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ANIMAL SPIRITS, FINANCIAL CRISES AND PERSISTENT UNEMPLOYMENT*

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This article uses a rational expectations model with multiple equilibrium unemployment rates to explain financial crises. The model has equilibria where asset prices are unbounded above. I argue that this is an important feature of any rational-agent explanation of a financial crisis, since for the expansion phase of the crisis to be rational, investors must credibly believe that asset prices could keep increasing forever with positive probability. I explain the sudden crash in asset prices that precipitates a financial crisis as a large shock to expectations that leads to a permanent increase in the unemployment rate.

This article develops a rational expectations model with multiple equilibrium unemployment rates, where the price of capital may be unbounded above. I argue that this property is an important feature of any rational-agent explanation of a financial crisis, since for the expansion phase of the crisis to be rational, investors must credibly believe that asset prices could keep increasing forever with positive probability.

The stock market boom of the 1920s, the Japanese land boom of the 1980s and the US housing bubble of the 2000s were all characterised by dramatic increases in the value of asset prices, a high growth rate of consumption and GDP, and a falling unemployment rate. Following each of these episodes, the economy entered a period of stagnation. The most severe of these was the Great Depression of the 1930s when the US unemployment rate increased from 2% to 25% and remained above 15% for a decade. The Japanese economy has still not fully recovered more than 20 years after Japanese property prices collapsed in 1989. A more recent example is provided by the Great Recession that followed the 2008 financial crisis. This recession was declared over by the NBER in June of 2009 but US unemployment has remained above 8% for 30 consecutive months.

For the past 30 years, economists have constructed dynamic stochastic general equilibrium models (DSGE) to explain business cycles. This agenda began with real business cycle theory, a framework that explains fluctuations in economic activity as the optimal response of a representative agent to random productivity shocks. Its close cousin, the new-Keynesian paradigm, adds additional shocks and nominal frictions. Both of these paradigms explain changes in asset prices, and changes in employment, as the equilibrium response of rational agents to changes in fundamentals.

Typically, financial crises are preceded by a period of rapid expansion in economic activity and rapid asset price appreciation followed by a crash in asset prices and a sharp persistent increase in the unemployment rate. Real business cycle models and

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conventional new-Keynesian DSGE models cannot explain these features of business cycles because there are no obvious candidates to explain which of the fundamentals was responsible either for the expansion or for the crash.

In contrast, in the model I develop in this article, booms and busts are caused by self-fulfilling bouts of optimism and pessimism. In a boom, it is rational for investors to keep bidding up asset prices because there are no physical or behavioural constraints that prevent the price from going even higher. The expansion phase of the crisis is fully rational.[1]

But although asset prices could continue to rise, there is nothing to ensure that they will continue to rise other than the collective beliefs of market participants. Asset prices are moved by what George Soros has called 'the mood of the market'. If market participants lose confidence in the markets, there are many other paths for asset prices that are consistent with alternative beliefs. I explain the end of the expansion, the *Minsky moment*, as a large self-fulfilling shock to beliefs about future asset prices that causes a permanent increase in the unemployment rate.[2]

1. Structure of the Paper

The article has four main Sections. I begin, in Section 2, by describing the physical environment and the structure of preferences and technology. Here, I solve the problem faced by a social planner and I show that there is a unique solution to the planning problem that defines an optimal unemployment rate.

In Section 3, I describe a decentralised equilibrium in which households and firms take prices as given and where unemployed workers must search for jobs. In contrast to standard search theory (Rogerson *et al.*, 2005), I drop the assumption that firms and workers bargain over the wage and I assume instead, as in Farmer (2010*b*, 2012*b*), that firms and workers are price takers in the labour market as well as in the product market.

In Section 4, I study the properties of equilibria and I prove two results. First, I show that there is a number $\mu < 1$ such that any unemployment rate in an interval $[0,\mu)$ is a steady state equilibrium. Second, I show that for a class of technologies that includes the ubiquitous case of a Cobb-Douglas production function, the steady state price of capital is a monotonically increasing function of the employment rate. I close the model by pinning down the price of capital with a belief function. As in Farmer (2002, 2012*a*), this function has the status of an independent fundamental equation that selects which of the many equilibria will prevail.

In Section 5, I study the quantitative properties of the model and I discuss the robustness of these properties to alternative modelling assumptions. First, I show that the steady state values of consumption, the relative price of capital, the real wage and the fraction of resources devoted to recruiting, are all approximately linear functions of employment over the range of unemployment rates that we have observed historically in US data. Over this range, all of these variables fluctuate within bounds

[1] This is in contrast to the popular notion that the expansion phase of a financial crisis is an asset price bubble, fuelled by 'irrational exuberance'.

[2] The term 'Minsky moment', named after the economist Hyman Minsky, was coined in 1998 by Paul McCulley of PIMCO, to describe the 1998 Russian financial crisis.

that are consistent with observation. Then I linearise the model around one of the many steady state equilibria and I show that belief shocks and productivity shocks have highly persistent effects on the unemployment rate.

The fact that temporary shocks have permanent effects implies that this model displays hysteresis.[3] The model generates a time series for the logarithm of the relative price of capital and for the logarithm of a transformation of the unemployment rate that follow cointegrated random walks.[4] I have argued elsewhere (Farmer, 2010*a*, 2012*d*) that this is exactly the behaviour we see in the data.

1.1. *Relationship to Previous Work*

A multiple equilibrium model that can account for the growth phase of a financial crisis must have two features. First, the model must have multiple equilibria. Second, the equilibria must be capable of explaining explosive growth in asset prices.

In Farmer (2012*b*), I constructed a model with search and matching frictions in the labour market. Although that model contains a continuum of steady state unemployment rates, it cannot explain the growth phase of the cycle because the asset price is bounded above and every bull market must come to an end at a predictable future date. As in conventional models, explosive growth in asset prices is ruled out by the assumption that actors are rational and forward looking.

In Farmer (2012*b*), I made the assumption that all labour is fired and rehired every period. I made that assumption for expository purposes, to highlight my main contribution: there is a continuum of steady state equilibria in models with incomplete factor markets (Farmer, 2006, p. 12). The assumption that all labour is fired and rehired every period allowed me to construct a simple model that conveniently illustrates that point.

In this article, I relax this assumption, and I model labour as a state variable as in standard models of labour search of the kind pioneered by Diamond (1982), Mortensen (1984) and Pissarides (1984). By modifying my model in this way, I am able to construct a calibrated example in which the values of the real wage, consumption, unemployment and the fraction of resources devoted to recruiting, all lie within empirically reasonable bounds. This modification is also responsible for the main result of the current article; that any positive price of capital can prevail in equilibrium and that every steady state unemployment rate is associated with a unique relative price of capital.

In my earlier work (Farmer, 2012*b*), I assumed that technology is Cobb-Douglas and preferences are logarithmic. In this article, I relax these two assumptions by allowing for the more general case of a constant elasticity of substitution (CES) production function with substitution parameter ρ and constant relative risk aversion (CRA)

[3] Blanchard and Summers (1986, 1987) have argued convincingly that unemployment is highly persistent and that persistence should be modelled by a dynamical system that displays hysteresis. Hysteresis means that a small perturbation of the initial conditions leads to a similar perturbation of the eventual steady state. In a system that displays hysteresis, the equilibrium is path dependent.

[4] The qualifier 'transformations' is necessary because a random walk is unbounded above and below. It is the logarithm of the relative price of capital and the logarithm of a logistic transformation of the unemployment rate that follow random walks in this model.

preferences with curvature parameter η. Although I maintain the assumptions of Cobb-Douglas technology and logarithmic preferences in my calibrated example, the extension to more general preferences and technologies is important since it demonstrates that the main results of the article do not rely upon special assumptions about the environment.

2. A Social Planning Problem

I begin by describing preferences and technology and solving the problem of a social planner whose goal is to maximise the welfare of a representative agent. The social planner is constrained by two technologies, one for moving unemployed workers from home to work; I call this the search technology, and one for transforming labour and capital into the consumption commodity; I call this the production technology.

2.1. *The Household's Preferences*

There is a continuum of identical households, each of whom derives utility from consumption of a unique commodity, C_t. Households maximise expected utility,

$$J = \mathrm{E}_s\left(\sum_{t=s}^{\infty} \beta^{t-s} \frac{C_t^{1-\eta}}{1-\eta}\right). \tag{1}$$

I assume that leisure does not yield utility and hence the participation rate will be constant and equal to 100%.[5]

The representative household has a measure 1 of workers that may be employed or unemployed. I represent the measure of workers that are unemployed and searching for a job with the symbol U_t and I represent the measure of workers engaged in production at the beginning of the period with the symbol L_t. These variables are related to each other by the constraint,

$$U_t = 1 - L_t. \tag{2}$$

2.2. *The Production Technology*

The consumption commodity is produced using the technology

$$C_t = \begin{cases} [bS_t^{\rho}(X_t)^{\rho} + aK_t^{\rho}]^{\frac{1}{\rho}}, & \text{if} \quad \rho \neq 0, \\ C_t = S_t^b X_t^b K_t^a, \quad a + b = 1, & \text{if} \quad \rho = 0, \end{cases} \tag{3}$$

where, X_t is labour used in production, K_t is capital and S_t is a technology shock. I assume that the representative firm has a measure L_t of available workers at date t. A measure X_t of these workers is allocated to the activity of production and a measure V_t is allocated to the activity of recruiting new workers. V_t and X_t are related by the constraint

[5] In an online technical appendix, (available at http://www.rogerfarmer.com), I show that this assumption is not essential and I demonstrate in a simple example, based on the static model from Farmer (2012*b*), that the major results of the article can be extended to the case of endogenous leisure.

$$X_t + V_t = L_t. \tag{4}$$

Capital is in fixed supply and

$$K_t = 1. \tag{5}$$

I have chosen to model capital as fixed, because I am interested in the connection between the relative price of capital and the unemployment rate. The assumption that capital is inelastically supplied allows me to discuss asset pricing without the need to construct a more complicated environment with multiple goods.[6]

2.3. *The Search Technology*

Each period, a fraction δ of workers separates exogenously from employment and a measure m_t of workers is hired. The separation and hiring processes are governed by the equations,

$$L_{t+1} = L_t(1 - \delta) + m_t, \tag{6}$$

and

$$m_t = (\Gamma V_t)^{\theta}(1 - L_t)^{1-\theta}, \tag{7}$$

where m_t is the measure of workers hired in period t, V_t is the measure of employed workers allocated to recruiting and $1 - L_t$ is the measure of unemployed workers searching for a job in period t.

Here, Γ measures the efficiency of the match process and θ measures the elasticity of the recruiting effort by firms. The parameter, θ, can be identified in data from estimates of the Beveridge curve. Using US data, Blanchard and Diamond (1990) found estimates of θ to be between 0.5 and 0.7. Since setting $\theta = 0.5$ will simplify some of the algebra of the model, I will make that assumption from this point on.[7]

2.4. *The Planner's Problem*

This economy satisfies all of the assumptions of standard general equilibrium theory. As the two technologies are convex and preferences are concave, the programming problem defined as

$$\max_{\{V_t, L_{t+1}\}} \mathrm{E}_s\left(\sum_{t=s}^{\infty} \beta^{t-s}\left\{\frac{[bS_t^{\rho}(L_t - V_t)^{\rho} + a]^{\frac{1-\eta}{\rho}}}{1-\eta} + \psi_t\left[L_t(1-\delta) + (\Gamma V_t)^{\frac{1}{2}}(1 - L_t)^{\frac{1}{2}} - L_{t+1}\right]\right\}\right), \tag{8}$$

has a unique solution.

[6] The simplest extension of this model would add produced capital with a one-sector technology where the produced good can be consumed or invested. That model is not a suitable vehicle with which to investigate unemployment and its connection to the stock market because the ability to produce the investment good implies that the price of capital, relative to the consumption good, is always equal to one. I leave the more general model, in which the consumption good and the investment good are produced from two different technologies, for future research.

[7] See the online technical appendix (available at http://www.rogerfarmer.com), where I relax this assumption and I show that Propositions 2 and 3 of the article can be extended to the case where θ is in the open interval (0,1). It is also possible to prove a version of Proposition 1, but the equation that defines the social planning solution is no longer quadratic and does not have a simple closed form expression.

Proposition 1. *Define the constants* A, B *and* C *as follows,*

$$A = \frac{\beta\Gamma^{\frac{1}{2}}}{2}, \quad B = 1 - \beta(1-\delta), \quad C = \frac{\beta\Gamma^{\frac{1}{2}}}{2}. \tag{9}$$

Let $\bar{X}$ *be the unique positive root of the quadratic*

$$AX^2 + BX - C = 0, \tag{10}$$

where $\bar{X}$ *is given by the expression*

$$\bar{X} = \frac{-[1-\beta(1-\delta)] + \sqrt{[1-\beta(1-\delta)]^2 + \Gamma\beta^2}}{\beta\Gamma^{\frac{1}{2}}}. \tag{11}$$

For values of β *close to* 1, *the optimal sequences* $\{V_s, L_s\}_{s=t}^{\infty}$ *that solve* (8) *converge asymptotically to a pair of numbers* $\{L, V\}$ *where*

$$L = \frac{\Gamma^{\frac{1}{2}}\bar{X}}{\delta + \Gamma^{\frac{1}{2}}\bar{X}}, \quad V = \left(\frac{\delta}{\delta + \Gamma^{\frac{1}{2}}\bar{X}}\right)\bar{X}^2. \tag{12}$$

For a proof of this Proposition, see Appendix A.

3. A Dynamic Equilibrium Model

In Section 3, I extend the equilibrium concept from Farmer (2010*b*, 2012*b*) to the dynamic model where labour is a state variable.

3.1. *Households*

The representative household solves the problem,

$$J = \mathrm{E}_s\left(\sum_{t=s}^{\infty} \beta^{t-s} \frac{C_t^{1-\eta}}{1-\eta}\right), \tag{13}$$

subject to the constraints

$$p_{k,t}K_{t+1} + p_t C_t \leq (p_{k,t} + r_t)K_t + w_t L_t, \tag{14}$$

$$L_{t+1} = L_t(1-\delta) + \tilde{q}_t(1-L_t). \tag{15}$$

Here, w_t is the money wage, p_t is the money price of commodities, $p_{k,t}$ is the money price of capital and r_t is the rental rate. Equation (15) represents the assumption that if $1 - L_t$ unemployed workers search, $\tilde{q}_t(1 - L_t)$ of them will find a job, where the fraction $\tilde{q}_t$ is determined in equilibrium by the aggregate search technology.

Since I need to value streams of payments I assume that a complete set of Arrow securities exists, one for each realisation of S_t. The price in units of goods at date t of a commodity delivered for certain at date τ in history $S^\tau \equiv \{S_t, S_{t+1}, \ldots S_\tau\}$ is given by the expression

$$Q_t^\tau = \frac{\beta^{\tau-t} p_t}{p_\tau}\left(\frac{C_\tau}{C_t}\right)^{-\eta}, \tag{16}$$

where I have suppressed the dependence of Q_t^τ on the history of shocks.

Using this definition, the transversality condition can be written as

$$\lim_{T\to\infty} Q_t^T p_{k,T} K_{T+1} = 0, \quad \text{for all histories } S^T. \tag{17}$$

In addition, the household will allocate resources through time optimally. That assumption leads to the following consumption Euler equation,

$$C_t^{-\eta} = \mathrm{E}_t\left[\beta C_{t+1}^{-\eta} \frac{p_t}{p_{t+1}}\left(\frac{p_{k,t+1} + r_{t+1}}{p_{k,t}}\right)\right]. \tag{18}$$

3.2. *Firms*

In a decentralised equilibrium, the technology is operated by a large number of competitive firms, each of which solves the problem,

$$\max_{\{K_t,V_t,X_t,L_t\}} \mathrm{E}_s\left[\sum_{t=s}^{\infty} Q_s^t\left(C_t - \frac{w_t}{p_t}L_t - \frac{r_t}{p_t}K_t\right)\right], \tag{19}$$

subject to the constraints,

$$C_t = \begin{cases} [bS_t^\rho (X_t)^\rho + aK_t^\rho]^{\frac{1}{\rho}}, & \text{if } \rho \neq 0, \\ C_t = S_t^b X_t^b K_t^a, \quad a + b = 1 & \text{if } \rho = 0, \end{cases} \tag{20}$$

$$L_t = X_t + V_t, \tag{21}$$

$$L_{t+1} = L_t(1-\delta) + q_t V_t. \tag{22}$$

Constraints (20), (21) and (22) hold for all $t = s,\ldots$. The sequences of money prices $\{p_t\}$, money wages $\{w_t\}$, money rental rates $\{r_t\}$ and the present value prices $\{Q_s^t\}$, are taken as given. In addition, the firm takes the sequence of search efficiencies of a recruiter, $\{q_t\}$, as given. All of these sequences are functions of the possible future histories of shocks.

Using (19)–(22), we may write the following Lagrangian for problem (19) as

$$\max \mathrm{E}_s \sum_{t=s}^{\infty}\left(Q_s^t\left\{[bS_t^\rho(L_t - V_t)^\rho + aK_t^\rho]^{\frac{1}{\rho}} - \frac{w_t}{p_t}L_t - \frac{r_t}{p_t}K_t + \psi_t[(1-\delta)L_t + q_t V_t - L_{t+1}]\right\}\right).$$

This expression is maximised when

$$a\left(\frac{C_t}{K_t}\right)^{1-\rho} = \frac{r_t}{p_t}, \tag{23}$$

$$bS_t^\rho\left(\frac{C_t}{L_t - V_t}\right)^{1-\rho} = \psi_t q_t, \tag{24}$$

and

$$\psi_t = \mathrm{E}_s\left(Q_t^{t+1}\left\{bS_{t+1}^{\rho}\left[\frac{C_{t+1}}{(L_{t+1}-V_{t+1})}\right]^{1-\rho} - \frac{w_{t+1}}{p_{t+1}} + \psi_{t+1}(1-\delta)\right\}\right). \tag{25}$$

The equations

$$C_t = \begin{cases} [bS_t^{\rho}(X_t)^{\rho} + aK_t^{\rho}]^{\frac{1}{\rho}}, & \text{if } \rho \neq 0, \\ C_t = S_t^b X_t^b K_t^a, \quad a+b=1 & \text{if } \rho = 0, \end{cases} \tag{26}$$

and

$$L_{t+1} = L_t(1-\delta) + q_t V_t, \tag{27}$$

must also hold. In addition, any optimal path must satisfy the transversality condition

$$\lim_{T\to\infty} Q_t^T \psi_T = 0 \quad \text{for all histories } S^T. \tag{28}$$

3.3. *Search*

The variables $\tilde{q}_t$ and q_t, are determined in equilibrium by market clearing in the markets for search inputs. Let a variable with a bar denote an economy-wide average. Using this notation, $\bar{L}_t$ is the measure of aggregate employment and L_t is the measure of workers hired by the representative firm. These variables are conceptually distinct although they turn out to be equal in equilibrium.

Each period I assume that in aggregate, a measure

$$\bar{m}_t = (\Gamma \bar{V}_t)^{\frac{1}{2}}(1-\bar{L}_t)^{\frac{1}{2}}, \tag{29}$$

of workers is hired and a measure $\delta \bar{L}_t$ of workers lose their jobs for exogenous reasons.

Together, these assumptions imply that the labour force in period $t+1$ will be given by the expression

$$\bar{L}_{t+1} = \bar{L}_t(1-\delta) + (\Gamma \bar{V}_t)^{\frac{1}{2}}(1-\bar{L}_t)^{\frac{1}{2}}. \tag{30}$$

Since (15) and (27) must also hold in a symmetric equilibrium it follows that

$$q_t = \Gamma^{\frac{1}{2}}\left(\frac{1-\bar{L}_t}{\bar{V}_t}\right)^{\frac{1}{2}}, \tag{31}$$

and

$$\tilde{q}_t = \Gamma^{\frac{1}{2}}\left(\frac{\bar{V}_t}{1-\bar{L}_t}\right)^{\frac{1}{2}}. \tag{32}$$

4. Characterising Equilibria

In this Section, I lay out the equations that characterise behaviour in a symmetric equilibrium of the model and I prove two Propositions.

In Proposition 2, I prove that there is an open set of stationary equilibria and that equilibria selected from this set have the property that the relative price of capital is unbounded above. This result is important because it implies that there will also be equilibria in which there is a positive probability that this price will grow without bound.

In Proposition 3, I show that for a class of technologies that includes the Cobb-Douglas case, the steady state relationship between employment and the relative price of capital is monotonically increasing. This property implies that for every steady state price of capital, there is a unique steady state unemployment rate.

4.1. *The Equations of the Model*

The following eight equations characterise the competitive equilibrium conditions. Equations (33) and (34) represent the Euler equation and the pricing kernel.

$$C_t^{-\eta} = \mathrm{E}_t\left[\beta C_{t+1}^{-\eta}\frac{p_t}{p_{t+1}}\left(\frac{p_{k,t+1}+r_{t+1}}{p_{k,t}}\right)\right], \tag{33}$$

$$Q_t^{t+1} = \frac{\beta p_t}{p_{t+1}}\left(\frac{C_{t+1}}{C_t}\right)^{-\eta}. \tag{34}$$

The next four equations combine optimising behaviour by firms with the search equilibrium condition (31),

$$\psi_t = \mathrm{E}_t\left\{Q_t^{t+1}\left[\psi_{t+1}q_{t+1} - \frac{w_{t+1}}{p_{t+1}} + \psi_{t+1}(1-\delta)\right]\right\}, \tag{35}$$

$$\frac{r_{t+1}}{p_{t+1}} = a(C_{t+1})^{1-\rho}, \tag{36}$$

$$bS_t^{\rho}\left(\frac{C_t}{L_t - V_t}\right)^{1-\rho} = \psi_t q_t, \tag{37}$$

$$L_{t+1} = L_t(1-\delta) + (\Gamma V_t)^{\frac{1}{2}}(1-L_t)^{\frac{1}{2}}. \tag{38}$$

Here, ψ_t is the shadow price of labour and q_t is given by the labour market search technology as

$$q_t = \Gamma^{\frac{1}{2}}\left(\frac{1-L_t}{V_t}\right)^{\frac{1}{2}}. \tag{39}$$

Finally, since I assume that $K_t = 1$, the production function,

$$C_t = \begin{cases} [bS_t^{\rho}(X_t)^{\rho} + a]^{\frac{1}{\rho}}, & \text{if } \rho \neq 0, \\ C_t = S_t^b X_t^b, & \text{if } \rho = 0, \end{cases} \tag{40}$$

must hold in aggregate.

These eight equations must determine the nine unknowns,

$$y_t \equiv \left\{C_t, L_t, V_t, \frac{r_t}{p_t}, \frac{w_t}{p_t}, \frac{p_{k,t}}{p_t}, Q_t, q_t, \psi_t\right\}. \tag{41}$$

The fact that there is one less equation than unknown arises from the absence of markets to allocate search intensity between the time of searching workers and the recruiting activities of firms, a point first made by Greenwald and Stiglitz (1988).

To close the model, I will assume that beliefs about the future value of asset prices, measured relative to the wage, are determined by an equation that I call a *belief function* (Farmer, 2002, 2010*a,b*, 2012*a,b*). This function describes how the variable x_t, defined as

$$x_t = \mathrm{E}_t\left(\frac{p_{k,t+1}}{w_{t+1}}\right), \tag{42}$$

depends on current and past observable variables.

The variable x_t is an independent state variable that selects one of the many possible equilibria. In Section 3, I provide a theory of how the sequence $\{x_t\}$ is determined, based on the idea that agents update their expectations of future asset prices using information from current asset prices. Before discussing this important additional equation, I first show that for every constant sequence $\{x\}$, the model possesses an equilibrium in which the nine endogenous variables, $C_t, L_t, V_t, r_t/p_t, w_t/p_t, p_{k,t}/p_t, Q_t, q_t$ and ψ_t are well defined.

4.2. *Steady State Equilibria*

In Farmer (2012*b*), I showed, in a version of this model where labour is fired and rehired every period, that there is a steady state equilibrium for any value of L in the interval [0,1]. In that model, for each equilibrium value of L, there is a different real asset price $p_{k,t}/w_t$ but asset prices are bounded above.

The following Definitions and Propositions extend my previous work to the dynamic model with CRA preferences and CES technology and show that, in equilibrium, asset prices are unbounded. I begin by defining a steady state equilibrium.

Definition 1. *A Non-stochastic Steady State Equilibrium is a vector* $\left\{C, L, V, r/p, w/p, p_k/p, Q, q, \psi\right\}$ *that solves the equations*

$$\frac{p_k}{w} = x, \tag{43}$$

$$\frac{C^{1-\rho}p}{p_k} = \frac{1-\beta}{a\beta}, \tag{44}$$

$$Q = \beta, \tag{45}$$

$$\psi[1-\beta(1-\delta)] = \beta q\psi - \beta\frac{w}{p}, \tag{46}$$

$$\frac{r}{p} = aC^{1-\rho}, \tag{47}$$

$$b\left(\frac{C}{L-V}\right)^{1-\rho}=\psi q, \tag{48}$$

$$\delta^2 L^2=\Gamma V(1-L), \tag{49}$$

$$q=\frac{\Gamma(1-L)}{\delta L}, \tag{50}$$

$$C_t=\begin{cases}[b(L-V)^\rho+a]^{\frac{1}{\rho}}, & \text{if } \rho\neq 0,\\ C_t=(L-V)^b, & \text{if } \rho=0.\end{cases} \tag{51}$$

These equations are derived from (33)–(40) and (42) by assuming that $S_t = 1$ for all t and solving the resulting non-stochastic equations for a steady state.

Proposition 2. *Define the constants λ, μ and Ω as follows:*

$$\lambda=\frac{\Gamma}{\Gamma+\delta^2},\quad \mu=\frac{\beta\Gamma}{\beta\Gamma+\delta(1-\beta(1-\delta))},$$

$$\Omega=\left(\frac{a\beta}{1-\beta}\right)\frac{\Gamma^\rho(\Gamma+\delta^2)^{1-\rho}}{\beta\Gamma+\delta[1-\beta(1-\delta)]}\left(\frac{\beta}{b}\right). \tag{52}$$

For all $L\in[0,\mu)$, there exists a steady state equilibrium. The values of the endogenous variables Q, C, V and q, for each value of L are given by the expressions

$$Q=\beta,\quad C=\left\{bL^\rho\left[1-\frac{\delta^2L}{\Gamma(1-L)}\right]^\rho+a\right\}^{\frac{1}{\rho}},$$

$$V=\frac{\delta^2L^2}{\Gamma(1-L)},\quad q=\frac{\Gamma(1-L)}{\delta L}, \tag{53}$$

and the values of the variable (r/p), ψ and (w/p) are computed from (35), (36) *and* (37). *The price of capital, measured in wage units is described by a continuous function: $g(L):[0,\mu)\rightarrow\tilde{P}\subset R_+$ where*

$$\frac{p_k}{w}=g(L)\equiv\frac{\Omega L^{1-\rho}(1-L)^\rho(\lambda-L)^{1-\rho}}{\mu-L}. \tag{54}$$

Proposition 3. *If $0\le\rho\le1$, $\tilde{P}\equiv R_+$, and the function g is strictly increasing with*

$$g(0)=0,\quad g(\mu)=\infty. \tag{55}$$

By the inverse function theorem there exists a function $h(x) = R_+ \to [0, \mu)$ *such that for all* $x \in R_+$ *there exists a steady state equilibrium, where*

$$L = h(x). \tag{56}$$

The steady state value of the vector of variables y, defined in (41) *is determined as in Proposition* 2.

Proposition 2 establishes that the equations that define a steady state equilibrium have a solution for a set of values of L less than some maximum value μ.[8] Proposition 2 is proved in Appendix B.

Proposition 3 goes further. It shows that, if $0 \le \rho \le 1$, L and p_k/w are related by a monotonically increasing function. When $L = 0$, $p_k/w = 0$ and p_k/w becomes infinite as L attains its upper bound. Proposition 3 is proved in Appendix C.

In my calibrated model I assume that the technology is Cobb-Douglas and, from Proposition 3, it follows that the function that links steady state asset prices with steady state employment, is invertible.[9]

4.3. *Closing the Model with a Belief Function*

The model I have described has a continuum of steady state equilibria. If this is to be a good description of the real world, I must take a stand on how agents form beliefs. As I have argued in my previous work, (Farmer, 2002, 2010*a,b*, 2012*a,b*), a model of multiple equilibria is an incomplete model. It must be closed by specifying a belief function. This is an independent equation that maps observations of current and past prices to expectations about future prices.

Applying that idea to this model, I make the assumption that beliefs, defined as

$$x_t \equiv \mathrm{E}_t\left(\frac{p_{k,t+1}}{w_{t+1}}\right), \tag{57}$$

are determined by the function,

$$x_t = x_{t-1}^{\lambda}\left(\frac{p_{k,t}}{w_t}\right)^{1-\lambda}\exp\left(s_t^b\right), \tag{58}$$

where s_t^b is a shock with distribution D, mean 0 and variance σ_b^2,

$$s_t^b \sim D\left(0, \sigma_b^2\right). \tag{59}$$

Equation (58) has the same form as the adaptive expectations equations first used by Friedman (1957) and Nerlove (1958) but, unlike their work, I am using adaptive

[8] The parameter Γ measures the efficiency of the match process. As Γ approaches ∞, the set of sustainable equilibrium employment rates approaches the interval [0,1].

[9] In an earlier version of this article, I asserted that restriction, $0 \le \rho \le 1$ is necessary in order for there to be an equilibrium in which the asset price is unbounded. I am indebted to Mingming Jiang of the University of California Riverside, for pointing out that this assertion is incorrect. In the model where labour is a state variable, there is always an equilibrium with an unbounded asset price. However, when ρ is less than 0, and large enough in absolute value, the function $g(L)$ becomes non-monotonic at sufficiently high employment rates.

expectations as a fundamental structural equation that replaces the labour supply equation in a model with incomplete factor markets (Farmer, 2006, p. 12).[10]

As is well known from the adaptive expectations literature (Nerlove, 1958), the adaptive expectations equation can be represented, up to a first order approximation, as follows[11]

$$\log(x_t) = \log(x_{t-1}) + (1-\lambda)\left\{\log\left(\frac{p_{k,t}}{w_t}\right) - \mathrm{E}_{t-1}\left[\log\left(\frac{p_{k,t}}{w_t}\right)\right]\right\} + s_t^b. \tag{60}$$

The parameter λ measures the influence of the current price of capital on expectations of the future price of capital. Last period's belief about the price of capital is updated by a fraction $(1 - \lambda)$ of last period's forecast error plus a random shock s_t^b which I assume has zero mean. Under this specification of beliefs, the expectation of the future price of capital is a non-stationary process that is hit by two kinds of shocks. When $\lambda = 1$, shocks are independent of the state of the economy. When $\lambda \neq 1$, there is an endogenous component to the shock that depends on the current realisation of the price of capital.

I also impose the normalisation,

$$w_t = 1, \quad \text{for all } t, \tag{61}$$

which represents the choice of the money wage as numeraire. When real magnitudes are defined relative to the money wage I say that these variables are measured in *wage units* (Farmer, 2010*b*, Chapter 5).

Although the choice of numeraire is innocuous, the specification of the belief function in wage units is not. By forming expectations this way, the functional form of the belief function remains invariant to changes in both inflation and growth. If households were to form beliefs about future asset prices defined relative to the consumption good, the parameters of that function would not remain invariant to changes in the growth process. Since growth can differ substantially from one decade to the next, the ability to make accurate forecasts, conditional on forecasts of the wage, provides an important planning advantage in a non-stationary world.[12]

It is important to recognise that (58) does not replace the rational expectations assumption. It is an independent equation that anchors beliefs in a world of multiple rational expectations equilibria. I will still maintain the rational expectations assumption which implies, in the log-linearised model, that the forecast errors, defined as

$$\begin{aligned} \eta_{ct} &\equiv \log(C_t) - \mathrm{E}_{t-1}(C_t), \quad & \psi_{yt} &\equiv \log(\psi_t) - E_{t-1}(\psi_t), \\ \eta_{pt} &\equiv \log(p_t) - \mathrm{E}_{t-1}(p_t), \quad & \eta_{pkt} &\equiv \log(p_{kt}) - E_{t-1}(p_{kt}), \end{aligned} \tag{62}$$

all have zero conditional means.

[10] In Farmer (2002), I show that adaptive expectations can be used to close a model with dynamic indeterminacy. In his Ph.D. thesis, Plotnikov (2013) uses the same idea to close a model with steady state indeterminacy. Farmer (2012*c*) provides a discussion of the role of dynamic and steady state indeterminacy in the history of economic thought. Dynamic and steady state indeterminacy are associated with what I call first and second generation models of endogenous business cycles.

[11] The equation is approximate because I have replaced the log of the expectation with the expectation of the log.

[12] In experiments on a related model of Plotnikov (2013), Plotnikov and I have found that the model generates counterfactual impulse response functions when it is closed with adaptive expectations defined in units of the consumption good. In contrast, adaptive expectations formed over wealth (in this case permanent income) defined in wage units provides a more accurate fit to the US data.

5. Quantitative Implications of the Model

Section 5 has four subsections. In subsection 5.1, I study the steady state connections between employment and the other key variables of the model. This is an appropriate exercise if one is interested in understanding persistent movements in unemployment and asset prices since these movements are governed by the steady state equations. The steady state relationships can potentially be uncovered from estimates of the cointegrating equations in the data.

In subsection 5.2, I study impulse responses to a linearised model. I demonstrate here that temporary shocks have persistent effects on all of the endogenous variables of the model and that the transition dynamics last for at most two quarters. This implies that it is movements along the steady state equations that I study in Subsection 1, that govern the persistent movements in asset prices and unemployment that characterise financial crises.

To check the conjecture that the model explains low frequency facts, in subsection 5.3 I graph the relationship between unemployment and the S&P500, measured in wage units and I compare it with the model equivalent.

Finally, in subsection 5.4, I show that belief shocks can act as an independent engine that drives business cycles. In the expansion phase of the cycle, households correctly forecast that the relative price of capital will continue to increase but they are aware that this process will end with positive probability. The end of the expansion is triggered by a large shock to expected future asset prices that leads to a self-fulfilling collapse in wealth and an increase in the unemployment rate.

5.1. *Properties of Different Steady State Equilibria*

Table 1 reports the parameter values used in a calibrated example of the model and Table 2 reports the implied solution to the social planning problem in an economy parameterised in this way.

In my example, technology is Cobb-Douglas and preferences are logarithmic. Under these assumptions, the parameter b represents labour's share of income which I set at 0.67. For the Cobb Douglas-logarithmic case, the parameter ρ is equal to 0 and $\eta = 1$.

The parameters δ and β are both dependent on the period and I calibrated them to quarterly data. I chose the quarterly separation rate, δ, to be 0.1 based on the interpretation of the JOLTS data from Shimer (2005) and I chose the quarterly discount factor β, to be 0.985. This implies an annual interest rate of 6%.

The parameter λ is important in determining the degree to which temporary shocks feed into persistent changes in employment and the price of capital. I chose, somewhat arbitrarily, a value of $\lambda = 0.75$ and I experimented by checking the robustness of the results to variations in λ. For values of λ in a range from 0.25 to 0.99, I found no change in the qualitative features of the impulse responses that I report below although lower values lead to larger long-run effects of temporary shocks and a longer period of adjustment.

The parameter Γ affects the optimal unemployment rate, the optimal measure of recruiters and the efficiency of an individual recruiter. I chose a value for this parameter of $\Gamma = 10$. Table 2 shows that this choice, in conjunction with the other

Table 1
Calibrated Values

Parameter	Symbol	Value
Labour's share	b	0.67
Production elasticity	ρ	0
Consumption elasticity	η	1
Separation rate	δ	0.1
Discount factor	β	0.985
Belief persistence	λ	0.75
Recruiting efficiency	Γ	10

Table 2
Values of Variables in the Planning Optimum

Variable	Symbol	Value
Employment rate	L^*	0.97
Capital price	p_k^*	27
GDP	C^*	0.96
Measure of recruiters	V^*	0.03
New hires per recruiter	q^*	3.3
Real wage	$1/p^*$	0.68

parameter choices, implies an optimal employment rate of 97%. In the social planning optimum, approximately 3% of all workers are engaged in recruiting activities, 94% are production workers and the remaining 3% are unemployed. This allocation implies that each recruiter can hire approximately 3.3 new workers every quarter. This seems low but the model neglects on-the-job hires which are a significant fraction of all transitions and allowing for a model with on the job search is beyond the scope of the current exercise.[13]

To check the robustness of my results to different parameterisations of the technology, I computed steady state values of all the endogenous variables for values of ρ of -10, -2, 0, 0.5 and 1. With the exception of the price of capital and the real wage, none of the reported results in Table 2 are sensitive to alternative choices of ρ. The price of capital varies from 16.3, when $\rho = -10$ to 28.4, when $\rho = 1$ and the real wage varies from 0.78 to 0.67. All of the steady state values are invariant to alternative parameterisations of the preference parameter η which does not appear in the equations that determine the steady state.

The unemployment rate in US data between 1948 and 2011, has varied from a low of 2.5% in May of 1953 to a peak of 10.8% in November of 1982. My calibrated example sets the optimal unemployment rate to 3%, which implies that the US economy has operated at or below capacity for most of the past the past 60 years.

[13] I do not have a strong feel for the 'right' value of Γ. The main impact of this parameter is on the value of the optimal employment rate. Taking Γ up to 100, for example, increases the optimal employment rate to 0.99 and the optimal measure of recruiters falls to 0.001. This implies that each recruiter can hire 27 new workers each quarter.

Figure 1 graphs the connections between employment, consumption, the price of capital, the real wage and the fraction of workers devoted to recruiting in a steady state equilibrium. The four panels of this Figure demonstrate that the steady state relationships amongst these five variables, over the observed range of employment rates, are all approximately linear.[14]

In Farmer (2012*b*), I demonstrated that the logarithm of the price of capital, measured in wage units, and the logarithm of a logistic transformation of the unemployment rate, are cointegrated random walks. I explained these data with a model in which exogenous movements in the price of capital, driven by self-fulfilling beliefs, cause movements in the unemployment rate. Under this interpretation, low frequency movements in the real price of capital, employment, the real wage, consumption and the number of recruiters are represented as movements along the curves depicted in Figure 1.

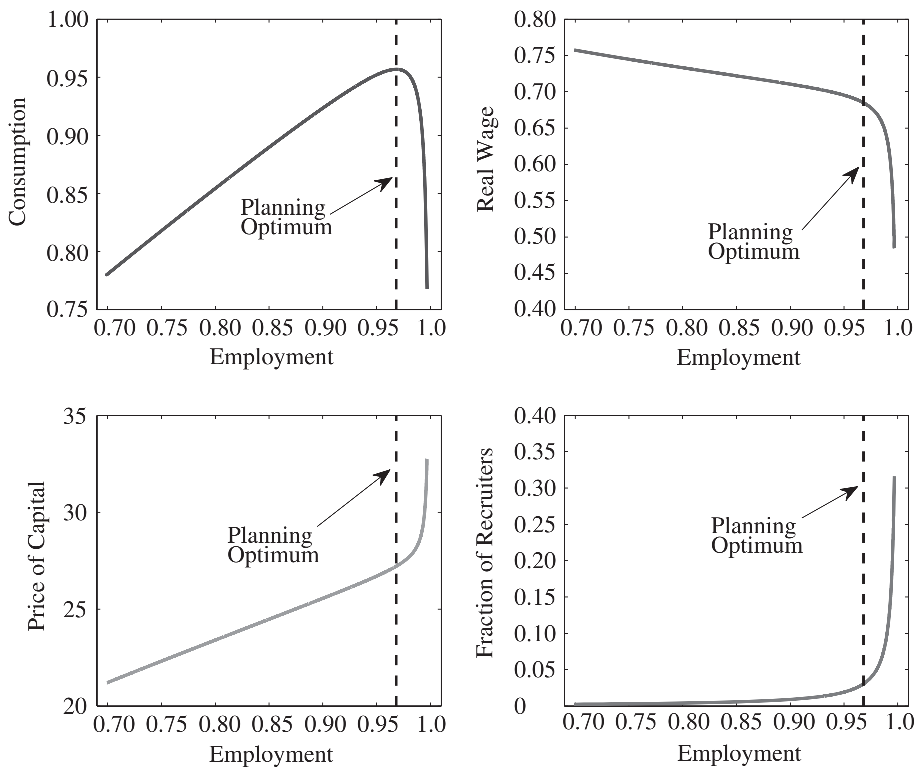

Fig. 1. *Steady State Relationships Between Some Key Variables*

[14] The qualitative features of these graphs are invariant to changes in ρ for values of ρ between 0 and 1. When ρ is less than 0, a case that Klump *et al.* (2004) have argued is empirically relevant, the price of capital becomes non-monotonic in employment once employment exceeds the planning optimum. Between the optimal employment level L^*, and the upper bound μ, there is a region in which increases in employment are associated with a falling price of capital before the graph turns around and p_k asymptotes to infinity as L approaches μ.

5.2. *Response to Shocks in a Linearised Model*

It is typical to study the properties of conventional DSGE models by linearising the model around the unique steady state equilibrium and studying the properties of the linear approximation. Since the model developed in this article is highly non-linear, the choice of a point around which to linearise the model is important. However, it is apparent from Figure 1 that over the observed range of unemployment in post-war US data, the key structural equations are approximately linear.[15] This approximate linearity suggests that, over this range, the choice of a point around which to linearise the model does not make much difference. I conducted a series of computational experiments to confirm that this is the case.

Figure 2 reports impulse response functions for four of the nine endogenous variables, consumption, employment, the price of capital and the real wage. The Figure was generated by linearising the model around an employment rate that is 95% of the social planning optimum. I experimented with values from 60% to 97% of the social planning optimum and, over this range, there is no change to either the qualitative or the quantitative features of this Figure.

In addition to the steady state indeterminacy of the non-linear model, the linearised model displays dynamic indeterminacy. I have argued elsewhere, Farmer (1991, 2000), that dynamic indeterminacy is a pervasive feature of DSGE models that can and should be exploited to explain why prices are slow to adjust in aggregate data. I have used that feature in the reported results in Figure 2 by setting the one-step ahead forecast error of the real wage equal to zero. Since I chose the money wage as numeraire, this assumption implies that the nominal price level is known one quarter in advance.

There are several features of Figure 2 that I want to draw attention to. First, notice that both TFP shocks (left hand panels) and belief shocks (right hand panels) have permanent effects on all of the four variables reported in the Figure. The permanent effect of temporary shocks occurs because agents update their beliefs using (60) and shocks to this equation have permanent effects on expectations of future wealth.

A 1% TFP shock causes consumption to increase by 0.8% on impact and approximately 0.05% permanently. The effect on employment is zero on impact because the current labour force is determined one quarter in advance. However, the prospect of higher future demand causes firms to shift workers from production to recruiting and, as a consequence, employment increases permanently by 0.12% beginning in quarter 2. It is important to notice that I have not assumed that TFP is autocorrelated. The permanent effect of a TFP shock is endogenous and is driven by the self-fulfilling belief that future wealth will be higher.

The third panel on the left of Figure 2 shows that a TFP shock causes the price of capital to increase by 0.65% on impact and 0.12% permanently. This increase in the relative price of capital occurs because investors rationally anticipate that future dividends will be higher. Finally, the bottom left panel shows that the real wage does not move on impact. This is a consequence of selecting an equilibrium in which the

[15] To save space, I have not graphed the relationships between employment and the remaining four structural variables, Q, q, V and r/p. These variables are also approximately linear for the observed range of unemployment data.

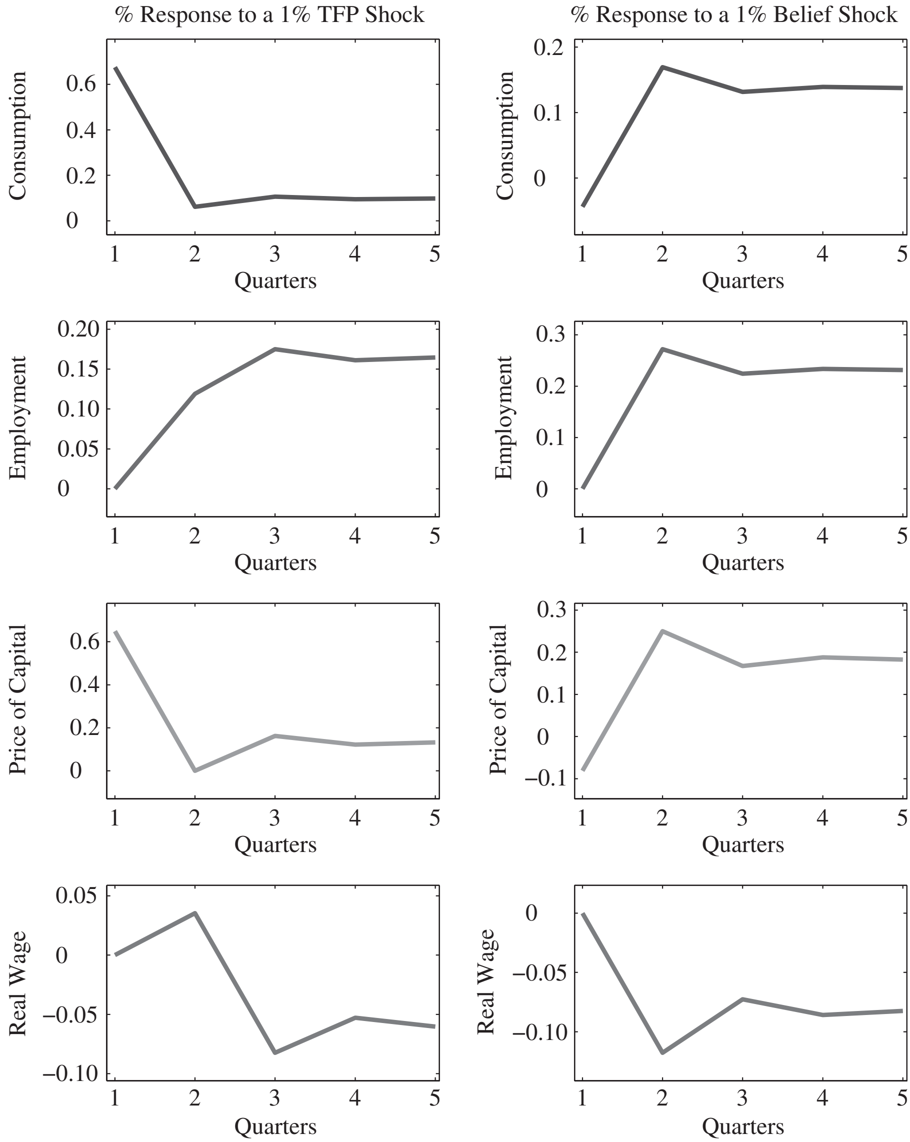

Fig. 2. *Impulse Responses to Two Kinds of Shocks*

price is predetermined. There is, however, a permanent reduction in the real wage of approximately 0.05%. This comes from the assumption that the economy moves up a concave production function in response to a permanent increase in aggregate demand.

The right panels of Figure 2 show the effects of a pure shock to beliefs. The top panel shows that a positive 1% shock to the expected future price of capital causes a small (approximately 0.05%) negative impact effect on consumption. This occurs as firms prepare for a future increase in demand by reallocating workers from production to recruiting. The effect is reversed one quarter after impact and by the fourth quarter, the effects on employment and consumption of a 1% belief shock settle down to their long-run values of 0.21% and 0.2% respectively. The effect of the 1% belief shock is to raise the price of capital permanently by 0.2% and to cause a small drop in the real wage, of the same order of magnitude as that of a TFP shock.

5.3. *Comparing the Model to Data*

Figure 2 shows that the model variables return to their long-run paths within two quarters following a shock. But because temporary shocks have permanent effects, unemployment and the price of capital do not return to a unique point. Data generated by this model will be I(1) series that are connected by a set of cointegrating equations. These cointegrating equations are the long-run relationships graphed in Figure 1.

To check the plausibility of this implication, the left panel of Figure 3 graphs the value of the S&P500, measured in wage units, against the unemployment rate. The right panel graphs the same variables generated by the model for a value of $\rho = -10$, which implies an elasticity of substitution between labour and capital of 0.1. The regression line in the left panel has a slope of -1.39 with a standard error of 0.23 and the slope of the line generated from the model is -1.1.[16]

5.4. *Financial Crises, Minsky Moments and Asymmetry*

Not all business cycles are generated by financial crises but, those that are, are characterised by protracted bursts of asset price appreciation and real GDP growth

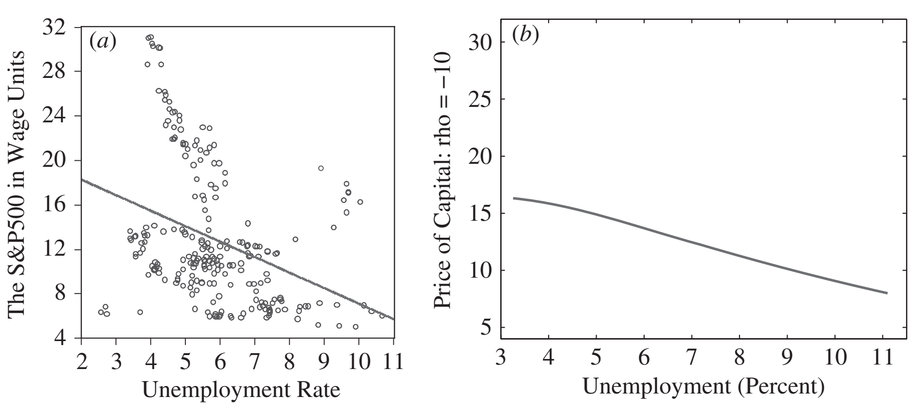

Fig. 3. *Unemployment and the Relative Price of Capital*

[16] I do not want to claim too much for this exercise, since production function estimates (Klump *et al.*, 2004) suggest that the elasticity of substitution is closer to 0.3 than to 0.1. However given the simplicity of the model, it is encouraging that there is a calibrated value of ρ for which it is able to capture this aspect of the time series data.

followed by steep falls in asset prices and large increases in the unemployment rate that sometimes lasts for a decade or more (Reinhart and Rogoff, 2009).

Why should the model described in this article lead to asymmetries of this kind? There are two possible answers. First, asymmetries may be built into the expectations mechanism itself. I assumed that the shock s_t^b that hits beliefs has a zero mean because, although the real value of the price of capital is a random walk, there is no evidence that it is a random walk with positive drift. If that were the case, we would expect to see an upward trend in a stock market index, measured in wage units, and a downward trend in the unemployment rate. There is no evidence of drift in these variables in the data.

But although there is no evidence of drift in wage deflated asset prices or the unemployment rate, there is evidence of an asymmetry in unemployment (Neftci, 1984). That asymmetry can be captured, in the model described in this article, by assuming that the distribution of shocks to beliefs is skewed. For example, shocks to beliefs might be driven by the mixture distribution,

$$s_t^b \sim p\mathrm{N}(1,\sigma^2) + (1-p)\mathrm{N}\left(\frac{-p}{1-p},\sigma^2\right). \tag{63}$$

By construction this shock has a zero mean; however, a model driven by sequences of shocks drawn from the distribution represented by (63) will experience asymmetric booms and busts. For $p > 0.5$, this shock process generates long expansions and short sharp contractions, much as we see in data. When ρ is close to 1, negative shocks will be infrequent but large. It is these rare large busts that are the model analogue of the 'Minsky moment' that I alluded to in the introduction.

A second possible reason why business cycles are asymmetric and why unemployment persists following a financial crisis is that the mechanisms of fiscal and monetary control that regulate modern market economies are constrained by government indebtedness or by monetary impotence as the nominal interest rate hits its lower bound. This explanation is consistent with the vision of business cycles described by Minsky (2008) in his widely acclaimed book, *Stabilizing an Unstable Economy*. In Minsky's view, the natural tendency of a free market economy is to swing between bouts of expansion and stagnation and it is the stabilising forces of fiscal and monetary interventions by government that have prevented post-war business cycles from replicating the worst excesses of nineteenth century capitalism.[17]

6. Conclusion

The two most recent recessions look a lot more like the 1929 contraction than any of the other post-war recessions. Each of them was accompanied by a boom and subsequent bust in asset prices, a feature that was not present in the other nine post-war recessions. In my view, the deregulation of financial markets in the 1990s had a

[17] In my view, this view is correct but Minsky's implementation of his vision is overly dismissive of conventional economic theory. I do not believe that we must jettison two hundred and fifty years of economic thought to accommodate his ideas.

lot to do with that. But what allowed asset prices to grow so fast in the first place and why was asset price growth not arbitraged away by efficient financial markets?

The answer I have given in this article is that a rapid expansion in asset prices is part of a rational expectations equilibrium in a world of multiple equilibria. It is not an example of 'irrational exuberance'. There is nothing in the economic environment to dictate that a bull market *must* come to an end in any given time period. However, equally, there is no reason why it should persist forever. Financial crises result from changing moods in the financial markets. Although they are equilibrium phenomena, in the sense of modern macroeconomic theory, they are not socially optimal. In the model I have constructed, not all equilibria are efficient, and that has important implications. My work suggests that economic policies designed to reduce the volatility of asset market movements will significantly increase welfare.

Appendix A. Proof of Proposition 1

Proof. A solution to (8) must satisfy the following first order conditions,

$$b\frac{[bS_t^{\rho}(L_t-V_t)^{\rho}+a]^{\frac{1-\eta-\rho}{\rho}}S_t^{\rho}}{(L_t-V_t)^{1-\rho}}=\frac{1}{2}\psi_t\Gamma^{\frac{1}{2}}\left(\frac{1-L_t}{V_t}\right)^{\frac{1}{2}}, \tag{A.1}$$

$$\psi_t=\mathrm{E}_t\left(\beta\left\{b\frac{[bS_{t+1}^{\rho}(L_{t+1}-V_{t+1})^{\rho}+a]^{\frac{1-\eta-\rho}{\rho}}S_{t+1}^{\rho}}{(L_{t+1}-V_{t+1})^{1-\rho}}+\psi_{t+1}\left[(1-\delta)-\frac{1}{2}\Gamma^{\frac{1}{2}}\left(\frac{V_{t+1}}{1-L_{t+1}}\right)^{\frac{1}{2}}\right]\right\}\right) \tag{A.2}$$

$$L_{t+1}=L_t(1-\delta)+(\Gamma V_t)^{\frac{1}{2}}(1-L_t)^{\frac{1}{2}}. \tag{A.3}$$

These equations must be obeyed by the optimal path $\{L_{s+1},V_s,\psi_s\}_{s=t}^{\infty}$, where L_t is given by an initial condition. Since the problem is concave, the solution is unique.

Let $\{L, V, \psi\}$ be a non-stochastic steady state solution of (8), defined as a solution to the equations,

$$b\frac{[b(L-V)^{\rho}+a]^{\frac{1-\eta-\rho}{\rho}}}{(L-V)^{1-\rho}}=\frac{\psi}{2}\Gamma^{\frac{1}{2}}\left(\frac{1-L}{V}\right)^{\frac{1}{2}}, \tag{A.4}$$

$$\psi=\beta b\frac{[b(L-V)^{\rho}+a]^{\frac{1-\eta-\rho}{\rho}}}{(L-V)^{1-\rho}}+\beta\psi(1-\delta)-\beta\psi\frac{1}{2}\Gamma^{\frac{1}{2}}\left(\frac{V}{1-L}\right)^{\frac{1}{2}}. \tag{A.5}$$

Rearranging these expressions, defining

$$X=\left(\frac{V}{1-L}\right)^{\frac{1}{2}}, \tag{A.6}$$

gives

$$AX^2+BX-C=0, \tag{A.7}$$

where,

$$A=\frac{\beta\Gamma^{\frac{1}{2}}}{2},\quad B=1-\beta(1-\delta),\quad C=\frac{\beta\Gamma^{\frac{1}{2}}}{2}. \tag{A.8}$$

This establishes the quadratic defined in the Proposition. The values of L and V are found by combining (A.6) with the steady state value of (A.3), given by,

$$\delta L = (\Gamma V)^{\frac{1}{2}}(1 - L)^{\frac{1}{2}}. \tag{A.9}$$

The local existence and convergence of dynamic paths, when β is 'close enough' to 1, are a consequence of the turnpike property of optimal growth models. For example, Cass (1966).

Appendix B. Proof of Proposition 2

Proof. Since only real variables are determined in equilibrium, I am free to choose the normalisation $w = 1$. In a steady state equilibrium it follows from (43) and (44) that,

$$p_k = \frac{a\beta}{1-\beta} Z, \tag{B.1}$$

where

$$Z \equiv pC^{1-\rho}. \tag{B.2}$$

I now seek an expression for Z as a function of L.

Combining (46) with (48), using the normalisation $w = 1$, gives,

$$\frac{[1-\beta(1-\delta)]}{q} b\left(\frac{C}{L-V}\right)^{1-\rho} = \beta b\left(\frac{C}{L-V}\right)^{1-\rho} - \beta\frac{1}{p}. \tag{B.3}$$

Combining (49) and (50) gives

$$q = \frac{\Gamma(1-L)}{\delta L}, \tag{B.4}$$

and substituting for q from (B.4) in (B.3) gives

$$\frac{1}{p} = b\left(\frac{C}{L-V}\right)^{1-\rho}\left\{\frac{\beta\Gamma(1-L) - \delta[1-\beta(1-\delta)]L}{\beta\Gamma(1-L)}\right\}. \tag{B.5}$$

Note that prices are non-negative whenever $L < \mu$, where

$$\mu = \frac{\beta\Gamma}{\beta\Gamma + \delta(1-\beta(1-\delta))}.$$

I next seek an expression for V as a function of L. Substituting from (B.4) into (50) gives

$$V = \frac{\delta^2 L^2}{\Gamma(1-L)}, \tag{B.6}$$

and hence

$$L - V = L\left[1 - \frac{\delta^2 L}{\Gamma(1-L)}\right]. \tag{B.7}$$

Substituting from (B.7) into (B.5) and rearranging terms gives

$$\begin{aligned} p_k &\equiv \frac{a\beta}{1-\beta} pC^{1-\rho} \\ &= \left(\frac{a\beta}{1-\beta}\right) \frac{\beta L^{1-\rho}\Gamma^{p}(1-L)^{\rho}[\Gamma(1-L) - \delta^2 L]^{1-\rho}}{b\{\beta\Gamma(1-L) - \delta[1-\beta(1-\delta)]L\}} \equiv g(L). \end{aligned} \tag{B.8}$$

Finally, using the definitions of Ω, μ and λ from (52), gives

$$g(L) = \frac{\Omega L^{1-\rho}(1-L)^{\rho}(\lambda - L)^{1-\rho}}{\mu - L},$$

which establishes the form of the function g.

Appendix C. Proof of Proposition 3

Proof. I must show that, for $\rho \geq 0$, g is strictly increasing. First notice from (52) that, since $0 < \beta < 1$,

$$\mu < \lambda < 1. \tag{C.1}$$

Taking the logarithmic derivative of g gives

$$\frac{L}{g}\frac{\partial g'}{\partial L}\bigg|_L = (1-\rho) - \rho\frac{L}{1-L} - (1-\rho)\frac{L}{\lambda - L} + \frac{L}{\mu - L}. \tag{C.2}$$

Rearranging terms

$$\overbrace{(1-\rho)}^{A_1} + \rho\,\overbrace{L\left(\frac{1}{\lambda - L} - \frac{1}{1-L}\right)}^{A_2} + \overbrace{L\left(\frac{1}{\mu - L} - \frac{1}{\lambda - L}\right)}^{A_3} > 0 \tag{C.3}$$

where

$$A_1 \geq 0, \quad A_2 > 0 \text{ and } A_3 > 0 \text{ for all } L \leq \mu. \tag{C.4}$$

The first inequality follows since $0 \leq \rho \leq 1$, and the second two inequalities follow from the additional facts that $\mu < \lambda < 1$ and $L < \mu$.

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