P U M : C

Number Theory B

1. Suppose that the greatest common divisor of n and 5040 is equal to 120. Determine the sum of the four smallest possible positive integers n.

2. Find the sum of the 23 smallest positive integers that are 4 more than a multiple of 23 and whose last two digits are 23.

3. Find the sum of all prime numbers p such that p divides

$$(p^2+p+20)^{p^2+p+2}+4(p^2+p+22)^{p^2-p+4}.$$

4. Compute the sum of all positive integers whose positive divisors sum to 186.

5. Given $k \geq 1$, let p_k denote the k-th smallest prime number. If N is the number of ordered 4-tuples (a, b, c, d) of positive integers satisfying $abcd = \prod_{k=1}^{2023} p_k$ with $a < b$ and $c < d$, find $N \pmod{1000}$.

6. Find the number of ordered pairs (x, y) of integers with $0 \leq x < 2023$ and $0 \leq y < 2023$ such that $y^3 \equiv x^2 \pmod{2023}$.

7. A positive integer $\ell \geq 2$ is called *sweet* if there exists a positive integer $n \geq 10$ such that when the leftmost nonzero decimal digit of n is deleted, the resulting number m satisfies $n = m\ell$. Let S denote the set of all sweet numbers ℓ. If the sum $\sum_{\ell \in S} \frac{1}{\ell-1}$ can be written as $\frac{A}{B}$ for relatively prime positive integers A, B, find $A + B$.

8. Given a positive integer ℓ, define the sequence $\{a_n^{(\ell)}\}_{n=1}^{\infty}$ such that $a_n^{(\ell)} = \lfloor n + \sqrt[\ell]{n} + \frac{1}{2} \rfloor$ for all positive integers n. Let S denote the set of positive integers that appear in all three of the sequences $\{a_n^{(2)}\}_{n=1}^{\infty}$, $\{a_n^{(3)}\}_{n=1}^{\infty}$, and $\{a_n^{(4)}\}_{n=1}^{\infty}$. Find the sum of the elements of S that lie in the interval $[1, 100]$.

Name:

Team:

Write answers in table below:

Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8