1. The product of the positive factors of a positive integer $n$ is 8000. What is $n$?

2. The least common multiple of two positive integers $a$ and $b$ is $2^5 \times 3^5$. How many such ordered pairs $(a, b)$ are there?

3. Let $f$ be a function over the natural numbers so that
   
   (a) $f(1) = 1$
   
   (b) If $n = p_1^{e_1} \ldots p_k^{e_k}$ where $p_1, \ldots, p_k$ are distinct primes, and $e_1, \ldots, e_k$ are non-negative integers, then $f(n) = (-1)^{e_1+\ldots+e_k}$.

   Find $\sum_{i=1}^{2019} \sum_{d|i} f(d)$.

4. Let $n$ be the smallest positive integer which can be expressed as a sum of multiple (at least two) consecutive integers in precisely 2019 ways. Then $n$ is the product of $k$ not necessarily distinct primes. Find $k$.

5. Consider the first set of 38 consecutive positive integers who all have sum of their digits not divisible by 11. Find the smallest integer in this set.

6. Let $f$ be a polynomial with integer coefficients of degree 2019 such that the following conditions are satisfied:

   (a) For all integers $n$, $f(n) + f(-n) = 2$.
   
   (b) $101^2 | f(0) + f(1) + f(2) + \ldots + f(100)$.

   Compute the remainder when $f(101)$ is divided by $101^2$.

7. For a positive integer $n$, let $f(n) = \sum_{i=1}^{n} \lfloor \log_2 i \rfloor$. Find the largest $n < 2018$ such that $n \mid f(n)$.

8. Call a positive integer $n$ compact if for any infinite sequence of distinct primes $p_1, p_2, \ldots$ there exists a finite subsequence of $n$ primes $p_{x_1}, p_{x_2}, \ldots, p_{x_n}$ (where the $x_i$ are distinct) such that

   $p_{x_1}p_{x_2}\cdots p_{x_n} \equiv 1 \pmod{2019}$

   Find the sum of all compact numbers less than $2 \cdot 2019$. 