1. Let $\gamma_1$ and $\gamma_2$ be circles centered at $O$ and $P$ respectively, and externally tangent to each other at point $Q$. Draw point $D$ on $\gamma_1$ and point $E$ on $\gamma_2$ such that line $DE$ is tangent to both circles. If the length $OQ = 1$ and the area of the quadrilateral $ODEF$ is $520$, then what is the value of length $PQ$?

2. Hexagon $ABCDEF$ has an inscribed circle $\Omega$ that is tangent to each of its sides. If $AB = 12$, $\angle FAB = 120^\circ$, and $\angle ABC = 150^\circ$, and if the radius of $\Omega$ can be written as $m + \sqrt{n}$ for positive integers $m, n$, find $m + n$.

3. Let $ABCD$ be a cyclic quadrilateral with circumcenter $O$ and radius $10$. Let sides $AB, BC, CD,$ and $DA$ have midpoints $M, N, P,$ and $Q$, respectively. If $MP = NQ$ and $OM + OP = 16$, then what is the area of triangle $\triangle OAB$?

4. Let $C$ be a circle centered at point $O$, and let $P$ be a point in the interior of $C$. Let $Q$ be a point on the circumference of $C$ such that $PQ \perp OP$, and let $D$ be the circle with diameter $PQ$. Consider a circle tangent to $C$ whose circumference passes through point $P$. Let the curve $\Gamma$ be the locus of the centers of all such circles. If the area enclosed by $\Gamma$ is $1/100$ the area of $C$, then what is the ratio of the area of $C$ to the area of $D$?

5. Triangle $ABC$ is so that $AB = 15, BC = 22$, and $AC = 20$. Let $D, E, F$ lie on $BC, AC,$ and $AB$, respectively, so $AD, BE, CF$ all contain a point $K$. Let $L$ be the second intersection of the circumcircles of $BFK$ and $CEK$. Suppose that $\frac{AK}{KD} = \frac{11}{7}$, and $BD = 6$. If $KL^2 = \frac{a}{b}$, where $a, b$ are relatively prime integers, find $a + b$.

6. Triangle $ABC$ has side lengths $13, 14,$ and $15$. Let $E$ be the ellipse that encloses the smallest area which passes through $A, B,$ and $C$. The area of $E$ is of the form $\frac{a \sqrt{b} \pi}{c}$, where $a$ and $c$ are coprime and $b$ has no square factors. Find $a + b + c$.

7. Let $ABC$ be a triangle with sides $AB = 34, BC = 15, AC = 35$ and let $\Gamma$ be the circle of smallest possible radius passing through $A$ tangent to $BC$. Let the second intersections of $\Gamma$ and sides $AB, AC$ be the points $X, Y$. Let the ray $XY$ intersect the circumcircle of the triangle $ABC$ at $Z$. If $AZ = \frac{p}{q}$ for relatively prime integers $p$ and $q$, find $p + q$.

8. $A_1A_2A_3A_4$ is a cyclic quadrilateral inscribed in circle $\Omega$, with side lengths $A_1A_2 = 28, A_2A_3 = 12\sqrt{3}, A_3A_4 = 28\sqrt{3},$ and $A_4A_1 = 8$. Let $X$ be the intersection of $A_1A_3, A_2A_4$. Now, for $i = 1, 2, 3, 4$, let $\omega_i$ be the circle tangent to segments $A_iX, A_{i+1}X,$ and $\Omega$, where we take indices cyclically (mod 4). Furthermore, for each $i$, say $\omega_i$ is tangent to $A_1A_3$ at $Y_i$, and $\Omega$ at $T_i$. Let $P_1$ be the intersection of $T_1X_1$ and $T_2X_2$, and $P_3$ the intersection of $T_3X_3$ and $T_4X_4$. Let $P_2$ be the intersection of $T_2Y_2$ and $T_3Y_3$, and $P_4$ the intersection of $T_1Y_1$ and $T_4Y_4$. Find the area of quadrilateral $P_1P_2P_3P_4$. 


