Technical Appendix
to
Animal Spirits, Financial Crises and Persistent Unemployment

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Part 1

This part extends the proof of propositions 2 and 3 to the case where $\theta \in (0, 1)$. The numbering mirrors the numbers from Appendix B in the paper.

Proof. Since only real variables are determined in equilibrium we are free to choose the normalization $w = 1$. In a steady state equilibrium„

$$p_k = \frac{a\beta}{1-\beta} Z, \tag{B1}$$

where

$$Z \equiv pC^{1-\rho}. \tag{B2}$$

We now seek an expression for Z as a function of L.

Using the normalization $w = 1$, and using the steady state equations from the paper, we have,

$$\frac{(1-\beta(1-\delta))}{q} b \left(\frac{C}{L-V}\right)^{1-\rho} = \beta b \left(\frac{C}{L-V}\right)^{1-\rho} - \beta \frac{1}{p}. \tag{B3}$$

Amending the search technology to allow for $\theta \in (0,1)$ implies,

$$q = \Gamma \left(\frac{1-L}{\delta L}\right)^{\frac{1-\theta}{\theta}}, \tag{B4'}$$

and substituting for q from (B4') in (B3) gives

$$\frac{1}{p} = b \left(\frac{C}{L-V}\right)^{1-\rho} \left(\frac{\beta\Gamma(1-L)^{\frac{1-\theta}{\theta}} - (1-\beta(1-\delta))(L\delta)^{\frac{1-\theta}{\theta}}}{\beta\Gamma(1-L)^{\frac{1-\theta}{\theta}}}\right). \tag{B5'}$$

Substituting from (B4') into the amended version of Equation (50) from the paper gives

$$V = \frac{(\delta L)^{\frac{1}{\theta}}}{\Gamma(1-L)^{\frac{1-\theta}{\theta}}}, \tag{B6'}$$

and hence

$$L - V = L\left(1 - \frac{(\delta L)^{\frac{1}{\theta}}}{\Gamma(1-L)^{\frac{1-\theta}{\theta}}}\right). \tag{B7'}$$

Substituting from (B7') into (B5') and rearranging terms gives

$$p_k \equiv \frac{a\beta}{1-\beta} pC^{1-\rho} = \frac{\Omega L^{1-\rho}(\lambda - z)^{1-\rho}}{(\mu - z)}, \tag{B8'}$$

where

$$\Omega = \left(\frac{\alpha\beta}{1-\beta}\right)\left(\frac{\beta}{b}\right)\frac{\delta^{\frac{\theta-\rho}{\theta}}}{(1-\beta(1-\delta))\,\Gamma^{(1-\rho)\left(\frac{1-\theta}{\theta}\right)-1}}, \qquad \lambda = \frac{\Gamma}{\delta^{\frac{1}{\theta}}},$$

$$\mu = \frac{\beta\Gamma}{(1-\beta(1-\delta))\,\delta^{\frac{1-\theta}{\theta}}}, \qquad z = \left(\frac{L}{1-L}\right)^{\frac{1-\theta}{\theta}}.$$

Finally, using thee definitions of Ω, μ, λ and z,

$$g(L) = \frac{\Omega L^{1-\rho}(\lambda - z)^{1-\rho}}{(\mu - z)},$$

which establishes the form of the function g. Notice that as $z \to \mu$, $p_k \to \infty$. Since z is strictly increasing in L for $\theta \in (0,1)$ this extends Proposition 2 to the case of a general Cobb-Douglas matching function. Notice that when $\theta = 1$, this argument breaks down. It is straightforward to extend the proof of Proposition 3 using the same argument used in Appendix C. ■

Part 2

This part amends the static model from Farmer (2012) by allowing for variable labor force participation. The extension to the dynamic model studied in this paper is straightforward but lengthy.

Let preferences be given by the expression

$$U = \log(C) + \chi \log(1-H), \tag{E1}$$

where the household sends H worker to search for jobs and L of them are successful. L and H are related by the expression

$$L = \tilde{q}H, \tag{E2}$$

where $\tilde{q}$ is taken as given by households. The budget constraint of the household is given by

$$pC \leq wL = wH\tilde{q}. \tag{E2}$$

This specification assumes that unemployed workers who search give the same disutility to the household as those who find jobs.

The firm's problem in the static economy is to allocate workers between search and recruiting to maximize profit, given by

$$pC - wL, \tag{E4}$$

subject to the constraints

$$C = (L - V)^{\alpha}, \quad L = qV. \tag{E5}$$

The aggregate matching technology is given by

$$L = H^{\frac{1}{2}} V^{\frac{1}{2}}. \tag{E6}$$

Combining (E2), (E5) and (E6) gives

$$\tilde{q} = \frac{1}{q}. \tag{E7}$$

The first order conditions for the two problems yield

$$\frac{\alpha q C}{L - V} = \frac{w}{p} = \frac{\chi C q}{\tilde{q}(1 - H)}. \tag{E8}$$

Combining (E8) with (E2), (E5) and (E7) yields an expression linking q with L.

$$q = \frac{\alpha + bL}{L(b + \alpha)}. \tag{E9}$$

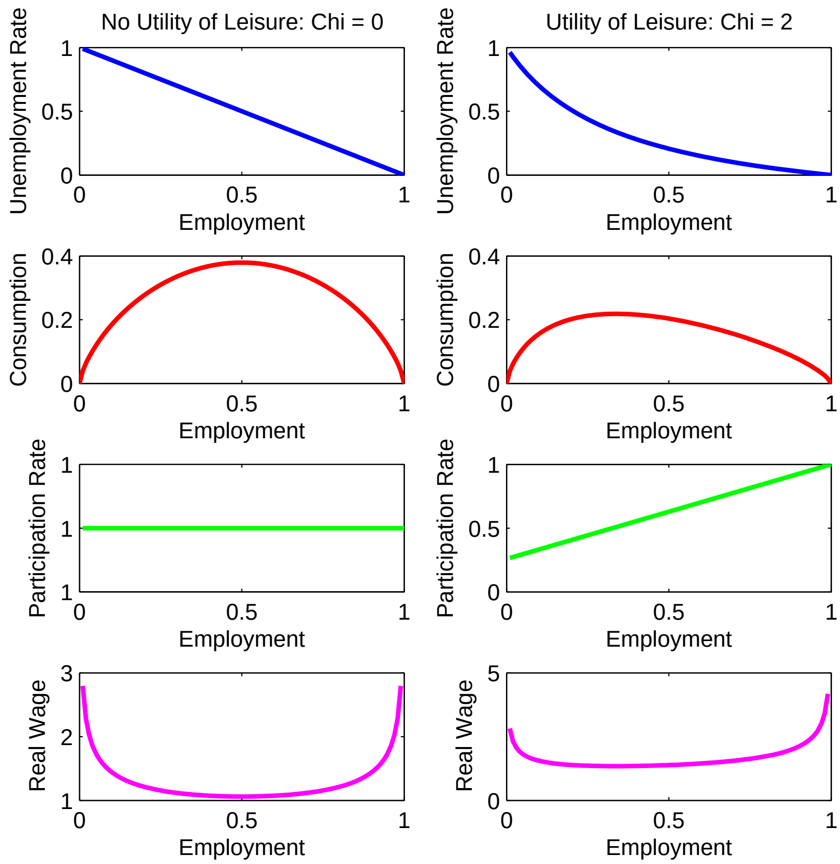

Figure 1: A Comparison of two Cases

The model with variable leisure collapses to the static model studied in Farmer (2012) in the case when the leisure parameter χ is set equal to 0. Figure 1 compares the case of no disutility of leisure in the four left panels with the case where leisure does yield disutility in the four right panels. In each case, I have graphed the unemployment rate, consumption, the participation rate and the real wage against employment. As in the dynamic model, any employment rate can be a steady state equilibrium of this model. The main qualitative difference of allowing for variable leisure is that the model now makes a prediction for how the participation rate varies with employment.

There is also a qualitative difference of the static model from the dynamic model studied in the body of the paper regarding the behavior of the real wage. In the dynamic model the real wage is decreasing in employment over its entire range. In the static model, the real wage is increasing in employment once the employment rate exceeds the social planning optimum. By comparing the left and right panels of the figure it is clear that this difference arises as a consequence of differences between the static and dynamic models and not as a consequence of assuming the leisure yields utility.

References

Farmer, R. E. A. (2012): "Confidence, Crashes and Animal Spirits," *Economic Journal*, 122(559).