# Old World Econometrics and New World Theory

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**Abstract**

This paper summarizes recent work on the econometrics of identification in rational expectations models by myself and Andreas Beyer. In this work, we discuss the relationship between structure and reduced form in linear rational expectations models and we apply our analysis to a class of new Keynesian models that are frequently used to inform the policy debate. We argue that the identifying assumptions used in this literature are invalid and that linear rational expectations models are themselves subject to the Lucas critique of econometric policy evaluation. We introduce an alternative approach to identification that uses breaks in policy as natural experiments to help identify the private sector structural equations of a linear model.

# 1 Introduction

Since this conference is about Post-Walrasian economics I must confess at the outset to feeling somewhat of an interloper. I take the defining characteristic of Walrasian economics to be the assumption that data can be well described by competitive markets with continuous market clearing. Much of my work over the past decade has been on the implications of indeterminacy and sunspots in Walrasian economies and, although the economies that I study are often not competitive, I have never felt confined by the assumption of continuous market clearing - on the contrary, in my view, market clearing is an irrefutable proposition once markets are defined broadly enough. I am nevertheless grateful to the organizers of the conference for the opportunity to collect together some ideas that I have been working on over the past few years in joint research with Andreas Beyer of the European Central Bank.

My work with Beyer begins with the observation that North American macroeconomists and European time-series econometricians often have difficulty communicating with each other. The main points of disagreement concern the way that theory should be confronted with data and the way that economists should treat expectations. Roughly speaking, the dominant paradigm in North America favors calibration and data is passed through a two-sided filter. The dominant trend in European econometrics points out that theoretical models often have rich implications for both low and high frequency components of the data; filtering data removes the low frequency component and throws away information that can potentially discriminate between competing theories. On this point we come down squarely on the side of the Old World. But not all of our sympathies lie with the way that European econometricians have applied formal econometric methods. David Hendry (1995), for example, has argued that there is little merit in the rational expectations hypothesis. In contrast we see merit in the argument that expectations are forward looking and we see no difficulty with pushing this idea to its logical extreme and assuming that expectations are rational. Our research agenda is to combine the general-to-specific methodology favored by the Hendry school of European econometrics with parsimonious theory-driven rational expectations models. This paper summarizes the progress we have made in pursuit of this approach.

# 2 Background

In the 1950's a group of economists at the Cowles Commission developed a framework for studying time-series data that dealt with the difficulty of conducting experiments. Cowles Commission methodology proposed a distinction between *structural models* that might involve systems of simultaneous equations and *reduced form* models in which each endogenous variable is explained as a function of predetermined and exogenous variables. The role of the theorist was to find a set of assumptions that would allow the econometrician to recover a unique structural model from estimates of the reduced form parameters.[1] This approach remained unchallenged for twenty years.

In the 1970's, Lucas (1976) published a critique of Cowles-Commission methodology based on the proposition that agents' expectations are forward looking. He argued that the Keynesian identification assumptions that were commonly used in the 1960's were mis-specified since they failed to account for cross-equation restrictions that arise naturally in models in which expectations are rational. He pointed out that although Keynesian models might fit well within a given sample, they should not be used to inform policy analysis since mis-specified structural parameters would not remain invariant to a change in the rule followed by a policy maker.

Following Lucas's critique, macroeconomists moved in two directions. One group continued to construct structural models but made different identifying assumptions. Specifically, they assumed that the data is generated by a dynamic general equilibrium model (possibly with sticky prices) in which forward looking agents have rational expectations of future prices.

A second group of economists, following the arguments of Sims (1980), gave up entirely on structural models and concentrated on identifying the effects of policy shocks in reduced form vector autoregressions (VARs). This enterprise is very different from the goal of structural modelers who seek to design optimal policy rules by estimating the "deep parameters" of the "true structure" of the economy. Those on the side of VAR models sometimes defend their methodology by arguing for a kind of meta-model in which all possible changes in policy rules have already been rationally anticipated by agents and are built into expectations. A second defense, and perhaps a more persuasive one, is that identification is difficult or impossible in rational expectations models since there are no credible instruments.

In my work with Beyer, we are on the side of the VAR modelers in recognizing the difficulty of finding credible identifying restrictions. We explore this theme in Beyer and Farmer (2003b) and Beyer and Farmer (2004) in which we argue that the identifying assumptions used by rational expectations modelers are difficult to defend and that the lack of identification may have serious consequences for a research agenda that seeks to find an optimal policy rule. But although identification is difficult, it is not impossible. In Beyer and Farmer (2002) and Beyer and Farmer (2003a) we argue that occasional changes in policy regime can be used as a natural experiment to help to identify a subset of the parameters of a structural model.

[1] See the collection of papers "Statistical Inference in Dynamic Economic Models", Koopmans, ed (1950). The introduction, Marschak (1950), provides a particularly clear summary of the Cowles Commission approach.

# 3 Structure and Reduced Form in Rational Expectations Models

The Cowles Commission distinguished between a structural model that might involve simultaneous relationships between variables, and a reduced form model in which each endogenous variable has been reduced to a function of exogenous and predetermined variables. The purpose of this section is twofold. First, I will discuss the amendments to Cowles Commission methods that arise from the possibility that data may be non-stationary but cointegrated. Second, I will discuss the amendments to these methods that arise in rational expectations models as a consequence of the existence of forward looking expectations.

## 3.1 The Old-World Approach

My work with Beyer has been conducted in the framework of U.S. monetary policy using quarterly data from 1970 through 1999 on the Fed. funds rate, unemployment and the inflation rate. Although many of the ideas I will discuss are more general, this environment is rich enough to introduce my main points and specific enough to make them with a minimum of notation.

Let $\{y_t\}_{t=1}^{T}$ be a vector-valued time series of 3 variables, partitioned as $\{y_t^s, y_t^p\}_{t=1}^{T}$. $y_t^s$ is a $2-$vector of private sector structural variables that consists of the first difference of the unemployment rate and the first difference of the inflation rate and $y_t^p$ is a scalar that represents the first difference of the federal funds rate. I have separated the variables in this way because I will need a notation that allows me to discuss the behavior of an economic system in which a policy maker may design a rule that sets a policy variable, (in my case this will be the federal funds rate), as a function of other variables in the system.

Because the data that Beyer and I have studied is non-stationary but cointegrated, I need a rich enough model to allow me to discuss the problems of inference that arise in this case. Lets begin with an approach that I call "Old World" econometrics since its major practitioners are European. An econometrician following this approach might construct the following structural model

$$\underset{2\times 2}{\overline{A}_{11}}\,\underset{2\times 1}{y_t^s} + \underset{2\times 1}{\overline{A}_{12}}\,\underset{1\times 1}{y_t^p} = \underset{2\times m}{\overline{B}_1}\,\underset{m\times 1}{z_{t-1}} + \underset{2\times 1}{v_t^s}, \tag{1}$$

$$\underset{1\times 2}{\overline{A}_{21}}\,\underset{2\times 1}{y_t^s} + \underset{1\times 1}{y_t^p} = \underset{1\times m}{\overline{B}_2}\,\underset{m\times 1}{z_{t-1}} + \underset{1\times 1}{v_t^p}. \tag{2}$$

$v_t$ is a $3-$vector of $i.i.d.$shocks partitioned conformably as $\{v_t^s, v_t^p\}$ with covariance matrix $\Omega_v$ and $z_{t-1}$ is an $m-$vector of exogenous and predetermined variables. $z_{t-1}$ contains a constant, $k$ lags of $y_t$, and the levels of the first lags of unemployment, inflation and the federal funds rate.

I have separated the model into two blocks. The first, Equation (1), represents the behavior of the private sector; the second, Equation (2), is a policy reaction function. Although it is not necessary to break up the model in this way in order to discuss the relationship of structural and reduced form models, the partition of the model into a private sector block and a policy rule will play an important role in my discussion of identification.

The Cowles Commission economists assumed that the data they studied were stationary. More recently there has been an explosion of work on the properties of dynamic systems of non stationary variables.[2] Recent advances in this area permit valid inferences to be made in structural and reduced form time series models when the level variables are non-stationary but cointegrated.

Any vector autoregression in levels has an equivalent representation as a Vector Error Correction model, (VECM). A VECM is a VAR in first differences in which first lags of the levels appear as additional explanatory variables. If the data are non-stationary but cointegrated then the matrix of level coefficients has reduced rank. If this matrix has full rank, the parameters of the VECM can be consistently estimated by ordinary least squares. But if it has reduced rank, the regressors will include non-stationary variables and statistical inference using classical methods will be invalid.

A solution to the problem of cointegrated regressors in systems of $n$ equations was provided by Johansen (1995) who showed how to consistently estimate the rank of the cointegrating matrix using a maximum likelihood approach. In Beyer and Farmer (2003a) we take a two-step approach to the problem of inference. In the first step we use Johansen's procedure to estimate the rank of the cointegrating matrix which we find to be equal to 2 for our U.S. data set. In the second step we replace the full set of three level variables, that might potentially be included in $z_{t-1}$, by two stationary linear combinations of these variables. The weights we use to construct these linear combinations are the cointegrating vectors estimated in step 1.

## 3.2 A New-World Twist

The class of models described in Equations (1) and (2) is rich, but not rich enough to include the structural models of rational expectations econometrics. To allow for the influence of expectations, the system must be expanded by adding the date $t$ vector $E_t [y_{t+1}]$ as an additional set of explanatory variables;

[2]See the survey by Watson (1994) for a detailed exposition of these developments.

$$\underset{2\times2}{\overline{A}_{11}}\underset{2\times1}{y_t^s} + \underset{2\times1}{\overline{A}_{12}}\underset{1\times1}{y_t^p} + \underset{2\times3}{\overline{F}_1} E_t \underset{3\times1}{[y_{t+1}]} = \underset{2\times m}{\overline{B}_1}\underset{m\times1}{z_{t-1}} + \underset{2\times1}{v_t^s}, \tag{3}$$

$$\underset{1\times2}{\overline{A}_{21}}\underset{2\times1}{y_t^s} + \underset{1\times1}{y_t^p} + \underset{1\times3}{\overline{F}_2} E_t \underset{3\times1}{[y_{t+1}]} = \underset{1\times m}{\overline{B}_2}\underset{m\times1}{z_{t-1}} + \underset{1\times1}{v_t^p}. \tag{4}$$

Since the three components of the vector $E_t[y_{t+1}]$ are endogenously determined at date $t$; one must add three additional equations to close the structural model. Under the rational expectations assumption, these equations define the one-step-ahead forecast errors of $y_t$,

$$\underset{3\times1}{y_t} = E_{t-1}\underset{3\times1}{[y_t]} + \underset{3\times1}{w_t}. \tag{5}$$

The set of models defined by Equations (3)–(5) may be classified into two types, *determinate* or *indeterminate,* depending on the values of the model's parameters. Determinate models allow the economist to find a unique reduced form that describes the evolution of $y_t$ as a vector autoregression driven by the vector of shocks $v_t$. The shocks $v_t$ are called *fundamental* to distinguish them from the *non-fundamental* forecast errors $w_t$. In determinate models, the $w_t$ are endogenously determined as a function of $v_t$ as part of the solution procedure. In indeterminate models the properties of $w_t$ must be independently chosen as part of the specification of the model. In this case the random variables $w_t$ have the interpretation of non-fundamental shocks to agents' beliefs that may independently influence the evolution of $y_t$.

The following definitions of the variance-covariance parameters of $w_t$ and $v_t$ will provide us with a language that is general enough to cover both determinate and indeterminate models,

$$E\left[(v_t^s)(v_t^s)'\right] = \Omega_v^s,\ E\left[\left(v_t^p\right)^2\right] = \Omega_v^p,\ E\left(v_t^s v_t^p\right) = \Omega_v^{sp},$$
$$E\left[w_t w_t'\right] = \Omega_w,\ E\left[v_t w_t'\right] = \Omega_{vw}.$$

In the determinate case the parameters $\Omega_w$ and $\Omega_{vw}$ are endogenously determined as functions of the fundamental parameters. In the indeterminate case, these matrices may contain independent elements that index a particular reduced form. In either case, by including the possibility that $\Omega_w$ and $\Omega_{vw}$ may be prespecified as part of the definition of the structural model, there will be a unique mapping from the structure to the reduced form.

## 3.3 Structure and Reduced Form

The model described by Equations (3)–(5) is well suited to the discussion of policy design. But to illustrate the concept of structure and reduced form it will help to simplify it in two ways. I will study a version of the model in which the data is stationary and in which only one lag appears on the right-hand-side. The purpose of introducing this stripped-down model is to show that the classification into structural and reduced form models introduced by the Cowles Commission can also be applied to linear rational expectations models. It will also allow me to discuss a complication that occurs in rational expectations models but is not present in standard models: This arises from the possibility that a structural rational expectations model may be determinate or indeterminate.

Let upper case $Y_t$ be a 3−vector of level data (I will retain lower case $y_t$ for differenced data) and write the structural rational expectations model as follows

$$\overset{\tilde{A}}{\begin{bmatrix} \underset{3\times3}{A} & \underset{3\times3}{F} \\ \underset{3\times3}{I} & \underset{3\times3}{0} \end{bmatrix}} \overset{X_t}{\begin{bmatrix} \underset{3\times1}{Y_t} \\ \underset{3\times1}{E_t\left[Y_{t+1}\right]} \end{bmatrix}} = \overset{\tilde{C}}{\begin{bmatrix} \underset{3\times1}{C} \\ \underset{3\times1}{0} \end{bmatrix}} + \overset{\tilde{B}}{\begin{bmatrix} \underset{3\times3}{B_1} & \underset{3\times3}{B_2} \\ \underset{3\times3}{0} & \underset{3\times3}{I} \end{bmatrix}} \overset{\tilde{X}_{t-1}}{\begin{bmatrix} \underset{3\times1}{Y_{t-1}} \\ \underset{3\times1}{E_{t-1}\left[Y_t\right]} \end{bmatrix}}$$
$$+ \overset{\tilde{\Psi}_v}{\begin{bmatrix} \underset{3\times3}{\Psi_v} \\ \underset{3\times3}{0} \end{bmatrix}} \underset{3\times1}{v_t} + \overset{\tilde{\Psi}_w}{\begin{bmatrix} \underset{3\times3}{0} \\ \underset{3\times3}{I} \end{bmatrix}} \underset{3\times1}{w_t}. \tag{6}$$

More compactly,

$$\underset{6\times6}{\tilde{A}} \underset{6\times1}{X_t} = \underset{6\times1}{\tilde{C}} + \underset{6\times6}{\tilde{B}} \underset{6\times1}{X_{t-1}} + \underset{6\times3}{\tilde{\Psi}_v} \underset{3\times1}{v_t} + \underset{6\times3}{\tilde{\Psi}_w} \underset{3\times1}{w_t}. \tag{7}$$

In Cowles Commission methodology, the mapping from structure to reduced form is found by premultiplying (7) by the inverse of $\tilde{A}$ to give the following expression.[3]

$$\underset{6\times1}{X_t} = \underset{6\times6}{\Gamma^*} \underset{6\times1}{X_{t-1}} + \underset{6\times1}{C^*} + \underset{6\times3}{\Psi_v^*} \underset{3\times1}{v_t} + \underset{6\times3}{\Psi_w^*} \underset{3\times1}{w_t}. \tag{8}$$

A structural rational expectations model also has a reduced form represented by Equation (8) although the mapping from the structural to the reduced form model is more complicated than in the Cowles Commission case. To derive the reduced form of a structural rational expectations model, one must eliminate the influence of unstable roots of the matrix $\tilde{A}^{-1}B$ by ensuring that the system always remains in the linear subspace associated with the stable roots of this matrix. This is achieved by allowing the endogenous random variables $w_t$ to adjust in each period to offset the effect of the fundamental shocks $v_t$.

[3]This transformation requires that the matrix $\tilde{A}$ have full rank. Although it is not necessary to make this assumption in order to find the reduced form of the linear rational expectations model, I will maintian the assumption here for expository purposes.

The classes of reduced form models that might be associated with the structural model can be indexed by the number of unstable roots of $\tilde{A}^{-1}B$. If this matrix has three unstable roots the solution is said to be determinate. In this case $\Gamma^*$ has rank $3, \Psi_w^*$ is identically zero and the endogenous errors $w_t$ do not appear in the reduced form. At the other extreme, if all of the roots of $\tilde{A}^{-1}B$ are inside the unit circle, the model has three degrees of indeterminacy. In this case $\Gamma^*$ has rank 6, $\Psi_w^*$ has rank 3 and $w_t$ is an arbitrary vector of shocks with zero conditional mean and arbitrary covariance matrix. The vectors $v_t$ and $w_t$ may also be correlated.

In addition to the determinate case and the case of three degrees of indeterminacy, the three equation linear rational expectations model may also have one or two degrees of indeterminacy. For example, the case of one degree of indeterminacy occurs when two roots of $\tilde{A}^{-1}B$ are outside the unit circle. In this case $\Gamma^*$ has rank 4, $\Psi_w^*$ has rank 1 and a one dimensional arbitrary nonfundamental shock $w_t$ influences the behavior of the system. As in the case of complete indeterminacy, $w_t$ has zero mean and may be correlated with $v_t$.

# 4 The Lucas Critique and New-World Theory

We now have the necessary notation in place to discuss how one might pursue an agenda that combines Old World econometrics with New World theory. Consider the class of linear rational expectations models represented by Equations (3)-(4) which we reproduce below;

$$\underset{2\times 2}{\overline{A}_{11}}\underset{2\times 1}{y_t^s}+\underset{2\times 1}{\overline{A}_{12}}\underset{1\times 1}{y_t^p}+\underset{2\times 3}{\overline{F}_1}E_t\underset{3\times 1}{[y_{t+1}]} = \underset{2\times m}{\overline{B}_1}\underset{m\times 1}{z_{t-1}}+\underset{2\times 1}{v_t^s}, \tag{9}$$

$$\underset{1\times 2}{\overline{A}_{21}}\underset{2\times 1}{y_t^s}+\underset{1\times 1}{y_t^p}+\underset{1\times 3}{\overline{F}_2}E_t\underset{3\times 1}{[y_{t+1}]} = \underset{1\times m}{\overline{B}_2}\underset{m\times 1}{z_{t-1}}+\underset{1\times 1}{v_t^p}. \tag{10}$$

Following the critique of Lucas (1976) much of macroeconomic theory is conducted using a linear rational expectations model in this class as an organizing framework. But before a model like this can be calibrated or estimated, its parameters must be restricted through identifying assumptions.

One approach to identification proceeds by deriving restrictions from microeconomic theory. For example, a theorist might assume that data is generated by the actions of a representative agent with a time separable utility function defined over current consumption and the real value of money balances. This agent has access to a technology for producing commodities using labor and a fixed factor and the agent may hold money or government bonds as stores of wealth. The technology might use intermediate inputs produced by monopolistically competitive firms that face frictions in the adjustment of their prices. Government behavior might be modeled with the assumption that the central bank forms rational expectations of future inflation and responds to these expectations by raising or lowering the federal funds rate. If the modeler is sophisticated then the parameters of this rule will themselves be derived from the assumption of maximizing behavior on the part of the central bank.

It is often asserted that the approach described above is superior to the Cowles Commission methods that preceded it since the parameters of a linear rational expectations model, restricted in this way, represent the "deep economic structure" and they would not therefore be expected to change if the central bank were to change its policy rule. This argument is surely correct if the economist has identified the true structural model, but this is a very big if. It is more likely that the theorist has used false identifying restrictions and, if this is the case, the model he believes to be structural cannot be used to guide policy. The argument of Lucas (1976) against the use of Keynesian econometric models as guides to policy evaluation applies to any mis-specified model, whether or not the model pays lip service to rational expectations by including expected future endogenous variables as right-hand-side variables. This is the point made in Beyer and Farmer (2003b) and Beyer and Farmer (2004) and, in the following section, I describe some of the examples we constructed in those papers to illustrate how serious the lack of identification may be.

# 5 Lack of Identification and its Consequences

In central banks and universities throughout the world, researchers are engaged in an agenda that hopes to uncover the properties of a "good" monetary policy. The thrust of this agenda is to construct economic models that are based on solid microfoundations where microfoundations means that aggregate data can be modeled 'as if' they were chosen by a set of identical representative agents operating in competitive markets with rational expectations of the values of future variables. The restrictions imposed by theory are used to identify the parameters of a linear model, similar to the one described by Equations (3)–(5). These parameters are given numerical values, either through calibration or through more formal estimation methods, and the resulting quantified model is used to inform policy analysis. Beyer and Farmer (2004) argue that this is a futile exercise for the following reason.

Using Equations (3)–(5) as a benchmark, consider the following partition of the model parameters into two sets. The elements of $\bar{A}_{11}, \bar{A}_{12}, \bar{F}_1, \bar{B}_1$, and $\Omega_v^s$ are parameters that describe private sector behavior: I will refer to them as $\theta^s$.[4] Since we have already included future expectations as endogenous regressors, $\theta^s$ is determined

[4] In the case of an indeterminate model, some or all of the elements of the matrices $\Omega_w$ and $\Omega_{vw}$ must also be included in this set.

by the specification of the technology, preferences and endowments of the agents in a microfounded model; in the language of Lucas and Sargent (1981), the elements of $\theta^s$ are functions of the 'deep structure'. The elements of $\bar{A}_{21}, \bar{A}_{22}, \bar{F}_2, \bar{B}_2, \Omega_v^p$ and $\Omega_v^{sp}$ are parameters that describe the policy rule of the government: I will refer to them as $\theta^p$. By definition, $\theta = \theta^s \cup \theta^p$ is a structural model.

Every structural model, $\theta$, has a unique reduced form $\Gamma^*$; but the converse is not true. In Beyer and Farmer (2004) we construct an algorithm that finds equivalence classes of exactly identified models. We begin with a "true" model $\theta$ that we refer to as the data generating process (DGP) and we use our algorithm to find its associated reduced form $\Gamma^*(\theta)$. We feed the reduced form parameters to a fictional econometrician who supplements $\Gamma^*(\theta)$ with a set of linear restrictions, described by a matrix $R$ and a vector $r$, that identifies some different model $\bar{\theta}$. Since $\theta$ and $\bar{\theta}$ both have the same reduced form, they obey the equivalence relationship,

$$\Gamma^*\left(\bar{\theta}\right) = \Gamma^*(\theta).$$

We call $\bar{\theta}$ the *equivalent model*. The equivalent model is identified by the restrictions

$$R\bar{\theta} = r.$$

Beyer and Farmer (2004) use this algorithm to construct four examples, each of which is progressively more destructive to a research agenda that hopes to design a "good" economic policy. The following two subsections discuss these example in more detail.

## 5.1 Lack of Identification in Determinate Models

The first two of Beyer and Farmer's examples exploit the fact that one cannot identify the unstable roots of a rational expectations model since the solution to the model is restricted to lie in a convergent subspace. To understand this idea, consider the following simple example,

$$y_t + fE_t[y_{t+1}] = by_{t-1} + v_t, \quad E[v_t] = \sigma_v^2. \tag{11}$$

If there is a unique solution to this model then its characteristic polynomial must have two roots, $\lambda_1$ and $\lambda_2$ that obey the inequality $|\lambda_1| < 1 < |\lambda_2|$ and the reduced form of the model will take the form;

$$y_t = \lambda_1 y_{t-1} + \psi v_t. \tag{12}$$

The econometrician can learn the value of $\lambda_1$ by regressing $y_t$ on $y_{t-1}$ but he can never recover the value of $\lambda_2$ unless he has prior knowledge of the variance of $v_t$. Although the reduced form parameter $\psi$ is a known function of $\lambda_2$, it cannot be disentangled from $\sigma_v^2$. In Beyer and Farmer (2004) we draw two implications from this point. First, in simple new-Keynesian models, the parameters of the policy rule are unidentified. Second, this lack of identification extends to the private sector equations.

## 5.2 Discriminating between Determinate and Indeterminate Models

One of the goals of the new-Keynesian agenda is to distinguish periods in which the policy regime led to a determinate outcome, from periods when it the outcome was indeterminate. It has been argued that central banks should avoid policy rules that induce indeterminacy since rules of this kind permit the influence of non-fundamental shocks to create additional undesirable uncertainty.[5]

Clarida et al. (2000) use single equation methods to estimate a policy rule before and after 1980 and they claim that Fed. policy before 1980 led to an indeterminate equilibrium that was influenced by non-fundamental shocks. After 1980, when Paul Volcker took over as chairman of the Board of Governors of the Fed., they find that there was a change in policy that led to a determinate equilibrium. Although the Clarida-Gali-Gertler paper has been criticized for use of single equation methods, subsequent studies using a full information approach, (Boivin and Giannoni (2002) and Lubik and Schorfheide (2003)), have confirmed their result.

In Beyer and Farmer (2004) we construct two examples that demonstrate the fragility of the Clarida-Gali-Gertler findings. In each of our examples the DGP is a standard new-Keynesian model in which the Fed. responds to its own expectations of future inflation by raising the interest rate by more than one percent for every one percent increase in expected inflation.[6] We present two equivalent models. The first is driven by a model of private sector behavior based on a model of supply due to Benhabib and Farmer (2000); this example is explained in more depth in subsection 5.3. In our second example we present an equivalent model in which the econometrician is assumed to know the true structure of the private sector but he is unable to determine the exact form of the policy rule followed by the Fed. Although, in the DGP, the Fed.

[5] Michael Woodford puts the point this way; "...I demand that the policy rule be not merely consistent with the desired equilibrium, but also that commitment to the rule imply a *determinate* equilibrium, so that the rule is not consistent with other, less desirable equilibria." Woodford (2003), page 536. (Emphasis in original).

[6] Following Leeper (1991), rules with this property are called *active*. If a one percent increase in expected inflation leads to a less than one percent increase in the federal funds rate the rule is called *passive*. In many simple new-Keynesian models an active rule is known to be associated with a determinate equilibrium while a passive rule leads to indeterminacy. Following Taylor (1999), the proposition that monetary policy should be active is referred to as the *Taylor Principle*.

responds to its expectation of future inflation, the econometrician incorrectly identifies an equivalent model in which the Fed. responds to contemporaneous inflation. Both of our examples of equivalent models have reduced forms that are indistinguishable from the DGP, but in each case the structural model is indeterminate. Whereas the DGP is driven by three fundamental shocks, the equivalent models are driven by two fundamental shocks and one, possibly correlated, non-fundamental.

To understand the mechanism behind our two examples, consider the following one equation model. Let the DGP be given by Equation (11): This is a determinate model driven by a fundamental shock, $v_t$. Suppose that the economist mistakenly estimates the following equivalent model,

$$y_t + \tilde{f} E_t [y_{t+1}] = 0. \tag{13}$$

The equivalent model has a family of solutions;

$$y_t = \tilde{\lambda}_1 y_{t-1} + w_t, \tag{14}$$

indexed by $\tilde{\lambda}_1$ and the variance of the non-fundamental shock, $\sigma_w^2$.

If $\left|\tilde{f}\right| < 1$ the model has a unique reduced form given by

$$y_t = 0.$$

This is a special case of (14) for which $\tilde{\lambda}_1 = 0$, and $\sigma_w^2 = 0$. If $\left|\tilde{f}\right| > 1$, the model has a family of reduced forms in which $\tilde{\lambda}_1 = 1/\tilde{f}$, and the value of $\sigma_w^2$ is arbitrary.

An economist using the methods of Lubik and Schorfheide (2003) would conclude that the model is determinate if the data follows the degenerate process $y_t = 0$ and indeterminate if it follows an $AR(1)$. Their conclusion follows from an identifying restriction that excludes $y_{t-1}$ from the structural model. If one is unwilling to make an assumption of this kind it is no longer possible to distinguish determinate from indeterminate models.

## 5.3 A More Concrete Example

Although the one equation example is useful for expository purposes, perhaps it is always possible to find identifying assumptions, based on sound microeconomic principles, that make the Lubik and Schorfheide method applicable. Unfortunately, this is not the case since there will typically be uncertainty as to the exact specification of the private sector equations.

In addition to the difficulty of finding the right equations for the private sector, there is a more fundamental problem that arises from the inability to identify the parameters of the policy rule. We show in Beyer and Farmer (2004), by means of an example, that the economist cannot distinguish a determinate from an indeterminate model, even if he knows the structure of the private sector. Our conclusion rests on the assumption that the economist does not know whether the Fed. responded to current past or expected future values of inflation. We show that these three possibilities may lead to different conclusions for the question: Was the data generated by a determinate or an indeterminate model?

The following example, drawn from Beyer and Farmer (2004), illustrates that very different structural models may have the same reduced form. We assume that the DGP is a relatively standard three-equation model and we refer to the equations as the IS curve, the Phillips curve and the policy rule. This terminology is inherited from Keynesian models but the theoretical structure of the new Keynesian model is based on a modification of an equilibrium model with a representative agent.[7]

The IS curve is given by the structural equation;

$$y_{1t} + a_{13} y_{3t} + f_{11} E_t [y_{1t+1}] + f_{12} E_t [y_{2t+1}] = b_{11} y_{1t-1} + c_1 + v_{1t}. \tag{15}$$

In this notation $y_{1t}$ is unemployment, $y_{2t}$ is inflation and $y_{3t}$ is the interest rate. The symbols $a_{ij}$,$f_{ij}$ and $b_{ij}$ represent coefficients on variable $j$ in equation $i$. We use $[a_{ij}]$ to represent coefficients on contemporaneous variables, $[f_{ij}]$ for coefficients of expected future variables and $[b_{ij}]$, to represent coefficients of lagged variables.

The second equation of the New-Keynesian model is the Phillips Curve also known as the New-Keynesian aggregate supply curve, (AS-curve). This equation takes the form;

$$y_{2t} + a_{21} y_{1t} + f_{22} E_t [y_{2t+1}] = c_2 + v_{2t}. \tag{16}$$

To close the New Keynesian model we assume that policy is generated by a reaction function of the form

$$y_{3t} + f_{32} E_t [y_{2t+1}] = b_{33} y_{3t-1} + c_3 + v_{3t}. \tag{17}$$

The Taylor principle, widely discussed by policy makers, is the condition $|f_{32}| > 1$. In simple forms of this model (when $b_{33} = 0$ and when there is no habit formation in preferences) the Taylor principle implies that there exists a unique equilibrium. In this case policy is said to be active.

In an alternative theory of the transmission mechanism, Benhabib and Farmer (2000) have argued that prices are slow to adjust because equilibria are typically indeterminate. The Benhabib-Farmer model has the same IS curve as the new-Keynesian model, but a different supply curve that takes the form

[7] For a more complete explanation of our use of these terms see Beyer and Farmer (2004).

$$y_{1t} + a_{23} y_{3t} = c_2 + b_{21} y_{1t-1} + v_{2t}. \tag{18}$$

In this model, the parameter $a_{23}$ is negative and the mechanism by which the nominal interest rate, $y_{3t}$, influences unemployment, $y_{1t}$, operates through liquidity effects in production; a lower nominal interest rate causes agents to hold more money, measured in real units. The increase in the ratio of money to output causes an expansion in economic activity because, in the Benhabib Farmer model, money is a productive asset. In a simple version of this model, with the additional simplifications of no habit formation, no lagged adjustment of the policy rule, and a policy rule of the form

$$y_{3t} + a_{32} y_{2t} = c_3 + v_{3t}, \tag{19}$$

equilibrium is determinate if $|a_{32}| < 1$ and indeterminate if $|a_{32}| > 1$. In this version of the policy rule, the Fed. responds to current inflation and not to expected future inflation.

The Benhabib-Farmer model with a contemporaneous policy rule can be shown to lead to a reduced form of the same order as the new-Keynesian model with a forward looking policy rule. However, In the Benhabib-Farmer (BF) model, the Taylor principle is reversed in the sense that active policy implies indeterminacy and passive policy implies determinacy.

In Beyer and Farmer (2004) we give a numerical version of this model. We compute its reduced form and combine the reduced form parameters with a set of linear restrictions that exactly identifies an observationally equivalent Benhabib-Farmer model. In our example, the DGP is given by the equations

$$\begin{aligned} y_{1t} &= 0.5 E_t [y_{1t+1}] + 0.5 y_{1t-1} + 0.05 (y_{3t} - E_t [y_{2t+1}]) - 0.0015 + v_{1t}, \\ y_{2t} &= 0.97 E_t [y_{2t+1}] - 0.5 y_{1t} + 0.0256 + v_{2t}, \\ y_{3t} &= 1.1 E_t [y_{2t+1}] - 0.012 + 0.8 y_{3t-1} + v_{3t}. \end{aligned} \tag{20}$$

This structural model has a unique equilibrium represented by Equation (21).

$$X_t = \Gamma^* X_{t-1} + C^* + \Psi_v v_t. \tag{21}$$

The equivalent Benhabib-Farmer model is parametrized as follows

$$\begin{aligned} y_{1t} &= 0.5 E_t [y_{1t+1}] + 0.5 y_{1t-1} + 0.05 (y_{3t} - E_t [y_{2t+1}]) - 0.0015 + v_{1t}, \\ y_{1t} &= 0.2038 y_{3t} + 0.859 y_{1t-1} - 0.0031 + v_{2t}, \\ y_{3t} &= 0.0626 y_{2t} + 0.7129 y_{3t-1} + 0.0018 + v_{3t}. \end{aligned} \tag{22}$$

We show that this equivalent model has a reduced form with identical parameters matrices $\Gamma^*$ and $C^*$ as the DGP. However, equilibria of the BF model are indeterminate and the model may be driven by a combination of fundamental and sunspot shocks. We were able to compute a variance covariance matrix for the fundamental and non-fundamental shocks to the BF model such that the New-Keynesian determinate model and the indeterminate Benhabib-Farmer model are observationally equivalent.

A considerable amount of recent literature in new-Keynesian economics has been directed towards estimating either policy rules (Clarida et al. (2000)) or private sector equations (Gali and Gertler (1999), Fuhrer and Rudebusch (2003), Fuhrer (1997)). The ultimate purpose of this literature is to draw inferences that can be used to design optimal economic policy rules. The argument made in Beyer and Farmer (2003b) and Beyer and Farmer (2004) is that much of this literature rests on identifying assumptions that are tenuous at best and it follows that policy conclusions drawn from new-Keynesian models should be treated with caution.

# 6 Policy Change as a Natural Experiment

In an insightful early comment Sargent (1971) pointed out that identification is difficult or impossible if policy makers pursue a single policy regime. In Beyer and Farmer (2002) and Beyer and Farmer (2003a) we pursue the idea that it might be possible to make headway on the issue of identification by exploiting information from multiple regimes.

In these papers we argue that there are frequently instances of real world events in which macroeconomic data changes at a discrete instant. Often, the change can credibly be attributed to an unanticipated change in economic policy regime. Examples include the collapse of the fixed exchange rate system in the 1970's or the change in chairmanship of the Fed. in 1980 when Paul Volcker instigated a set of disinflationary policies. If one is willing to make the identifying assumption that these regime changes are exogenous unpredictable events, they may be used to identify other equations of the macroeconomic system.

Consider the model represented by Equations (3)–(5) that we reproduce below,

$$\underset{2\times 2}{\overline{A}_{11}}\,\underset{2\times 1}{y_t^s} + \underset{2\times 1}{\overline{A}_{12}}\,\underset{1\times 1}{y_t^p} + \underset{2\times 3}{\overline{F}_1} E_t \underset{3\times 1}{[y_{t+1}]} = \underset{2\times m}{\overline{B}_1}\,\underset{m\times 1}{z_{t-1}} + \underset{2\times 1}{v_t^s}, \tag{23}$$

$$\underset{1\times 2}{\overline{A}_{21}}\,\underset{2\times 1}{y_t^s} + \underset{1\times 1}{y_t^p} + \underset{1\times 3}{\overline{F}_2} E_t \underset{3\times 1}{[y_{t+1}]} = \underset{1\times m}{\overline{B}_2}\,\underset{m\times 1}{z_{t-1}} + \underset{1\times 1}{v_t^p}, \tag{24}$$

$$\underset{3\times 1}{y_t} = \underset{3\times 1}{E_{t-1}[y_t]} + \underset{3\times 1}{w_t}. \tag{25}$$

The New-Keynesian and the Benhabib-Farmer models that we discussed above are both special cases of this general linear structure. For the reasons discussed in Section 5.3, this model is not identified unless one is willing to place considerably more structure on the coefficients of Equation (23). Several recent papers do exactly this by generating this equation from a primitive structural model based on optimizing behavior by forward looking agents. Examples include Boivin and Giannoni (2002), Ireland (2001) and Lubik and Schorfheide (2003).

Although we disagree with the identification methods of earlier authors, we do find evidence of a structural break in the data around 1980. In Beyer and Farmer (2003a) we argue, in the context of U.S. monetary data, that 1979 quarter 4 represents a natural break point and we show that it is possible to estimate two separate stable parameter VAR's; one for a period from 1970 quarter 1 through 1979 quarter 3 and the second for the period 1979 quarter 4 through 1999 quarter 4. We assume that this break in the parameters of the VAR was caused by a change in the parameters of Equation (24) that we attribute to a change in the chairman of the Fed. board of governors. In contrast, we assume that the parameters of Equation (23) remained constant across the break. Using this assumption we define new variables that are equal to zero over the first sub-period and equal to $\{y_t\}$ over the second. We use these variables as instruments to identify a subset of the parameters of Equation (23). The following section explains which parameters are identified by this assumption and which are not.

## 6.1 Which Parameters are Identified?

We defined a structural model to be the union of the private sector parameters, $\theta^s$ and the policy parameters $\theta^p$. In Beyer and Farmer (2003a) we show that if there is a break in the parameters $\theta^p$, one can identify a subset of the parameters of $\theta^s$ that are found by defining a rotation of the structure that we refer to as the quasi-reduced form.

To define the recoverable structure, premultiply Equation (23) by the matrix $\bar{A}_{11}^{-1}$ to generate the equivalent model

$$\underset{2\times1}{y_t^s} + \underset{2\times1}{A_{12}}\,\underset{1\times1}{y_t^p} + \underset{2\times3}{F_1}\,\underset{3\times1}{E_t\left[y_{t+1}\right]} = \underset{2\times m}{B_1}\,\underset{m\times1}{z_{t-1}} + \underset{2\times1}{\tilde{v}_t^s}, \tag{26}$$

$$\underset{1\times2}{\overline{A}_{21}}\,\underset{2\times1}{y_t^s} + \underset{1\times1}{y_t^p} + \underset{1\times3}{\overline{F}_2}\,\underset{3\times1}{E_t\left[y_{t+1}\right]} = \underset{1\times m}{\overline{B}_2}\,\underset{m\times1}{z_{t-1}} + \underset{1\times1}{v_t^p}, \tag{27}$$

$$\underset{3\times1}{y_t} = \underset{3\times1}{E_{t-1}\left[y_t\right]} + \underset{3\times1}{w_t}, \tag{28}$$

We refer to the parameters of (26) as, $\bar{\theta}^s$, where the bar denotes the parameters of the quasi-reduced form. One can show that, although the quasi-reduced form has less parameters than the structural model, it is a member of the equivalence class of reduced forms; that is,

$$\Gamma^*\left(\bar{\theta}^s \cup \theta^p\right) = \Gamma^*\left(\theta^s \cup \theta^p\right).$$

The quasi-reduced form, $\bar{\theta}^s$, has two important properties. First, like the true structural model $\theta^s$, $\bar{\theta}^s$ is invariant in the sense of Engle et al. (1983), to changes in $\theta^p$. Second, $\bar{\theta}^s$ can be identified by changes in $\theta^p$, whereas the DGP, $\theta^s$ cannot.

To identify $\theta^s$, the policy maker could change the way his instrument, the interest rate, responds to lagged variables and to current realizations of unemployment and inflation. By varying $\theta^p$ in this way he can elicit information about the way that unemployment and inflation respond to changes in the contemporaneous interest rate. Notice however, that changes of this kind can never elicit information about the way that the unemployment rate responds to inflation in the IS curve or the way that inflation responds to unemployment in the Phillips curve. The parameters that describe how unemployment and inflation respond to the interest rate are elements of $\bar{\theta}^s$, the quasi-reduced form; these parameters can be identified through changing the policy rule. The parameters that describe how unemployment and inflation react to each other in the private sector equations are elements of $\theta^s$, the deep structure; these parameters remain unidentified through experiments that are under the control of the policy maker.

# 7 Conclusion

The calibration methodology that dominates much of North American macroeconomics is an advance over the macroeconomic theory that preceded it since some quantitative discipline on the set of plausible theories is better than none. But if a calibrated model is to provide a suitable guide for the design of good policies, one must be certain that one has used the correct identifying assumptions to calibrate key parameters. In our view, the methods currently used to achieve identification are arbitrary at best. Although we have made this point in the context of new Keynesian monetary models, it applies equally to any macroeconomic model based on dynamic stochastic general equilibrium theory.

If models cannot be identified through the choice of microfoundations then how is a quantitative macroeconomist to proceed? Our suggestion is inspired by the general-to-specific methodology promoted by David Hendry.[8] Like Hendry, we are in favor of beginning with a model that is rich enough to endogenously explain the key features of the dynamics that characterize the data. Unlike Hendry, we embrace the rational expectations revolution in macroeconomics and have no difficulty with models that include expectations of future variables among the set of endogenous regressors. In our work so far, we have shown how to exploit natural experiments to identify the parameters of a structural rational expectations model. In future work, we hope to exploit our identification method to help in the design of optimal policy rules.

[8] Hendry (1995).

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