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HONORARY LECTURE

TWO NEW KEYNESIAN THEORIES OF STICKY PRICES

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Two alternative theories of aggregate supply, both with a New Keynesian "flavor," are compared. The first assumes that prices are rigid due to the existence of menu costs. The second derives price stickiness endogenously as one equilibrium in an economy with multiple equilibria. In both cases I show that the Ball–Romer concept of real rigidities is essential to explain why monetary policy has real persistent effects. I argue that dynamic menu cost models are determinate because they make special assumptions about the way that money enters the economy. For example, most authors assume either a cash-in-advance constraint or that money enters separably into utility or production functions. Once one moves beyond these special cases, menu cost models that display real rigidity are also likely to display indeterminacy.

Keywords: Menu Costs, Indeterminacy, Sticky Prices

1. INTRODUCTION

Macroeconomic research, based on the real business-cycle (RBC) model, has been relatively successful at explaining comovements between real variables in aggregate time-series data. This success has led researchers to extend the model by adding a motive for agents to hold money, so that it also may be used to understand comovements between real and nominal variables. Three motives that have been widely studied are cash-in-advance constraints [Svensson (1985), Lucas and Stokey (1987)], money in the utility function [Patinkin (1965), Brock (1974)], and money in the production function [Patinkin (1965)]. All three methods of introducing money into an RBC model lead to similar conclusions. General equilibrium models with money, in the absence of significant frictions, cannot explain important features of the observed correlations between money, prices, and income. These features include the observation that, when the nominal money supply increases,

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real variables increase temporarily and return asymptotically to their steady-state values. The price level is slow to respond to a shock to the nominal money supply, but when a shock of this nature occurs, its effects on the price level are cumulative and permanent. In this paper, I refer to these features of the data as the *monetary transmission mechanism*.

I refer to RBC models, amended with a motive for holding money, as first-generation models of money. The apparent failure of these models to explain the monetary transmission mechanism has led to the development of a second generation of models, also based on general equilibrium theory, which build in explicit nominal rigidities of one kind or another. Second-generation theories include the limited participation model of Christiano et al. (1996); models with costly price adjustment such as that of Rotemburg (1996); models with nominal contracting such as those of Taylor (1979), Chari et al. (1996), and Garcia and Ascari (1999); and, more recently, a group of models with staggered price adjustment based on the work of Calvo (1983).[1] It is this latter group that I concentrate on in this survey.[2] These models have become extremely popular and are now widely used as tools to evaluate the past impact of monetary policy and as guides to the construction of sound future policies.

As an alternative to models that incorporate nominal rigidities, an alternative literature argues that purely classical (first-generation models) are fully capable of explaining the monetary transmission mechanism when they are amended to allow for the possibility that equilibria may depend not only on fundamentals, but also on the self-fulfilling beliefs of households and firms. The argument for purely first-generation models relies on that fact that equilibria of monetary models may be indeterminate.[3] In models with indeterminacy, one of the many possible equilibria is characterized by comovements between real and nominal variables that closely resemble those that one finds in the data. Indeterminacy has been used extensively to model the idea that "animal spirits" may be important causes of business cycles but its use in monetary theory is less widely accepted, in part because the mechanisms that lead to indeterminacy in monetary economies are not widely understood. Except for an important recent paper by Kiley (1997), most of the literature on persistence in staggered-price-setting models has proceeded independently of the literature on indeterminacy. However, as Kiley points out, the same assumption that leads to indeterminacy is also necessary to generate nominal persistence in staggered-price models. This raises the obvious question of why staggered-price models do not also display indeterminate equilibria in the examples that have become common in the literature. This paper aims to clarify the connections between models of indeterminacy and models of staggered price setting and, in so doing, to increase the acceptance of models that explain the monetary transmission mechanism with the indeterminacy approach.

My argument unfolds as follows: First, I define the concept of a real rigidity, introduced by Ball and Romer (1990), and I explain the importance of this concept in a model with a neoclassical labor market. Second, I develop a dynamic equilibrium model that nests the Benhabib–Farmer (1999) and Calvo (1983) models as alternative special cases. Third, I show that both special cases require the assumption that there must be a large real rigidity, to explain why shocks to nominal money have persistent real effects. Since real rigidities are required to explain the persistent effects of monetary policy, I argue that one can (and should) dispense with the assumption of staggered price setting.

2. THE NEW KEYNESIAN MODEL

In classical models, prices are assumed to adjust instantly to equate quantities of all commodities demanded and supplied. Many commentators have suggested that the instantaneous market-clearing assumption may be unrealistic because it implies that a positive increase in the money supply will have no effect on employment or output. Instead, in the classical model, an increase in the nominal quantity of money will immediately raise prices in proportion to the magnitude of the increase. Since time-series data suggest that there are important real effects when nominal money first enters an economy, these commentators propose that one should construct an alternative theory of aggregate demand and supply that adds realistic "frictions" that can account for these effects.

A first step toward modeling frictions is to modify the competitive model by introducing price-setting agents. This section describes a simple way of accomplishing this modification, based on Chamberlin's (1933) model of monopolistic competition.

2.1. Market Structure in a Model of Monopolistic Competition

In the model of monopolistic competition, output Y is assembled from a continuum of intermediate goods $Y_i,\ i \in [0, 1]$:

$$Y = F(\tilde{Y}_i), \qquad (1)$$

and each intermediate good is assembled from raw labor using the technology

$$Y_i = f(L_i). \qquad (2)$$

In this notation, $\tilde{Y}_i$ is a linearly homogeneous measurable function $[0, 1] \rightarrow R_+$ representing production opportunities, and $f(L_i)$ is a neoclassical production function with constant or decreasing returns to scale.[4] The production of final commodities is competitive and final-goods producers are assumed to maximize profits. Free entry implies that there will be zero profits. These assumptions allow one to describe demand for the ith intermediate commodity as a function of the ith relative price and of aggregate output. Details are provided in Appendix A, where I show how to derive the ith producer's demand function in a widely used example of a technology originally from Dixit and Stiglitz (1977).

The ith intermediate producer is modeled as a monopolistic competitor that exploits its market power by recognizing that the price of its commodity depends on how much it sells. Its revenue (measured in units of final commodities) is given by a function R,

$$R = R\left(Y, \frac{P_i}{P}\right), \tag{3}$$

where Y is aggregate output and P_i/P is the relative price of the ith producer.

The firm's costs are determined by the quantity of labor that it demands. These also depend on aggregate demand and relative price P_i/P, two variables that reflect the firm's scale of operations through their influence on sales. I model the firm's labor requirement with the function L:

$$L_i = L\left(Y, \frac{Pi}{P}\right), \tag{4}$$

and, in Appendix A, I derive functional forms for R and L in the Dixit–Stiglitz example.

2.2. Utility and the Representative Consumer

Given the market structure described in the preceding section, I assume that firms are owned by a single representative household that maximizes a utility function defined over consumption and labor[5]:

$$U(C, L). \tag{5}$$

When the household decides to increase the scale of operation of the ith firm, it incurs a utility cost that arises from the additional labor that it must supply to the firm. Labor supply of the household is equal to the integral of the labor required to run all of the intermediate industries, and these labor input requirements depend on the pricing policy of the ith firm and on aggregate demand[6]:

$$L = \int L\left(Y, \frac{P_i}{P}\right) di. \tag{6}$$

An increase in the scale of operation also yields a benefit in the form of additional revenues that are available to be spent on consumption goods:

$$C = \int R\left(Y, \frac{P_i}{P}\right) di. \tag{7}$$

Putting these pieces together leads to the maximization problem

$$\max_{\tilde{P}_i/P} U\left(\int R\left(Y, \frac{P_i}{P}\right) di, \int L\left(Y, \frac{P_i}{P}\right) di\right), \tag{8}$$

which must be solved for each of the continuum of industries in the interval [0, 1].

To add money to this model, following Ball and Romer (1990), I assume that aggregate output Y is equal to real balances M/P:

$$Y = M/P. \tag{9}$$

These assumptions allow one to replace Y by M/P in equation (8) to generate the utility function

$$\max_{\tilde{P}_i/P} U\left(\int R\left(\frac{M}{P}, \frac{P_i}{P}\right) di, \int L\left(\frac{M}{P}, \frac{P_i}{P}\right) di\right). \tag{10}$$

3. STICKY PRICES

The New Keynesian literature begins with the monopolistic structure laid out in the preceding sections and modifies it by adding a cost of changing prices. This section discusses the main issue that has arisen in this literature: a search for conditions under which a small cost of changing a nominal price could have big effects on aggregate output.[7]

3.1. Ball and Romer's Concept of a Real Rigidity

I begin with a concept introduced by Ball and Romer (1990), who formalize the issue of the importance of nominal rigidities in the following way. Suppose that the money supply and the equilibrium price level are both equal to 1 (we can always define units to make this so). Now let the money supply increase by some amount Δ to a new level, M, where $\Delta \equiv M - 1$. Given this structure, Ball and Romer ask the following question: If there is a small cost of changing price, could there be a Nash equilibrium in which nominal prices are rigid? To address this question they define the concept of a *real rigidity*.

To simplify notation, I define a reduced-form utility function $W(M/P, \tilde{P}_i/P)$ with the identity

$$W\left(\frac{M}{P}, \frac{\tilde{P}_i}{P}\right) \equiv U\left(\int R\left(\frac{M}{P}, \frac{P_i}{P}\right) di, \int L\left(\frac{M}{P}, \frac{P_i}{P}\right) di\right). \tag{11}$$

The price-setting agent will choose P_i/P to maximize utility. This requires that the derivative of W with respect to $\tilde{P}_i/P$ be set equal to zero:

$$W_2\left(\frac{M}{P}, \frac{\tilde{P}_i}{P}\right) = 0. \tag{12}$$

It follows, from the implicit function theorem, that one can write the relative price of the ith firm as a function of real balances:

$$\frac{P_i}{P} = \phi\left(\frac{M}{P}\right), \tag{13}$$

where the derivative of ϕ is given by the expression

$$\phi'\left(\frac{M}{P}\right) = -\frac{W_{21}\left(\frac{M}{P}, \phi\left(\frac{M}{P}\right)\right)}{W_{22}\left(\frac{M}{P}, \phi\left(\frac{M}{P}\right)\right)}. \tag{14}$$

Ball and Romer propose that π, defined as $\pi \equiv \phi'(1)$, be used as an index of real rigidity. This concept measures the sensitivity of the optimal relative price with respect to a change in aggregate demand. If $\phi(M/P)|_{M/P=1}$ is flat [$\phi'(1)$ is small in absolute value], agents would be willing to tolerate big changes in M/P without changing their price by much. In an economy with real rigidities, if there is a small cost to changing a nominal price, households might decide not to adjust their relative price even if there are substantial changes in aggregate demand.

3.2. Real Rigidity and Indeterminacy

In a recent paper, Kiley (1997) argued that real rigidity in dynamic models makes indeterminacy more likely. Even in static models, there is a sense in which real rigidity is a move "toward" indeterminacy since, when there is a high degree of real rigidity, small changes in the fundamentals of the economy cause very large changes in the equilibrium level of real balances.

Figure 1 illustrates an economy in which utility is influenced by a parameter S that represents a productivity or taste shock. The figure depicts the function $\phi(M/P; S)$ for two different values of S. Equilibrium occurs when $P_i/P = \phi(M/P, S) = 1$, and real rigidity is represented by the fact that the ϕ locus is flat; this implies that small shifts in S (movements up and down of ϕ) cause very big shifts in M/P. In the limiting case of a real rigidity, ϕ is independent of M/P and, in this case, if an equilibrium exists, $\phi(M/P; S)$ is identically equal to 1 for all values of S, and equilibrium is indeterminate. Flat ϕ implies that big changes in

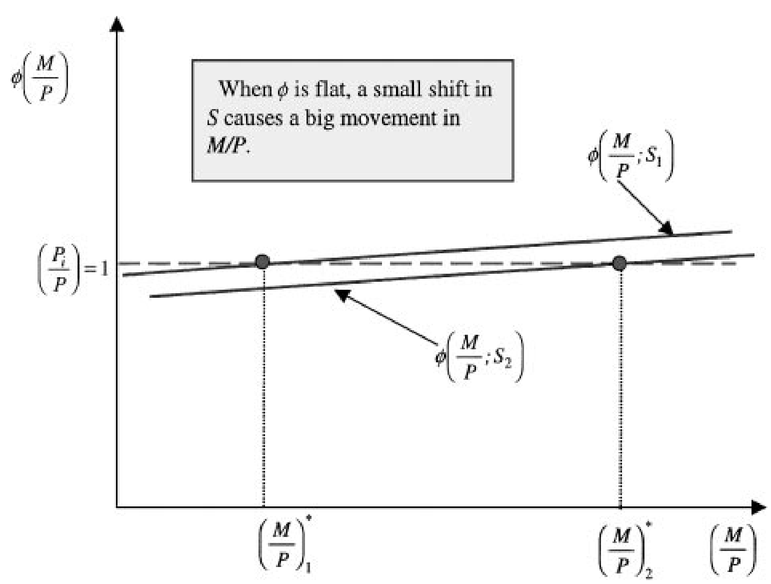

FIGURE 1. Real rigidity and indeterminacy.

M/P do not change utility by very much and, in this case, small menu costs may support big changes in real balances.

3.3. Real Rigidity in the Model of Monopolistic Competition

The concept of real rigidity is relatively abstract and can be applied to a large class of economies for which there exists some function $W(M/P, P_i/P)$ that relates the utility of a price-setting agent to an aggregate variable M/P and its relative price P_i/P. To understand more clearly the consequences of this assumption, this section derives an explicit expression for the function $\phi(M/P)$ for the model of monopolistically competitive industries.

In the Ball–Romer model, the ith price-setting agent sets W_2 equal to zero. For the model of monopolistic competition, this leads to following expression:

$$W_2\left(\frac{M}{P}, \frac{\tilde{P}_i}{P}\right) \equiv U_c(C, L) R_2\left(Y, \frac{P_i}{P}\right) + U_L(C, L) L_2\left(Y, \frac{P_i}{P}\right) = 0, \qquad \mathbf{(15)}$$

where $R_2(Y, P_i/P)$ and $L_2(Y, P_i/P)$ are partial derivatives of the revenue function and the labor input function with respect to P_i/P. For the class of Dixit–Stiglitz technologies, the ratio R_2/L_2 is proportional to the marginal product of labor, and condition (15) can be written as[8]

$$\frac{-U_L(C, L)}{U_C(C, L)} = \lambda f_L(L_i) \frac{P_i}{P}, \qquad \mathbf{(16)}$$

where λ is a constant that reflects the degree of competitiveness of the intermediate-goods market. Rearranging this expression leads to an equation that defines the relative price of a price-setting agent as a function of the ratio of the marginal cost of employing an extra unit of labor (this is the term $-U_L/U_C$) to its marginal product in industry i [this is the term $f_L(L_i)$]:

$$\frac{P_i}{P} = \frac{1}{\lambda} \frac{-U_L(C, L)}{U_C(C, L)} \frac{1}{f_L(L_i)}. \qquad \mathbf{(17)}$$

In a symmetric equilibrium, all firms will choose to employ the same quantity of labor and, in that case, L_i can be replaced by aggregate labor L. It also will be true, in equilibrium, that the quantity equation of money will hold, $C = Y = M/P$, and firms will produce on their production functions. It follows that, in equilibrium, $L = f^{-1}(M/P)$. We can use these facts to derive the following explicit expression for the function $\phi(M/P)$:

$$\phi\left(\frac{M}{P}\right) = \frac{1}{\lambda} \frac{-U_L(f(L), L)}{U_C(f(L), L)} \frac{1}{f_L(L)}, \qquad \mathbf{(18)}$$

where

$$L = f^{-1}\left(\frac{M}{P}\right).$$

Equation (18) describes $\phi(M/P)$ as the ratio of the marginal cost of hiring an extra worker to his marginal product. I refer to the ratio $-U_L/U_C$ as marginal cost because, in a competitive labor market, this term would be equated to the real wage. In the following section, I pursue this idea by characterizing real rigidities in terms of slopes of demand and supply curves of labor.

3.4. Real Rigidity and the Demand and Supply of Labor

Consider the way that a decentralized labor market would operate. Households would choose to supply labor to the point where the marginal rate of substitution was equal to the real wage,

$$\frac{-U_L(C, L)}{U_C(C, L)} = \frac{w}{P}, \tag{19}$$

and firms would choose to demand labor to the point where the marginal product of labor was proportional to the real wage,[9]

$$f_L(L) = \frac{1}{\lambda}\frac{w}{P}. \tag{20}$$

If one were to analyze a competitive labor market in this model, one could represent a log-linearized version of equation (19) as a labor supply curve,

$$\omega = k_1 + a_1 l + a_2 c, \tag{21}$$

where k_1 is a constant, ω is the log of the real wage, and c and l are logarithms. Similarly, a log-linear version of equation (20) represents labor demand:

$$\omega = k_2 + b_1 l. \tag{22}$$

Since consumption in equilibrium is equal to output $f(l)$, equations (21) and (22) can both be written as functions of l alone. The case in which the slopes of these functions are equal coincides with the maximum degree of real rigidity in the Ball–Romer definition and, in this case, if an equilibrium exists, the labor demand and supply curves must coincide and hence the equilibrium will be indeterminate.

The characterization of real rigidity in terms of labor demand and supply suggests a geometric characterization of real rigidity. Consider Figures 2 and 3, which plot the supply price of labor (the curve $\omega = k_1 + a_1 l + a_2 f_l l$) and its demand price (the curve $\omega = k_2 + b_1 l$) for two different economies. Economy 2 is one in which demand and supply curves have standard slopes, Economy 3 is one in which labor supply slopes downward.[10] Since real balances are increasing in L, real rigidity can be represented on the figure as the vertical distance between supply and demand curves—the gap between the supply price and the demand price of labor. In Economy 2, this gap gets big quickly as the economy moves away from the equilibrium. In Economy 3, the gap increases slowly and the economy can tolerate big deviations of labor from its equilibrium value without generating large pressures for relative prices to adjust.

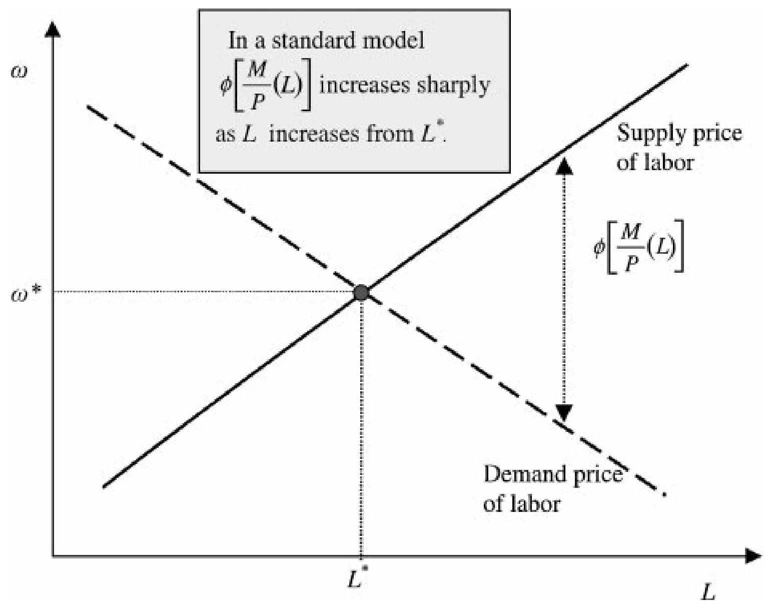

FIGURE 2. Labor demand and supply in the standard case.

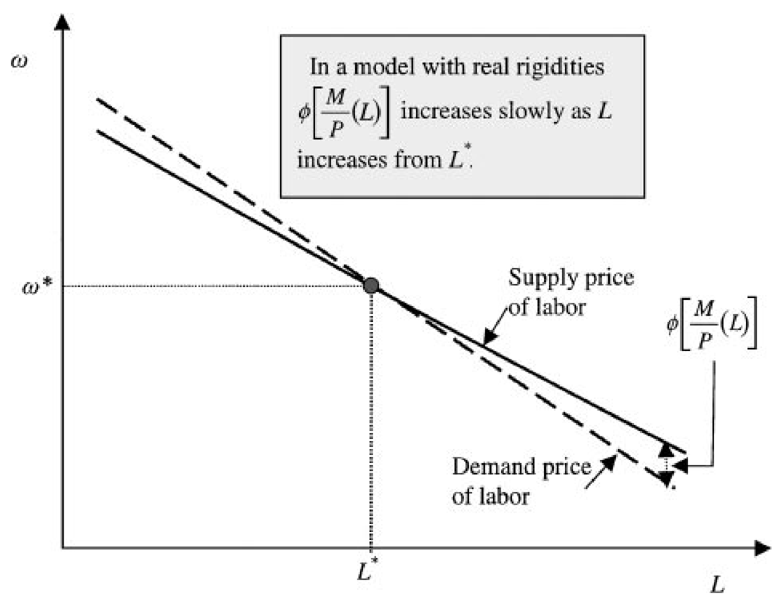

FIGURE 3. Labor demand and supply in an economy with real rigidities.

4. DIFFERENT ROUTES TO REAL RIGIDITY

Although the importance of real rigidities for nominal persistence is widely recognized in the literature, many authors shy away from providing explicit theories of the labor market that can generate real rigidity. For example, it has become common following Ball and Romer to specify a wage equation as a primitive of a model without enquiring as to how this may be consistent with optimizing behavior by households. In the following analysis, in contrast, I concentrate on real rigidities that are associated with fully competitive spot markets for labor.[11]

4.1. Benhabib–Farmer Conditions

In spot labor markets, real rigidities require the slopes of labor demand and supply equations to be similar; this is related to the condition that Benhabib and Farmer (1994) derive as necessary for indeterminacy in an RBC model. The Benhabib–Farmer condition is that the labor demand and supply curves should cross with "wrong slopes." Although there is nothing in the statement of this condition that requires the slopes to be similar, much of the work that has implemented the Benhabib–Farmer condition in calibrated models has used calibrations of the labor market that would satisfy the Ball–Romer definition of real rigidities.

There are two ways that labor demand and supply curves can have similar slopes. The first is that aggregate labor demand may slope upward as in Benhabib and Farmer (1994) or Farmer and Guo (1994) due to externalities in production.[12] Although this condition initially seemed promising as a description of aggregate data, recent empirical work by Basu and Fernald (1994), has cast doubt on its empirical relevance in the U.S. economy. An alternative possibility is that the constant consumption labor supply curve (labor supply as a function of the real wage holding constant consumption) slopes downward. This assumption is the one exploited in recent work by Benhabib and Farmer (1999) in which they construct a simple monetary model with indeterminacy.[13]

Downward-sloping labor supply requires the assumption that consumption is an inferior good, an assumption that seems a priori implausible. Nevertheless, a downward-sloping labor supply curve is required to explain data if one maintains the assumption of a competitive labor market. It is also a property of estimated models of the labor market. The data force one to infer that leisure or consumption is inferior because, at business-cycle frequencies, consumption and average hours supplied to the market are both procyclical variables. If the representative household chooses to supply more labor and to consume more output as a result of a demand-driven movement along a neoclassical production function, then either consumption or leisure must be inferior. If they were both normal goods, then the household would choose more leisure (fewer hours worked) at the same time that it chose more consumption.[14]

One does not need to believe that consumption is inferior in practice in order to use the assumption to explain data; models do not need to be correct to be useful.

One could believe that the spot labor market assumption is incorrect, but a convenient way of summarizing data. If one follows this route, one would hope that the assumption of a competitive labor market eventually could be replaced by a more accurate approximation to the real world. The main reason to be skeptical of the inferiority assumption is that, at business-cycle frequencies, most movements in aggregate hours occur as workers are fired or rehired, not as they move into and out of the labor force as households optimally choose to spend more or less time in leisure. To account for these facts, one would need a model that recognizes at least three activities: employment, leisure, and search (time spent in unemployment). The following simple model, based on unpublished research with Nicola Giammarioli, makes this case.

4.2. A Search Model with Real Rigidities and Normal Leisure

This section uses the idea of a matching function, made popular by Mortenson and Pissarides (1994), to explicitly model unemployment in an equilibrium business-cycle model.[15] Suppose that utility is given by

$$U = U(C), \tag{23}$$

and that firms produce with the technology

$$Y = F(L) - J, \tag{24}$$

where J represents real resources used in recruiting. Now let the number of workers hired every period be given by

$$H = m(S, J), \tag{25}$$

where $m(S, J)$ is a constant returns-to-scale matching function in which H is the number of new matches, S is the number of labor hours that workers spent searching for employment, and J represents the real resources used up in search by firms.

The simplest case to study is an extreme one in which the entire workforce is rehired every period. In that case, since workers get no disutility from search, they will supply all of their time to the search activity. One can normalize this and set $S = 1$. Since the entire workforce is rehired every period, employment will be equal to the number of matches, $L = H$. In a representative-agent economy, this simple search technology leads to the equilibrium conditions

$$F_L(L^*) m_J(1, J^*) = 1, \tag{26}$$

$$L^* = m(1, J^*). \tag{27}$$

Would this economy exhibit a significant real rigidity? This depends on the properties of the matching function $m(S, J)$. Remember that in the spot market for labor, in which leisure gives disutility, the requirement for real rigidity is that

$$F_L(L^*) \cong \frac{-U_L(F(L^*), L^*)}{U_C(F(L^*), L^*)}. \quad \textbf{(28)}$$

In my search economy, one can find a similar condition in which the properties of the utility function are replaced by the properties of the matching function:

$$F_L(L^*) \cong \frac{1}{m_J(1, J(L^*))}, \quad \textbf{(29)}$$

where the function $J(L^*)$ is defined implicitly from equation (27). In the special case in which the production function is linear, real rigidity requires m_J to be constant, an assumption that implies that hours spent working by workers and real resources spent in recruiting by firms are good substitutes for each other. This may not be an implausible description of the actual search process, but clearly one would like a more realistic model. It also would be desirable to know something about the elasticity of substitution of real-world matching functions.[16]

5. A DYNAMIC CLASSICAL MODEL

In Sections 5 and 6, I construct dynamic classical and Keynesian models and show how each has been used to explain the monetary transmission mechanism. My aim is to provide a structure in which the two models can be compared easily. I develop the idea that they are both special cases of a more general model that makes different simplifying assumptions.

The classical model abstracts from staggered price setting and hence the only dynamic equation of the model is the Euler equation that explains how the representative agent allocates real balances over time. The New Keynesian model assumes a cash-in-advance constraint and hence the Euler equation is trivial. Here, the only dynamic equation is one that follows from staggered price setting, but, although the two models explain the dynamics of inflation in different ways, there is an important connection between them. In both kinds of models, there must be large real rigidities in order to explain why nominal shocks are so persistent in the data.

5.1. Assumptions About Technology and Preferences

In dynamic monetary models one thinks of a continuum of identical families, each of which maximizes the expected value of a utility function:

$$\max \sum_{t=1}^{\infty} \beta^{t-1} U(C_t, L_t) \quad \textbf{(30)}$$

subject to the constraints

$$M_t = M_{t-1} + P_t F\left(L_t, \frac{M_{t-1}}{P_t}\right) - P_t C_t + T_t, \tag{31}$$

$$\lim_{s\to\infty} Q_t^s \frac{M_s}{P_s} \geq 0, \tag{32}$$

where T_t is a lump-sum nominal transfer from the government, and Q_t^s is the price of a unit of currency in period t for delivery at date s.[17] Equation (31) is a period-by-period intertemporal budget constraint and (32) is a "no-Ponzi-scheme" constraint that requires the agent to be solvent at every date.

To introduce money into the classical model, I have modeled it as a productive asset. I assume that output is produced by the function

$$Y_t = F\left(L_t, \frac{M_{t-1}}{P_t}\right), \tag{33}$$

where Y_t is output, L_t is labor input, and M_{t-1}/P_t are money balances accumulated at date $t-1$ and used in production at date t.[18]

A simple way of solving problems in this class is to substitute the budget constraint into the objective function to yield the problem

$$\max_{\{L_t, M_t\}_{t=1}^{\infty}} \sum_{t=1}^{\infty} \beta^{t-1} U\left(\frac{M_{t-1}}{P_t} + F\left(L_t, \frac{M_{t-1}}{P_t}\right) - \frac{M_t}{P_t} + \frac{T_t}{P_t}, L_t\right). \tag{34}$$

Agents choose sequences of labor supply and real balances to maximize the discounted value of utility.

5.2. Alternative Ways of Including Money in the Model

Although I have included money in the production function, I could equally well have added money to the utility function or specified a cash-in-advance constraint. Cash-in-advance is more restrictive than the alternative approaches, although it can be modeled as a special case of either of them by allowing the elasticity of substitution between money and labor (in the case of money in production) or money and consumption (in the case of money in the utility function) to approach minus infinity.

In the model of money in the production function, the first-order condition for labor supply in each period takes the form

$$\frac{-U_L(C_t, L_t)}{U_C(C_t, L_t)} = F_L\left(L_t, \frac{M_t}{P_t}\right). \tag{35}$$

If, instead, one models money in the utility function, this equation would take the form

$$\frac{-U_L\left(C_t, L_t, \frac{M_t}{P_t}\right)}{U_C\left(C_t, L_t, \frac{M_t}{P_t}\right)} = F_L(L_t). \tag{36}$$

In either case, by substituting the expression $C_t = F(L_t)$ into the appropriate first-order condition, one can find reduced-form expressions linking output and employment to real balances:

$$L_t = h\left(\frac{M_t}{P_t}\right), \qquad Y_t = H\left(\frac{M_t}{P_t}\right). \tag{37}$$

5.3. Money and Aggregate Supply

The preceding discussion is related to textbook derivations of the aggregate supply curve that typically are derived in static models by assuming that the labor market is in equilibrium. Since money usually is added to these models as an afterthought by specifying a quantity equation of money, money typically is excluded from the labor-market-clearing conditions. One can make this omission rigorous but it requires the assumption that money is separable in production or utility, or that there is a cash-in-advance constraint. More generally, one would find that a labor-market-clearing equation can be solved to find output as a function of real balances. In the present model, the expression that follows from imposing labor market clearing *does* include real balances and I refer to the expression

$$Y_t = H(M_t/P_t) \tag{38}$$

as the aggregate supply curve.

5.4. Money and Aggregate Demand

In addition to the first-order condition for the choice of labor, to solve the dynamic classical model, one must specify an Euler equation that follows from optimally choosing sequences of real balances[19]:

$$\frac{1}{P_t}U_C(C_t, L_t) = \beta\frac{1}{P_{t+1}}U_C(C_{t+1}, L_{t+1})\left[1 + F_m\left(L_{t+1}, \frac{M_t}{P_{t+1}}\right)\right]. \tag{39}$$

To solve the classical model, one combines the labor-market condition with the Euler equation to find a difference equation that determines real balances in equilibrium. Substituting the functions $h(M/P)$ and $H(M/P)$ into equation (39) leads to the following equation in the single state variable, $m \equiv M/P$:

$$G(m_t) = \frac{\beta}{\mu_{t+1}}G(m_{t+1})X(m_{t+1}), \tag{40}$$

where $G(m)$ and $X(m)$ are defined by the expressions

$$G(m) \equiv mU_C(H(m), h(m)), \qquad X(m) \equiv 1 + F_m(h(m), m), \tag{41}$$

and μ_t is the money growth factor M_t/M_{t-1}. The following section discusses the kinds of equilibria that can arise in this economy and relates them to the concept of real rigidities.

5.5. Determinacy of Equilibrium

The case in which equation (40) is locally unstable around the steady state is one in which the equilibrium of the model is determinate.[20] In the case of no shocks to the system, this solution would correspond to real balances remaining at the steady state. When equilibrium is determinate, the classical model is unable to explain the monetary transmission mechanism because the real equilibrium of the economy is invariant to a change in the scale of monetary policy. Most interpreters of time-series data have concluded that if one could conduct an experiment in which one moved from the monetary policy $\{M_t^1\}_{t=1}^{\infty}$ to the policy $\{\lambda M_t^1\}_{t=1}^{\infty}$, one would expect that real output and employment would be affected at the date that the change occurred. For this reason, economists have searched for models that exhibit nonneutralities in the short run.[21]

In a recent paper, Benhabib and Farmer (1999) showed that when the labor market exhibits significant real rigidity, the classical model is fully capable of explaining nonneutralities. Their work hinges on the idea that the difference equation (40) may switch stability and, in this case, there may be equilibria in which new money entering the economy affects quantities in the short run and feeds asymptotically into prices. The following subsection explains the Benhabib–Farmer argument.

5.6. Indeterminacy and the Monetary Transmission Mechanism

Figure 4 illustrates equation (40) for the case in which the steady-state equilibrium of a monetary model is locally determinate. Much of the standard economic

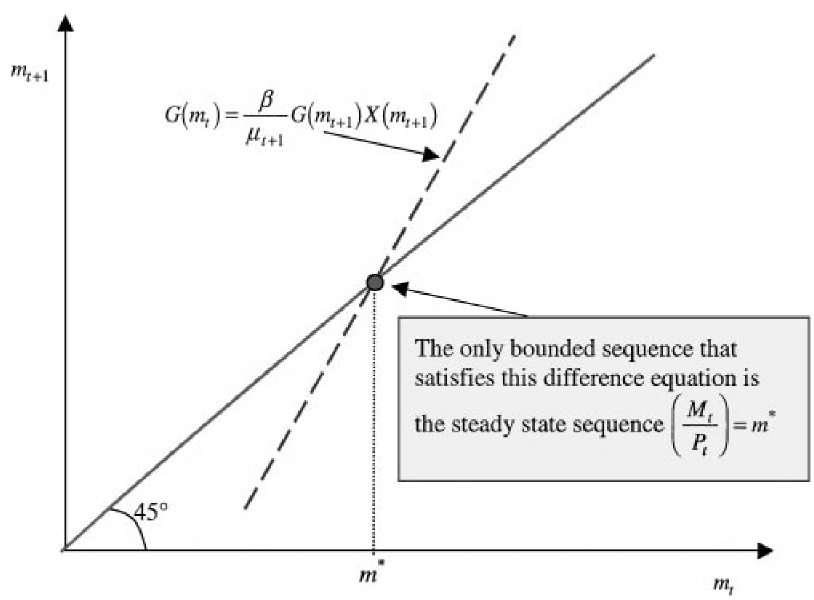

FIGURE 4. A determinate equilibrium in a monetary model.

intuition about the effects of monetary policy interventions is implicitly built on this case.

To understand the implications of determinacy for the monetary transmission mechanism, consider a thought experiment. Assume an economy with a history in which nothing has ever changed. Now, at date T, let there be an unanticipated increase in the money supply, distributed to households in proportion to existing money balances, and suppose that after date T, all households correctly realize that this event will never be repeated. If agents inhabit the world depicted in Figure 4, the only possible equilibrium is one in which the price level at date T increases immediately in proportion to the increase in real balances and real balances remain at their steady-state equilibrium level, m^*.

Suppose instead that agents inhabit the world depicted in Figure 5. This figure depicts the case in which the difference equation (40) is locally stable, a situation that can occur if money enters the production function or the utility function in a nonseparable way and if there are important real rigidities. When equation (40) is stable, there are multiple bounded sequences that satisfy it. Since any such sequence is a valid equilibrium, the economy contains a set of indeterminate equilibria. If the nominal money supply were to increase at date T, the nominal price need not adjust immediately. Instead, no price responses in the immediate period is consistent with equilibrium. In this equilibrium, real balances would increase from m^* to m_T and, unlike the economy in Figure 4, the lack of price response would be fully consistent with rational expectations and market clearing.

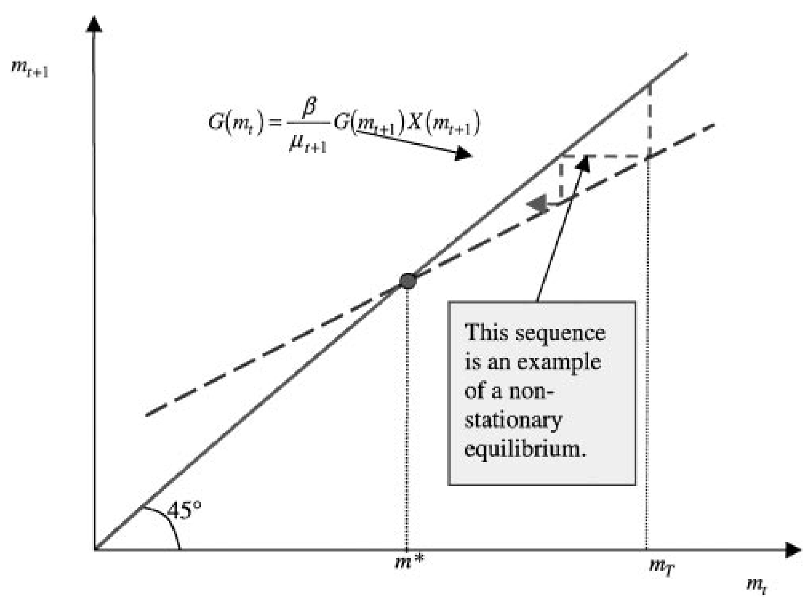

FIGURE 5. An indeterminate steady set of equilibria in a monetary model.

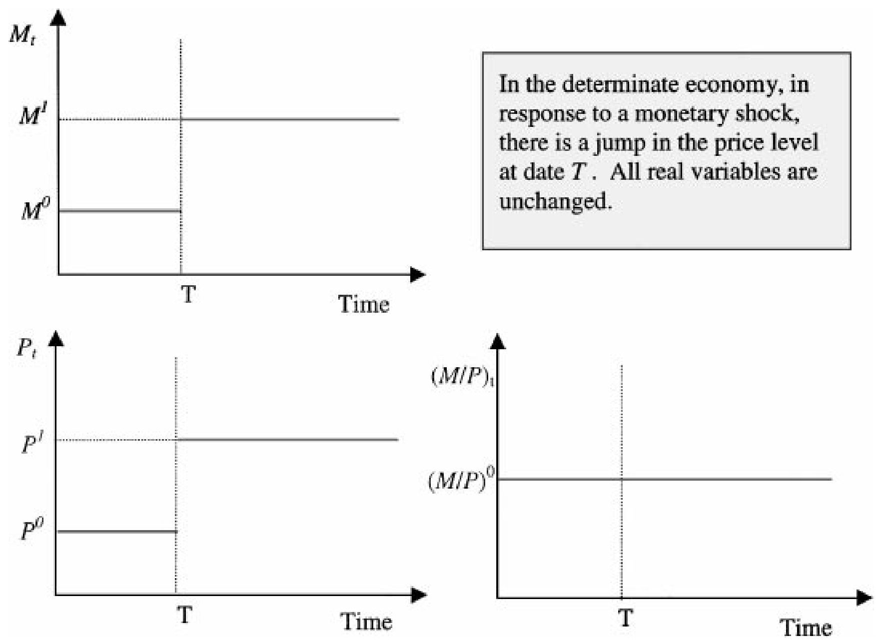

FIGURE 6. Predicted impulse responses in a determinate model.

To validate nominal rigidity, as a rational expectations equilibrium, the price level must increase in period $T + 1$. As prices rise, real balances fall and move back over time, converging asymptotically to m^*. Since the money stock does not increase further after it jumps in period T, the decrease in real balances must be accomplished by a slow increase in the price level. Figures 6 and 7 illustrate the time path of the price level, the nominal money supply, and real balances in these two economies. In the equilibrium described in Figure 5, as in Figure 4, all markets clear and agents have rational expectations of future prices. Money has real effects in this economy because nominal rigidity is the unique equilibrium response when the model is supplemented by a complete description of the way that agents form beliefs.

6. A DYNAMIC NEW KEYNESIAN MODEL

I have described how indeterminacy models explain sticky prices. In this section, I describe an alternative approach based on recent papers by a group of economists writing in the New Keynesian tradition. Each of these papers is based on the staggered-price-setting paper of Calvo (1983) and provides a dynamic generalization of the static menu cost model studied by Akerlof and Yellen (1985), Mankiw (1985), and Ball and Romer (1990).[22]

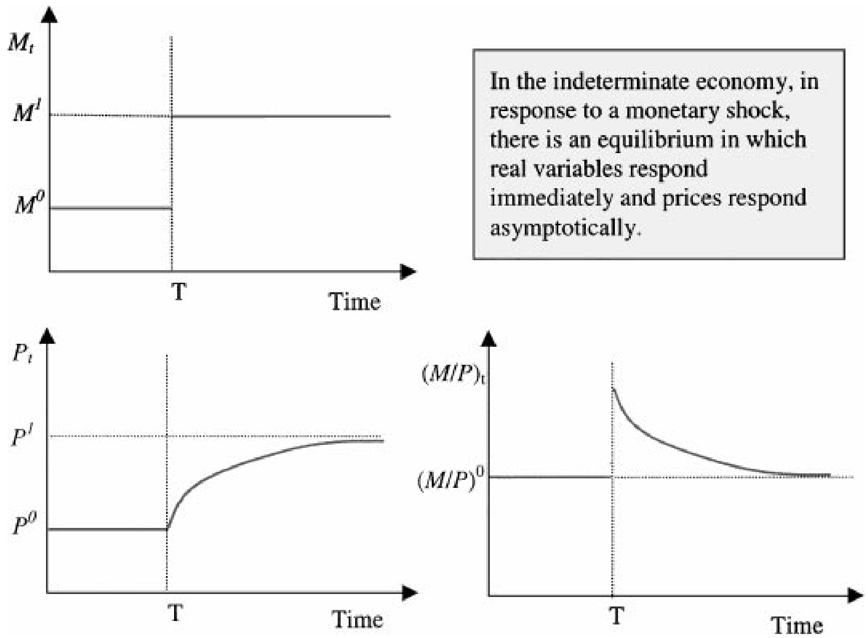

FIGURE 7. Predicted impulse responses in an indeterminate model.

6.1. Choosing the Price Level

The dynamic menu cost literature can be used to derive a version of the aggregate supply curve that resembles the Phillips curve in traditional models. In the dynamic menu cost model, a fraction $1-\alpha$ of price-setting agents is able to change its price each period but the remaining fraction, α, must keep its price fixed. In this way, one can model menu costs by assuming them to be infinite for some subset of firms and zero for the others. By modeling the ability to change price as an exogenous i.i.d. random variable, one maintains the flavor of the idea that price setting is costly, while keeping the algebra of the model manageable.

As in the classical model, one begins by assuming that there exists a representative agent that solves the problem

$$\max_{\{L_t, M_t\}_{t=1}^{\infty}} \sum_{t=1}^{\infty} \beta^{t-1} U\left(\frac{M_{t-1}}{P_t} + Y_t - \frac{M_t}{P_t} + \frac{T_t}{P_t}, L_t\right). \qquad \textbf{(42)}$$

The menu cost model differs from the classical approach by adopting an industrial structure based on the static model of monopolistic competition. It assumes that the final-goods sector is competitive and that the household earns revenues from its ownership of intermediate industries. These revenues must be spent on consumption commodities or used to accumulate money balances.

To introduce money into the model, I impose a cash-in-advance constraint. This assumption is one of two that are used frequently in the New Keynesian literature; the other is to model real balances as separable in utility. Since, in equilibrium, new money entering the economy will exactly equal nominal transfers, T_t, the household must choose to spend its revenues on consumption. Using these assumptions, one can define the household's indirect period utility function $W(M/P, \tilde{P}_i/P)$ as follows:

$$W\left(\frac{M}{P}, \frac{\tilde{P}_i}{P}\right) \equiv U\left(\frac{M}{P}\int\left(\frac{P_i}{P}\right)^{\frac{\lambda}{\lambda-1}} di, \int f^{-1}\left(\frac{M}{P}\left(\frac{P_i}{P}\right)^{\frac{1}{\lambda-1}} di\right)\right), \quad \textbf{(43)}$$

Using the definition of $W(M/P, \tilde{P}_i/P)$, one can rewrite the maximization problem of the representative agent as follows:

$$\max_{\{P_{it}\}} U = E_1\left\{\sum_{t=1}^{\infty}\beta^{t-1}W\left(\frac{M_t}{P_t}, \frac{\tilde{P}_{it}}{P_t}\right)\right\}. \quad \textbf{(44)}$$

In equation (44), E_1 is the expectation operator conditional on Date 1 information and $\tilde{P}_{it}$ is the price chosen in period t by a randomly chosen subset of firms.

A couple of observations greatly simplify the analysis of this model. First, it follows from symmetry that all agents who change their price at date t will make the same decision. Following Kimball (1995), I refer to the price that would be chosen as the *optimal reset price* denoted $\hat{P}_t$. Second, the state space for this problem is potentially infinite since, at any point in time, there exists a continuum of firms with pricing policies, some of which will have been in place since $t = -\infty$. However, the consumer cares only about aggregate output, and for this there is a convenient scalar state variable that summarizes the impact of histories on welfare: this variable is the price level that ruled at date $t-1$. In the following subsection, I exploit these observations to find two equations that characterize the dynamics of the model.

6.2. Sticky Prices and the New Keynesian Phillips Curve

This section derives a linear approximation to the first-order condition for price-setting firms that bears a close resemblance to the Phillips curve in textbook Keynesian models. This approximation, referred to as the New Keynesian Phillips curve, differs from more standard Phillips curves in that it contains expected future inflation, as well as lagged inflation, as a determinant of aggregate supply.

The first step toward the derivation of the Phillips curve is to describe how the fixed prices of the firms that do not change their prices can be combined in a price aggregator with those of the firms that do adjust. This aggregator, given by equation (45) is the factor price frontier and, for the Dixit–Stiglitz aggregator, it has the following form:

$$1 = \int \left(\frac{P_{it}}{P_t} \right)^{\frac{\lambda}{\lambda-1}} di. \qquad \textbf{(45)}$$

Recall that all firms that change their price will choose to set it to the same level, the optimal reset price $\hat{P}_t$. Using equation (45), one can derive the following equation that links inflation with the optimal reset price[23]:

$$1 = \alpha \left(\frac{P_{t-1}}{P_t} \right)^{\frac{\lambda}{1-\lambda}} + (1-\alpha) \left(\frac{\hat{P}_t}{P_t} \right)^{\frac{\lambda}{1-\lambda}}. \qquad \textbf{(46)}$$

A firm that does change its price in period t will choose $\hat{P}_t$ to maximize the discounted present value of expected utility. This leads to the following first-order condition:

$$E_t \sum_{s=0}^{\infty} (\alpha\beta)^s \frac{1}{P_{t+s}} W_2 \left(\frac{M_{t+s}}{P_{t+s}}, \frac{\hat{P}_t}{P_{t+s}} \right) = 0. \qquad \textbf{(47)}$$

Equation (47) contains an infinite weighted sum of future marginal utilities because the firm takes account of the fact that there is positive (but declining) probability that the price it sets in period t will prevail for the infinite future.[24]

Equations (46) and (47), together with a specification of monetary policy (a rule for determining $\{M_t\}_{t=1}^{\infty}$), completely characterize an equilibrium in the staggered-price model. The simplest example of a monetary policy rule is the given by the equation

$$M_t = M + u_t, \qquad \textbf{(48)}$$

where u_t is a random variable with zero mean that represents a money supply shock.

In Appendix B, I show that by combining equations (46), (47), and (48), and linearizing the resulting expression around a steady state, one can derive the following linear expression for the New Keynesian Phillips curve:

$$y_t = k_0 + bX\pi_t - bX\beta E_t[\pi_{t+1}], \qquad \textbf{(49)}$$

where the variables y_t and π_t are defined as

$$y_t \equiv \log(Y_t) \quad \text{and} \quad \pi_t \equiv \log(P_t) - \log P_{t-1}, \qquad \textbf{(50)}$$

k_0 is a constant, and the coefficients b and X are given by the expressions

$$b \equiv \frac{\alpha}{1-\alpha} \frac{1}{1-\alpha\beta} \quad \text{and} \quad X \equiv \frac{-W_{22}}{W_{21}}. \qquad \textbf{(51)}$$

The parameters b and X are important determinants of the dynamic response of real variables to nominal shocks: b is a measure of the importance of nominal rigidities in the economy, and X is the Ball–Romer measure of real rigidities. The parameter b is itself determined by α, the probability that the price will remain fixed, and β, the rate of time preference; b is related to α by the formula

$$b \equiv \frac{\alpha}{1-\alpha}\frac{1}{1-\alpha\beta}. \tag{52}$$

For quarterly data, α in the vicinity of $3/4$ implies an expected duration of price stickiness of four quarters. For $\alpha = 3/4$, and a discount rate of 1% per quarter, b is approximately equal to 11. As α gets close to 0, nominal rigidities become small and b tends to 0, and as α gets close to 1, nominal rigidities become important and b tends to ∞.

The absolute value of X is the Ball–Romer measure of real rigidity. When $|X|$ is big, $\phi'(M/P)$ is small and the relative price of the agent is relatively insensitive to big changes in aggregate demand. The sign of X is also important in the following discussion and, for the model of monopolistic competition, X is positive.[25] The following section uses these facts to characterize the dynamics of price adjustment around the steady state of this economy.

6.3. Dynamics of the New Keynesian Model

The dynamics of the New Keynesian model are greatly simplified by the cash-in-advance assumption,

$$\frac{M_t}{P_t} = Y_t, \tag{53}$$

which also represents the aggregate demand curve. Taking first differences of equation (53) and combining it with the Phillips curve (49) leads to the following pair of equations that characterize equilibrium:

$$\begin{bmatrix} 1 & \beta bX \\ 1 & 0 \end{bmatrix}\begin{bmatrix} y_t \\ \pi_{t+1} \end{bmatrix} + \begin{bmatrix} 0 & -bX \\ -1 & 1 \end{bmatrix}\begin{bmatrix} y_{t-1} \\ \pi_t \end{bmatrix} = \begin{bmatrix} k_0 + w_{t+1} \\ \mu_t \end{bmatrix}, \tag{54}$$

where μ_t is the money growth rate and w_{t+1} is the expectational error

$$w_{t+1} \equiv bX[\pi_{t+1} - E_t(\pi_{t+1})]. \tag{55}$$

This can be rewritten in the reduced form

$$\begin{bmatrix} \tilde{Y}_t \\ \tilde{\pi}_{t+1} \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ \dfrac{-1}{\beta bX} & \dfrac{(1+bX)}{\beta bX} \end{bmatrix}\begin{bmatrix} \tilde{Y}_{t-1} \\ \tilde{\pi}_t \end{bmatrix} + \begin{bmatrix} v^1_{t+1} \\ v^2_{t+1} \end{bmatrix}, \tag{56}$$

where tildes denote deviations from the steady state and v^i_{t+1}, $i = \{1, 2\}$ are linear combinations of the shocks w_{t+1} and μ_t.

The dynamics of this system depend on the roots of the matrix

$$A \equiv \begin{bmatrix} 1 & -1 \\ \dfrac{1}{\beta bX} & \dfrac{(1+bX)}{\beta bX} \end{bmatrix}, \tag{57}$$

which has a trace equal to

$$\mathrm{Tr} = 1 + \frac{1}{\beta} + \frac{1}{\beta b X}, \qquad (\mathbf{58})$$

and a determinant equal to

$$\mathrm{DET} = \frac{1}{\beta}. \qquad (\mathbf{59})$$

Notice that the roots depend on three parameters: b, which is determined by the degree of nominal rigidities; X, which is the Ball–Romer definition of real rigidities; and β, which is the discount factor. The following argument establishes that the matrix A has two positive real roots that split around unity, implying that the steady state is a saddle. First note that the trace of a matrix is equal to the sum of its roots and the determinant is the product of its roots. Consider the case in which $X = \infty$. In that case, the roots are equal to 1 and $1/\beta$. Now consider the case in which bX is a finite positive number. As $(\beta b X)^{-1}$ increases from 0, the sum of the roots increases but their product is unchanged. To increase the sum of two numbers while preserving their product, one must become larger and the other smaller. It follows that both roots are positive, one root is always greater than $1/\beta$, and the other is smaller than 1.

Since expected inflation is a jump variable, the unique rational expectations equilibrium in the staggered-price model is found by eliminating the unstable root. The dynamics of adjustment of the real variables, in response to a nominal shock, are governed by the stable root and, for monetary shocks to be persistent, this root must be relatively close to 1. It is instructive to consider the case in which X approaches $+\infty$ since in this case the two roots are equal to 1 and $1/\beta$. In this case, the root $1/\beta$ is solved forward and the smaller root, unity, governs the dynamics of the system. This is the case where nominal shocks are infinitely persistent. For $X < \infty$, the larger root of A increases above $1/\beta$ and the smaller root falls below 1. It is the magnitude of this smaller root that governs the persistence of nominal shocks, and as X increases, this root is pushed further away from 1 toward 0. In practice, X must be very large for the root to remain close to 1, so that the model is able to explain the degree of nominal persistence that one finds in data.

7. LESSONS FOR LINEAR MODELS

One of the themes that has been stressed by the New Keynesians is that the staggered-price-setting model bears a strong resemblance to the textbook IS-LM model, supplemented by a price-setting equation. Since there are many dimensions in which the IS-LM model does a relatively good job of describing data, a version of the model with sound microfoundations has become something of a Holy Grail. In the following discussion, I have no quarrel with the IS-LM part of the New Keynesian argument. A dynamic version of the IS curve has a perfectly sensible interpretation as a representation of the Euler equation in a simple economy with logarithmic preferences. Similarly, an LM curve emerges as a first-order condition for optimal money holding in almost any dynamic model that includes a well-defined motive for holding real balances. It is the price-setting equation that I wish to question and, in this section, I study a linear model that can account for either the indeterminacy or the staggered-price-setting explanations of the Phillips curve.

7.1. IS-LM and Maximizing Models

I begin by writing down a linearized Euler equation in a model with logarithmic preferences over consumption. All lowercase variables represent logarithmic deviations from balanced growth paths, y_t is output, i_t is the nominal interest rate, and p_t is the price level. The expectational error w_{t+1}^1 is defined as the realization of $[a_0 + y_t + a_1(i_t - p_{t+1} + p_t)]$ minus its expectation:

$$y_{t+1} = y_t + a_1(i_t - p_{t+1} + p_t) + w_{t+1}^1. \quad \textbf{(60)}$$

Equation (60) is what McCallum and Nelson (1999) have called an optimization-based IS curve.

A second equation that follows from money in the production function, or from money in the utility function, is an LM curve. In the money in the production-function model, there is a first-order condition of the form

$$F_m\left(L, \frac{M}{P}\right) = i, \quad \textbf{(61)}$$

where $F(L, M/P)$ is a neoclassical production function and F_m is its derivative with respect to M/P. By combining this first-order condition with the production function $Y = F(L, M/P)$ to eliminate labor, one arrives at a linearized equation of the form

$$m_t - p_t = y_t - a_2 i_t. \quad \textbf{(62)}$$

In the case of money in utility, one has a similar condition:

$$\frac{-U_m\left(C, L, \frac{M}{P}\right)}{U_C\left(C, L, \frac{M}{P}\right)} = i. \quad \textbf{(63)}$$

For the case of a separable utility function, one combines this condition with the production function and the equilibrium condition, $C = Y = F(L)$, to find an equation linking real balances, output, and the interest rate with the same form as equation (62). When utility is nonseparable, one must also exploit the first-order condition for the labor market,

$$\frac{-U_L\left(C, L, \frac{M}{P}\right)}{U_C\left(C, L, \frac{M}{P}\right)}, \quad \textbf{(64)}$$

to eliminate L. Once again, one arrives at an equation linking output, real balances, and the interest rate—an LM curve.

7.2. The Phillips Curve and Optimizing Models

This brings us to the critical equation: the price-setting equation of the model. In the static version of the New Keynesian model, optimal price setting leads to the equation

$$\frac{P_i}{P} = \frac{\left[-\dfrac{U_L\left(C, L, \dfrac{M}{P}\right)}{U_C\left(C, L, \dfrac{M}{P}\right)}\right]}{\left[\lambda f_L\left(L, \dfrac{M}{P}\right)\right]}. \tag{65}$$

The numerator of this expression is the marginal rate of substitution, and the denominator is proportional to the marginal rate of transformation. The constant of proportionality, λ, is a measure of the degree of competitiveness in this economy. I have included money in both the production function and the utility function in this expression, although most models will include only one of these motives for holding money. Equation (65) is solved, in the static version of the New Keynesian model, by setting $P_i = P$ for all price setters. This then leads to a standard labor-market-clearing condition. Notice, however, that, in general, real balances will enter the labor-market equations unless money enters the economy in a separable way.

In the static model, a linearized version of the price-setting equation would lead to the aggregate supply curve

$$y_t = a_3(m_t - p_t). \tag{66}$$

It is equation (66) that I exploit in the indeterminacy model in which the fact that real balances affect aggregate supply is central to the indeterminacy explanation of the monetary transmission mechanism. Real rigidity is important to the explanation because the demand and supply curves of labor must have similar slopes in order for real balances to have a big effect on output.[26]

In the menu cost model, the price-setting equation is dynamic and not all price setters are allowed to reoptimize every period. The inability to reoptimize implies $P_i \neq P$. Instead, this identity is replaced by an equation linking the optimal reset price $\hat{P}/P$ with inflation. The first-order condition for the optimal reset price involves an infinite discounted sum of ratios of marginal rates of substitution and marginal rates of transformation and it is this infinite sum that can be transformed to lead to the New Keynesian Phillips curve,

$$y_t = a_4(p_t - p_{t-1}) - a_5(p_{t+1} - p_t) + w_{t+1}^2, \tag{67}$$

where a_4 and a_5 are parameters and w_{t+1}^2 is an expectational error.

To derive equation (67), one of two conditions must hold. Either money must enter through a cash-in-advance constraint so that the demand-for-money equation (62) does not depend on the interest rate, or money must enter the model in a separable way so that money does not enter the utility or production functions. The former method is employed by Jeanne (1998), who was then able to study equations (62) and (67) independently of the Euler equation (60), which we refer to as the dynamic IS curve. The alternative approach, followed by Rotemburg and Woodford (1997a,b), assumes money enters separably into utility. They study monetary policies that fix the interest rate and, in their model, the dynamics of output and inflation are determined by equations (60) and (67) and are independent of the LM curve. However, the cases studied by Jeanne (1998) and Rotemburg and Woodford (1997a,b) are nongeneric because they each rely on special assumptions that allow them to write down an aggregate supply curve that is independent of real balances.

In almost all parameterizations of monetary economies, money will enter the aggregate supply equation and the demand for money will be interest elastic. It follows that the aggregate supply equation generally will be of the form

$$y_t = a_3(m_t - p_t) + a_4(p_t - p_{t-1}) - a_5(p_{t+1} - p_t) + w^2_{t+1}. \qquad \textbf{(68)}$$

The fact that a_3 is generically nonzero does not imply that the New Keynesian model will necessarily display indeterminacy. However, recall that real rigidities are essential for the staggered-price model to display persistence of monetary shocks. Real rigidity implies that the slopes of labor demand and supply are similar and in generic monetary models, one or the other of these curves will be shifted by a change in real balances. If the curves have similar slopes, then small shifts in real balances will have big effects on output. In other words, real rigidities imply that as soon as one moves away from the separable case, real balances are likely to have big effects on output and for the same calibrations that are required to generate persistence in staggered-price models, a_3 will not just be nonzero but also will be large.

Without analyzing a complete model that subsumes both the indeterminacy and staggered-price-setting model as special cases, one cannot say for sure that such a system will display indeterminate equilibria. However, the pointers from the two models that I have analyzed in this paper point in this direction.

8. CONCLUSIONS

Although the issue is not fully resolved, many observers agree that nominal changes in the quantity of money have real short-run effects on output and employment. This has led to a search for the microfoundations of macroeconomics. The most recent class of explanations for the real effects of money are based around the assumption of staggered price setting—for some reason (unexplained in the models) not all firms are able to adjust nominal prices in every period. The study of staggered-price-setting models has revealed, however, that nominal rigidities are necessary but not sufficient to explain the observed persistent effects of monetary policy. In addition to nominal rigidities, there must be significant real rigidities. The definition of real rigidity is that price-setting agents can tolerate large changes in aggregate demand without altering relative prices. Its implication, in a model with an equilibrium labor market, is that the slopes of labor demand and supply curves must be close.

A second explanation for the persistent effects of nominal rigidities is that equilibrium is indeterminate. There are many possible equilibrium responses to an increase in the nominal quantity of money. One of these is agents' rational expectation that change will take time. This explanation leads to a separate set of intellectual challenges: How are expectations formed? How are they coordinated? Why this equilibrium rather than another? It is, however, a logically consistent explanation of the monetary transmission mechanism and one that can be shown to be consistent with all of the known features of the comovements between variables in aggregate data. Just like the staggered-price-setting model, the indeterminacy approach requires that the economy display significant real rigidities.

In almost all recent research in monetary theory, the indeterminacy approach and the staggered-price approach have been pursued independently. The one exception is the recent work by Kiley (1997) who points out the connections between them. Indeterminacy models and staggered-price models both need to assume large real rigidities to explain the persistence of monetary shocks. Indeterminacy models leave out the staggered-price route to persistence because it is superfluous. Staggered-price models avoid indeterminacy by making special, nongeneric assumptions about the way that money enters the economy, which allow them to study a subset of the equilibrium equations of their models independently from the others. When these assumptions are relaxed, staggered-price-setting models are likely also to display indeterminacy. Since indeterminacy is a generic problem in models with real rigidities, the questions that it raises for theories of expectations formation will have to be addressed at some stage. It is possible that adding staggered price setters may provide additional explanatory power; ultimately that is for the data to decide. My own guess is that it will not.

NOTES

1. Work in this literature includes papers by Fuhrer and Moore (1995), Kimball (1995), Roberts (1995), Yun (1996), Kiley (1997), Rotemburg and Woodford (1997a,b), Gali and Gertler (1998), Jeanne (1998), Clarida et al. (1999), and King and Wolman (in press).

2. The reader is referred to the work of Christiano et al. (1996) for a more comprehensive discussion of the limited-participation model and its relationship to the sticky-price literature.

3. The possibility of indeterminacy in monetary models was pointed out by Black (1974) and was studied in early work by Calvo (1979). The first paper to point out the possibility of Keynesian-style price stickiness arising from indeterminacy in an overlapping generations model is by Geanakoplis and Polemarchakis (1986) and there are a number of papers that pursue this idea in the context of the overlapping generations model. These include Azariadis and Cooper (1985), Chiappori and Guesnerie (1990, 1992), and Farmer (1991, 1992). More recently, a number of authors have studied the indeterminacy explanation for monetary transmission in the context of infinite-horizon models, for example, Woodford (1986, 1988), Matsuyama (1991), Matheny (1992, 1998), Beaudry and Devereux (1993), Lee (1993), Benhabib and Farmer (1994), and Bennett (1997).

4. If we had assumed that the number of firms was finite, we would have been able to represent output as follows:

$$Y = F(Y_1, Y_2, \ldots Y_n).$$

Instead, our economy has a continuum of firms indexed by $i \in [0, 1]$. To represent the dependence of final output on the continuum of intermediate inputs, we use the notation, $\tilde{Y}_i$, where $\tilde{Y}_i$ is a measurable function $[0, 1] \to R_+$ that describes the output of the firm located at position i in the interval $[0, 1]$.

5. $U(C, L)$ is assumed to be at least twice continuously differentiable, increasing in C, decreasing in L, and concave.

6. For the model in this section to make sense, it is important that the decision problems of households and intermediate firms be solved separately from those of final firms. We maintain the fiction that all firms, including those in the competitive sector, are owned by a single representative household. However, we do not allow the household to recognize interdependencies between the maximization problems of the firms that it owns.

7. If private agents face a small cost of changing prices, the social cost of failing to fully adjust nominal prices may exceed the private cost. This was pointed out independently by Akerlof and Yellen (1985), who think of the private behavior as "not fully rational," and Mankiw (1985), who introduced the concept of "menu costs." Ball and Romer's (1990) contribution was to point out that the gap between private and social costs of adjustment is small in most calibrated models of money. They show that small private menu costs cannot have large aggregate effects unless the economy exhibits significant real rigidities.

8. See Appendix A for details.

9. In a competitive economy the same equation would hold, except that the markup parameter λ^{-1} would be equal to 1.

10. An example of an economy like this would be provided by the preferences $U = -(L + B)/C$, where B is a constant. In that case, consumption is an inferior good.

11. In an influential paper, Chari et al. (1996) argue that models with overlapping contracts cannot explain the monetary transmission mechanism. Their argument applies equally to models of staggered price setting and it is based on the implicit assumption that real rigidities are implausible descriptions of labor markets.

12. This is the route emphasized by Kiley (1997).

13. In related papers, Pelloni and Waldmann (1997) derive conditions for indeterminacy in an endogenous growth model with inferior leisure and Matheny (1998) studies the effect of money on cash-in-advance economies when consumption and leisure are Pareto substitutes.

14. Mankiw et al. (1985) point out that inferior leisure is required to explain the facts. They estimate a classical model of the U.S. economy and in several of their specifications find that preferences are nonconvex. Farmer and Guo (1995) estimate a similar model on annual data and find evidence of downward-sloping labor supply [see also the discussion by Aiyagari (1995)]. Farmer and Ohanian (1999) use the same data set as Farmer and Guo and are able to fit the data relatively well with a nonseparable utility function. Their estimates are consistent with convex preferences but imply inferior consumption as in the work by Mankiw et al. (1985) and Farmer and Guo (1995).

15. Recent literature that incorporates search into an RBC economy using a Mortenson–Pissarides matching function includes Mertz (1995) and Andolfatto (1996). Cooley and Quadrini (1998) study inflation and unemployment using the same device.

16. Most existing work of which I am aware maintains the assunption of Cobb–Douglas matching functions, which imposes the restriction that the elasticity of substitution is equal to unity. In contrast, the model sketched in this paper suggests that real rigidity occurs as the elasticity of substitution of the matching function approaches infinity (the matching function itself becomes close to linear).

17. It is possible to add government debt to this model without changing the equilibrium of the model, providing one assumes that debt is in zero net supply. The advantage of adding debt is that it allows one to define the nominal interest rate in equilibrium. I have left the bond market out of the model to keep the notation to a minimum.

18. This mirrors the specification in Benhabib and Farmer (1999), with the exception that they allow money transfers T_t to enter the production function so that monetary shocks may have contemporaneous effects on output.

19. For the model of money in the utility function, there is an analog of this condition that takes the form

$$\frac{1}{P_t}U_C\left(C_t, L_t, \frac{M_{t-1}}{P_t}\right) = \beta\frac{1}{P_{t+1}}U_C\left(C_{t+1}, L_{t+1}, \frac{M_t}{P_t}\right)\left[1 + \frac{U_m\left(C_{t+1}, L_{t+1}, \frac{M_t}{P_{t+1}}\right)}{U_C\left(C_{t+1}, L_{t+1}, \frac{M_t}{P_{t+1}}\right)}\right].$$

20. An equilibrium is completely characterized by a bounded solution to equation (41), and instability of this equation around the steady state implies that there is only one such solution.

21. Even in the determinate version of the classical model, it is not true that the real equilibrium of the economy is invariant to arbitrary changes in the monetary policy sequence $\{M_t\}_{t=1}^{\infty}$. For example, a change in monetary policy from the constant money growth rule $M_t = (1+\mu_1)M_{t-1}$ to some new rule $M_t = (1+\mu_2)M_{t-1}$ will, in general, change steady-state real balances, output, and employment. This property of equilibrium is referred to as failure of the economy to display *superneutrality*. Although the existence of nonsuperneutralities is interesting, it is not enough to explain the monetary transmission mechanism.

22. These include the papers by Fuhrer and Moore (1995), Kimball (1995), Roberts (1995, 1997), Yun (1996), Kiley (1997), Rotemburg and Woodford (1997a,b), Gali and Gertler (1998), Jeanne (1998), Clarida et al. (1999), and King and Wolman (in press).

23. Let $J_t \subset [0, 1]$ be the set of all firms that change price in period t and define a variable q_t as follows:

$$\frac{q_t}{P_t} \equiv \frac{1}{\alpha}\left[\int_{i\notin J_t}\left(\frac{P_{it}}{P_t}\right)^{\frac{\lambda}{1-\lambda}}\right]^{\frac{1-\lambda}{\lambda}} di,$$

where q_t is a geometrically weighted average of the prices of all those firms that *do not* change price in period t. This variable can be observed because the average price of all firms that do not change price must be the same as average price of *all* firms in the previous period. Using this fact, equation (46) follows from the factor price frontier and the definition of q_t.

24. In each period $t+s$, for $s \geq 0$, the firm will be allowed to reset its price with probability $(1-\alpha)$ and will be forced to maintain $\hat{P}_t$ with probability α. Maximization of expected utility implies that utility in period $t+s$ will be discounted at rate $(\beta\alpha)^s$, which reflects the rate at which date utility is discounted, captured by the term β^s and the probability that the price $\hat{P}_t$ will still prevail in period $t+s$, captured by the term α^s.

25. $X \equiv |\phi'(M/P)|^{-1}$. For the model of monopolistic competition, ϕ' has the same sign as the change in the gap between the supply price and the demand price of labor. As long as the supply curve slopes up more steeply than the demand curve, ϕ' will be positive. This is the case in all calibrated models in the literature. It is also true in the Benhabib–Farmer (1994) model since their definition of labor supply does not include the general equilibrium effect that arises from the impact of labor supply on consumption. This term *does* appear in the definition of real rigidity used in this paper.

26. It would also be possible for money to have a big effect on output if the direct elasticity of money in the production function was big. This can be ruled out by reasonable calibrations that place the elasticity of m in production at less than 1%.

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APPENDIX A: SOME DETAILS OF THE MONOPOLISTICALLY COMPETITIVE MODEL

This appendix describes a simple version of the monopolistically competitive model based on the technology studied by Dixit and Stiglitz (1977).

In the Dixit–Stiglitz model, the final technology is given by the function

$$Y = \left(\int_0^1 Y_i^{\lambda} \right)^{1/\lambda} di, \tag{A.1}$$

where Y is final output and Y_i is input of the ith intermediate good. (We assume that there is a continuum of monopolistic competitors distributed uniformly on the interval [0, 1].)

The inverse demand function faced by intermediate producers is obtained by assuming that the final technology is operated in a competitive market and that final-goods producers choose their input mix to minimize cost. Final-goods producers solve

$$\min_{\tilde{Y}_i} \int \frac{P_i}{P} Y_i \, di \quad \text{such that} \quad \left(\int_0^1 Y_i^{\lambda} \right)^{1/\lambda} di \geq Y, \tag{A.2}$$

taking intermediate-goods prices and final-goods prices as given. The solution to their decision problem, plus the assumption of zero profits, leads to the inverse demand function

$$P_i = P \left(\frac{Y_i}{Y} \right)^{\lambda - 1}, \tag{A.3}$$

which is taken as parametric by each of the many intermediate-goods producers. To make the problem interesting, we must assume that $0 < \lambda \leq 1$, which implies that intermediate goods are substitutes for each other, rather than complements. Notice that the economy becomes competitive when $\lambda = 1$. In this case the intermediate goods are perfect substitutes and the inverse demand curve is horizontal. When $0 < \lambda < 1$, however, each of the intermediate-goods producers has some monopoly power.

Each intermediate producer is a monopolistic competitor that produces the ith intermediate good from labor using the technology

$$L_i = f^{-1}(Y_i), \tag{A.4}$$

where $f^{-1}(Y_i)$ is the labor required to produce the output Y_i.

The revenue function $R(Y, P_i/P)$ is given by the expression

$$R\left(Y, \frac{P_i}{P}\right) \equiv Y \left(\frac{P_i}{P}\right)^{\frac{\lambda}{\lambda-1}}, \tag{A.5}$$

and the labor demand function is given by

$$L\left(Y, \frac{P_i}{P}\right) \equiv f^{-1}\left(Y\left(\frac{P_i}{P}\right)^{\frac{1}{\lambda-1}}\right). \qquad \textbf{(A.6)}$$

The derivative of the revenue function with respect to price is

$$R_2 \equiv Y\frac{\lambda}{\lambda-1}\left(\frac{P_i}{P}\right)^{\frac{1}{\lambda-1}} = Y_i\frac{\lambda}{\lambda-1}, \qquad \textbf{(A.7)}$$

and the derivative of the labor input function is

$$L_2 \equiv \frac{1}{f_L}Y\frac{1}{\lambda-1}\left(\frac{P_i}{P}\right)^{\frac{1}{\lambda-1}-1} = \frac{1}{\left(\frac{P_i}{P}\right)f_L}Y_i\frac{1}{\lambda-1}, \qquad \textbf{(A.8)}$$

where the second equalities in each expression exploit the inverse demand function. Taking the ratio of equations (A.7) and (A.8) leads to the expression

$$\frac{R_2}{L_2} = \lambda f_L(L_i)\frac{P_i}{P}, \qquad \textbf{(A.9)}$$

which is expression (16) in the text.

APPENDIX B: DERIVING THE PHILLIPS CURVE IN THE STAGGERED-PRICE MODEL

This appendix shows how to derive the New Keynesian Phillips curve as a linear approximation to the dynamics of equation (47) in the neighborhood of the nonstochastic stationary state, $\{P^*, \hat{P}^*, M\}$. First, define the variable μ_t^s as follows:

$$\mu_t^s \equiv \frac{M_{t+s}}{M_t}, \qquad \textbf{(B.1)}$$

and let $m_t \equiv M_t/P_t$. Using these definitions, write equation (47) as

$$E_t\sum_{s=0}^{\infty}(\alpha\beta)^s\frac{m_{t+s}}{m_t\mu_t^s}W_2\left(m_{t+s}, \frac{\hat{P}_t}{P_t}\frac{m_{t+s}}{m_t\mu_t^s}\right) = 0. \qquad \textbf{(B.2)}$$

Since in the steady state,

$$\frac{1}{1-\alpha\beta}W_2(m^*, 1) = 0, \qquad \textbf{(B.3)}$$

it follows that $W_2(m^*, 1) = 0$. Now define the numbers b_s and c_s,

$$b_s \equiv (\alpha\beta)^s W_{21}(m^*, 1),$$
$$c_s \equiv (\alpha\beta)^s W_{22}(m^*, 1),$$

and notice that

$$b_s = b_{s-1}\alpha\beta, \qquad c_s = c_{s-1}\alpha\beta. \tag{B.4}$$

Using these definitions, take a first-order Taylor-series approximation to equation (B.2) in the neighborhood of the steady state. Letting dx_t be the logarithmic deviation of a variable x_t from its steady-state value leads to the linearized expression

$$E_t \sum_{s=0}^{\infty} \left[c_s\left(dm_{t+s} - dm_t - d\mu_t^s\right) + b_s dm_{t+s} + c_s(d\hat{P}_t - dP_t)\right] = 0.0 \tag{B.5}$$

Since the monetary policy is stationary and since I assume i.i.d. innovations to the money supply, this expression can be simplified further by exploiting the fact that $E_t d\mu_t^s = 0$:

$$E_t \sum_{s=0}^{\infty} [c_s(dm_{t+s} - dm_t) + b_s dm_{t+s} + c_s(d\hat{P}_t - dP_t)] = 0. \tag{B.6}$$

Equation (B.6) is a linear approximation to the first-order condition for the optimal reset equation at date t. This equation also must hold at date $t + 1$:

$$E_t \sum_{s=0}^{\infty} [c_s(dm_{t+s+1} - dm_{t+1}) + b_s dm_{t+s+1} + c_s(d\hat{P}_{t+1} - dP_{t+1})] = 0, \tag{B.7}$$

where I have evaluated the expectation in equation (B.7) at t using the law of iterated expectations, $E_t[E_{t+1}(x_{t+s})] = E_t(x_{t+s})$. Now write out equation (B.6) in two parts:

$$b_0 dm_t + c_0(d\hat{P}_t - dP_t) + E_t \sum_{s=1}^{\infty} [c_s(dm_{t+s} - dm_t) + b_s dm_{t+s} + c_s(d\hat{P}_t - dP_t)] = 0. \tag{B.8}$$

Using the recursive relationship between coefficients, one can rewrite equation (B.8) as follows:

$$\begin{aligned} &b_0 dm_t + c_0(d\hat{P}_t - dP_t) + \alpha\beta E_t \sum_{s=0}^{\infty} [c_s(dm_{t+s+1} - dm_t) + b_s dm_{t+s+1} \\ &\quad + c_s(d\hat{P}_t - dP_t)] = 0. \end{aligned} \tag{B.9}$$

Subtracting $\alpha\beta$ times equation (B.7) from equation (B.9) leads to the expression

$$\begin{aligned} &b_0 dm_t + c_0(d\hat{P}_t - dP_t) + \frac{\alpha\beta c_0}{(1-\alpha\beta)}(d\hat{P}_t - dP_t) + \frac{\alpha\beta c_0}{1-\alpha\beta}(dm_{t+1} - dm_t) \\ &\quad - E_t \frac{\alpha\beta c_0}{(1-\alpha\beta)}(d\hat{P}_{t+1} - dP_{t+1}) = 0, \end{aligned} \tag{B.10}$$

where I have used the fact that

$$\sum_{t=0}^{\infty} c_s = \frac{c_0}{(1-\alpha\beta)}. \tag{B.11}$$

I use two additional facts to simplify this expression further. First, the fact that money supply shocks are i.i.d. implies that

$$E_t(dm_{t+1} - dm_t) = -E_t(dP_{t+1} - dP_t). \tag{B.12}$$

Second, one can linearize the price equation to write the real value of the optimal reset price as a function of lagged inflation,

$$(d\hat{P}_t - dP_t) = \frac{\alpha}{1-\alpha}(dP_t - dP_{t-1}), \tag{B.13}$$

where the coefficient $\alpha/(1-\alpha)$ follows from linearizing the price equation (46) around the steady state. Substituting equations (B.12) and (B.13) back into equation (B.10) leads to the expression

$$\log(m_t) = k_0 + bX[\log(P_t) - \log(P_{t-1})] - bX\beta E_t[\log(P_{t+1}) - \log(P_t)], \tag{B.14}$$

where

$$b \equiv \frac{\alpha}{1-\alpha}\frac{1}{1-\alpha\beta} \quad \text{and} \quad X = \frac{-W_{22}}{W_{21}}. \tag{B.15}$$

From the cash-in-advance equation $m_t = Y_t$, this can be written as

$$\log(Y_t) = k_0 + bX[\log(P_t) - \log(P_{t-1})] - bX\beta E_t[\log(P_{t+1}) - \log(P_t)], \tag{B.16}$$

which is the equation that appears in the text.