

Testing for Indeterminacy: An Application to U.S. Monetary Policy: Comment

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The equilibrium of a rational expectations model is determinate if it is locally unique; it is indeterminate if many other equilibria are arbitrarily close to the first equilibrium. If equilibria are indeterminate, nonfundamental shocks may contribute to the variance of economic fluctuations and, if agents are risk averse, these fluctuations will reduce welfare. Hence, it is of some importance to a policymaker to ensure that his actions do not induce indeterminacy.

In an influential article, Richard Clarida, Jordi Galí, and Mark Gertler (2000) have argued that US monetary policy led to an indeterminate equilibrium in the period from 1950 through 1979 and to a determinate equilibrium in the period since 1980. Their work has been criticized by Thomas Lubik and Frank Schorfheide (2004) who point out that determinacy is a property of a system that cannot be established using single-equation methods. Lubik and Schorfheide write down a fully specified rational expectations model based on a representative agent economy. Using a Bayesian approach, they specify a prior probability distribution over parameters that places equal weight on determinate and indeterminate regions of the parameter space. Using data for the US economy on the output gap, the interest rate and the inflation rate, they compute posterior odds ratios for these regions and are able to strongly confirm Clarida, Galí, and Gertler's (2000) findings.

In this note we point out, by means of a simple example, that it is not possible to decide whether real world data are generated

by a determinate or an indeterminate process without making untestable assumptions about the dynamic structure of the underlying model. We construct two models that generate the same likelihood function and, hence, are observationally equivalent. Model 1 displays an indeterminate equilibrium driven purely by nonfundamental (sunspot) shocks. Model 2 displays a determinate equilibrium driven purely by fundamental shocks.

I. Background

The possibility that the equilibria of infinite-horizon monetary economies may be indeterminate has been recognized at least since the 1970s. More recently, attention has been drawn to indeterminacy in real economies: Jess Benhabib and Farmer (1994) provide a simple version of a real business-cycle model, with increasing returns to scale, which displays indeterminate equilibria, and Farmer and Jang-Ting Guo (1994) calibrate this model and simulate data that mimic the properties of a real business cycle model. Two papers by Takashi Kamihigashi (1996) and Harold L. Cole and Lee Ohanian (1999) point to an observational equivalence between sunspot and nonsunspot models, but there has been very little work that we are aware of on the econometrics of this issue. Farmer and Guo (1995) is the first paper we know of that attempts to test for indeterminacy in a fully specified econometric model. Hashem Pesaran (1987) points out that restrictions on lag-length will play an important role in deciding the issue of indeterminacy in linear rational expectations models, although the consequences of this point for policy analysis do not seem to have been widely recognized. Both Farmer and Guo (1995) and Lubik and Schorfheide (2004) rely on a priori restrictions of this kind.

II. Two Equivalent Models

This section constructs an example to illustrate our main point. We write down two single equation models that govern the behavior of a

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scalar variable, p_t . In Model 1, p_t depends only on its own future expectation, and we choose parameters such that the model has an indeterminate equilibrium that is driven by nonfundamental noise. For simplicity, we assume that there is no fundamental uncertainty in this economy, although the example could easily be complicated to allow for this possibility. In Model 2, p_t depends on its own expected future values, and it also depends on p_{t-1} . We choose parameters to ensure that there is a unique rational expectations equilibrium.

A. Model 1

This model has a single structural equation that takes the form

$$(1) \quad p_t = aE_t[p_{t+1}],$$

and we impose the parameter restriction $|a| > 1$. We write the system as a first-order, matrix-difference equation in the two endogenous variables p_t and $E_t[p_{t+1}]$:

$$(2) \quad \begin{matrix} \mathbf{A} & \mathbf{Y}_t \\ \begin{bmatrix} 1 & -a \\ 1 & 0 \end{bmatrix} & \begin{bmatrix} p_t \\ E_t[p_{t+1}] \end{bmatrix} \end{matrix} \\ \\ \begin{matrix} \mathbf{B} & \mathbf{Y}_{t-1} & \mathbf{\Psi}_w \\ \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} & \begin{bmatrix} p_{t-1} \\ E_{t-1}[p_t] \end{bmatrix} & + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \end{matrix} w_t.$$

Equation (2) contains a nonfundamental error, w_t , which is defined in the second row of equation (2) to be the difference between p_t and its date $t - 1$ expectation. In a determinate rational expectations model, this nonfundamental shock would be endogenously determined as a function of the fundamental shocks to the system in a way that removes the influence of any explosive root. In the case of indeterminate equilibria, there are not enough explosive roots to uniquely determine the endogenous variables of the model. This is the case in our example, since we make the assumption $|a| > 1$. In our example, there are no fundamental shocks, and w_t represents an independent nonfundamental shock.

The reduced form of equation (2) is found by eliminating the influence of the unstable roots.

In our example, the matrix \mathbf{A} is invertible and one can compute the roots of $\mathbf{A}^{-1}\mathbf{B}$ by hand¹: they are equal to 0 and λ , where $\lambda \equiv a^{-1}$. The reduced form is given by the expression

$$(3) \quad \begin{bmatrix} p_t \\ E_t[p_{t+1}] \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & \lambda \end{bmatrix} \begin{bmatrix} p_{t-1} \\ E_{t-1}[p_t] \end{bmatrix} + \begin{bmatrix} 1 \\ \lambda \end{bmatrix} w_t.$$

Appendix A shows how to rewrite this equation, using p_{t-1} instead of $E_{t-1}[p_t]$ as the state variable, to obtain the following expressions for p_t and $E_t[p_{t+1}]$ as functions of the observable variable p_{t-1} and the sunspot shock w_t :

$$(4) \quad p_t = \lambda p_{t-1} + w_t,$$

$$(5) \quad E_t[p_{t+1}] = \lambda^2 p_{t-1} + \lambda w_t.$$

B. Model 2

For the case of Model 2, we assume again that there is a single structural equation given by the expression

$$(6) \quad p_t = aE_t[p_{t+1}] + bp_{t-1} + v_t.$$

Equation (6) differs from (1) in three respects. First, the lagged state variable p_{t-1} enters the equation; second, there is a fundamental shock, v_t ; and third, we choose a and b such that the equilibrium of the model is determinate.

Equation (6) can be written as a vector system by writing the definition of w_t as a separate equation. This form of the model is represented below:

$$(7) \quad \begin{matrix} \mathbf{A} & \mathbf{Y}_t \\ \begin{bmatrix} 1 & -a \\ 1 & 0 \end{bmatrix} & \begin{bmatrix} p_t \\ E_t[p_{t+1}] \end{bmatrix} \end{matrix} \\ \\ \begin{matrix} \mathbf{B} & \mathbf{Y}_{t-1} & \mathbf{\Psi}_v & \mathbf{\Psi}_w \\ \begin{bmatrix} b & 0 \\ 0 & 1 \end{bmatrix} & \begin{bmatrix} p_{t-1} \\ E_{t-1}[p_t] \end{bmatrix} & + \begin{bmatrix} 1 \\ 0 \end{bmatrix} v_t & + \begin{bmatrix} 0 \\ 1 \end{bmatrix} w_t. \end{matrix}$$

¹ Christopher Sims (2002) provides code in matlab to compute the reduced form of a linear model of this kind in which the dimension of the system is arbitrary and the matrices A and B may be singular.

Model 2 has two shocks: v_t is a fundamental shock and w_t is a nonfundamental shock; w_t is defined in the second row of equation (7) to be the difference between p_t and its date $t - 1$ expectation. Since we choose parameters such that there is a unique equilibrium, the nonfundamental shock, w_t , will be determined endogenously as a function of v_t .

Premultiplying equation (7) by \mathbf{A}^{-1} leads to the expression

$$(8) \quad \begin{bmatrix} p_t \\ E_t[p_{t+1}] \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\frac{b}{a} & \frac{1}{a} \end{bmatrix} \begin{bmatrix} p_{t-1} \\ E_{t-1}[p_t] \end{bmatrix} + \begin{bmatrix} 0 \\ -\frac{1}{a} \end{bmatrix} v_t + \begin{bmatrix} 1 \\ \frac{1}{a} \end{bmatrix} w_t.$$

It is convenient for the following analysis to reparameterize the model in terms of the two roots of

$$\mathbf{A}^{-1}\mathbf{B} = \begin{bmatrix} 0 & 1 \\ -\frac{b}{a} & \frac{1}{a} \end{bmatrix},$$

which we call θ and λ . The parameters a and b are given by the expressions $a = 1/(\theta + \lambda)$ and $b = \lambda\theta/(\theta + \lambda)$. If the equilibrium is unique, there must be one unstable root that allows one to pin down the nonpredetermined variable $E_t[p_{t+1}]$ as a function of the lagged-state variable p_{t-1} and the fundamental shock v_t . Without loss of generality, we assume that θ is the unstable root such that

$$|\theta| > 1, \quad |\lambda| < 1.$$

In Appendix B, we show how to solve explicitly for the reduced form, which can be written as follows:

$$(9) \quad p_t = \lambda p_{t-1} + \frac{(\lambda + \theta)}{\theta} v_t,$$

$$(10) \quad E_t[p_{t+1}] = \lambda^2 p_{t-1} + \frac{\lambda(\lambda + \theta)}{\theta} v_t.$$

III. Models 1 and 2 Compared

The reduced form of Model 2 is given by equations (9) and (10). Recall that the reduced

form for Model 1 is given by equations (4) and (5), which we repeat below:

$$(11) \quad p_t = \lambda p_{t-1} + w_t,$$

$$(12) \quad E_t[p_{t+1}] = \lambda^2 p_{t-1} + \lambda w_t.$$

An econometrician who observes p_t can consistently estimate λ and the variance of the error term; but in the absence of independent information on the true variance of w_t or v_t , there is no way to distinguish w_t from $((\lambda + \theta)/\theta)v_t$. Suppose that Model 2 is the data-generating process and that v_t has distribution D_v with mean 0 and standard deviation σ_v . Then, there exists a sunspot error with distribution D_w and standard deviation σ_w , where

$$\sigma_w = \frac{(\lambda + \theta)}{\theta} \sigma_v$$

such that the likelihood functions of Models 1 and 2 are identical. If D_v is normal (as is often assumed), then D_w is also normal. We have provided an example of a determinate model and an indeterminate model that are observationally equivalent.

IV. Conclusion

If our result is correct, then how are Lubik and Schorfheide able to distinguish determinate and indeterminate regions of the parameter space in US data? Their method hinges on prior restrictions over lag length that exclude certain models from consideration. To see how this might work, suppose that a Bayesian econometrician were to be confronted with data generated by Model 2 in which the equilibrium was determinate. Let the Bayesian choose a prior probability distribution over parameters that places zero weight on the possibility that $b \neq 0$; hence, no amount of evidence will allow him to revise this prior in favor of a model with $b \neq 0$. This individual would conclude, incorrectly, that the data were generated by Model 1 with an indeterminate equilibrium. As Pesarán pointed out in his 1987 book, prior restrictions on lag length are likely to be extremely important in deciding between determinate and indeterminate models.

There is no reason to think that our example is special. Beyer and Farmer (2004) exhibit more general examples based on a three-equation model. Our results imply that, at a very fundamental level, distinguishing between determinate and indeterminate models must be based on untestable restrictions about the dynamic structure of an economic model. The examples that we have constructed in (2) suggest that the task of separating determinate from indeterminate theories of the data is much harder than one might first suspect.

The Lubik and Schorfheide (2004) paper is an important contribution to the literature and their method for distinguishing between alternative regions of the parameter space of an econometric model will no doubt provide a useful addition to the toolkit of applied researchers. Our message is one of caution in the interpretation of results. Any attempt to categorize an observed data series as arising from a determinate or an indeterminate model is determined as much by subtle choices over the way to model the dynamics as it is by the data themselves.

APPENDIX A

This Appendix derives equations (4) and (5). The first row of equation (3) implies

$$(A1) \quad E_{t-1}[p_t] = p_t - w_t$$

and, hence, (leading this equation one period)

$$(A2) \quad E_t[p_{t+1}] = p_{t+1} - w_{t+1}.$$

The second row of equation (3) gives

$$(A3) \quad E_t[p_{t+1}] = \lambda E_{t-1}[p_t] + \lambda w_t.$$

Replacing $E_{t-1}[p_t]$ and $E_t[p_{t+1}]$ from (A1) and (A2) gives

$$(A4) \quad (p_{t+1} - w_{t+1}) = \lambda(p_t - w_t) + \lambda w_t,$$

which simplifies to

$$(A5) \quad p_{t+1} = \lambda p_t + w_{t+1}.$$

This equation must also hold at date t , which gives equation (4) in the paper.

Equation (5) is derived as follows. Combining (A1) with (A3) gives

$$(A6) \quad E_t[p_{t+1}] = \lambda(p_t - w_t) + \lambda w_t = \lambda p_t.$$

Now substitute from (A5), lagged one period, to give

$$(A7) \quad E_t[p_{t+1}] = \lambda^2 p_{t-1} + \lambda w_t.$$

APPENDIX B

This Appendix shows how to solve Model 2 in terms of the roots λ and θ . The reduced form of this model is given by the expression

$$(B1)$$

$$\begin{aligned} \begin{bmatrix} p_t \\ E_t[p_{t+1}] \end{bmatrix} &= \begin{bmatrix} 1 & -a \\ 1 & 0 \end{bmatrix}^{-1} \begin{bmatrix} b & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} p_{t-1} \\ E_{t-1}[p_t] \end{bmatrix} \\ &+ \begin{bmatrix} 1 & -a \\ 1 & 0 \end{bmatrix}^{-1} \begin{bmatrix} 1 \\ 0 \end{bmatrix} v_t + \begin{bmatrix} 1 & -a \\ 1 & 0 \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ 1 \end{bmatrix} w_t, \end{aligned}$$

where

$$\begin{aligned} \begin{bmatrix} 1 & -a \\ 1 & 0 \end{bmatrix}^{-1} \begin{bmatrix} b & 0 \\ 0 & 1 \end{bmatrix} &= \begin{bmatrix} 0 & 1 \\ -\frac{1}{a} & \frac{1}{a} \end{bmatrix} \begin{bmatrix} b & 0 \\ 0 & 1 \end{bmatrix} \\ &= \begin{bmatrix} 0 & 1 \\ -\frac{b}{a} & \frac{1}{a} \end{bmatrix}. \end{aligned}$$

Since this is a two-parameter model, we can completely characterize the system in terms of the two roots, λ and θ . The characteristic polynomial of $\mathbf{A}^{-1}\mathbf{B}$ is given by

$$(B2) \quad F^2 - \frac{1}{a}F + \frac{b}{a} = 0,$$

and the roots λ and θ are related to the parameters a and b by the equations,

$$\theta + \lambda = \frac{1}{a},$$

$$\theta\lambda = \frac{b}{a},$$

from which it follows that

$$a = \frac{1}{\lambda + \theta} \quad \text{and} \quad b = \frac{\lambda\theta}{\lambda + \theta}.$$

We can rewrite the matrix $\mathbf{A}^{-1}\mathbf{B}$ in terms of λ and θ as

$$\begin{aligned} \mathbf{A}^{-1}\mathbf{B} &= \begin{bmatrix} 0 & 1 \\ -(\lambda + \theta) & \lambda + \theta \end{bmatrix} \begin{bmatrix} \frac{\lambda\theta}{\lambda + \theta} & 0 \\ 0 & 1 \end{bmatrix} \\ &= \begin{bmatrix} 0 & 1 \\ -\theta\lambda & \lambda + \theta \end{bmatrix}. \end{aligned}$$

The eigenvectors of $\mathbf{A}^{-1}\mathbf{B}$ associated with the roots λ and θ are given by the expressions

$$\theta \rightarrow \begin{bmatrix} 1 \\ \theta \end{bmatrix} \quad \text{and} \quad \lambda \rightarrow \begin{bmatrix} 1 \\ \lambda \end{bmatrix},$$

and, hence, $\mathbf{A}^{-1}\mathbf{B}$ can be diagonalized as

$$\begin{aligned} \mathbf{A}^{-1}\mathbf{B} &= \mathbf{Q}\mathbf{\Lambda}\mathbf{Q}^{-1} \\ &= \begin{bmatrix} 1 & 1 \\ \theta & \lambda \end{bmatrix} \begin{bmatrix} \theta & 0 \\ 0 & \lambda \end{bmatrix} \begin{bmatrix} \mathbf{Q}^{-1} \\ \mathbf{\Lambda} \end{bmatrix} \begin{bmatrix} \lambda & -1 \\ -\theta & 1 \end{bmatrix}, \end{aligned}$$

where the columns of \mathbf{Q} are eigenvectors.

We now write the system as a pair of scalar equations by introducing the following definitions:

$$\mathbf{z}_t = \begin{bmatrix} z_{1t} \\ z_{2t} \end{bmatrix} = \mathbf{Q}^{-1} \begin{bmatrix} p_t \\ E_t[p_{t+1}] \end{bmatrix},$$

$$\boldsymbol{\xi}_t = \begin{bmatrix} \xi_{1t} \\ \xi_{2t} \end{bmatrix} = \mathbf{Q}^{-1}\mathbf{A}^{-1} \begin{bmatrix} 1 \\ 0 \end{bmatrix} v_t,$$

$$\boldsymbol{\eta}_t = \begin{bmatrix} \eta_{1t} \\ \eta_{2t} \end{bmatrix} = \mathbf{Q}^{-1}\mathbf{A}^{-1} \begin{bmatrix} 0 \\ 1 \end{bmatrix} w_t.$$

Using the definitions of \mathbf{Q}^{-1} , \mathbf{A}^{-1} and $\mathbf{Q}^{-1}\mathbf{A}^{-1}$,

$$\mathbf{Q}^{-1} = \begin{bmatrix} \frac{\lambda}{\lambda - \theta} & \frac{-1}{\lambda - \theta} \\ -\theta & 1 \end{bmatrix},$$

$$\mathbf{A}^{-1} = \begin{bmatrix} 0 & 1 \\ -(\lambda + \theta) & (\lambda + \theta) \end{bmatrix},$$

$$\mathbf{Q}^{-1}\mathbf{A}^{-1} = \begin{bmatrix} \frac{\lambda + \theta}{\lambda - \theta} & \frac{-\theta}{\lambda - \theta} \\ -\frac{\lambda + \theta}{\lambda - \theta} & \frac{\lambda}{\lambda - \theta} \end{bmatrix},$$

we can write the expressions for z_{1t} , z_{2t} , ξ_{1t} , ξ_{2t} , η_{1t} , and η_{2t} in terms of the parameters θ and λ :

$$(B3) \quad \begin{cases} z_{1t} = \frac{\lambda}{\lambda - \theta} p_t - \frac{1}{\lambda - \theta} E_t[p_{t+1}], \\ z_{2t} = \frac{-\theta}{\lambda - \theta} p_t + \frac{1}{\lambda - \theta} E_t[p_{t+1}], \\ \xi_{1t} = \frac{\lambda + \theta}{\lambda - \theta} v_t, \\ \xi_{2t} = -\frac{\lambda + \theta}{\lambda - \theta} v_t, \\ \eta_{1t} = \frac{-\theta}{\lambda - \theta} w_t, \\ \eta_{2t} = \frac{\lambda}{\lambda - \theta} w_t. \end{cases}$$

Using these definitions, the system can be decomposed into the following pair of scalar difference equations:

$$(B4) \quad z_{1t} = \theta z_{1t-1} + \xi_{1t} + \eta_{1t},$$

$$(B5) \quad z_{2t} = \lambda z_{2t-1} + \xi_{2t} + \eta_{2t}.$$

Since $\theta > 1$, we must set

$$z_{1t} = \theta z_{1t-1} = 0,$$

to eliminate the influence of the explosive root. It follows from (B4) that

$$(B6) \quad \xi_{1t} + \eta_{1t} = 0,$$

i.e., the sum of fundamental and nonfunda-

mental errors must add up to zero. From (B3) it also follows that

$$\frac{\lambda}{\lambda - \theta} p_t - \frac{1}{\lambda - \theta} E_t[p_{t+1}] = 0,$$

and, hence,

$$(B7) \quad E_t[p_{t+1}] = \lambda p_t.$$

Using (B7) and the definition of z_{2t} from (B3) yields

$$(B8) \quad z_{2t} = p_t,$$

and using the expression (B6) and the definitions of η_{1t} and ξ_{1t} from (B3), it follows that

$$(B9) \quad w_t = \frac{\theta + \lambda}{\theta} v_t.$$

Finally, substituting (B8) in (B5) and eliminating ξ_{2t} and η_{2t} using (B3) and (B9) yields the following reduced form expression for p_t :

$$p_t = \lambda p_{t-1} - \left(\frac{\lambda + \theta}{\lambda - \theta} \right) v_t \\ + \left(\frac{\lambda}{\lambda - \theta} \right) \left(\frac{\theta + \lambda}{\theta} \right) v_t,$$

which simplifies to give

$$p_t = \lambda p_{t-1} + \left(\frac{\theta + \lambda}{\theta} \right) v_t.$$

Finally, from (B7),

$$E_t[p_{t+1}] = \lambda^2 p_{t-1} + \lambda \left(\frac{\theta + \lambda}{\theta} \right) v_t.$$

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