

A THEORY OF BUSINESS CYCLES

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This paper constructs a complete dynamic general equilibrium model of a macroeconomy that is similar in many respects to the IS/LM model that dominated the thinking of most macroeconomists for a generation. Unlike the IS/LM model all markets are modeled as in equilibrium at all points in time. Since the model is set in an overlapping generations structure in which there is incomplete participation in insurance markets, we are able to model business fluctuations that are driven by the self-fulfilling beliefs of investors. These fluctuations are Pareto inefficient since agents are risk averse and would prefer a non-stochastic allocation to an allocation that fluctuates. Our model is in contrast to the real business cycle approach that also uses a general equilibrium model but in which all fluctuations are Pareto efficient. Since the framework of our model is a complete intertemporal maximizing model we are able to explain why there may be a role for government in stabilizing business fluctuations.(JEL E32, D51)

1. Introduction

This paper introduces a simple model of equilibrium business cycles that is similar in many ways to the IS/LM model that dominated our thinking of the subject for thirty years. Unlike the IS/LM model it is based on intertemporal general equilibrium theory and it is, therefore, explicitly dynamic. The model is well suited to ask questions of interest to policy makers. How will a specific fiscal or monetary policy affect the dynamic response of prices, output, and employment? Should government conduct countercyclical fiscal policies? What are the major sources of business cycle fluctuations. Although I do not answer any of these questions directly in the paper; I do present a framework that can form the foundation for quantitative assessments of the economy. Although it is an equilibrium framework, it is not

one that biases the answers to the effectiveness of policy in advance. The model incorporates potential sources of market failure that may, or may not, be quantitatively important. By constructing more complete general equilibrium models, based on the framework in this paper, I believe that we may achieve a consensus structural model that is capable of rationalizing the reduced form estimates of the economy that arise from the VAR methodology of Sims (1980).

The model in the paper is a variant of a dynamic general equilibrium model in which »sunspots matter». ¹ There have been theoretical examples of these models in the literature now for some time but none of the published

¹ I shall use the terms »sunspots» (Cass and Shell, 1983), »self-fulfilling prophecies», (Azariadis, 1981) and »Animal Spirits» (Keynes, 1936) interchangeably.

examples has been capable of providing a quantitative explanation of the data at the same level as real business cycle theory. These examples also suffer from a credibility problem. They typically generate recessions as a consequence of voluntary reductions in labor supply.² In reality we see recessions that are associated with countercyclical layoffs- not countercyclical quits. One of the main contributions of this paper is to explain this phenomenon within the context of an equilibrium business cycle model – as a consequence of the self-fulfilling expectations of investors.

A second feature of the model that is very different from either the standard Keynesian framework, or from more familiar examples of equilibrium business cycle models, is its explanation of the monetary transmission mechanism. The paper constructs a general equilibrium model that contains multiple rational expectations equilibria. I have argued elsewhere³ that indeterminacy of a general equilibrium model is not a problem to be dispensed with by making clever assumptions; it is a virtue that permits one to explain many features of the data that are otherwise puzzling; for example, the prevalent phenomenon of »sticky prices«. One might believe that indeterminacy is a feature that should be avoided since it precludes the ability to explain.⁴ The model in this paper resolves this issue by arguing that agents use a parameterized rule, called a *belief function*, to forecast the future. I argue that the parameters of the belief function are to be treated as primitives in the same way as the parameters of the utility function and the technology.⁵

² This is true of much of the sunspot literature that I am aware of. Examples are Azariadis (1981), Azariadis and Guesnerie (1986), Beaudry and Devereux (1993), Benhabib and Farmer (1994), Farmer and Guo (1994), Farmer and Woodford (1984), Rotemberg and Woodford (1992), and Gali (1994) are exceptions. These papers contain mechanisms that operate through countercyclical markups which cause sunspot shocks to shift labor demand.

³ Farmer (1991b), (1992), (1993).

⁴ See Woodford's (1988) rebuttal of this argument.

⁵ The »Lucas Critique« does not apply in economies in which beliefs can independently influence outcomes. See my paper »The Lucas Critique, Policy Invariance and Multiple Equilibria«, Review of Economic Studies (1991). The concept of a belief function used in this paper picks out one particular rational expectations equilibrium. We do not con-

The paper is organized in the following way. Section 2 contains a preview of the complete general equilibrium model by summarizing the main equations. Section 3, discusses the role of the intertemporal substitution mechanism in equilibrium models and it explains how this mechanism is modified in the paper. Section 4 explains a theory of production in some detail. This theory explores the dynamic implications of Cooper and John's (1988) concept of *strategic complementarities*⁶ and much of these three sections are spent explaining how aggregate technologies that are non convex can be made consistent with a competitive theory of distribution. The end result of this discussion is contained in a theory of aggregate supply that explains why output falls when real interest rates rise. This theory of supply is embedded, in section 6, into a complete general equilibrium model based on a two-period overlapping generations model of the economy.

For the reader who is interested in a quick preview of the major results- sections 2, 5 and 6 are self-contained. These sections develop the general equilibrium model conditional on accepting the idea that aggregate supply depends on the real rate of interest. Section 6 describes a theory of expectations and it traces out the effects on the model of two »typical« shocks. The first of these shocks is a drop in investor confidence; the second is a monetary policy shock in which the Central Bank raises the nominal interest rate. The effects of both of these shocks are traced out in the context of a simple diagram that bears a strong resemblance to the IS/LM model of textbook macroeconomics.

2. A Preview of the Main Arguments

This paper constructs an intertemporal general equilibrium model.

sider the issue of learning as in the papers by Evans (1985) or Marcet and Sargent (1989). However, in related work John Duffy (1994) has shown that out of equilibrium learning mechanisms can converge to equilibria of the class that we consider in this paper.

⁶ See also Howitt and Macafee (1988), (1992) for work that uses strategic complementarities to generate sunspot equilibria. Bryant, (1983) uses a similar idea.

(1) $Y_t = f(r_t)$,
Aggregate Supply (AS)

(2) $\frac{M_t}{P_t} = Y_t k(i_t)$,
Liquidity Preference (LM)

(3) $\frac{B_t}{P_t} + \frac{M_t}{P_t} + I(r_t) = sY_t$.
Asset Market Equilibrium (IS)

The notation is fairly standard, Y is domestic product, r is the real rate of interest, i is the nominal rate of interest, P is the price level, M is the stock of money, B is the stock of government debt, I is new investment and S is savings. The dynamics of the model are represented by the rational expectations assumption:

(4) $(1+r_t) = E_t \left[\frac{(1+i_t)P_t}{P_{t+1}} \right]$,

where E_t is the expectations operator conditional on date t information. The model is closed by the government budget constraint:

(5) $\frac{B_t}{P_t} + \frac{M_t}{P_t} = \frac{B_{t-1}}{P_{t-1}}(1+i_{t-1})$
 $+ \frac{M_{t-1}}{P_{t-1}} + G_t$

I have left taxes out of this exposition and so this final equation serves only to define the value of government spending for any given level of debt and money. Since I will be phrasing my argument in the context of the two period overlapping generations model the third equation that represents »investment equals savings» will also represent asset market equilibrium. I shall also be assuming 100% depreciation of capital. These two oversimplifications imply that the young people in my model will hold the entire capital stock plus the entire stock of government debt.

The Role of Intertemporal Substitution

An important element of equilibrium business cycle theory is the idea that households substitute leisure intertemporally. Figure 1 illus-

trates this idea in the context of a diagram that depicts labor market equilibrium. The labor demand curve is a decreasing function of the real wage and an increasing function of the existing stock of capital, K . The labor supply function is increasing in the real wage and increasing in the real rate of interest as a consequence of intertemporal substitution of leisure between periods in response to perceived changes in intertemporal prices. An increase in the real interest rate from r_1 to r_2 causes an increase in labor supply, (an outward shift of the labor supply schedule), a fall in the equilibrium real wage from $(W/P)_1$ to $(W/P)_2$ and an increase in the equilibrium level of employment from L_1 to L_2 . It is a mechanism of this kind that underlies the early attempts of equilibrium business cycle theory to explain the economic consequences of monetary fluctuations.⁷

The intertemporal substitution mechanism has been widely criticized for its implication that employment fluctuations over the business cycle are generated by voluntary movements in labor supply, rather than by fluctuations in labor demand. It is typical during recessions for the economy to experience a reduction in employment as a consequence of an increase in layoffs; separations are initiated by firms rather than by workers. It is difficult to reconcile this

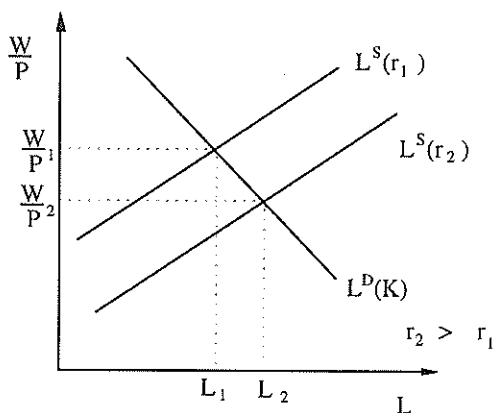


Figure 1. Labor demand and supply in a »standard» rational expectations model.

⁷ The 1972 JET paper by Robert Lucas, »Expectations and the Neutrality of Money» is the most famous example of a rational expectations model of this kind.

evidence with an equilibrium model in which employment variations are described by a shift of the labor supply curve.

As a response to the criticism that the intertemporal substitution mechanism is unrealistic, equilibrium business cycle theory changed direction. In the 1980's equilibrium theorists developed the idea that most economic fluctuations are the consequence of persistent shocks to the technology and for the last decade this idea, »real business cycle theory», (RBC) has been the major focus of macroeconomic research in the United States. Whilst the RBC mechanism is relatively successful at capturing contemporaneous movements in quantities, it does not do a good job of explaining why movements in nominal magnitudes can affect output. It is also difficult to find convincing explanations for sources of real disturbances that are coordinated across sectors in the way that is necessary to understand the comovements that we observe in US output. In a typical business cycle all sectors of the economy contract and expand together. To explain a movement of this kind with a real business cycle model one must understand how productivity shocks can affect all industries at the same time. In the absence of a well documented source of such coordinated shocks, many economists remain skeptical of the RBC explanation.

This paper modifies the intertemporal substitution mechanism in a way that *can* account for

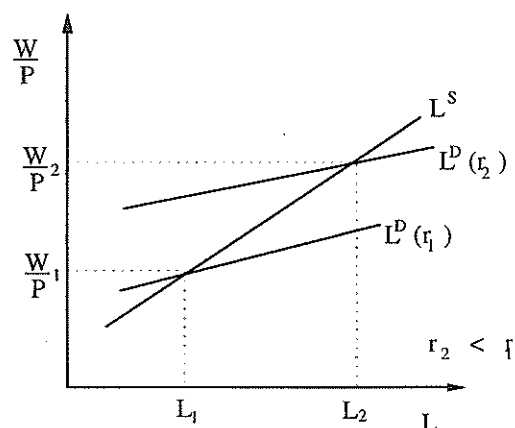


Figure 2. Labor demand and supply in the model developed in this paper.

the correlation between real and nominal magnitudes that we observe in time series data. There are two main threads to the argument. The first is a development of recent research on »coordination failures» which suggests that the aggregate technology may be non-convex and that labor demand curves for industries as a whole may be mildly increasing as a function of the real wage.⁸ The second thread is the idea that the capital goods industries and the consumption goods industries do not produce perfect substitutes. Since the output of the capital goods industries typically produce a return in the future; the relative price of capital goods to consumption goods will be linked to the real rate of interest. In the model that I will describe, fluctuations in the rate of interest will cause movements of labor demand rather than of labor supply.

Figure 2 depicts labor market equilibrium. Unlike the intertemporal substitution that drives familiar examples of rational expectations models, the idea in this paper is that a shift in the expected real rate of interest may increase labor demand. In terms of figure 2 this change in expectations is depicted as an upward shift of the labor demand curve which drives up the wage from $(W/P)_1$ to $(W/P)_2$ and the level of employment from L_1 to L_2 .

I have drawn Figure 2 with an upward sloping labor demand curve to reflect the assumption of increasing returns to scale. In a competitive model without this assumption, an expansion of labor demand emanating in the investment goods industries would induce an offsetting contraction in the production of consumption goods as competitive firms cut back on their demand for labor in the face of an increase in the real wage. But the presence of increasing returns to scale implies that an increase in the demand for labor in one sector may generate an increase in employment in all sectors simultaneously as the increased incomes generated by an increase in employment permit the economy to make more efficient use of economies of scale.

⁸ This work includes the evidence of Baxter and King (1991), Caballero and Lyons (1992), Domowitz, Hubbard and Petersen (1988), and Hall (1986), (1987), (1988).

3. Industrial Structure

Consumption Goods and Capital Goods

In section (3), I am going to derive two functions that I refer to as the aggregate supply curve, and the investment schedule. These are the functions

$$(6) \quad Y_t = f(r_t), \text{ and } I = I(r_t),$$

that appear in the capsule summary of the model in section 2. Section 3 describes assumptions about the production technology and it explains the implications of a theory of monopolistic competition for the labor and product markets. The payoff to this discussion is contained in section 4 that uses a theory of labor market equilibrium to derive the two schedules referred to above.

The model has two productive sectors that I refer to as the consumption goods industry and the capital goods industry. The output of the consumption goods industry will be represented as Y_t^C and the output of the capital goods industry as Y_t^K and I will use the symbol P_t (without a superscript) to mean the money price of a consumption good and P_t^K (with a K superscript) to mean the money price of a capital good.

What is a Capital Good?

Standard macroeconomic models impose the assumption that capital goods and consumption goods are perfect substitutes in production and, as a consequence of this assumption, the relative price of consumption goods and capital goods is unity. As an alternative to the standard approach, in this paper I will adopt a set of assumptions that allows me to recognize that the relative price of capital goods to consumption goods may differ from unity. The technology that I will work with can be written as:

$$(7) \quad C_t = K_t + f(L_t^C)$$

$$(8) \quad Y_t^K = g(L_t^K)$$

$$(9) \quad Y_t^K = K_{t+1}$$

where L^C denotes labor in the consumption sector and L^K is labor in the capital goods sector. The first equation says that a consumption good can be produced either by using labor or by using capital, the second says that a capital good can be produced using only labor and the third equation combined with the first equation implies that a capital good produces exactly one consumption good in the subsequent period. These assumptions amount to assuming that there are two techniques for producing final output. One technique produces consumption goods immediately. The other technique is a roundabout production method that takes an additional period. This roundabout technique produces an inventory of goods-in-process in the first period (this is the stock of goods, K). In the second period these goods-in-process turn into perfect substitutes for the consumption good.⁹

Measuring Economic Activity

Gdp is one variable that one might use to represent economic activity. Rather than use gdp, however, I am going to use an alternative measure that bears a simple relationship to wage income. This alternative measure is the net domestic product, Y_t or domestic income.¹⁰ Since I have maintained the assumption of 100% depreciation, the net domestic product is equal to the value of gdp minus the value of the capital stock:

$$(10) \quad Y_t = \text{GDP}_t - K_t = Y_t^C + \frac{P_t^K}{P_t} Y_t^K.$$

⁹ This technology is a special case of a two-sector economy that is represented, in its most general form by the equations: (1) $C_t = F(K_t^C, L_t^C)$, (2) $K_{t+1} - (1 - \delta)K_t = G(K_t^K, L_t^K)$, and (3) $K_t = K_t^C + K_t^K$, where the superscripts K and C denote industry, and the functions $F(\bullet)$ and $G(\bullet)$ are the sectoral technologies in the consumption goods and capital goods industries.

¹⁰ Since there are no indirect taxes in this economy the (net) domestic product is also equal to domestic income. One could also define the gross domestic income as:

$$\text{GNI}_t = \frac{W_t}{P_t} L_t + \frac{rr_t}{P_t} K_t$$

where rr_t is the rental rate on a machine. Since a machine produces one unit of the consumption good rr_t must equal P_t in equilibrium from which it follows that rental income is identically equal to depreciation.

To arrive at Gdp one would add in the value of the consumption goods produced from inventories of capital (assumed to equal K). The main benefit of using Y_1 to measure economic activity, rather than Gdp, is provided by the fact that it is identically equal to the wage bill.

$$(11) \quad Y_1 = \frac{W_1}{P_1} L_1.$$

This convenient simplification follows from two assumptions. The first is that there is free entry in all industries which implies that, in equilibrium, economic profit will be equal to zero. The second is that the only primary input in either sector is labor.

Industrial Structure in the Two Industries

This section discusses industrial organization in the two industries. I will show that fixed costs may imply that industry demand curves for labor are increasing in the real wage. This will be an important component of the theory of economic fluctuations developed in the paper since it will explain how disturbances in one sector may be amplified through comovements across all sectors. The end result of the discussion will be summarized by two diagrams that depict the relationship between labor input and industry output and between the product wage (defined below) and industry demand-for-labor.

Specialization and the Role of Fixed Costs

A good deal of recent work on *strategic complementarities* has explored the idea that the comovements across sectors that we see during business cycles may be transmitted through non-competitive elements in production.¹¹ In this paper these non competitive elements will arise from the presence of fixed costs that create an incentive to concentrate production into relatively large units. Although producers of individual commodities have some monopoly power, the extent of this power is limited by

¹¹ Some of the work in this area includes Beaudry and Devereux (1993), Chatterjee and Cooper (1993), Cooper and John (1988), Cooper and Haltiwanger (1993), Hall (1986), (1990), Klenow (1991), and Shea (1993).

competition with other industries that produce imperfect substitutes. To capture this idea formally I will think of the consumption good that enters utility, Y^C , as a composite good that is produced from a range of intermediate inputs, C^i using a technology $F(\bullet)$ ¹²:

$$(12) \quad Y^C = F(C^1, C^2, \dots, C^M) = \left[\sum_{i=1}^M (C^i)^\epsilon \right]^{\frac{1}{\epsilon}}$$

The term M represents the number of intermediate goods, and ϵ measures the degree of substitutability between inputs.¹³ When $\epsilon = 1$, the intermediate goods are perfect substitutes, when $0 < \epsilon < 1$, (the case I consider here) they are imperfect substitutes and when $\epsilon < 0$ they are complements.

A value of ϵ between zero and one captures the idea of the benefits of the division of labor since it implies that the total product is increasing in the number of intermediate goods. Suppose that we think of each intermediate good as hours of labor devoted to a single task and suppose that there are L hours of labor in total to be used to produce the final product Y^C . If the hours of labor are divided equally amongst M tasks then the total quantity of output produced will be equal to:

$$(13) \quad Y^C = \left(M \left(\frac{L}{M} \right)^\epsilon \right)^{\frac{1}{\epsilon}} = M^{\frac{1-\epsilon}{\epsilon}} L$$

which, as long as ϵ is between zero and one, is increasing in M. If there were no limit to the division of labor, efficient operation of this technology would require an infinite degree of specialization across different tasks. A natural limit to the division of tasks is suggested by the

¹² Logically, this way of posing the problem has the same structure as one in which the »intermediate goods» directly enter utility and the function $F(\bullet)$ represents a utility aggregator. For expositional purposes, however, it will be simpler to think of the final package of commodities as assembled by a competitive final goods industry.

¹³ M is chosen endogenously and may be a real number (integer constraints are ignored). To be formally correct one could think of a continuum of commodities, each being infinitesimally small.

existence of fixed costs: learning the necessary skills or moving from one location to another for example. To incorporate this idea we could modify the technology in the following way:

$$(14) \quad Y^C = \left(M \left(\frac{L}{M} - A \right)^\varepsilon \right)^{\frac{1}{\varepsilon}} = \frac{1-\varepsilon}{M^\varepsilon L} - \frac{1}{M^\varepsilon A}$$

where A is a fixed setup cost associated with each task. When we incorporate setup costs in this way, the production of the final good is a concave function of M with a well defined maximum. For example, when $\varepsilon = 1/2$ the function is a quadratic in M with a maximum at $M = A L/2$.

Since each task involves the payment of a fixed cost there will be a natural tendency for tasks to be specialized in the hands of a single producer. This idea is modeled by thinking of production as organized into two layers. Final goods are produced by a competitive industry that uses the technology described in equation (14). Each intermediate good is produced by a monopolistic competitor using the technology:

$$(15) \quad C^i = L^i - A.$$

In the following subsection I will show how an industry that is organized in this way would operate in equilibrium. I will impose the assumption that free entry of firms drives the profits in the final goods industry, and in each of the intermediate industries, to zero, and I will derive a relationship between the real wage and the employment in the industry that I refer to as the *labor demand curve*.

Industry Production in Equilibrium

In this subsection I am going to ask three questions; what determines the number of intermediate producers, M? what determines the volume of labor employed in the industry, L? and what determines industry output Y^C ? To answer these questions I will impose the assumption that the industry is in continuous equilibrium in every period in the sense that there

are no unexploited profit opportunities.

To explore the implications of the zero profit assumption I will make use of the profit function of a producer of consumption goods:

$$(16) \quad \Pi(P^i, P; Y^C) = P Y^C \left(1 - M^{\frac{\varepsilon-1}{\varepsilon}} \left(\frac{P^i}{P} \right) \right).$$

This function, which is derived in the appendix, describes the profit that would be made by a firm that produces Y^C units of the consumption good by choosing an optimal mix of M inputs when all inputs have the same price, equal to P^i and when the output price is equal to P.

A second property that I derive in the appendix is that the profit maximizing intermediate goods producer will choose to set price as a fixed markup over marginal cost:

$$(17) \quad P^i = \frac{W}{\varepsilon}.$$

Using this pricing rule one can describe the profit of an intermediate producer as an increasing function of scale:

$$(18) \quad \Pi^i = \frac{W}{\varepsilon} \left((1-\varepsilon)L^i - A \right).$$

Suppose, for a moment, that the number of intermediate producers is fixed at some number, M. In that case the final goods producers will make positive profits whenever

$$(19) \quad \frac{P^i}{P} < M^{\frac{1-\varepsilon}{\varepsilon}}$$

and negative profits whenever the reverse inequality holds. This argument implies that the final sector will display a horizontal input demand schedule as a function of the relative input price. But since P^i is set as a fixed multiple of the wage this demand schedule will translate into a horizontal demand curve for labor by the industry as a whole. This industry demand for labor is depicted in figure 3.

If the industry were competitive, the horizontal labor demand curve would be the end of the story. But there is a relationship between scale and marginal product that is not captured by the horizontal demand schedule. This relationship

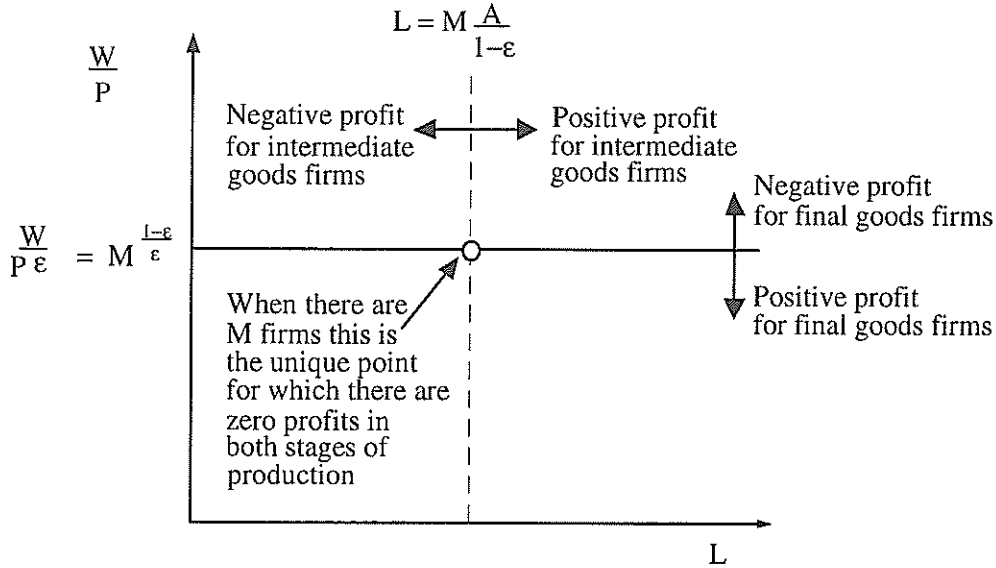


Figure 3. Industry demand for labor when the number of intermediate producers is fixed.

is reflected in the profit of an intermediate producer (equation (18)) which is an increasing function of scale. Since each intermediate producer faces fixed set-up costs, there will be a *minimum efficient scale* of operation, defined by the equation

$$(20) \quad L^i = \frac{A}{1-\epsilon}$$

that will just permit the producer to cover its costs. Whenever intermediate producers are employing more labor than $A/(1-\epsilon)$, they will make a positive profit and whenever they are employing less they will make a negative profit. If the industry as a whole hires L units of labor, and if there are M intermediate producers of equal size, the total industry use of labor must be at least equal to

$$(21) \quad M \frac{A}{1-\epsilon}$$

for the individual producers to make a profit. If L exceeds $MA/(1-\epsilon)$ then intermediate producers will make positive profits and there will be entry into the industry. If L is less than $MA/(1-\epsilon)$ then intermediate firms will make a loss and there will be pressure for firms to leave.

The above arguments suggest that one can

divide the plane into four regions as in Figure 3. Points below (above) the horizontal line

$$(22) \quad \frac{W}{P} = \frac{\epsilon}{M^{1-\epsilon}}$$

are points of positive (negative) profits in the final goods industry. Points to the right (left) of the vertical line

$$(23) \quad L = M \frac{A}{1-\epsilon}$$

are points of positive (negative) profits in the intermediate goods industry. For any number, M , there is a unique pair $(W/P, L)$ with the property that profit in both stages of the industry is equal to zero. By parametrically increasing M one may trace out a locus of points that is associated with zero profit equilibrium in the industry as a whole: this locus, depicted in the lower panel of Figure 4, is referred to as the *industry labor demand schedule*. Algebraically it is defined by the expression:

$$(24) \quad L^D = k_1 \left(\frac{W}{P} \right)^{\frac{\epsilon}{1-\epsilon}}$$

where k_1 is a function of the parameters A and ϵ . One may also derive an equation linking

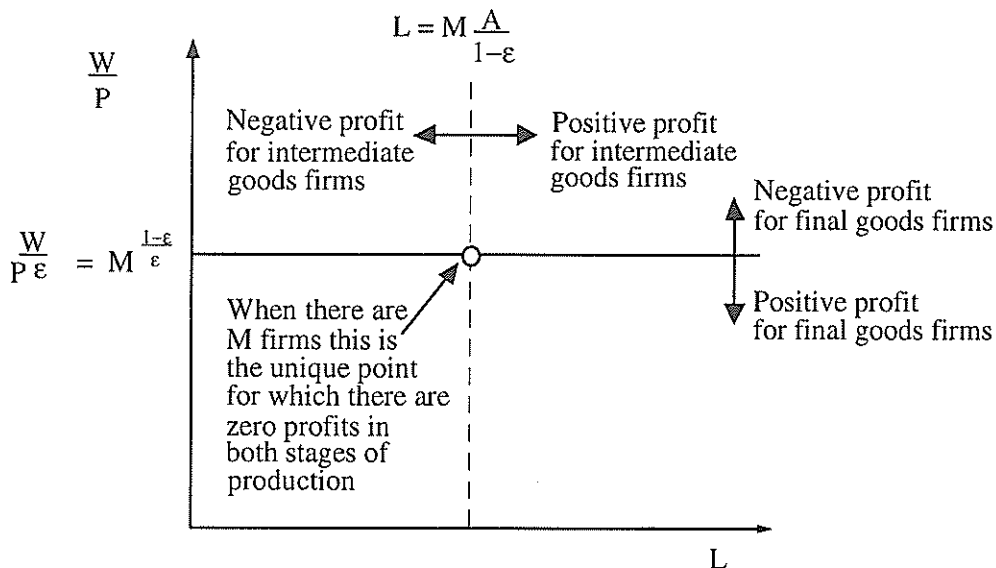


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the production of final commodities, Y^C , to industry use of labor. This equation is derived by recognizing that, in a zero profit equilibrium, each intermediate firm will produce:

$$(25) \quad C^i = A \frac{\epsilon}{1-\epsilon}$$

and the number of intermediate producers must equal:

$$(26) \quad M = L \frac{1-\epsilon}{A}$$

Replacing these expressions in the final goods production function:

$$(27) \quad Y^C = M^\epsilon C^i$$

leads to the reduced form *industry production function*

$$(28) \quad Y^C = k_2 \left(L^D\right)^\frac{1}{\epsilon}$$

where the constant k_2 also depends on the parameters A and ϵ . It is equations (27) and (24) that are graphed in the top and bottom panels of Figure 4.

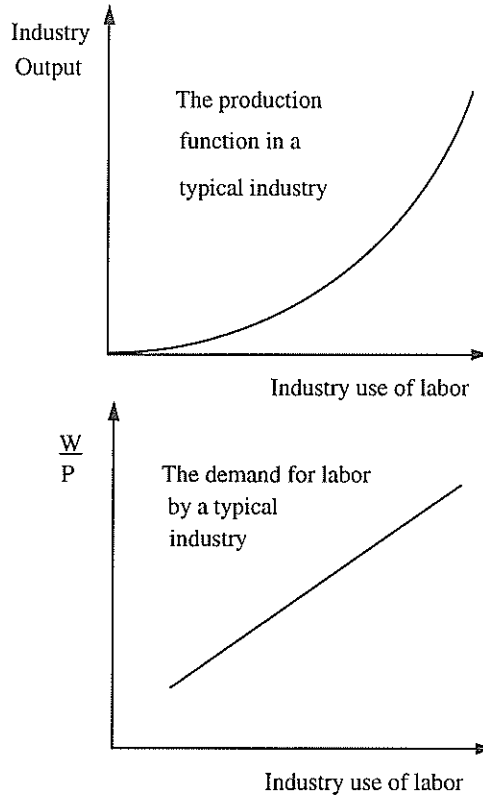


Figure 4. The Aggregate Production of a typical industry.

The Capital Goods Industry

I have described the functioning of the consumption goods industry under the assumption that there are fixed costs associated with the production of intermediate goods. In this subsection I am going to show how the capital goods industry can be modeled in the same way. The results for the capital goods industry will mirror those of the consumption goods industry- with one exception; the product wage in the capital goods industry is the money wage divided by the price of capital goods. This fact implies that if one derives the demand for labor in this industry, as a function of the wage measured in consumption goods, that this schedule will be shifted by changes in the relative price of capital goods. This relative price is itself related to the real rate of interest.

I will assume that the structure of the capital goods industry is identical to the structure of the

consumption goods industry. There is a final capital good that is produced from a range of N intermediate capital goods according to the technology:

$$(29) \quad Y^K = B \left(\sum_{i=1}^N \left(K^i\right)^\epsilon \right)^\frac{1}{\epsilon}$$

I have maintained the assumption that the capital goods technology uses the same aggregator as the consumption goods industry since this makes some of the algebra more manageable; nothing of importance hinges on this assumption. The constant B is important, however, since it reflects the relative productivity of the two technologies. When we describe the equilibrium of a complete economy, B will help to determine the relative size of the two sectors.

Each of the N intermediate capital goods is produced by a monopolistic competitor using the technology:

$$(30) \quad K^i = L^i - A.$$

As with the consumption goods industry, the demand for labor will be a function of the real wage. But since the price of capital goods may differ from the price of consumption goods, the output of the industry will be sensitive to relative prices. The relationship between the real wage in the capital goods industry and number of firms that is consistent with zero profits in the industry is defined by the expression:

$$(31) \quad N = \left(\frac{W}{B \varepsilon P^K} \right)^{\frac{\varepsilon}{1-\varepsilon}},$$

where P^K is the price of capital goods. Since these capital goods will turn into consumption goods in the following period, the price of a capital good will be related to the rate of interest by the expression:

$$(32) \quad \frac{P_t}{P_t^K} = E_t \left[\frac{P_t(1+i_t)}{P_{t+1}} \right] \equiv (1+r_t)$$

where r_t is the real rate of interest between periods t and $t+1$. The denominator of the left hand side of the equation, P_t^K , is the cost of buying one unit of the capital good in period t . In the subsequent period this good will produce one unit of consumption that can be sold for price P_{t+1} . This is the denominator of the right hand side of the equality which must be weighted by the term that appears in the numerator, the interest factor $(1+i_t)$.

By the same arguments that were used to construct the aggregate technology in the consumption sector; this technology defines an aggregate demand for labor in the capital goods industry of the form:

$$(33) \quad L^D = k_3 \left(\frac{W(1+r)}{P} \right)^{\frac{\varepsilon}{1-\varepsilon}}$$

and an aggregate relationship between production and labor use of the form:

$$(34) \quad Y^K = k_4 \left(L^D \right)^{\frac{1}{\varepsilon}}.$$

4. The Aggregate Supply Curve and the Supply of Capital Goods

In this section I am going to derive two relationships that are building blocks for the general equilibrium model that I laid out in the beginning of the paper. The first of these relationships is between the real rate of interest, r_t , and net domestic product, Y_t , and I refer to it as the *aggregate supply curve*. The second is between the value of investment goods produced in any period and the real rate of interest and I refer to this relationship as the *investment schedule*.

Deriving the Aggregate Supply Curve

The aggregate supply curve describes the domestic product of an economy that has the industrial structure described in the previous section, for any given value of the real rate of interest, *assuming a particular form for the labor supply schedule*. The italics in the previous sentence emphasize the fact that the domestic product cannot be determined in a general equilibrium model without making some assumptions about the behavior of households. In this section I am going to make the assumption that the labor supply in the entire economy is an increasing function of the real wage. In the subsequent section I will write down a utility function for a two period lived household that is consistent with this assumption and I will show how to construct a two period overlapping generations model that allows the aggregate supply curve to be written as part of a fully specified general equilibrium model.

Figure 5 depicts a graphical derivation of aggregate supply. The center panel of this figure represents the demand-for-labor in the consumption goods industry,

$$(35) \quad L^D = k_1 \left(\frac{W}{P} \right)^{\frac{\varepsilon}{1-\varepsilon}}$$

and the far right panel depicts the labor demand schedule in the capital goods industry;

$$L^D = k_3 \left(\frac{W(1+r)}{P} \right)^{\frac{\varepsilon}{1-\varepsilon}}$$

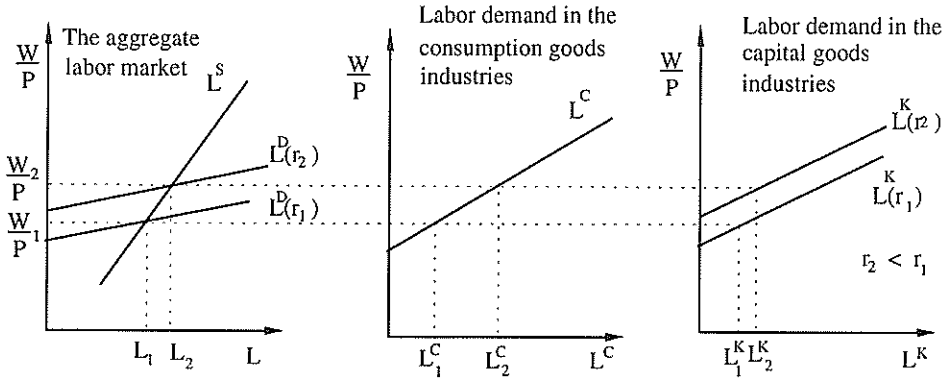


Figure 5. Equilibrium in the Labor Market when there are two monopolistically competitive industries.

Notice that the labor demand schedule in the capital goods industry can be written as a function of the real wage, (W/P) and of the real rate of interest. In other words, the schedule in the right hand panel of Figure 5 that is depicted as function of the real wage alone is shifted by changes in the real interest rate. The left hand panel of Figure 2 depicts the economy wide labor demand schedule that is derived by adding up the labor demand in the two industries:

$$(36) \quad L^D = k_1 \left(\frac{W}{P} \right)^{\frac{\epsilon}{1-\epsilon}} + k_3 \left(\frac{W(1+r)}{P} \right)^{\frac{\epsilon}{1-\epsilon}},$$

together with a labor supply schedule of the form:

$$(37) \quad L^S = \left(\frac{W}{P} \right)^{\frac{1}{\gamma}}.$$

The parameter γ is the inverse of the labor supply elasticity with respect to the real wage. The equality of the demand and supply of labor determines the real wage as a function of the real rate of interest and domestic product is equal to the real wage times labor supply. The aggregate supply curve is derived by equating the demand and supply for labor and writing the real wage, W/P as a function of the rate of interest. W/P is the solution to the equation,

$$(38) \quad \frac{W}{P} \left(\frac{1}{\gamma} - \frac{\epsilon}{1-\epsilon} \right) = \left(k_1 + k_3 (1+r) \right)^{\frac{\epsilon}{1-\epsilon}}$$

which I will write as a function

$$(39) \quad \frac{W}{P} = \omega(r).$$

Notice that $\omega(r)$ is a decreasing function as long as $(1/\gamma) < (\epsilon/1-\epsilon)$ which is the condition that the slope of the labor demand curve with respect to the real wage should be flatter than the slope of the labor supply curve.

One of the features of the US data is that the real rate of interest is not strongly correlated with aggregate economic activity. A theory that relies on a strongly cyclical interest rate to generate a transmission mechanism from policy to employment is not likely to be successful. It is therefore interesting to ask, under what conditions the theory that we are discussing will generate an aggregate supply curve that is relatively flat. Equation (38) holds the answer to this question: when the slope of the labor demand curve, $(\epsilon/1-\epsilon)$, is equal to the slope of the labor market determines the real rate of interest. In this special case, any real wage and any level of employment is consistent with labor market equilibrium. When the slopes of the two schedules are close, a small change in the real rate of interest will, have a large effect on the real wage and a large effect on employment.

Using the function $\omega(r)$ it is possible to derive the aggregate supply schedule that we described at the beginning of this section. The aggregate supply curve is the relationship between net domestic product (equal to the wage bill) and the real rate of interest:

$$(40) \quad Y = f(r) = \omega(r)L[\omega(r)] = [\omega(r)]^{1+\frac{1}{\gamma}}$$

Notice that the slope of the aggregate supply schedule has the same sign as the slope of the wage schedule $\omega(r)$ and it inherits similar properties. In particular, aggregate supply is a decreasing function of r as long as labor demand does not slope up »too steeply» and the schedule is flatter the closer are the demand and supply elasticities in the labor market.

Deriving the Investment Schedule

I have described how the domestic product of the economy depends on the rate of interest under the assumption that the labor market is in equilibrium. It is also possible to describe the value of the output of the investment sector under the same assumptions about household behavior. The relationship between the value of investment, $P^K Y^K$, and the real interest rate, r , will be an important component of the complete equilibrium model since it determines the demand for savings on the part of the corporate sector.

It should be apparent from Figure 5 that a fall in the rate of interest will not necessarily generate an increase in employment in the investment goods industries since there are two effects going on that work in opposite directions. Formally, the demand for labor in equilibrium can be described as a function of the rate of interest by substituting the function $\omega(r)$ into equation (33):

$$(41) \quad L^K = k_3 \left(k_1 + k_3 (1+r)^{\frac{\epsilon}{1-\epsilon}} \right)^{\frac{\epsilon\gamma}{1-\epsilon(1+\gamma)}} (1+r)^{\frac{\epsilon}{1-\epsilon}} = L^K(r).$$

The interest rate appears twice in this expression. The first appearance reflects the effect of the interest rate on the real wage and it is unambiguously negative. The second appearance reflects the upward shift of the investment demand schedule which tends to reduce employ-

ment for any given real wage. In this paper I will impose the assumption that the wage effect dominates and that the demand for labor by capital goods producers rises when the interest rate falls. The closer are slopes of labor demand and labor supply, (the closer is $\epsilon(1+\gamma)$ to 1), the more likely it is that this condition will hold.

The investment demand schedule is defined as the relationship between the value of investment, $(P^K/P)Y^K$, and the rate of interest.

$$(42) \quad I(r) = \frac{P^K}{P} Y^K = \frac{1}{1+r} k_4 (L^K(r))^{\frac{1}{\epsilon}}$$

Since the relative price of capital goods is equal to $1/(1+r)$ the first part of this expression is unambiguously decreasing in r . The assumption that L^K is a decreasing function of r is sufficient to imply that $I(r)$ is decreasing in r .

The Relative Slopes of I(r) and f(r)

A final property of the schedules $f(r)$, the aggregate supply schedule, and $I(r)$, the investment schedule, concerns their relative slopes. This property will be an important determinant of the dynamics of the complete model that we will explore by drawing a diagram that bears a strong resemblance to the IS/LM model. To derive the relative slopes, notice that:

$$(43) \quad Y = \frac{1}{1+r} Y^K + Y^C.$$

If we define a function $Y^C(r)$ that relates the output of the consumption goods industry to the real rate of interest, this definition of domestic product can be written in terms of the functions $f(r)$, $I(r)$ and $Y^C(r)$:

$$(44) \quad f(r) = I(r) + Y^C(r).$$

The slopes of these functions are related in the following way:

$$(45) \quad \frac{\partial f}{\partial r} = \frac{\partial I}{\partial r} + \frac{\partial Y^C}{\partial r}.$$

Since $Y^C(r)$ is a decreasing function,¹⁴ it follows that

¹⁴ Y^C is increasing in the real wage and the real wage is a decreasing function of r .

$$(46) \quad \left| \frac{\partial f}{\partial r} \right| > \left| \frac{\partial I}{\partial r} \right|.$$

Both numbers are negative, but a fall in the interest rate will have a larger effect on domestic product than on investment.

5. The Behavior of Households

So far I have described the behavior of an economy in which labor supply depends only on the real wage. In this section I am going to close the model by embedding the production sector in a two period overlapping generations economy. I will derive the labor supply curve from assumptions about preferences of the agents, and I will show how the model behaves in the face of alternative kinds of shocks to beliefs and to policy.

Utility Functions

I am going to populate the model with two period lived agents with preferences represented by:

$$(47) \quad U = E_t[C_{t+1}^t] \left(C_t^t - \frac{L_t^{1+\gamma}}{1+\gamma} \right) m_t$$

where $m_t \equiv M_t/P_t$, L_t is labor supply and C_t^s is the consumption at date t of the generation born at date s . I have chosen a specification for which the agents of the model are risk neutral since nothing critical in the argument will hinge on risk. A second point to note is the way that I have dealt with money. Although this is an overlapping generations model, the money of the model will not be a perfect substitute for government debt. Real balances are included as an argument of the utility function to capture the exchange motive for holding money.

A third feature of these preferences involves the fact that consumption when young and labor supply enter utility in a separable form to simplify the labor supply function (labor supply will depend only on the real wage). Preferences of this kind are often criticized in the context of representative agent economies since

they are not consistent with growth that arises from exogenous productivity increases. It is not difficult, however, to modify the preferences to allow for growth to arise from, for example, increases in the productivity of a labor endowment.

A more serious criticism of the model might focus on the fact that it assumes two period lives. I have chosen this structure for expositional purposes; I want to focus on a particular class of dynamic adjustment in the economy that arises as a consequence of expectational adjustments. I have chosen the two period OG model as a device to exposit the main idea because it is possible, in this model, to strip away the additional sources of economic dynamics that arise from wealth effects in consumption and from the accumulation of capital. The three equations that describe equilibria in this model are not designed to form the foundation for an empirical investigation of the US economy; however, I would not expect that it would be very difficult to expand the model in a way that would be suitable for such a purpose.

Budget Constraints

The budget constraints of the agents in the model are represented by the equations:

$$(48) \quad C_t^t P_t + M_t + B_t + P_t^K K_t \leq W_t L_t$$

Constraint when young

$$(49) \quad C_{t+1}^t P_{t+1} \leq P_{t+1} K_t + B_t(1+i_t) + M_t$$

Constraint when old

Putting these two equations together leads to the expected life cycle constraint

$$(50) \quad C_t^t + \frac{E_t[C_{t+1}^t]}{(1+r_t)} + \frac{i_t}{1+i_t} m_t \leq \omega_t L_t.$$

Expected life-cycle constraint

Recall that r_t is defined as the *expected* real rate of interest. The risk neutrality assumption implies that the young individual is indifferent between the certain return from holding capital, P_t^K/P_t and the uncertain return from lending to the government, $(1+i_t)P_t/P_{t+1}$. The term

$\omega_t = W_t/P_t$ is the real wage, i_t is the nominal rate of interest on one period bonds issued in period t for redemption in period $t+1$, and B_t is the quantity of these bonds held by the young agent. The term K_t represents a capital good which costs P_t^K in period t . Capital goods in this economy produce a single unit of the consumption commodity in the subsequent period.

Demand and Supply Functions

The solutions to the household's problem are represented by the functions:

$$(51) \quad L_t = (\omega_t)^\frac{1}{\gamma},$$

$$(52) \quad C_t^l = \frac{(3+\gamma)}{3(1+\gamma)} L_t \omega_t,$$

$$(53) \quad \frac{E_t[C_{t+1}^l]}{(1+r_t)} = \frac{\gamma}{3(1+\gamma)} L_t \omega_t,$$

$$(54) \quad \frac{i_t}{1+i_t} m_t = \frac{\gamma}{3(1+\gamma)} L_t \omega_t.$$

Since the wage bill is equal to domestic product, $L_t \omega_t$, in each of these demand functions may be replaced by Y_t .

6. The Complete Intertemporal General Equilibrium Model

At the beginning of this paper I wrote down a capsule summary of a complete general equilibrium model. In this section I am going to show where the four equations that summarize this model come from; and I am going to trace out the effects of two disturbances to this model. The first of these disturbances is designed to form the foundation for a theory of business cycles based on »sunspots» or »animal spirits». The second is designed to show how a theory of fluctuations that is based on animal spirits can explain the monetary transmission mechanism in a fully specified equilibrium model without invoking unexplained barriers to price adjustment.

The Equations of the Model

Equations (55), (56) and (57) form the core of the model. Equation (55) is the aggregate supply curve that we derived in some detail in sections (4) and (5). Equation (56) is a liquidity preference schedule that arises from the equality of the

$$(55) \quad Y_t = f(r_t),$$

Aggregate Supply (AS)

$$(56) \quad \frac{M_t}{P_t} = Y_t k(i_t),$$

Liquidity Preference (LM)

$$(57) \quad \frac{B_t}{P_t} + \frac{M_t}{P_t} + I(r_t) = s Y_t.$$

Asset Market Equilibrium (IS)

demand and supply of money where the function $k(i)$ is defined as the constant k multiplied by $(1+i)/i$:

$$(58) \quad k(i_t) = k \frac{1+i_t}{i_t},$$

$$\text{and } k = \frac{\gamma}{3(1+\gamma)}.$$

Equation (57) equates the savings of young agents

$$\text{Savings} = Y_t - C_t^l = s Y_t,$$

$$\text{where } s = \frac{2\gamma}{3(1+\gamma)}$$

with the two uses of these savings; private investment, $I(r_t)$ and loans to the public sector $(M_t+B_t)/P_t$.

To determine the dynamics of the model, the real rate of interest is defined by the rational expectations assumption:

$$(59) \quad (1+r_t) = E_t \left[\frac{(1+i_t)P_t}{P_{t+1}} \right].$$

I am going to assume that fiscal policy in this economy is defined by a sequence of values of $M+B$ that are set by the fiscal authority. In line with the monetary policy that has been followed

for much of this century in the US I will assume that the central bank manages the mix of debt and money in order to fix an interest rate target; in other words, the Central Bank pegs the rate of interest.

The Nature of Equilibrium in the Model

I have set up the model in a way that permits the beliefs of agents in the model to affect outcomes in a rational expectations equilibrium. In a standard rational expectations model the beliefs of the agents are pinned down by the economic fundamentals. In this economy there are many possible ways that agents might form their beliefs, each of which is consistent with the rational expectations assumption. These alternative sets of beliefs are reflected in a multitude of Markov equilibria each of which converges to an invariant probability measure.¹⁵

To describe sequences of variables that constitute equilibria in this economy, notice that for any value of $(B+M)/P$, there will be a unique solution to equations (55) and (57) considered as simultaneous equations in Y and r . By replacing the function $f(r)$ into equation (57) and defining the function

$$(60) \quad \psi(r) = sf(r) - I(r),$$

one may write the equilibrium solution for P as a function of r and $(M+B)$:

$$(61) \quad P = \frac{M+B}{\psi(r)}.$$

Now replace this function in the definition of the real rate of interest:

$$(62) \quad (1+r_t) = E_t \left[\frac{(1+i_t)(M_t+B_t)\psi(r_{t+1})}{\psi(r_t)(M_{t+1}+B_{t+1})} \right].$$

¹⁵ The main idea in this paper, that beliefs should be a central determinant of a macroeconomic model, is discussed in depth in my book *The Macroeconomics of Self-Fulfilling Prophecies*, MIT Press, Cambridge MA 1993. For a discussion of the idea of sticky prices in equilibrium see Farmer (1991b), (1992), (1993). For an early exposition of the idea in a simple two period OG model see Farmer and Woodford (1984).

to arrive at a functional equation that must be satisfied by any sequence of probability distributions for r . A rational expectations equilibrium is a sequence of probability distributions for r that satisfies this functional equation. As an example of one such equilibrium, suppose that agents believe that the Wall Street Journal is always correct in its forecasts and let ε_t be a random variable with mean zero that represents forecasts of the Wall Street Journal about the behavior of real interest rates. Then one equilibrium in this economy will be represented by the Markov process in r_t that is generated by the equation:

$$(63) \quad r_t = \psi^{-1} \left(\frac{(1+r_{t-1} + \varepsilon_t)\psi(r_{t-1})(M_t+B_t)}{(1+i_{t-1})(M_{t-1}+B_{t-1})} \right).$$

This process satisfies equation (62) by construction and, as long ε_t has small support, one can show that it defines a Markov process that converges to an invariant distribution. This follows from the fact that the difference equation (63) that comes from setting ε_t to zero is stable. Familiar examples of rational expectations models lead to unstable difference equations that must be solved forwards to find r_t as a function of all future values of B and M . I have been careful to construct an example of a model with an indeterminate steady state. Since inflationary expectations are connected to the real rate of interest by the equation

$$(64) \quad \frac{P_{t+1}}{P_t} = \frac{(1+i_t)}{(1+r_t)},$$

the mechanism described in equation (64) determines one possible method by which agents in the model could forecast future prices. If they use this method, the economy will experience fluctuations in economic activity solely as a result of the beliefs of the investors in the model which are represented by different realizations of the »sunspot» variable, ε_t .

Although there are many possible equilibria, in the absence of a specification of beliefs, this should not disturb us as scientists. It means that one needs to specify a belief function, such as equation (64), in order to close the model. A working hypothesis is that this belief function remains stable through time which is an as-

sumption that should not prove any more disturbing than the heroic assumption that agents have stable preferences.

Beliefs and Business Cycles

Figure 6 depicts the path of a typical recession in this economy that might be induced by an unfavorable prediction of the Wall Street Journal. In particular, it depicts the outcome of a negative draw of ϵ in period 2, followed by a sequence of values of ϵ equal to zero. Figure 6 begins in period 1 at a long run steady state depicted as point A. In period 2 investors expect that the real rate of interest will be higher and this causes them to reduce their demand for labor in the capital goods industries. This change in expectations is represented in figure 6 as a leftward shift of the IS curve generated by an

increase in the contemporaneous price level. Prices movements that are induced by belief shocks would be expected to be *countercyclical* in this model in contrast to the standard Keynesian IS/LM transmission mechanism.

Figure 6 does not work quite like the familiar IS/LM model of textbook Keynesian economics. The level of economic activity is determined at a point in time by the intersection of the IS and AS schedules, for any given value of P and $(M+B)$. The LM curve is horizontal reflecting the assumption that the Central Bank picks the money rate of interest and it is drawn as a dashed line to emphasize that the LM schedule is picking the money rate of interest, not the real rate.

In period 2 of the model the equilibrium is at point B with a higher real rate of interest and a lower level of domestic product. In subsequent periods, prices fall as long as the real rate of interest exceeds the nominal rate at i . These price falls are associated with rightward movements in the IS curve that eventually returns to point A.

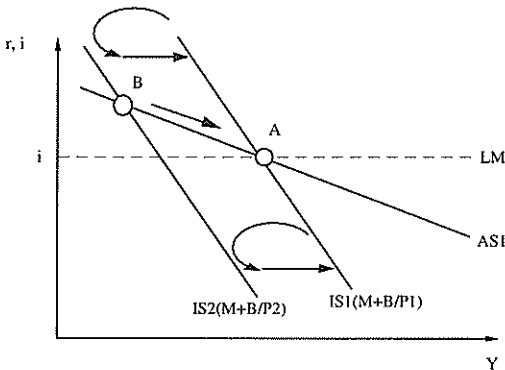


Figure 6. A recession induced by loss of investor Confidence

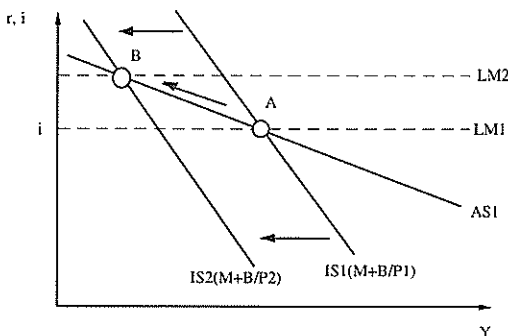


Figure 7. A recession induced by a monetary contraction.

Business Cycles and Nominal Shocks

One of the most difficult features to capture in a macroeconomic model is the response of the economy to a nominal shock. Impulse response functions in US data indicate that an increase in the nominal rate of interest has little or no impact in the period at which it occurs. Over time, output contracts building up to a peak response about nine months later.¹⁶ Figure 7 depicts the effect of a nominal contraction in the model of this paper. In period 2 the LM shifts from LM1 to LM2. If agents use equation (64) to forecast real rates, and therefore inflation, this shift has no effect in the current period. In the subsequent period the increase in nominal rates opens up a gap between real and

¹⁶ There are several excellent sources documenting these patterns in the US data. See for example, Sims (1989), «Models and their Uses», in the American Journal of Agricultural Economics. More recently, Bernanke and Blinder (1992), Gertler and Gilchrist (1991), Christiano and Eichenbaum (1992a), (1992b), King and Watson (1993) have been engaged in systematic attempts to document the short run effects of monetary policy.

nominal rates that induces inflation. In terms of Figure 7, the IS curve shifts to the left and this leftward expansion continues until the economy reaches point B at a new higher real rate. Notice that business cycles that are associated with nominal shocks will generate pro-cyclical prices.

7. Conclusion

For many years, the IS-LM model provided a pivotal role in analyzing aspects of the role of policy in macroeconomics. The apparatus was widely discredited as a consequence of the apparent inability of the model to understand the appearance in the 1970's of the emergence of the simultaneous occurrence of inflation and unemployment, so called *stagflation*. However, the IS-LM apparatus is still widely used by policy makers and journalists. Proponents of the approach claim that some version of IS-LM, supplemented with a «Phillips curve» still does a reasonably good job of tracking data. In this paper I have tried to recover some of the ideas that drive the IS-LM model in the framework of an intertemporal maximizing model. The key is to replace the non-market clearing view of the labor market with an alternative model that rests on the idea of *indeterminacy* in a general equilibrium model. The ultimate success of the approach will rest on more realistic models that can be calibrated to data, and I am currently engaged in a research agenda that does just this. However, in order to make assessments of the effects of policy I still find a version of IS-LM a useful guide. This paper has tried to explain why I take this view and to explain exactly what variant of the model I believe may be useful.

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$$\Pi(P^1, P^2, \dots, P^N, P^K, Y^K) =$$

$$(A2) \quad \max_{K^1, \dots, K^N} \left(P^K Y^K - \sum_{i=1}^N P^i K^i \right)$$

such that $B \left[\sum_{i=1}^N (K^i)^{\epsilon} \right]^{\frac{1}{\epsilon}} \geq Y^K$

Assuming a binding constraint, the first order conditions for this problem are given by:

$$(A3) \quad -P^i + \lambda B \left[\sum_{i=1}^N (K^i)^{\epsilon} \right]^{\frac{1}{\epsilon}-1} (K^i)^{\epsilon-1} = 0$$

where λ is the multiplier associated with the inequality constraint. Rearranging terms gives:

$$(A4) \quad K^i = Y^K \left(\frac{P^i}{\lambda B^{\epsilon}} \right)^{-\frac{1}{\epsilon}}$$

Replacing this expression for the i’th intermediate demand into the production constraint yields the expression:

$$(A5) \quad Y^K = B \left[\sum_{i=1}^N (K^i)^{\epsilon} \right]^{\frac{1}{\epsilon}} = B \left[\sum_{i=1}^N \left(\frac{P^i}{\lambda B^{\epsilon}} \right)^{-\frac{\epsilon}{1-\epsilon}} (Y^K)^{\epsilon} \right]^{\frac{1}{\epsilon}}$$

which yields an expression for λ , the shadow value of increasing output:

$$(A6) \quad \left(\frac{1}{\lambda} \right)^{\frac{1}{1-\epsilon}} = B^{\frac{1}{1-\epsilon}} \left[\sum_{i=1}^N (P^i)^{-\frac{\epsilon}{1-\epsilon}} \right]^{\frac{1}{\epsilon}}$$

Replacing this expression in the i’th demand function leads to the expression:

$$(A7) \quad K^i = Y^K (P^i)^{-\frac{1}{1-\epsilon}} B^{-1} \left[\sum_{i=1}^N (P^i)^{-\frac{\epsilon}{1-\epsilon}} \right]^{\frac{1}{\epsilon}}$$

The profit function is found by replacing (A7) in (A1):

Appendix

This appendix derives the profit functions discussed in the body of the paper. Let Y^K be the production function of the capital goods industry. Then the profit of a final goods producer in this industry is given by:

$$(A1) \quad \Pi = Y^K P^K - \sum_{i=1}^N P^i K^i$$

Define the profit function of an arbitrary firm in this industry as:

$$(A8) \quad \Pi(P^1, P^2, \dots, P^N, P^K, Y^K) = \frac{P^K Y^K}{B} \left(B - \left[\sum_{i=1}^N \left(\frac{P^i}{P^K} \right)^{\frac{-\varepsilon}{1-\varepsilon}} \right]^{\frac{1-\varepsilon}{-\varepsilon}} \right)$$

The profit function in the consumption goods industry is identical to this expression with the exception that $B=1$ and P^K is replaced by P .

For the special case when all intermediate goods are identically priced (A8) reduces to:

$$(A9) \quad \Pi(P^i, P^K, Y^K) = \frac{P^K Y^K}{B} \left(B - N^{\frac{\varepsilon-1}{\varepsilon}} \left(\frac{P^i}{P^K} \right) \right)$$

which is the expression referred to in the body of the paper.

Consider the problem of a typical intermediate goods producer facing the demand curve:

$$(A10) \quad C^i = \lambda C \left(\frac{P}{P^i} \right)^{\frac{1}{1-\varepsilon}}$$

Profits are given by

$$(A11) \quad \Pi^i = P^i C^i - W L^i$$

and the technology is represented by:

$$(A12) \quad C^i = L^i - A.$$

The solution to the problem of maximizing profits, taking (A10) as given is to set price as a fixed markup over marginal cost:

$$(A13) \quad P^i = \frac{W}{\varepsilon}$$

and the profits of the i 'th producer are given by:

$$(A14) \quad \Pi^i = P^i (L^i - A) - W L^i$$

which, when firms use the optimal pricing rule, are equal to:

$$(A15) \quad \Pi^i = \frac{W}{\varepsilon} (L^i - A - \varepsilon L^i).$$