1. Given the graph $G$ and cycle $C$ in it, we can perform the following operation: add another vertex $v$ to the graph, connect it to all vertices in $C$ and erase all the edges from $C$. Prove that we cannot perform the operation indefinitely on a given graph.

2. Prove that for every positive integer $m$, every prime $p$ and every positive integer $j \leq p^{m-1}$, $p^m$ divides $(p^m)^j - (p^{m-1})^j$.

3. Let $ABCDEF$ be a convex hexagon with area $S$ such that $AB \parallel DE, BC \parallel EF, CD \parallel FA$ holds, and whose all angles are obtuse and opposite sides are not the same length. Prove that the following inequality holds: $A_{ABC} + A_{BCD} + A_{CDE} + A_{DEF} + A_{EFA} + A_{FAB} < S$, where $A_{XYZ}$ is the area of triangle $XYZ$. 