $AB$ trough $P$ intersect circumcircle of $BCP$ in $X$. Circumcircles of $BCP$ and $APD$ intersect at $P, Y$. Let $XY \cap BC = Z$. If $\angle ADC$ is obtuse, then $BZ = \frac{a}{b}$, where $a, b$ are coprime positive integers. Compute $a + b$. 

1