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Natural rate doubts

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Abstract

We study the low frequency comovements in unemployment, inflation and the federal funds rate in the U.S. From 1970 through 1979 all three series trended up together; after 1979 they all trended down. The conventional explanation for the buildup of inflation in the 1970s is that the Fed reacted to an increase in the natural rate of unemployment by conducting an overly passive monetary policy. We show that this explanation is difficult to reconcile with the observed comovement of the Fed funds rate and inflation. We argue instead that the source of the inflation buildup in the 1970s was a downward drift in the real interest rate that was translated into a simultaneous increase in unemployment and inflation by passive Fed policy. Our explanation relies on the existence in the data of a positive long-run unemployment–inflation relationship. This is consistent with non-superneutrality of money in the long-run, and hence the title of our paper 'natural rate doubts'.

JEL classification: C32; E3; E43; E58

Keywords: Natural rate; Phillips curve; Real interest rate; Cointegration

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[1]The views in this paper do not necessarily represent those of the European Central Bank.

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1. Introduction

Many macroeconomic time series are highly persistent and often these data are well modeled as cointegrated processes. Examples include the work by King et al. (1991). They link gross domestic product, consumption and investment by a pair of cointegrating equations that represent the 'great ratios'. King et al. have argued that inflation, the nominal interest rate and money growth are non-stationary but cointegrated in post-war U.S. data and that the unemployment rate is highly persistent and well approximated as an $I(1)$ variable in small samples. This paper shows how to use evidence from estimated cointegrating relationships like these to make inferences about the plausibility of alternative long-run structural models.

Typically, the data generating processes for macroeconomic time series are assumed to be autoregressive series whose joint distribution can be represented by vector autoregressions (VARs). However, it is well known that in practice VARs that are estimated from macroeconomic time series data very often suffer from parameter non-constancy, even if the samples are of modest length (see e.g. Rudebusch, 1998). A related issue in applied theoretical work is that assumptions about the underlying shocks – typically Gaussian – often do not hold when models are confronted with real world macro data. Several remedies have been proposed to this problem. The simplest, and one we follow in this paper, is to split a sample into sub-periods that are separated by one or more breaks and to model each sub-period separately by a stable parameter VAR (see e.g. Juselius (2001) for an early application).[2] In this paper we apply this method to a cointegrated VAR model, similar to that of King et al. We develop a reduced form that displays parameter shifts across sub-samples but parameter stability (including constant variance) within a given regime. We show how to use this estimated reduced form to distinguish between alternative structural models.

We illustrate our approach in the context of U.S. monetary policy and we show that the data can be used to shed light on the natural rate hypothesis, i.e., the assumption that the long-run unemployment rate is independent of monetary and fiscal policy.

In Section 2 we locate our work within the recent literature. Section 3 discusses the characteristics of the data and describes a statistical model that can account for these characteristics. Our main finding is that when a sample of U.S. data is split into two sub-samples, each sub-sample is well described by a cointegrated VAR with a single common trend and two cointegrating relationships. We estimate the parameters of the cointegrating relationships for each regime and find that one of them is stable across regimes but the other is different before and after 1979.

In Sections 4 and 5 we interpret our empirical results within a class of three equation new Keynesian models (NKM) each of which embodies the natural rate hypothesis. Under our identifying assumptions we are led to doubt the ability of this

[2]Alternative approaches to the issue of parameter instability includes work by Sims and Zha (2004) who apply the Markov switching approach and Cogley and Sargent (2005) who adopt a random coefficient approach. We discuss these alternatives further in Section 4.3.

class of models to explain the data. Section 6 discusses alternative theories put forward by Orphanides (2001, 2004), and by Ireland (1999), and Section 7 explains why the evidence from cointegration analysis presents a problem for these approaches. Although they can explain why inflation and unemployment move together, these alternative models have difficulty with low frequency movements in the interest rate and inflation. In Section 8 we offer an alternative explanation for the experience of the 1970s and 1980s using the idea that observed non-stationarity arises from drift in the underlying real rate of interest: we also outline the key assumptions on which our interpretation rests. Section 9 concludes.

2. Where our work fits in the literature

This section provides a lightening tour of the natural rate hypothesis and the use of non-stationary data to test for the existence of a long-run Phillips curve.

2.1. The history of the Phillips curve

The natural rate hypothesis is one of the most extensively studied issues in the recent history of macro econometrics. In 1958 Phillips (1958) pointed to the existence of a stable structural relationship between wage inflation and unemployment in the United Kingdom. King and Watson (1994) (KW) provide a comprehensive survey of the history of the debate over the nature of the Phillips curve and an analysis of the properties of the post-war history of the behavior of unemployment and inflation. Their goal is to provide, in their words, a ‘revisionist interpretation’ of the data. In the following section we summarize the KW approach and explain how our paper is related to their work.

2.2. The natural rate and non-stationarity

KW look at the post-war history of inflation and unemployment in the U.S. They pass the two series through a band-pass filter that decomposes each of them into three components; one at low frequencies, one at high frequencies and one at business cycle frequencies that they define to be cycles with periods between 18 months and 8 years. KW find that

> …there is a pronounced negative correlation of inflation and unemployment at business cycle frequencies, which is remarkably stable over the postwar period. Lower frequency comovements of inflation and unemployment, however, display links that are very unstable across time.

They go on to say that at low frequencies

> …there is evidence of $I(1)$ behavior in inflation and unemployment, but no evidence of cointegration. This corresponds to the idea that there are ‘stochastic

trends' in inflation and unemployment; it sets the stage for structural estimates relating these trends.[3]

The presence of non-stationarity in the inflation and unemployment series is an important finding since testing for the existence of a stable long-run trade-off between inflation and unemployment requires that one or other of the series in question should undergo a permanent change. This is the point made by Fisher and Seater (1993), in the context of tests for the superneutrality of money;

> …the restrictions implied by … [longrun superneutrality]… depend on the orders of integration [of the variables involved, because]… the consequences of an event cannot be inferred if the event has not occurred.[4]

But although the presence of non-stationarity is necessary to discriminate between alternative models of the long-run Phillips curve, presence of non-stationarity is not sufficient. One must also make an identification assumption. King and Watson (1997) take up this issue in their 1997 paper. After establishing that unemployment and inflation can each be modeled as $I(1)$ series they proceed to catalogue a range of possible prior identifying assumptions, some of which would lead one to accept the natural rate hypothesis and others that would lead one to reject it.

2.3. *What is new about our approach?*

In our study we begin with essentially the same data set as KW although we add data on the federal funds rate and we use quarterly rather than monthly data. Like KW, we find that one cannot reject the assumption that unemployment and inflation are $I(1)$ series and we establish the same for the Fed funds rate. But unlike KW we find that the low frequency comovements of inflation and unemployment are stable and cointegrated across the whole sample. In our empirical model below we explain this contrasting evidence. Our basic argument will be that a stationary and stable long-run relationship between unemployment and inflation can be established once time-varying coefficients of the inflation and interest rate relationship are allowed for. In Fig. 1 we plot data for the federal funds rate and the unemployment rate and in Fig. 2 we plot the federal funds rate and the rate of change in the GDP deflator. We want to draw attention to two features of this data that will be important for our arguments. The first is that although unemployment and the interest rate move in opposite directions at high frequency, they move together at low frequencies: in the period before 1980 both series are trending up; in the period after 1980 they are trending down. The same low frequency comovement can be seen clearly in the inflation and interest rate data.

A second feature of the data that will be significant for our later argument is that the difference between the inflation rate and the nominal interest rate is larger after 1980 than before. Since the difference in these series measures the real interest rate,

[3]King and Watson (1994, p. 209).

[4]Fisher and Seater (1993, pp. 402–403).

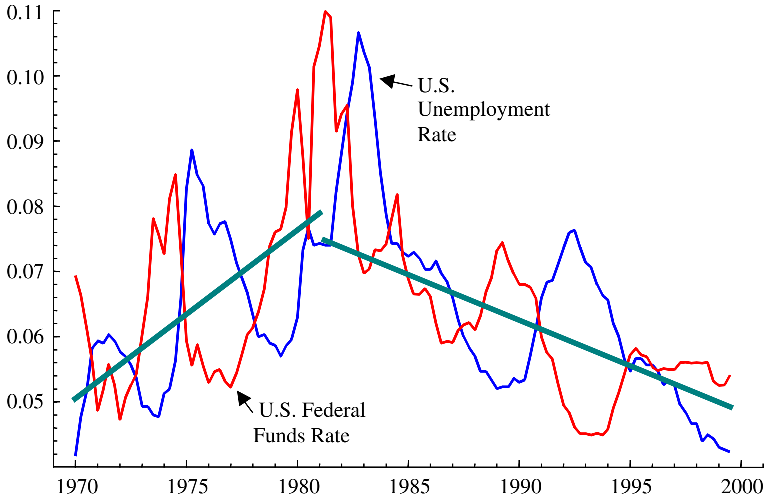

Fig. 1. Unemployment and the federal funds rate.

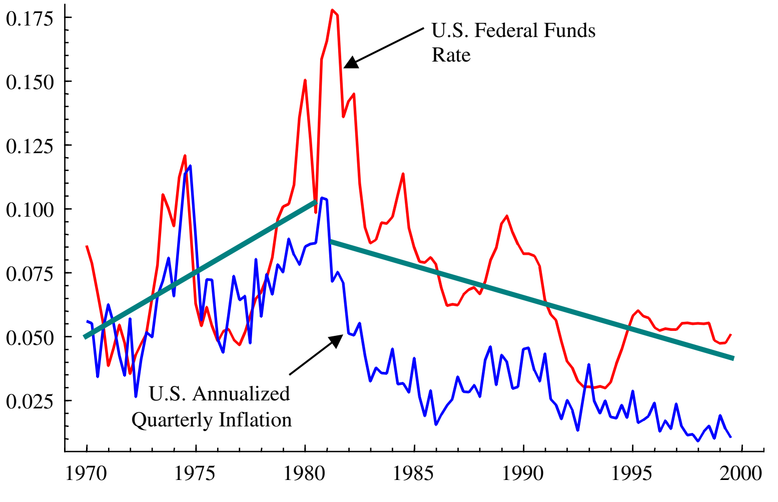

Fig. 2. Inflation and the federal funds rate.

this fact implies that the average real interest rate was higher in the second part of the sample than the first.

These pictures are suggestive but one would like to be able to provide quantitative assessments of the probabilities of alternative explanations of these facts. For this purpose we carried out a more formal analysis of the low frequency comovements in unemployment, inflation and the federal funds rate using cointegration analysis.

3. Modeling the data

This section describes an approach to low frequency movements in the data, based on cointegration analysis.

3.1. A statistical model

We adopt the notational convention that lowercase letters are scalars, uppercase letters are vectors, boldface uppercase letters are matrices and superscript T denotes the transpose operator. Let u_t, i_t, and $\dot{p}_t$ be the unemployment rate, the interest rate and the inflation rate, define the three-dimensional process $X_t = \{u_t, i_t, \dot{p}_t\}$ and consider the autoregressive representation

$$X_t = \mathbf{\Pi}_1 X_{t-1} + \mathbf{\Pi}_2 X_{t-2} + \cdots + \mathbf{\Pi}_k X_{t-k} + \mathbf{\Phi} D_t + \varepsilon_t, \quad t = 1, \ldots, T. \tag{1}$$

The terms $\mathbf{\Pi}_i$ and $\mathbf{\Phi}$ represent conformable matrices of coefficients and D_t is a $q \times 1$ vector of deterministic variables that might contain a constant, trend and possibly dummy variables. We assume that $\{\varepsilon_t\}$ is a sequence of independent Gaussian variables with zero mean and covariance matrix $\mathbf{\Omega}$ and we write the system as an observationally equivalent vector error correction model (VECM)

$$\Delta X_t = \sum_{i=1}^{k-1} \mathbf{\Gamma}_i \Delta X_{t-i} + \mathbf{\Pi} X_{t-1} + \mathbf{\Phi} D_t + \varepsilon_t, \tag{2}$$

where the matrices $\mathbf{\Pi} = \sum_{i=1}^{k} \mathbf{\Pi}_i - \mathbf{I}$ and $\mathbf{\Gamma}_i = -\sum_{j=i+1}^{k} \mathbf{\Pi}_j$ are each 3×3.

The key idea behind cointegration is that the matrix $\mathbf{\Pi}$ that premultiplies the levels variables in Eq. (2) may not be of full rank. If the rank of $\mathbf{\Pi}$ is r, where $r < 3$, $\mathbf{\Pi}$ can be decomposed as the product of two matrices $\boldsymbol{\alpha\beta}^{\mathrm{T}}$ where $\boldsymbol{\alpha}, \boldsymbol{\beta}$ are each $(3 \times r)$ and have rank r. The r columns of $\boldsymbol{\beta}$ are called cointegrating vectors and $\boldsymbol{\alpha}$ is referred to as the loading matrix (see e.g. Johansen, 1988).

3.2. Modelling the full sample: evidence for structural breaks

We began by fitting an unrestricted VAR for the period 1960Q1–1999Q3 to the three variables and we tested for parameter stability. Although we were unable to fit a stable parameter VAR to the whole period, once we allowed for a break in the coefficients in 1979, we *were* able to fit a well behaved VAR for the period 1970Q1–1999Q3. Since the dynamics and stochastic properties of data in the period from 1960Q1 through 1969Q4 behave quite differently from the other sub-periods, we chose to discard this period for the purposes of the current paper.

To establish formally that the data contain a break we conducted a test based on the work of Bai et al. (1998) (BLS). We used the framework for integrated data as described in Section 3, pp. 402–408, of their paper. This method allows for breaks in the short-term coefficients and in the cointegrating vectors of a VECM. The BLS point estimate of the break date coincides with the maximum of the likelihood function after sequentially estimating the model, allowing for all possible break dates, using a DOLS regression as described in Stock and Watson (1993). Fig. 3 graphs the recursive likelihood as a function of the break date, k. After applying the BLS procedure to the sample 1970Q1–1999Q4 we found a point estimate for k of 1979Q3 and a 90% confidence interval of 21 quarters on either side. Since this is a relatively wide confidence region we conducted a sensitivity analysis when estimating our model by checking the results for a range of alternative dates and found that our results are robust to the exact choice of break date. To gain further evidence, we conducted a version of Hansen and Johansen's (1999) version of Nyblom (1989) *sup Q* and *Mean Q* test. These are tests for a break in the estimated cointegrating vectors. We report the results in Table 1. The test statistics show that both tests reject the null of no break over the 70–99 sample. The test statistic for the *sup Q* test is 4.8 which exceeds the 95% critical value of 2.45. Similarly, the test statistic for the *Mean Q* test is 1.71 which exceeds the 95% critical value of 1.11. In

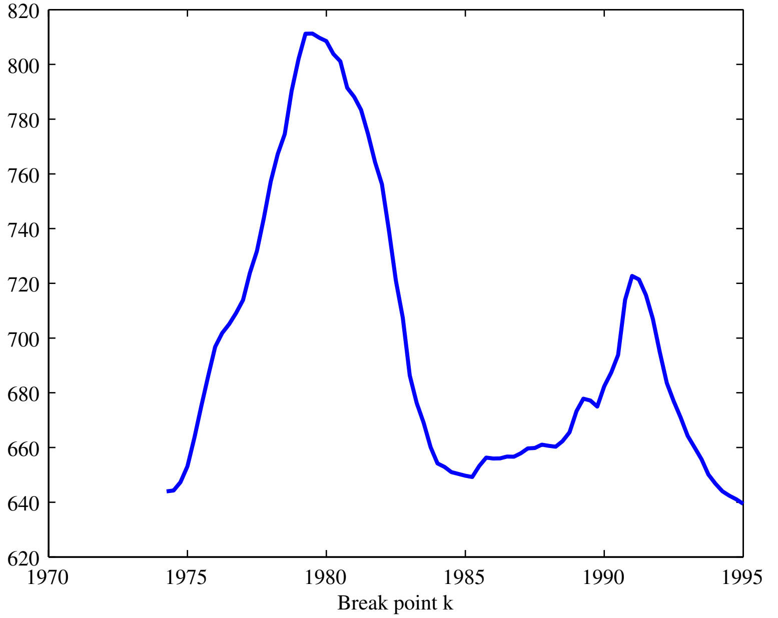

Fig. 3. Recursive likelihood function.

Table 1
The Hansen and Johansen Nyblom test

	70–99		70–79		79–99	
Test	Q	95% c.v.	Q	95% c.v.	Q	95% c.v.
sup Q	4.80	2.45	1.93	2.42	1.77	2.45
Mean Q	1.71	1.11	0.37	1.09	0.80	1.10

Critical values simulated using Warne's (2003) program "Structural VAR'.

contrast, the test statistics for the two sub-periods 70–79 and 79–99 fail to reject the null of a constant parameter model at the 95% level. Our next step was to split the sample in 1979Q3 and, for each sub-sample, to develop a separate well specified cointegrated VAR.

3.3. Empirical models for each sub-period

This subsection provides a brief summary of our main findings. A more complete analysis can be found in the working paper version of our paper (Beyer and Farmer, 2001).

For each sub-period we estimated a separate model based on Eq. (2) and we estimated the rank of $\mathbf{\Pi}$ to be two in both cases. For the three-dimensional process X_t this implies one common stochastic trend that is represented by a zero eigenvalue. Fig. 4 shows recursively estimated eigenvalues of $\mathbf{\Pi}$ for each sub-sample and demonstrates that there is strong evidence for one zero eigenvalue in each of the models. Next, we established the existence of two empirically stable cointegrating vectors for each sub-period.[5] One of these cointegrating vectors appears to be stable also across the two sub-samples; the other one breaks in 1979. To identify the cointegrating equations we used the exact identifying restrictions

$$(\boldsymbol{\beta}^i)^{\mathrm{T}} X_t = \begin{pmatrix} 0 & 1 & \beta^i_{13} \\ 1 & 0 & \beta^i_{23} \end{pmatrix} \begin{pmatrix} u_t \\ i_t \\ \dot{p}_t \end{pmatrix}, \quad i = 1, 2,$$

and for consistent estimation of the parameters β^i_{13} and β^i_{23}, we used the reduced rank maximum likelihood method (see e.g. Johansen, 1996 or Johansen and Juselius, 1994). The notation β^i_j refers to cointegrating vector j in sub-period i for $i, j \in \{1, 2\}$. $\hat{\beta}^i_j$ is our estimate of β^i_j.

Table 2 shows our estimates of $\boldsymbol{\beta}^i$ with standard errors in parentheses and Figs. 5–7 show the recursive estimates of the freely estimated coefficients $\hat{\beta}^i_j$ together with their ± 2 standard error bands.[6] Our estimates of the first cointegrating vector show that in the first sub-period the interest rate cointegrates with inflation with a coefficient of 0.76. In the second sub-period it cointegrates with inflation with a coefficient of 1.5. In contrast the low frequency comovements of unemployment with inflation are very similar in both periods.[7] The different stability properties of the two cointegrating vectors can be seen clearly in Fig. 7.

[5] In addition to the results presented in the previous section we used a battery of misspecification and parameter constancy tests, including tests for homoskedasticity, to ensure well specified models for both sub-samples.

[6] All calculations were done using PcGive, see Doornik and Hendry (2001). For the calculation of the standard errors see e.g. Johansen (1996, Theorem 13.4).

[7] The coefficients are 0.58 and 0.75 and a test of the hypothesis that these coefficients are equal across each sub-period would not be rejected. For example, restricting both coefficients to be 0.6 in each sub-period yields p-values of more than 80%.

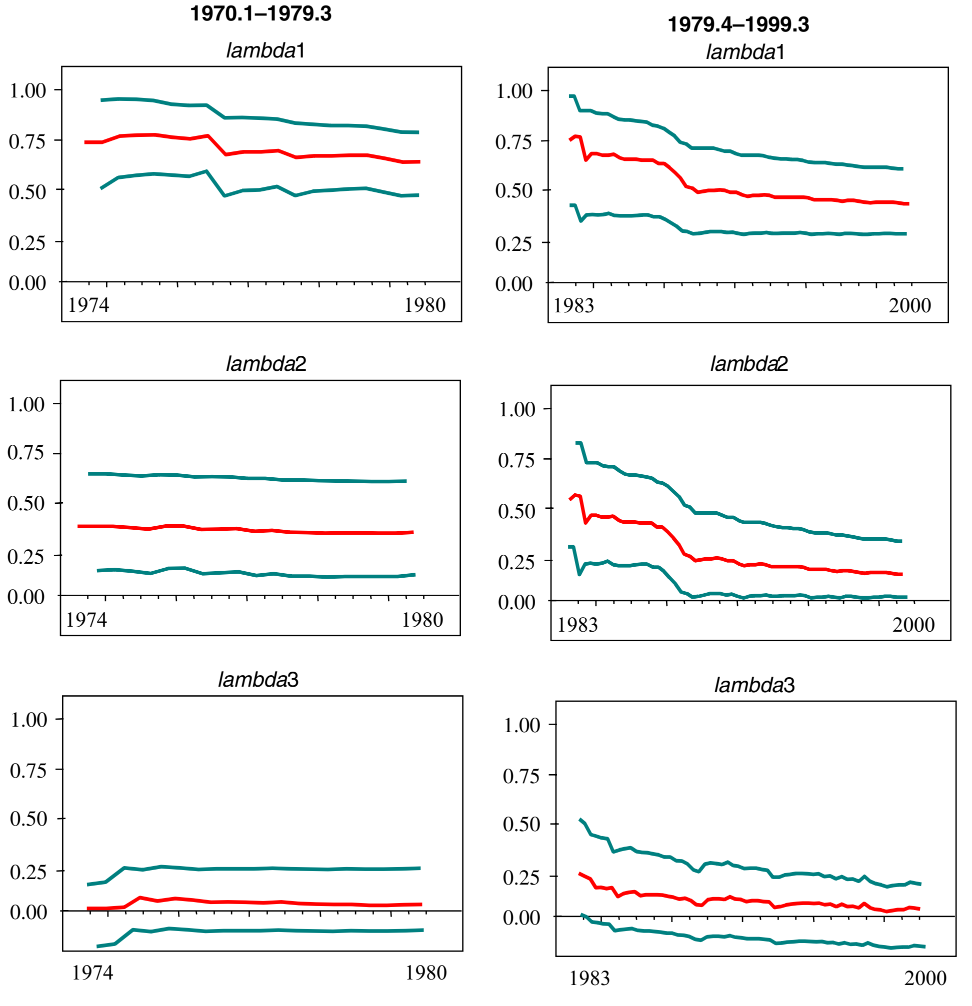

Fig. 4. Recursive estimates of the roots of $\mathbf{\Pi}$ with 2-standard error bounds.

Obviously our samples are of relatively modest size; however, it is still possible to gain information from these samples about the low frequency properties of the data. There are two reasons why we think this is possible.

First, our evidence from stability tests and recursive estimation is robust and gives us confidence that each sub-period can be represented by a stable parameter model. The graphs of the recursive eigenvalues in Fig. 4 clearly suggest that the smallest eigenvalue in both sub-samples can be assumed to be zero, indicating one unit root or common stochastic trend in the data. Of course one could assume that the process is highly persistent but stationary. However in a recent paper Johansen (2004) demonstrates that in this case inferences on $\boldsymbol{\beta}$ that are based on asymptotic distributions can be highly misleading and huge samples would be needed to get

Table 2
Identified cointegrating vectors

	u	i	$\dot{p}$	$const$[a]
$\hat{\beta}_1^1$	0	1	−0.76	−0.02
s.e.			(−0.17)	
$\hat{\beta}_2^1$	1	0	−0.58	−0.03
s.e.			(0.10)	
$\hat{\beta}_1^2$	0	1	−1.50	−0.03
s.e.			(−0.29)	
$\hat{\beta}_2^2$	1	0	−0.75	−0.04
s.e.			(0.15)	

[a] *const* is the deterministic mean of the cointegrating vector over the sample. It represents that part of the constant which lies in the cointegrating space, see Johansen (1994). Standard errors in round brackets.

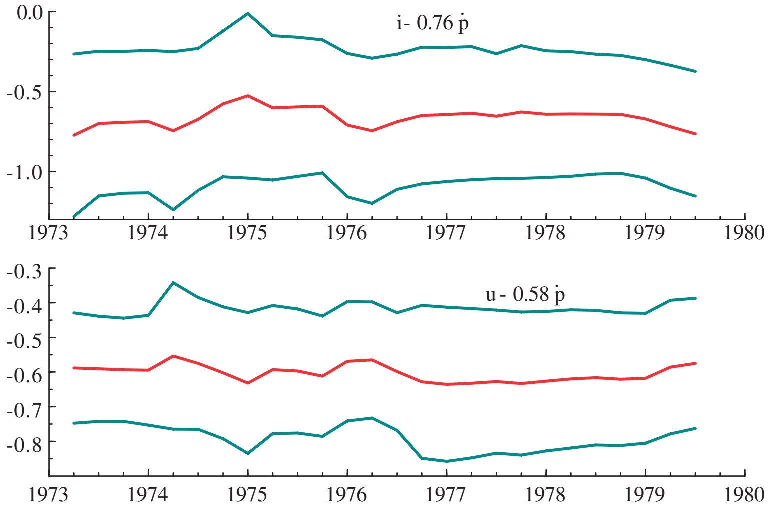

Fig. 5. Recursive estimates of the cointegrating vectors (first sub-sample).

close to the asymptotic distribution. By contrast, when imposing a unit root, if the process is highly persistent but stationary, gives reliable inference results for $\boldsymbol{\beta}$.

Second, the cointegrating relationships can be given a structural interpretation, even in small samples. Often cointegration relationships are called ‘long-run’ equations since they represent equations that would hold between variables if all shocks could be switched off and the dynamical system allowed to return to a stationary state or a balanced growth path. In order to conduct statistical inference about the cointegration vectors, Johansen’s method is based on asymptotic theory. However, valid parameter estimation of the coefficients, and a corresponding economic interpretation of the cointegrating relationships, does not require an infinite amount of data. Different kinds of long-run equations will be associated with

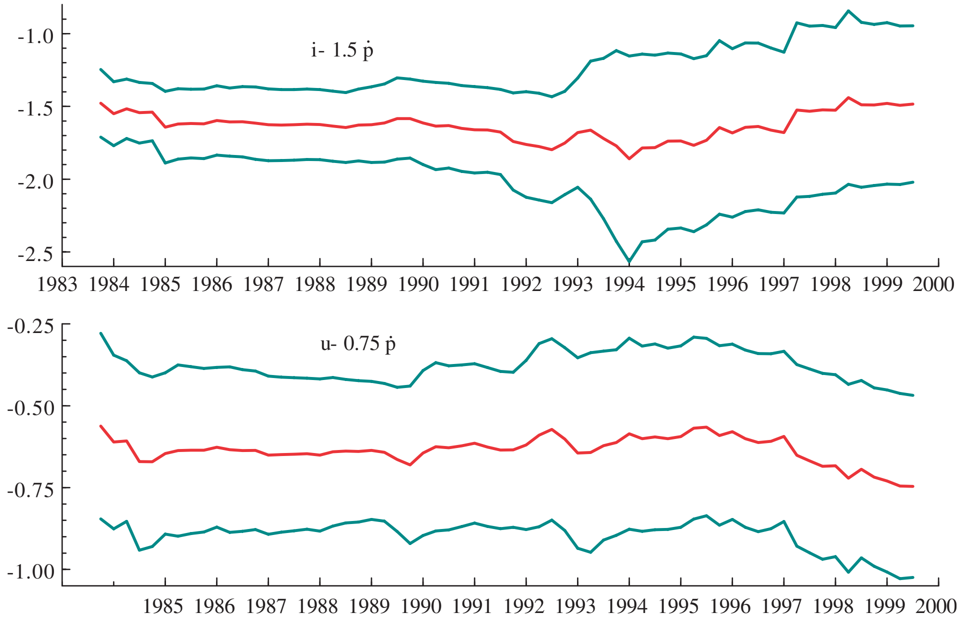

Fig. 6. Recursive estimates of the cointegrating vectors (second sub-sample).

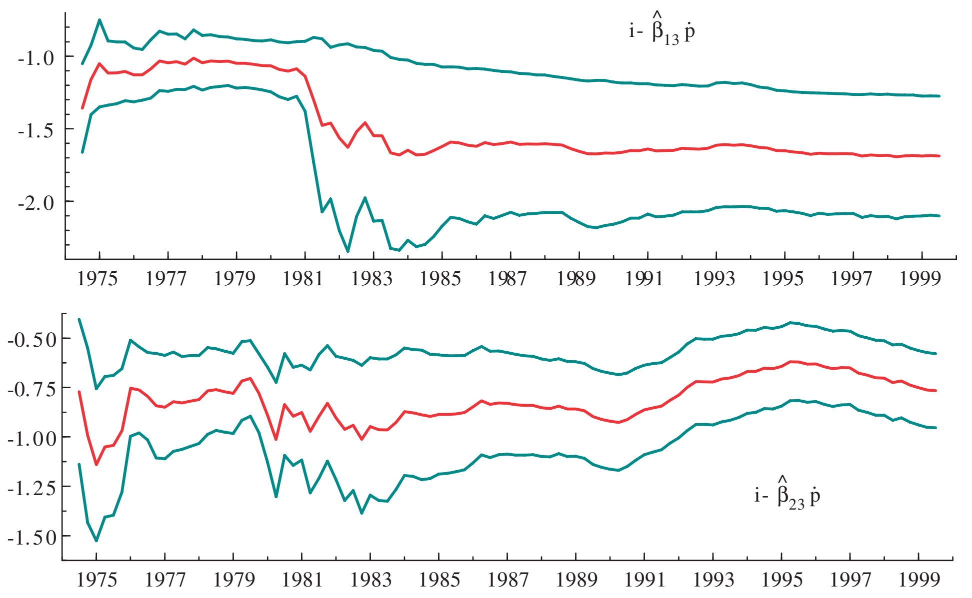

Fig. 7. Recursive estimates of the cointegrating vectors (full sample).

different parameters of the associated dynamical system even if the system only remains in a given state for a limited period of time. An obvious reason why the parameters of the system might change is as a consequence of an abrupt change in policy.

4. Developing a structural model

The purpose of this section and the following sections is to explain our findings using economic theory. We will ask the question: Are our estimates of the low frequency cointegrating relationships in the data consistent with a class of modern economic models that incorporate the natural rate hypothesis?

4.1. The theoretical background

Our idea is to map a subset of parameters from a backward looking cointegrated VAR into those of a structural rational expectations model based on the new Keynesian paradigm (see e.g. Galí and Gertler, 1999). In its standard format this model is based on three structural equations: an aggregate demand curve, an aggregate supply curve and a policy rule. The following equations represent a parameterized version of our approach[8]:

$$u_t + a_{13}(i_t - \mathrm{E}_t[\dot{p}_{t+1}]) + f_{11}\mathrm{E}_t[u_{t+1}] = b_{11}u_{t-1} + c_1 + v_{1t}, \tag{3}$$

$$a_{21}u_t + \dot{p}_t + f_{22}\mathrm{E}_t[\dot{p}_{t+1}] = c_2 + b_{22}\dot{p}_{t-1} + v_{2t}, \tag{4}$$

$$i_t + f_{32}\mathrm{E}_t[\dot{p}_{t+1}] = b_{33}i_{t-1} + c_3 + v_{3t}. \tag{5}$$

Formally the model can be written

$$\mathbf{A}X_t = \mathbf{F}\mathrm{E}_t[X_{t+1}] + \sum_{j=1}^{k} \mathbf{B}_j X_{t-j} + \mathbf{C}D_t + V_t, \tag{6}$$

$$\mathrm{E}_{t-1}(V_t V_{t+k}^{\mathrm{T}}) = \begin{cases} \boldsymbol{\Sigma}, & k = 0, \\ 0, & k \neq 0, \end{cases} \tag{7}$$

where, as in the empirical model, $X_t = \{u_t, i_t, \dot{p}_t\}$ is a 3×1 vector, $\mathbf{A}$, $\mathbf{F}$ and $\mathbf{B}_j$, $j = 1, \ldots, k$, are 3×3 matrices of coefficients, $\mathbf{C}$ is a $3 \times q$ matrix of parameters and D_t is a $q \times 1$ vector of deterministic variables.

V_t is a 3×1 vector of covariance stationary shocks with covariance matrix $\boldsymbol{\Sigma}$ and E_t is a conditional expectations operator. For model (6) to be given a structural interpretation we assume that $\boldsymbol{\Sigma}$ is diagonal. We assume further that the model has a

[8] Since we are not using output in our study we assume instead that it is inversely related to unemployment through a production function (Okun's, 1962, 'Law') and we replace output by the negative of unemployment.

unique reduced form and that it can be represented by the same autoregressive representation as in (1) and hence in (2).

We can re-parameterize the structural model to resemble an analogue of an error-correction-model

$$\tilde{\mathbf{A}}\Delta X_t = \tilde{\mathbf{F}}\Delta \mathrm{E}_t[X_{t+1}] + \sum_{j=1}^{k-1} \tilde{\mathbf{B}}_j \Delta X_{t-j} + \tilde{\mathbf{\Pi}} X_{t-1} + \tilde{\mathbf{C}} D_t + V_t, \tag{8}$$

where the matrices $\tilde{\mathbf{A}}$, $\tilde{\mathbf{F}}$, $\tilde{\mathbf{B}}_1, \ldots, \tilde{\mathbf{B}}_{k-1}$ and $\tilde{\mathbf{\Pi}}$ are related to the structural parameters by the transformations

$$\tilde{\mathbf{\Pi}} \equiv \tilde{\mathbf{F}} + \sum_{j=1}^{k} \tilde{\mathbf{B}}_j - \mathbf{A}, \quad \tilde{\mathbf{A}} \equiv \mathbf{A} - \mathbf{F},$$

$$\tilde{\mathbf{B}}_j \equiv -\sum_{i=j+1}^{k} \mathbf{B}_i, \quad \tilde{\mathbf{F}} = \mathbf{F}.$$

Since we are interested in the low frequency components of the data, we will simplify notation by incorporating stationary components of the dynamics into new error terms $\tilde{V}_t$ and $\tilde{\varepsilon}_t$ which we define by the identities

$$\tilde{V}_t \equiv V_t - \tilde{\mathbf{A}}\Delta X_t + \tilde{\mathbf{F}}\Delta \mathrm{E}_t[X_{t+1}] + \sum_{j=1}^{k-1} \tilde{\mathbf{B}}_j \Delta X_{t-j},$$

$$\tilde{\varepsilon}_t \equiv \varepsilon_t - \Delta X_t + \sum_{j=1}^{k-1} \mathbf{\Gamma}_j \Delta X_{t-j}. \tag{9}$$

Given Definition (9) we write the structural and reduced form models as follows:

$$\tilde{\mathbf{\Pi}} X_t + \mathbf{C} D_t = \tilde{V}_t,$$

$$\mathbf{\Pi} X_t + \mathbf{\Phi} D_t = \tilde{\varepsilon}_t.$$

4.2. Identifying the structural model

Row i of the matrix $\tilde{\mathbf{\Pi}}$ represents coefficients $\tilde{\pi}_{ij}$ of equation i, which an economic theorist would refer to as the long-run of the corresponding left-hand side variable of the economic model

$$\tilde{\mathbf{\Pi}} X_{t-1} = \begin{pmatrix} \tilde{\pi}_{11} & \tilde{\pi}_{12} & \tilde{\pi}_{13} \\ \tilde{\pi}_{21} & \tilde{\pi}_{22} & \tilde{\pi}_{23} \\ \tilde{\pi}_{31} & \tilde{\pi}_{32} & \tilde{\pi}_{33} \end{pmatrix} \begin{pmatrix} u \\ i \\ \dot{p} \end{pmatrix}_{t-1}. \tag{10}$$

A pre-condition for our theoretical model is that the structural matrix $\tilde{\mathbf{\Pi}}$, like the reduced form matrix $\mathbf{\Pi}$, must also have reduced rank and will have a representation in the form

$$\tilde{\mathbf{\Pi}} = \tilde{\boldsymbol{\alpha}}\boldsymbol{\beta}^{\mathrm{T}},$$

where $\tilde{\boldsymbol{\alpha}}$ are the structural loadings of $\boldsymbol{\beta}$, the cointegrating vectors. Notice that, for any given normalization of the cointegrating space, $\boldsymbol{\beta}$ is invariant between the structure and the reduced form, however, $\tilde{\boldsymbol{\alpha}}$ and $\boldsymbol{\alpha}$ are different across these representations (see Johansen, 1996).

The structural and reduced form parameter matrices $\{\tilde{\mathbf{\Pi}}, C\}$ and $\{\mathbf{\Pi}, \Phi\}$ are related by the identity

$$(\tilde{\mathbf{\Pi}} \;\; \mathbf{C}) \equiv \tilde{\mathbf{A}}(\mathbf{\Pi} \;\; \mathbf{\Phi}).$$

Structural models are usually identified by placing restrictions on the matrices $\tilde{\mathbf{A}}$ and $\tilde{\mathbf{\Pi}}$. In this paper, we propose to study the implications of restrictions of this kind for the estimated reduced form matrix $\mathbf{\Pi}$.

Our approach proceeds in the following steps. We take the 2×3 matrix of estimated cointegrating vectors $\boldsymbol{\beta}^{\mathrm{T}}$. Next we specify the parameters of (8) in order to identify the model as a NKM. We then study the restrictions which the coefficients in $\boldsymbol{\beta}^{\mathrm{T}}$ imply for the matrices $\tilde{\mathbf{\Pi}}$ and $\tilde{\mathbf{A}}$ in the structural NKM. Given this setup, we ask the question: What do the aggregate demand curve, the aggregate supply curve and the policy equation imply for the cointegrating equations? To answer this question one can conceptually set $X_t = X_s = \mathrm{E}_t[X_{t+1}]$ and set all of the shocks equal to zero in every period. This procedure leads to a system of three equations in the steady-state values of u, i and $\dot{p}$. If the equation in question is hit by a stationary error then this long-run equation should hold in the low frequency data. Since we see only two cointegrating equations, either the aggregate demand equation, the aggregate supply equation or the policy equation must *not* be hit by a stationary error. By studying the exactly identified cointegrating equations estimated in the data, we can infer which of the three equations is hit by a non-stationary error.

4.3. Dealing with structural breaks in the data

In our statistical analysis of the data we found that the period from 1970 through 1999 can be modeled as two separate stable parameter models. Furthermore, the date of the break in the data that separates these models coincides quite closely with the end of a period during which the Fed was chaired by Burns. Following Burns, Miller took over for a brief period from March 1978 through August 1979 when he was succeeded by Paul Volcker. After a short episode of turbulence (1979–1982), the Volcker years (1979–1987) and those of his successor Alan Greenspan (1987–), have been remarkably stable. Since the break date in 1979 coincides with a change in Fed chairman, and a dramatic and well documented change in monetary policy, it seems reasonable to assume that the changes in the behavior of the data were caused by changes in the way the Fed sets policy.[9]

[9]Supporting evidence for the case that 1979 marked a significant turning point in U.S. monetary policy can be found in work by Orphanides and Williams (2004) who analyze Fed minutes and argue that Fed operating policy changed dramatically when Paul Volcker came into office.

The existence of a break in 1979 has been well documented by other authors. In a seminal paper, Clarida et al. (2000) attributed the break to a change in the parameters of the monetary policy rule. In contrast, Sims and Zha (2004) estimated a reduced form Markov switching model and found evidence that parameter instability can be attributed to changes in the variance–covariance matrix of shocks across regimes. In a related paper (Beyer and Farmer, 2003), we estimated a structural model with a single break in 1979. In that work we showed that the data is consistent with a model in which the private sector structural equations (the parameters of the aggregate demand curve and of the Phillips curve) remain constant across the entire sample period and the only change that occurs in 1979 is in the parameters of the policy rule. Our empirical work in the current paper addresses the issue of heteroskedasticity by allowing the reduced form variance–covariance matrix to differ across each sub-period although we impose (and test for) the assumption of homoskedasticity within regimes.

To deal with breaks in our theoretical analysis, we estimate different cointegrating parameters for our model before and after the break. By applying exclusion restrictions to the structural model (i.e. restrictions on the parameter matrices $\tilde{\mathbf{A}}$ and $\tilde{\mathbf{\Pi}}$), we provide structural restrictions on the cointegrating space. We will study the implications of different assumptions about the source of the break in the cointegrating equations and we will argue that the most natural interpretation of the source of the break is that it can be attributed to a change in policy. Further, we will use our interpretation of the source of the break to learn about the structure of the long-run of the other equations of the model.

5. Mapping from an empirical model to a structural model

In this section we will explore the implications of the estimated matrix $\hat{\mathbf{\Pi}}$ of the cointegrated VAR on various schemes that identify $\tilde{\mathbf{\Pi}}$, the matrix of long-run coefficients of the structural NKM. These different identification schemes are associated with different theories. We ask: Which theoretical equilibrium relationships are compatible with $\hat{\mathbf{\Pi}}$? As we focus on the long-run coefficients we represent the short-run dynamics of (8) by $\tilde{V}_t$ as in (9).

5.1. The aggregate demand equation (strong form)

We begin with the aggregate demand curve. A common assumption is that demand arises from the Euler equation of a representative agent with separable preferences over consumption, leisure and real balances. In this case, the aggregate demand equation takes the form

$$\tilde{\pi}_{12} i_t - \tilde{\pi}_{12} \dot{p}_t + c_1 = \tilde{v}_{1t}, \quad \tilde{\pi}_{12} > 0. \tag{11}$$

If the aggregate demand curve is one of the equations hit by a stationary error, the following equation will describe the relationship of the estimated cointegrating vectors to the long-run parameters of the aggregate demand equation

$$[0 \;\; \tilde{\pi}_{12} \;\; -\tilde{\pi}_{12}] = [\tilde{\alpha}_{11} \;\; \tilde{\alpha}_{12}]\begin{bmatrix} 0 & 1 & \beta_{13} \\ 1 & 0 & \beta_{23} \end{bmatrix}. \tag{12}$$

This is a system of three equations

$$0 = \tilde{\alpha}_{12},$$

$$\tilde{\pi}_{12} = \tilde{\alpha}_{11},$$

$$-\tilde{\pi}_{12} = \tilde{\alpha}_{11}\beta_{13} + \tilde{\alpha}_{12}\beta_{23}.$$

Notice that since $\tilde{\alpha}_{12} = 0$ only the first cointegrating vector enters the strong form of the aggregate demand equation. Further, the above restrictions, taken together, imply

$$\beta_{13} = -1,$$

and hence the first cointegrating vector should be given by

$$(\beta_1)^{\mathrm{T}} = \begin{array}{cccc} u & i & \dot{p} & const \\ \Big[0 & 1 & -1 & -\tilde{c}_1 \Big]. \end{array} \tag{13}$$

Eq. (13) implies that the Fisher equation will be one of the two cointegrating vectors and it carries the implication that the real interest rate should be stationary around a constant mean.

5.2. The aggregate demand equation (weak form)

The implication of a stationary real rate with a constant mean is a strong one and there are a variety of alternative models that impose a weaker long-run restriction. For example, Keynesian macro-models imply that there should be an upward sloping relationship between unemployment and the real interest rate (a downward sloping IS curve in output-interest rate space). A similar implication follows from general equilibrium models in which Ricardian equivalence fails to hold. To allow for models in this class we consider a weaker specification of the aggregate demand equation

$$\tilde{\pi}_{11}u_t + \tilde{\pi}_{12}i_t - \tilde{\pi}_{12}\dot{p}_t + c_1 = \tilde{v}_{1t}, \quad \tilde{\pi}_{11} < 0, \;\; \tilde{\pi}_{12} > 0, \tag{14}$$

where the level of unemployment as well as differences of unemployment appear in the equation.

If the weak form of the aggregate demand equation holds then the relationship of the estimated cointegrating vectors to the long-run parameters of the aggregate demand equation would be given by

$$[\tilde{\pi}_{11} \;\; \tilde{\pi}_{12} \;\; -\tilde{\pi}_{12}] = [\tilde{\alpha}_{11} \;\; \tilde{\alpha}_{12}]\begin{bmatrix} 0 & 1 & \beta_{13} \\ 1 & 0 & \beta_{23} \end{bmatrix}. \tag{15}$$

This system of equations no longer imposes the Fisher equation as a cointegrating vector. Notice that since $\tilde{\alpha}_{11}$ and $\tilde{\alpha}_{12}$ are both different from zero, both cointegrating vectors enter the weak form of the aggregate demand equation. However, the assumption that $\tilde{\pi}_{11}$ and $\tilde{\pi}_{12}$ are positive will imply sign restrictions on the cointegrating vectors. In particular these restrictions imply that upward drift in the real interest rate should be associated with upward drift in unemployment. In Section 7.2 we show that these restrictions are inconsistent with the estimated cointegrating space over the first sub-period.

5.3. The aggregate supply equation

Standard forms of the aggregate supply equation are captured by an expectations augmented Phillips curve. This equation imposes no long relationship between unemployment and inflation and it implies a long-run supply curve of the form

$$\tilde{\pi}_{21}u_t - c_2 = \tilde{v}_{2t}, \quad \tilde{\pi}_{21} > 0. \tag{16}$$

Eq. (16) implies that expected inflation will increase when unemployment is below its natural rate, where the natural rate of unemployment which we denote u^{NR} is given by the expression,

$$u^{\mathrm{NR}} = \frac{c_2}{\tilde{\pi}_{21}}.$$

To address the fact that unemployment is a highly persistent process, several authors have argued that the natural rate of unemployment is time-varying. Prominent examples include Gordon (1997), Grant (2002), Staiger et al. (1997). More recently, Laubach and Williams (2003) have also proposed models in which the real rate of interest may be time-varying. To allow for a non-stationary natural rate in our theoretical model, we propose the following model of the natural rate:

$$u_t^{\mathrm{NR}} = c_2 + \zeta_t,$$

where ζ_t represents its time-varying component. If ζ_t is stationary, even if it is highly persistent, it can be incorporated into $\tilde{v}_{2t}$ and Eq. (16) will continue to hold. If it is a non-stationary random walk, as, for example, in Gordon (1997), the structural error term $\tilde{v}_{2t}$ will be $I(1)$ and, in this case, we would not expect to observe Eq. (16) as one of the cointegrating equations in the data. We return to this point below in Section 5.5 in which we discuss how our theoretical model can be used to interpret cointegrating relationships in the data.[10]

Our specification in Eq. (16) can be expanded to include most of the variants of the Phillips curve that have been discussed in the literature by adding additional polynomials in the lag operator premultiplying the expected difference in the Fed funds rate and the expected growth in unemployment. These variants would have the

[10] A similar argument will apply to the demand curve if, as in Laubach and Williams (2003), the 'natural rate of interest' is time-varying. A time-varying natural rate of interest can be incorporated into the error term of the demand equation. If its time variation is non-stationary, the error term to the demand equation will also be non-stationary.

same implication for low frequency movements in unemployment, inflation and the Fed funds rate as long as only differences or expected differences in endogenous variables are allowed to enter the equation; in other words, as long as the proposed equation is consistent with the natural rate hypothesis.[11]

If Eq. (16) is one of the structural equations in the data generating process, we can write the relationship of the estimated cointegrating vectors to the long-run parameters of the aggregate supply equation as

$$[\tilde{\pi}_{21} \;\; 0 \;\; 0] = [\tilde{\alpha}_{21} \;\; \tilde{\alpha}_{22}] \begin{bmatrix} 0 & 1 & \beta_{13} \\ 1 & 0 & \beta_{23} \end{bmatrix}. \tag{17}$$

Again this is a system of three equations

$$\begin{aligned} \tilde{\pi}_{21} &= \tilde{\alpha}_{22}, \\ 0 &= \tilde{\alpha}_{21}, \\ 0 &= \tilde{\alpha}_{21}\beta_{13} + \tilde{\alpha}_{22}\beta_{23}. \end{aligned}$$

Due to the restriction $\tilde{\alpha}_{21} = 0$, it follows that only the second cointegrating vector is allowed to enter the aggregate supply equation. Furthermore, if $\tilde{\pi}_{21} = \tilde{\alpha}_{22} \neq 0$ then since

$$\tilde{\alpha}_{22}\beta_{23} = 0,$$

it follows that β_{23} must be zero. This, however, is a strong implication that is contradicted by the data since it implies that unemployment must be stationary. We return to this point in Section 5.5 when we discuss the source of the common trend.

5.4. The policy rule

The final equation of System (6) is a policy rule of the form

$$\tilde{\pi}_{31}u_t + \tilde{\pi}_{32}i_t + \tilde{\pi}_{33}\dot{p}_t - c_3 = \tilde{v}_{3t}, \quad \tilde{\pi}_{31} > 0, \;\; \tilde{\pi}_{32} > 0, \;\; \tilde{\pi}_{33} < 0. \tag{18}$$

The ratios $\tilde{\pi}_{33}/\tilde{\pi}_{32}$ and $\tilde{\pi}_{31}/\tilde{\pi}_{32}$ measure the strength with which the Fed changes the Fed funds rate in response to low frequency movements in inflation and unemployment and c_3 is a constant in the policy rule that reflects both the Fed's target inflation rate and its estimate of the natural rate of unemployment.

If Eq. (18) is one of the structural equations in the data generating process the relationship of the estimated cointegrating vectors to the long-run parameters of the policy equation can be written as follows:

$$[\tilde{\pi}_{31} \;\; \tilde{\pi}_{32} \;\; \tilde{\pi}_{33}] = [\tilde{\alpha}_{31} \;\; \tilde{\alpha}_{32}] \begin{bmatrix} 0 & 1 & \beta_{13} \\ 1 & 0 & \beta_{23} \end{bmatrix}. \tag{19}$$

[11]There is a large literature on the microfoundations of aggregate supply that discusses whether lagged or expected future inflation should appear in this equation. This literature derives the influence of excess capacity (or unemployment) on inflation in a class of models in which there are nominal rigidities in price or wage setting (see the survey by Clarida et al., 2000).

Using the same method as above this can be written as a system of three equations which places sign restrictions on the cointegrating vectors but no exclusion restrictions:

$$\tilde{\pi}_{31} = \tilde{\alpha}_{32},$$
$$\tilde{\pi}_{32} = \tilde{\alpha}_{31},$$
$$\tilde{\pi}_{33} = \tilde{\alpha}_{31}\beta_{13} + \tilde{\alpha}_{32}\beta_{23}.$$

If the Fed raises the interest rate when unemployment is below the natural rate, and raises the interest rate when inflation is high, then $\tilde{\pi}_{33}/\tilde{\pi}_{32}$ will be negative and $\tilde{\pi}_{31}/\tilde{\pi}_{32}$ will be positive.

5.5. *Where does the common trend come from?*

Given that the rank of $\tilde{\mathbf{\Pi}}$ is reduced to two we must amend the set of structural models under consideration by introducing a source of non-stationarity.

To build a structural model that is consistent with cointegration and non-stationarity, we assume that the error term to *one* of its three equations is an $I(1)$ process. This assumption implies that we must choose to replace one of the Eqs. (11), (16) or (18) by its non-stationary analog. That means that one of the errors $\tilde{v}_{it}$ would need to be replaced by an $I(1)$ error such that

$$e_{it} - e_{it-1} = \tilde{v}_{it}, \quad i = 1, 2, 3. \tag{20}$$

Suppose, for example, that the source of the common trend is the aggregate supply equation. In this case we would retain Eqs. (11) and (18) in the system and replace Eq. (16) by its non-stationary analog. To recover the equation with a stationary error, we would first difference the analog, leading to the expression

$$\tilde{\pi}_{21}(\Delta u_t) = \tilde{v}_{2t}. \tag{21}$$

Since Eq. (21) does not contain levels of any of the variables, it will not contribute to the cointegrating space; instead, the error $\tilde{v}_{2t}$ represents through e_{2t} the common trend.

6. Interpreting the data

We have remarked that in the data a slow upward drift of the unemployment rate in the 1970s is accompanied by a similar upward drift in the inflation rate and the Fed funds rate. After 1980, all three series begin to drift back down. If the natural rate hypothesis is correct, then this drift must arise from a unit root in the aggregate supply equation: In effect, the natural rate itself is a random walk. This is the favored explanation in the literature. But if the natural rate drifted up in the 1970s and down in the 1980s, why was it accompanied by similar drifts in inflation and the Fed funds rate? The following subsections discuss two prominent explanations of these comovements.

6.1. A explanation based on a policy game

In a recent paper, Ireland (1999) constructs a bivariate model of inflation and unemployment using the Barro and Gordon (1983) model of time inconsistent monetary policy. In his work, the Fed plays a game against the public. In this policy game it directly picks the mean of the inflation rate in an attempt to minimize a quadratic loss function. Ireland shows that if the natural rate of unemployment is non-stationary then time inconsistency in the policy game will cause the equilibrium unemployment rate and the equilibrium inflation rate both to inherit non-stationarity from the natural rate of unemployment. However, a linear combination of unemployment and the inflation rate will be stationary. Hence the Barro–Gordon model can account for why the policy maker might transfer a unit root in unemployment into a unit root in inflation and it can also explain why these variables are cointegrated in the data. Notably, Ireland does not model the interest rate.

6.2. An explanation based on misperception

Orphanides (2001) has proposed a different mechanism to explain why inflation and unemployment both went up (and came down) together. His explanation relies on the fact that, during the 1970s, most economists did not know that the natural rate of unemployment had increased and he substantiates this claim by looking at real time estimates of potential output. These estimates were much more optimistic about the trend growth path of the economy than were subsequent revisions of the same series. According to the Orphanides explanation, the Fed overstimulated the economy in the 1970s by reducing the Fed funds rate because it mistook an increase in the natural rate of unemployment for a recession.

6.3. Are these explanations consistent with our empirical evidence?

In our discussion of results in Section 7 we will make use of the Orphanides assumption that the Fed erroneously responded to the unemployment rate instead of to deviations of unemployment from its natural rate. In the absence of this assumption our model has no hope of explaining why inflation and unemployment appear cointegrated. With this assumption we will be able to replicate a version of Orphanides' argument and also explain why we find his argument unconvincing. We will show that if non-stationarity in the data arose because of a unit root in the natural rate of unemployment then one of the cointegrating equations in the data should be the Fisher equation. Our data analysis strongly rejects the hypothesis that the Fisher equation holds across the two sub-periods and so we are led to look for an alternative explanation of the facts.

Our dissatisfaction with Ireland's explanation is based on the same idea. We differ from Ireland's study since we have included the federal funds rate in our analysis whereas Ireland looked at a bivariate model of inflation and unemployment. Since Ireland did not directly model the interest rate, it is always possible that a richer version of his analysis might be able to account for all of the facts; but we find this unlikely since a richer version of Ireland's model is likely to include the Fisher equation just like the models we study in this paper.

7. Some implications of non-stationarity in the natural rate

What are the implications of the assumption that the natural rate of unemployment is a random walk? In the following two subsections we study this question using two alternative assumptions about the aggregate demand curve.

The mapping from the structural coefficients $\tilde{\mathbf{\Pi}}$ to the estimated cointegrating matrix $\boldsymbol{\beta}$ is given by the expression

$$\begin{bmatrix} \tilde{\pi}_{11} & \tilde{\pi}_{12} & \tilde{\pi}_{13} \\ \tilde{\pi}_{21} & \tilde{\pi}_{22} & \tilde{\pi}_{23} \\ \tilde{\pi}_{31} & \tilde{\pi}_{32} & \tilde{\pi}_{33} \end{bmatrix} = \begin{bmatrix} \tilde{\alpha}_{11} & \tilde{\alpha}_{12} \\ \tilde{\alpha}_{21} & \tilde{\alpha}_{22} \\ \tilde{\alpha}_{31} & \tilde{\alpha}_{32} \end{bmatrix} \begin{bmatrix} 0 & 1 & \beta_{13} \\ 1 & 0 & \beta_{23} \end{bmatrix}. \tag{22}$$

Under the Ireland and Orphanides explanations of the data, unemployment is non-stationary because the aggregate supply equation *does not* hold as a cointegrating equation. In these explanations the aggregate supply curve (16) is replaced by its non-stationary analog and the structural loading coefficients $\tilde{\alpha}_{21}$ and $\tilde{\alpha}_{22}$ are both equal to zero.

Under this assumption, we can eliminate the second row of (22):

$$\begin{bmatrix} \tilde{\pi}_{11} & \tilde{\pi}_{12} & \tilde{\pi}_{13} \\ \tilde{\pi}_{31} & \tilde{\pi}_{32} & \tilde{\pi}_{33} \end{bmatrix} = \begin{bmatrix} \tilde{\alpha}_{11} & \tilde{\alpha}_{12} \\ \tilde{\alpha}_{31} & \tilde{\alpha}_{32} \end{bmatrix} \begin{bmatrix} 0 & 1 & \beta_{13} \\ 1 & 0 & \beta_{23} \end{bmatrix}.$$

Solving the first 2×2 block of this expression for the $\tilde{\alpha}_{ij}$ and rearranging the last 2×1 block gives

$$\begin{bmatrix} \beta_{13} \\ \beta_{23} \end{bmatrix} = \begin{bmatrix} \tilde{\pi}_{12} & \tilde{\pi}_{11} \\ \tilde{\pi}_{32} & \tilde{\pi}_{31} \end{bmatrix}^{-1} \begin{bmatrix} \tilde{\pi}_{13} \\ \tilde{\pi}_{33} \end{bmatrix}. \tag{23}$$

The following subsections replace the numbers β_{13} and β_{23} by their sample estimates for each sub-sample and they impose the assumptions on the parameters $\tilde{\pi}_{11}$, $\tilde{\pi}_{12}$ and $\tilde{\pi}_{13}$ implied by the strong and weak forms of the aggregate demand curve.

7.1. Implications of the strong form of aggregate demand

The strong form of the aggregate demand curve implies that $\tilde{\pi}_{11} = 0$ and $\tilde{\pi}_{12} = -\tilde{\pi}_{13}$. These restrictions arise from the assumption that the model contains a representative consumer with time-separable preferences. Imposing them on (23) gives the expression

$$\begin{bmatrix} \beta_{13} \\ \beta_{23} \end{bmatrix} = \begin{bmatrix} -1 \\ (\tilde{\pi}_{32} + \tilde{\pi}_{33})/\tilde{\pi}_{31} \end{bmatrix}.$$

The estimates of β_{13} and β_{23} in the data are given by

$$\begin{array}{llll} \beta^1_{13} = -0.76, & \beta^1_{23} = -0.58, \\ \text{s.e.} \quad (-0.17) & \text{s.e.} \quad (0.10), \\ \beta^2_{13} = -1.50, & \beta^2_{23} = -0.75, \\ \text{s.e.} \quad (-0.29) & \text{s.e.} \quad (0.15). \end{array}$$

The estimates of β_{23} in each regime are consistent with the sign restrictions that $\tilde{\pi}_{31}$ and $\tilde{\pi}_{32}$ are positive and $\tilde{\pi}_{33}$ is negative. But, if the Fisher equation is to hold in the data, we should expect to see $\beta^1_{13} = \beta^2_{13} = -1$.

Fig. 8 illustrates the full sample estimates of the two cointegrating vectors for both sub-periods. The Fisher hypothesis requires that the coefficient on inflation in the first cointegrating vector should equal -1 in each sub-sample and it further requires that the two sub-sample estimates should be equal. Notice from the left panel of the figure that one cannot reject the Fisher hypothesis in either period since the dashed line indicating a coefficient of -1 is marginally within the 2-standard error confidence bound in each case. However, the recursive estimates in Figs. 6 and 7 show that for the second period this restriction could only just be accepted at the end of the sample period but is clearly rejected over the 1980s and in the beginning of the 1990s.

Further evidence against the Fisher hypothesis comes from the fact that the point estimates in each sub-sample in Fig. 8 lie well outside of the 2-standard error bounds for the other sub-sample. This implies that although one cannot reject the hypothesis that the Fisher equation holds over either separate sub-sample, one *can* reject the joint hypothesis that it holds in both sub-samples together. We are led to reject both the Orphanides and the Ireland explanations for the comovements of inflation and

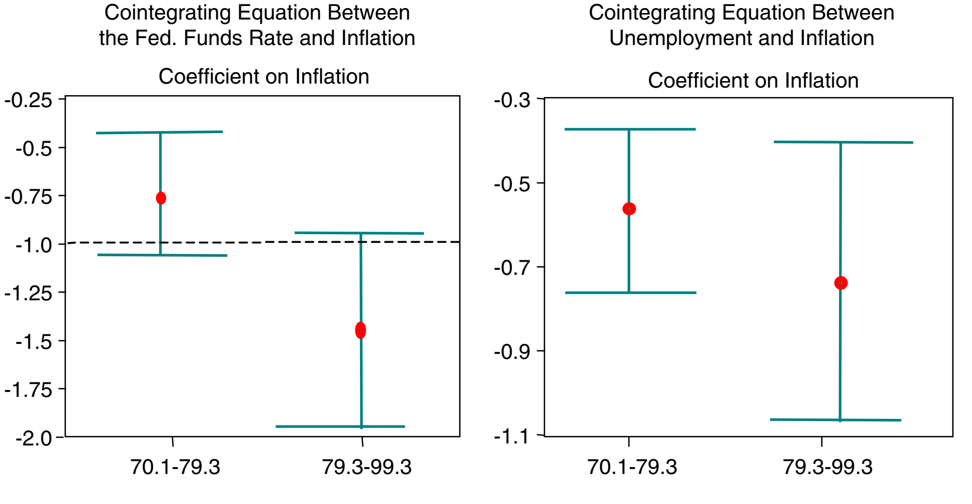

Fig. 8. Estimates of the cointegrating vectors with 2-standard error bands.

unemployment because their explanations are inconsistent with the observed comovements of the interest rate with inflation.

7.2. Implications of the weak form of aggregate demand

When the weak form of the aggregate demand equation holds, we also find an inconsistency with the estimated cointegrating vectors, but only for the first sub-period. Both the strong and weak forms of aggregate demand retain the restriction, $\tilde{\pi}_{12} = -\tilde{\pi}_{13}$. The strong form sets $\tilde{\pi}_{11}$ to zero while the weak form leaves it unrestricted. Under the weak form of aggregate demand the mapping from $\boldsymbol{\beta}$ to $\tilde{\boldsymbol{\Pi}}$ is found by imposing the restriction $\tilde{\pi}_{12} = -\tilde{\pi}_{13}$ on (22):

$$\begin{bmatrix} \beta_{13} \\ \beta_{23} \end{bmatrix} = \begin{bmatrix} (-\tilde{\pi}_{12}\tilde{\pi}_{31} - \tilde{\pi}_{11}\tilde{\pi}_{33})/(-\tilde{\pi}_{11}\tilde{\pi}_{32} + \tilde{\pi}_{12}\tilde{\pi}_{31}) \\ (\tilde{\pi}_{12}\tilde{\pi}_{32} + \tilde{\pi}_{12}\tilde{\pi}_{33})/(-\tilde{\pi}_{11}\tilde{\pi}_{32} + \tilde{\pi}_{12}\tilde{\pi}_{31}) \end{bmatrix}. \tag{24}$$

To analyze these restrictions it helps to define the following three positive numbers:

$$a^i = -\frac{\tilde{\pi}^i_{33}}{\tilde{\pi}^i_{32}} > 0, \quad b^i = -\frac{\tilde{\pi}^i_{11}}{\tilde{\pi}^i_{12}} > 0, \quad c^i = \frac{\tilde{\pi}^i_{31}}{\tilde{\pi}^i_{32}} > 0, \quad i = 1, 2.$$

a^i and b^i are the *negatives* of the long-run elasticities of the interest rate with respect to unemployment in the policy rule and the aggregate demand equation in sub-period i and c^i is the long-run elasticity of the interest rate with respect to unemployment in the policy rule, also in sub-period i. The fact that a^i is positive follows from the assumption that the Fed raises the interest rate in response to rising inflation. b^i is positive if the IS curve has the usual slope (this means that higher unemployment is associated with a higher real interest rate) and c^i is positive if the Fed lowers the interest rate in response to increasing unemployment. All of these responses describe low frequency relationships that are predicted to follow from assumptions about the structure of an economic model.

The following argument establishes that the estimates $\beta^1_{13} = -0.76$ and $\beta^1_{23} = -0.58$ are inconsistent with the theoretical restrictions that a^1, b^1 and c^1 should all be positive.[12] Using the definitions of a^i, b^i and c^i it follows that the estimated cointegrating parameters β^i_{13} and β^i_{23} are related to the positive numbers a^i, b^i and c^i (these numbers are ratios of the structural parameters $\tilde{\pi}^i_{ij}$) by the expressions

$$\beta^i_{13} = -(c^i + a^i b^i)/(b^i + c^i)$$

and

$$\beta^i_{23} = (1 - a^i)/(b^i + c^i).$$

Plugging in estimated numbers for β^1_{13} and β^1_{23} leads to the expressions

$$-0.76 = -(c^1 + a^1 b^1)/(b^1 + c^1) \quad \text{s.e.} \quad (0.17), \tag{25}$$

$$-0.58 = (1 - a^1)/(b^1 + c^1) \quad \text{s.e.} \quad (0.10). \tag{26}$$

[12] The same argument does not apply to the second sub-period.

The numerator and denominator of Expression (25) are both weighted sums of the same positive numbers b^1 and c^1 that differ only in the weight a^1 attached to b^1 in the numerator. For this expression to be equal to -0.76 it follows that a^1 must be a positive number between 0 and 1.

But if a^1 is between zero and 1, the coefficient on inflation in the second cointegrating vector, given by the expression

$$(1-a^1)/(b^1+c^1),$$

must be positive. Our point estimate of this parameter is equal to -0.58 with a standard error of 0.1 and, hence, under our maintained assumptions, the weak form of the aggregate demand curve is inconsistent with data from the first sub-period.

8. An alternative explanation of the data

We have argued that, if the natural rate hypothesis holds, non-stationarity must arise from a common trend that shifts the aggregate supply curve. In this section, we replace the natural rate by the following alternative aggregate supply equation in which there is a long-run upward sloping Phillips curve,

$$\tilde{\pi}_{21}u_t+\tilde{\pi}_{23}\dot{p}_t-c_2=\tilde{v}_{2t}, \quad \tilde{\pi}_{21}>0, \ \tilde{\pi}_{23}<0. \tag{27}$$

The idea of a long-run relationship between unemployment and inflation is consistent with a model in which money has non-superneutral effects. Non-superneutralities are relatively easy to identify in countries that experience hyperinflations since hyperinflation is typically accompanied by high unemployment and severe recessions. We will assess the ability of a model of this kind to also explain the low frequency movements in inflation and unemployment that occurred in the last 30 years in the United States.

There are many possible mechanisms that might cause inflation to be non-superneutral. One explanation is given by the monetary model of Benhabib and Farmer (2000) in which the effects of money on equilibrium output can be substantial. But there are many other possibilities. Non-neutralities in the tax code would cause changes in the equilibrium supply of labor in an equilibrium model like the one studied by Cooley and Hansen (1989). In search models with liquidity effects, such as those studied by Den Haan et al. (1999), or in models with hysteresis effects like the one studied by Ball (1997), one would expect there to be permanent effects on the equilibrium unemployment rate resulting from changes in monetary policy.

If (27) holds in the data then the common trend must arise either because the policy equation or the aggregate demand curve is non-stationary. Our preferred specification is one in which the common trend arises from a slowly drifting aggregate demand curve. The alternative explanation, that the policy rule is non-stationary, will meet the same objection that we leveled at the natural rate model – we would expect to see the Fisher equation in the data.

8.1. Why did inflation and unemployment move together?

If the aggregate demand curve is non-stationary then we must replace Eq. (11) with its non-stationary version that has non-stationary errors (20). In a representative agent model, our assumption would imply that the agent's rate of time preference is a random walk. In more complex general equilibrium models, the equilibrium real interest rate might be non-stationary as a result of cohort effects in an overlapping generations model or as a result of non-stationary fiscal policies.

If the aggregate demand equation is replaced by taking into account (20), the structural loading coefficients α_{11} and α_{12} will both be zero and the mapping from the cointegrating coefficients to the structural coefficients will be given by the expression

$$\begin{bmatrix} \tilde{\pi}_{21} & \tilde{\pi}_{22} & \tilde{\pi}_{23} \\ \tilde{\pi}_{31} & \tilde{\pi}_{32} & \tilde{\pi}_{33} \end{bmatrix} = \begin{bmatrix} \tilde{\alpha}_{21} & \tilde{\alpha}_{22} \\ \tilde{\alpha}_{31} & \tilde{\alpha}_{32} \end{bmatrix} \begin{bmatrix} 0 & 1 & \beta_{13} \\ 1 & 0 & \beta_{23} \end{bmatrix}.$$

Solving the first 2×2 block of this expression for the $\tilde{\alpha}_{ij}$ and rearranging the last 2×1 block gives

$$\begin{bmatrix} \beta_{13} \\ \beta_{23} \end{bmatrix} = \begin{bmatrix} \tilde{\pi}_{22} & \tilde{\pi}_{21} \\ \tilde{\pi}_{32} & \tilde{\pi}_{31} \end{bmatrix}^{-1} \begin{bmatrix} \tilde{\pi}_{23} \\ \tilde{\pi}_{33} \end{bmatrix}. \tag{28}$$

If the aggregate supply equation has a long-run Phillips curve, we have one exclusion restriction, $\tilde{\pi}_{22} = 0$, since, by assumption, the Fed funds rate does not enter the aggregate supply equation. Imposing this restriction leads to the following interpretation of the mapping from estimated cointegrating parameters β_{13} and β_{23} to the structural parameters, $\tilde{\pi}_{ij}$:

$$\begin{bmatrix} \beta_{13} \\ \beta_{23} \end{bmatrix} = \begin{bmatrix} (\tilde{\pi}_{33}/\tilde{\pi}_{32}) - (\tilde{\pi}_{31}/\tilde{\pi}_{32})(\tilde{\pi}_{23}/\tilde{\pi}_{21}) \\ (\tilde{\pi}_{23}/\tilde{\pi}_{21}) \end{bmatrix}.$$

In our model, $(\tilde{\pi}_{23}/\tilde{\pi}_{21})$ is a structural parameter that reflects the slope of the long-run Phillips curve. This parameter is identified and is equal to the cointegrating coefficient β_{23}. The parameters $(\tilde{\pi}_{33}/\tilde{\pi}_{32})$ and $(\tilde{\pi}_{31}/\tilde{\pi}_{32})$ are elasticities of the Fed's response of the interest rate to inflation and unemployment and these parameters are not separately identified. Hence, it is not possible to disentangle the effects of a policy response to inflation from the response to unemployment.

If our model were correct we would expect to see the same cointegrating relationship between unemployment and inflation in both sub-periods. According to our hypothesis, this equation is a structural equation representing the long-run Phillips curve. Since we cannot reject the hypothesis that the estimated value of $(\tilde{\pi}_{23}/\tilde{\pi}_{21})$ in the first period (equal to 0.58 ± 0.1) is equal to the estimated coefficient in the second period (0.75 ± 0.15), we conclude that the hypothesis of a long-run upward sloping Phillips curve is not contradicted by the data. Our model is also consistent with a change in the cointegrating equation between inflation and the federal funds rate from -0.76 to -1.5. We interpret this break in the cointegrating vector as the consequence of a change in Fed policy.

8.2. Some limitations of our explanation

Since the concept of a vertical long-run Phillips curve is an important pillar of modern macroeconomics, the reader may be skeptical of the evidence that we have presented. Hence, before concluding we should state clearly the assumptions on which our analysis rests. First, we have argued that the data can be well modeled by two separate statistical models separated by a break in 1979; the parameters of each sub-model are stable and the models display within period homoskedasticity. This analysis is open to dispute. For example, Cogley and Sargent (2005) have argued that this period of U.S. history can be more accurately represented by a random coefficient model and Sims and Zha (2004) favor an interpretation in which there are frequent switches between regimes. Since the samples are relatively short, the question of how best to model the data is open to alternative interpretations.

Our critique of the natural rate hypothesis also rests on a theoretical model which interprets each sub-period through the lens of the rational expectations assumption. We assume that the change that occurred in 1979 was abrupt and unanticipated and that expectations, after the break, adjusted rapidly to their equilibrium level. To the extent that expectations took time to adjust, perhaps as a consequence of adaptive learning, our analysis may confound short-run dynamics with long-run theoretical relationships.

Finally, our explanation rests on the assumption that the variance–covariance matrix of structural shocks is diagonal since we attribute a non-stationary shock to one and only one equation. If our model is misspecified, perhaps because of an omitted variable that enters more than one equation, then the conclusion that the long-run Phillips curve slopes up does necessarily follow from our interpretation of the data.

9. Conclusion

In this paper we have shown, by means of an example, how to move between reduced form and structural interpretations of cointegrating relationships to assess the plausibility of alternative economic theories. In our example, we argued that the inflation rate, the unemployment rate and the nominal interest rate can be well described as non-stationary but cointegrated variables in U.S. data from 1970 through 1999. The first cointegrating equation, linking inflation with the Fed funds rate, displays a much larger response of the interest rate to inflation after 1979 than before. The cointegrating equation linking unemployment with inflation has been stable over the entire period. Seen through the lens of the NKM, this evidence leads us to be skeptical of theories that incorporate superneutrality as a maintained assumption of an economic model, hence the title of our paper, 'natural rate doubts'.

Although our example is suggestive, our interpretation of the data rests on several maintained hypotheses and the reader is entitled to accept or reject our interpretation based on their plausibility. Whether one accepts our interpretation of the inflation and unemployment data, we believe that our approach can usefully be applied in alternative contexts. Our method requires that one identifies a cointegrated data set that contains one or more common trends. In practice, economic data often displays considerable parameter instability and, in this case, we recommend splitting the sample into separate stable parameter sub-periods. Over each sub-period, we have argued that the existence of a common trend can be used to make inferences about the properties of a subset of the equations of the structural theoretical model.

Since it is difficult, in finite data, to distinguish an integrated process from one that is highly persistent but stationary, it is often argued that one cannot learn much from cointegration analysis in small samples. We believe this argument to be incorrect. Cointegration relationships represent the steady-state (or balanced growth path) equations of a theoretical model and to learn about these steady-state equations one does not need access to an infinite data set. Large swings in the common trend trace out a subspace of the data that is orthogonal to the cointegrating relationships and by studying the character of this subspace one can learn much about the class of theoretical models that is consistent with it. From an econometric point of view it is more important that the assumptions of heteroskedasticity, normality and parameter constancy are correct than that the sample be long. It is for this reason that we recommend splitting a given sample into pieces, each of which satisfies the necessary assumptions necessary for valid statistical inference.

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