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Nominal price stickiness as a rational expectations equilibrium

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The *equilibrium* approach to price stickiness explains the apparent inflexibility of money prices as one possible equilibrium in a model that contains multiple equilibria. This paper points out that the equilibrium approach is robust to the assumption that agents may write contracts that are contingent upon the realization of the money supply.

1. Introduction

The literature on microfoundations attempts to explain macroeconomic phenomena by aggregating the choices of individual actors. One of the puzzles faced by advocates of this approach is the apparent inflexibility of prices in the face of aggregate disturbances. It seems that supposedly rational agents react to new information by adjusting quantities rather than prices. Most research in this area has attempted to explain the puzzle of 'price stickiness' by directly modelling frictions that prevent prices from adjusting quickly to their frictionless values. For example, the papers by Akerlof and Yellen (1985) and by Mankiw (1985) assume that there is a small cost of adjusting behavior in the face of new information, and the paper by Ball and Romer (1987) combines a friction of this kind with a coordination failure to generate a model with multiple rankable equilibria.[1]

An alternative *equilibrium* view of price stickiness has been put forward by Azariadis and Cooper (1985), Geanokoplos and Polemarchakis (1986), Farmer

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[1]The contract approach that began with papers by Azariadis (1975), Baily (1974), and Gordon (1974) is related to this literature. Cooper's book (1987) is an excellent survey of this field.

and Woodford (1984), and others.[2] According to the equilibrium approach one should model apparent price inflexibility as a rational expectations equilibrium. The idea is to exploit a model in which there are many equilibria each of which is indexed by a belief about what will occur in the future. Each of the possible equilibria is consistent with rational expectations and one of the equilibria is consistent with 'sticky prices'. The implication of this agenda is that 'sticky prices' occur because agents have come to believe that they will occur. Since each of the equilibria in models where beliefs matter has a unique implication for the time series properties of the data the equilibrium agenda is potentially capable of being confronted with formal econometric tests. The goal of this paper is much more modest.

I will address a criticism of the equilibrium approach to price stickiness which was first raised by Barro (1977) in his criticism of Fischer's (1977) work on nominal contracting. Barro argued that a contract in which money prices were set in advance was inconsistent with the existence of a complete set of Arrow–Debreu insurance markets. Most work which takes the equilibrium approach to price stickiness has proceeded by constructing artificial environments in which agents are not allowed to write insurance contracts that are contingent on the realization of monetary agreggates. This work would therefore seem to be subject to the same criticism that Barro levelled at contract theory. But Barro's argument rests on the presumption that the existence of a complete set of insurance opportunities will rule out 'sunspots'. In this paper I will exploit a distinction that was first made by Cass and Shell (1983) in their paper 'Do Sunspots Matter?'. Cass and Shell distinguish between complete markets and complete participation in markets. They show that, if agents are unable to insure against events that occur before they are born, equilibria may exist in which beliefs can independently influence allocations. This reasoning implies that equilibria in which purely nominal events have real consequences may persist in the face of complete insurance opportunities.[3]

2. The model structure

The environment in which I construct my argument is that of a one-good overlapping generations model with pure trade and a single identical agent in

[2] See also Farmer (1991) and Benhabib and Farmer (1991).

[3] Incomplete participation is a convenient way of introducing economies in which sunspots matter but it is not the only way. Cass and Shell (1989) construct an artificial overlapping generations economy in which sunspots matter but all agents are allowed to trade in contingent claims markets that open before they are born. Similarly, the paper by Benhabib and Farmer (1991) contains sunspot equilibria in an economy with a representative agent and complete markets. In both of these examples, however, it is important that there is some aspect of the model that causes the first welfare theorem to fail.

each generation.[4] Agents live for two periods and supply n_t units of labor in youth which are transformed one for one into n_t units of the unique perishable consumption commodity. Agents work when young and consume when old and each agent maximizes the utility function

$$U = U(c_{t+1}, \bar{n} - n_t),$$

where $\bar{n}$ represents the agent's endowment of leisure.

Assumption 1. U is a monotonically increasing, strictly concave, and twice continuously differentiable function of consumption and leisure.

Define $\hat{R}$ to be the 'autarkic interest factor' where

$$\hat{R} \equiv U_2(0, \bar{n}) / U_1(0, \bar{n}).$$

Assumption 2. $0 \leq \hat{R} < 1.$

Assumption 1 contains standard smoothness and differentiability conditions; the strict concavity assumption implies that agents are risk-averse. Assumption 2 implies that there will exist an equilibrium in which money has value. It is possible to construct overlapping generation economies where Assumption 2 is violated but they are not interesting models of *monetary* economies.

Uncertainty is introduced into the model by allowing a single stochastic disturbance to occur in period T of the model. This means that all generations born before period $T-1$ and after period T face no uncertainty. The uncertain event that occurs in period T involves a proportional transfer to the holders of money that takes one of two possible values, refered to as x^{α} and x^{β}. The probabilistic structure of the economy thus involves a single branch at date T.

As with most overlapping generations models I assume that the initial stock of money is owned by a generation, born in period 1, that faces a trivial decision problem. This generation, referred to as generation 0, spends its allocation of fiat money on commodities in period 1. All other generations born before date $T-1$ solve the problem

$$\max U(c_{t+1}, \bar{n} - n_t), \tag{1}$$

[4]The work of Kehoe and Levine (1985) demonstrates that indeterminate equilibria arise much more generally and Woodford (1986) has shown that sunspot equilibria can always be constructed in economies with indeterminate steady states.

such that

$$n_t p_t \geq M_t^{\mathrm{D}}, \tag{2}$$

$$c_{t+1} p_{t+1} \leq M_t^{\mathrm{D}}, \tag{3}$$

where M_t^{D} is the nominal demand for money between periods t and $t+1$ and p_t is the money price of commodities. Putting these two constraints together generates the single lifecycle constraint

$$n_t \geq c_{t+1}(p_{t+1}/p_t), \tag{4}$$

and the solution of the problem, under Assumptions 1 and 2, generates a continuous differentiable demand function for real balances, m_t, of the form

$$m_t = m(p_t/p_{t+1}), \qquad 1 < t < T-1, \tag{5}$$

where

$$\frac{\partial m}{\partial x}(x)\bigg|_{x=\hat{R}} > 0. \tag{6}$$

Agents born after date T face an identical problem to the one described above. But in a competitive equilibrium the branch at date T could generate one of two possible price sequences. The demand function for real balances of agents born after date T must therefore be indexed by the event $\{\alpha, \beta\}$:

$$m_t^i = m(p_t^i/p_{t+1}^i), \qquad i = \alpha, \beta, \quad t \geq T+1. \tag{7}$$

Eqs. (5) and (7) describe the demand for money in periods $1, \ldots, T-2$, before the realization of the monetary shock, and in periods $T+1, \ldots, \infty$, after its realization. In the following section I will describe how the money supply process works and in section 4 I explain how the demand for money is determined in periods $T-1$ and T. These intermediate periods are a little more complicated than the rest of the model since there is an insurance market operating that permits generations $T-1$ and T to write contracts contingent on the realization of a nominal disturbance. It is the operation of this insurance market that is the central focus of the paper.

3. Government behavior

To close the model it is assumed that the government prints enough money in each period to finance the real purchases g. I use the symbols M to refer to the nominal supply of money and M^{D} to refer to nominal demand. In all periods other than period T the generation of seignorage revenue is the only source of money creation. In period T there is an additional source of money creation (or destruction) that arises from a random proportional change to the existing stock of money. This stochastic transfer (or tax) amounts to a change in the unit of account.

These assumptions imply that the money supply M_t in each period before date T is endogenously determined by

$$M_t = M_{t-1} + gp_t \quad \text{if} \quad t < T. \tag{8}$$

In periods after date T, the money supply and the price level must be indexed by the event $\{\alpha, \beta\}$:

$$M_t^i = M_{t-1}^i + gp_t^i, \qquad t > T, \quad i = \alpha, \beta. \tag{9}$$

At the date at which uncertainty is resolved the money supply rule takes the form

$$M_T^i = x^i M_{T-1}^i + gp_T^i, \qquad i = \alpha, \beta. \tag{10}$$

I will refer to the term x^α as a tax and to x^β as a transfer and I assume specifically that

$$0 < x^\alpha < 1 < x^\beta, \tag{11}$$

$$\pi^\alpha x^\alpha + \pi^\beta x^\beta = 1, \tag{12}$$

$$\pi^\alpha + \pi^\beta = 1. \tag{13}$$

The purpose for introducing positive government spending is most easily understood by describing the equilibria of a related model in which

$$x^\alpha = x^\beta = 1.$$

This is the special case in which there is no uncertainty. In perfect foresight an equilibrium[5] is a sequence of inflation factors that satisfies the difference equation

$$m(p_t/p_{t+1}) = (p_{t-1}/p_t)m(p_{t-1}/p_t) + g \equiv f_g(p_{t-1}/p_t), \tag{14}$$

[5]This usage of the term conforms to general equilibrium theory in which an 'equilibrium' is a sequence of prices and a sequence of allocations that are feasible and jointly consistent with the assumptions of rationality and market clearing. In dynamic general equilibrium theory an equilibrium may be a sequence that is nonconstant through time.

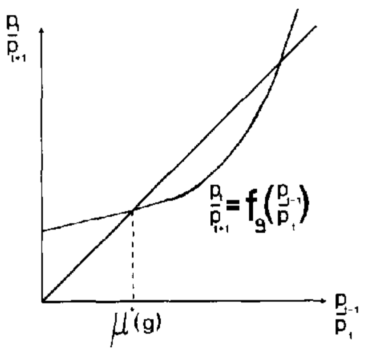

Fig. 1

together with the initial condition

$$m(p_1/p_2) = M_1/p_1 + g. \tag{15}$$

When government purchases equal zero the perfect foresight model possesses a unique *steady state* equilibrium in which fiat money has positive value. It also contains a steady state equilibrium in which fiat money has no value and there is no intergenerational trade. When government spending is positive the set of steady states is shifted. If one indexes a family of models by the level of government purchases, each member of the family contains two stationary equilibria with positive valued fiat money. These two steady state equilibria are depicted in fig. 1 as the intersections of the map f_g with the 45 degree line. One of the monetary steady state equilibria, labelled as $\mu^*(g)$, is 'indeterminate', that is, there exists a continum of equilibria each of which converges to that steady state. In the stochastic model, in which the transfer at date T is random, the existence of indeterminate equilibria in the perfect foresight economy is important to the method that is used to construct 'sticky price' equilibria.[6]

In nonstochastic overlapping generations models an equilibrium is characterized by a sequence of inflation factors that satisfies the difference equation (14). In the economy with a single random event at date T the description of equilibrium is more complicated. For dates up to and including $T-2$

[6]The particular method by which I arrive at indeterminacy is not, however, critical. Other examples of models that generate indeterminate steady states are provided by the work on search externalities [Howitt and McAfee (1988a)], dynamic examples of strategic complementarities [Howitt and McAfee (1988b)], or exchange externalities [Benhabib and Farmer (1991)]. Any one of these models would provide a suitable framework to make my point.

equilibrium is completely characterized by equality of the demand and supply of money. This is captured by eq. (14). The same is true for periods $T+2$ to infinity although one must index the money market equation by the event $\{\alpha, \beta\}$. Equilibrium for dates $T+2, \ldots, \infty$ is characterized by a pair of sequences that satisfy

$$m\left(p_t^i / p_{t+1}^i\right) = f_g\left(p_{t-1}^i / p_t^i\right), \qquad i = \alpha, \beta, \quad t > T. \tag{16}$$

At dates $T-1$ and T things are a little more complicated since one must take account of the securities markets that allow the agents to purchase insurance against the occurrence of the nominal disturbance. The following section describes the behavior of individuals in periods $T-1$ and T and it derives a set of demand functions that characterizes this behavior.

4. The insurance market

The additional source of money creation in period T complicates the decisions that must be taken by agents born in periods $T-1$ and T. I will assume that agents in both of these generations maximize expected utility and that they are able to participate in an insurance market that opens prior to the realization of the random monetary disturbance. This device amounts to the assumption that uncertainty is not resolved until *after* the birth of generation T and it allows me to describe the contracts that would be written by two sides of an insurance agreement in general equilibrium.

This section is notation-intensive but the results are simple to describe. Generations born before and after the resolution of uncertainty face decision problems that may be characterized by a single demand function. But the behavior of generation $T-1$ is characterized by *two* independent functions since it may pick different values for consumption in different states of nature. Generation T has a still more complicated decision that is characterized by *three* independent demand functions since both labor supply and consumption may be different in different states. The purpose of the following analysis is to develop this argument more formally.

Generation $T-1$ faces the problem

$$\max \mathrm{E}_{i=\alpha,\beta} U\left(c_T^i, \bar{n} - n_{T-1}\right), \tag{17}$$

such that

$$n_{T-1} p_{T-1} \geq M_{T-1}^{\mathrm{D}}, \tag{18}$$

$$q^\alpha b^{\alpha T-1} + q^\beta b^{\beta T-1} + M^{*T-1} \leq M_{T-1}^{\mathrm{D}}, \tag{19}$$

$$p_T^i c_T^i \leq b^{iT-1} + M^{*T-1} x^i, \qquad i = \alpha, \beta. \tag{20}$$

The superscripts T and $T-1$ are used to index generations and the subscripts $T-1$, T, and $T+1$ are used to index trading periods. Generation $T-1$ carries the nominal money stock M^{D}_{T-1} into period T, and it trades in securities with generation T. The terms $b^{\alpha T-1}$ and $b^{\beta T-1}$ represent the net demands for state-dependent securities and M^{*T-1} is the demand for money at the close of the securities market. The generation $T-1$ agent will, therefore, be holding the portfolio $(b^{\alpha T-1}, b^{\beta T-1}, M^{*T-1})$ at the time that the state is realized.

An α (β) security is a promise to pay one unit of money if and only if state α (β) occurs. It trades at price q^{α} (q^{β}) in the securities market. The constraints (19) and (20) represent trading opportunities before and after the realization of the monetary disturbance, and they are meant to capture the real world institution of an insurance contract that may be written contingent upon the realization of a monetary aggregate. Since the existence of these insurance opportunities completes the market structure, it is possible to describe the agent's lifetime trading opportunities by a single budget constraint which is found by substituting the inequalities (18) and (20) into (19):

$$q^{\alpha}p^{\alpha}_{T}c^{\alpha}_{T} + q^{\beta}p^{\beta}_{T}c^{\beta}_{T} + M^{*T-1}\left(1 - q^{\alpha}x^{\alpha} - q^{\beta}x^{\beta}\right) - n_{T-1}p_{T-1} \leq 0. \tag{21}$$

Maximization of expected utility subject to this constraint leads to a pair of independent demand functions that are described below:

$$n_{T-1} = m_{T-1}(P_{T-1}), \tag{22}$$

$$c^{\alpha}_{T} = f(P_{T-1}), \tag{23}$$

where the relative price vector P_{T-1} is defined by the identity

$$P_{T-1} \equiv \left\{ \frac{p_{T-1}}{p^{\alpha}_{T}q^{\alpha}}, \frac{p_{T-1}}{p^{\beta}_{T}q^{\beta}} \right\}. \tag{24}$$

The functions $m_{T-1}(\cdot)$ and $f(\cdot)$ are continuous differentiable maps from R^2_+ into R_+. The existence of two Arrow securities *and* money implies a redundancy in the market structure. It is clear from the budget constraint (21) that the following equality must hold if an equilibrium is to be characterized by the absence of arbitrage:

$$q^{\alpha}x^{\alpha}+q^{\beta}x^{\beta}=1. \tag{25}$$

If this relationship did not hold, an individual trader could make unbounded riskless profit by buying one security and selling a linear combination of the other two.

In addition to the two demand functions (22) and (23) and the no-arbitrage condition (25), the inequalities (18)–(20) must each hold with equality. One may use this information to find expressions for c_t^{β}, m_T-1, and for the purchasing power in states α and β in terms of the two functions $m_{T-1}(\cdot)$ and $f(\cdot)$.

A similar analysis may be conducted for generation T. The only difference here is that this generation may insure over labor supply in states α and β. Generation T agents maximize the expected value of a function $U(c^i_{T+1}, \bar{n}-n^i_T)$ subject to the period-by-period constraints:

$$q^{\alpha}b^{\alpha T}+q^{\beta}b^{\beta T}+M^{*T}\leq 0, \tag{26}$$

$$M^i_T\leq n^i_T p^i_T+b^{iT}+x^i M^{*T}, \qquad i=\alpha,\beta, \tag{27}$$

$$c^i_{T+1}p^i_{T+1}\leq M^{Di}_T, \qquad i=\alpha,\beta, \tag{28}$$

which may be combined to generate the life-cycle constraint:

$$\begin{aligned} &p^{\alpha}_{T+1}q^{\alpha}c^{\alpha}_{T+1}+p^{\beta}_{T+1}q^{\beta}c^{\beta}_{T+1}-p^{\alpha}_T q^{\alpha}n^{\alpha}_T-p^{\beta}_T q^{\beta}n^{\beta}_T \\ &+M^{*T}(1-q^{\alpha}x^{\alpha}-q^{\beta}x^{\beta})\leq 0. \end{aligned} \tag{29}$$

Maximization of expected utility subject to (26)–(28) generates three independent continuous differentiable demand/supply functions $m^{\alpha}_T(\cdot)$, $m^{\beta}_T(\cdot)$, and $h(\cdot)$, each of which maps from R^3_+ into R. These functions represent the demand for money between period T and period $T+1$ in states α and β and the supply of labor in period T state α:

$$m^{\alpha}_T=m^{\alpha}_T(P_T), \tag{30}$$

$$m^{\beta}_T=m^{\beta}_T(P_T), \tag{31}$$

$$n^{\alpha}_T=h(P_T), \tag{32}$$

where the relative price vector P_T is defined by

$$P_T \equiv \left\{ \frac{p_T^\alpha}{p_{T+1}^\alpha}, \frac{p_T^\beta q^\beta}{p_{T+1}^\alpha q^\alpha}, \frac{p_{T+1}^\beta q^\beta}{p_{T+1}^\alpha q^\alpha} \right\}. \tag{33}$$

Optimality implies that the inequality (29) and the period budget constraints (26)–(28) must each hold with equality. Using this information one may derive expressions for c_{T+1}^α, c_{T-1}^β, n_T^β, and for net demands for purchasing power in states α and β in terms of the functions $m_T^\alpha(\cdot)$, $m_T^\beta(\cdot)$, and $h(\cdot)$.[7]

5. Equilibrium

The following definition develops a formal statement of the characteristics of an equilibrium. The idea is to find a sequence of prices that characterizes equilibrium before the resolution of uncertainty and two sequences that characterize equilibrium after its resolution. Periods $T-1$ and T are characterized by commodity prices and securities prices that connect up the equilibrium price sequences before and after the monetary shock.

Definition 1. A *Price System* consists of:

1. A sequence of nonnegative real numbers $\{p_t\}_{t=0}^{T-1}$ that represent commodity prices in terms of money for periods before the realization of the money shock.
2. A pair of sequences of nonnegative real numbers $\{p_t^i\}_{t=T}^{\infty}$, $i=\{\alpha,\beta\}$, that represent commodity prices in terms of money, for period T and beyond, contingent on the occurrence of the monetary shock.
3. A pair of nonnegative real numbers $\{q^\alpha, q^\beta\}$ that represent security prices in the period T security market.

[7] I have chosen to define the functions $m_T^\alpha(\cdot)$ and $m_T^\beta(\cdot)$ as primitives, where

$$c_{T+1}^\alpha = m_T^\alpha(P_T)(p_T^\alpha/p_{T-1}^\alpha),$$

since it is convenient to define equilibrium in terms of the asset market clearing equations. It might help to clarify the analysis if one thinks of the transformed problem, maximizing expected utility subject to single constraint (29), which is closer to standard atemporal consumer theory. Under this representation it is clear that a solution defines three independent demand functions, two of which may be chosen to be consumption demands in states α and β. Call these functions $c^\alpha(P_T)$ and $c^\beta(P_T)$. One then defines the functions $m_T^\alpha(\cdot)$ and $m_T^\beta(\cdot)$ using the period constraints (28^α) and (28^β).

Definition 2. An *Equilibrium Price System* is a price system that satisfies the following equations:

1. $m(p_1/p_2) = M_1/p_1 + g$,
2. $m(p_t/p_{t+1}) = (p_{t-1}/p_t)m(p_{t-1}/p_t) + g,\ t = 1,\dots,T-2$,
3. $m(p_t^i/p_{t-1}^i) = (p_{t-1}^i/p_t^i)m(p_{t-1}^i/p_t^i) + g,\ i = \alpha,\beta,\ t = T+2,\dots,$
4. $m(p_{T+1}^i/p_{T+2}^i) = (p_T^i/p_{T+1}^i)m^i(P_T) + g,\ i = \alpha,\beta$,
5. $m_{T-1}(P_{T-1}) = (p_{T-2}/p_{T-1})m(p_{T-2}/p_{T-1}) + g$,
6. $m_T^i(P_T) = (p_{T-1}/p_T^i)x^i m_{T-1}(P_{T-1}) + g,\ i = \alpha,\beta$,
7. $x^\alpha q^\alpha + x^\beta q^\beta = 1$,
8. $f(P_{T-1}) + g = h(P_T)$,

where

$$P_{T-1} \equiv \left\{\frac{p_{T-1}}{p_T^\alpha q^\alpha}, \frac{p_{T-1}}{p_T^\beta q^\beta}\right\}, \qquad P_T \equiv \left\{\frac{p_T^\alpha}{p_{T+1}^\alpha}, \frac{p_T^\beta q^\beta}{p_{T+1}^\alpha q^\alpha}, \frac{p_{T+1}^\beta q^\beta}{p_{T+1}^\alpha q^\alpha}\right\}.$$

Definition 3. A *Predetermined Equilibrium price system* is an equilibrium price system with the property that

$$p_T^\alpha = p_T^\beta.$$

Eqs. (1)–(3) require that demand equals supply of money in periods 1 to $T-2$ and periods $T+2,\dots$ in states α and β. Goods market clearing for these periods is implied by Walras Law. In periods $T-1$, T, and $T+1$ one has seven independent market clearing equations to determine the eight prices, p_T^α, p_T^β, p_{T+1}^α, p_{T+1}^β, p_{T+2}^α, p_{T+2}^β, q^α, and q^β. These equations are represented by (4)–(8). Eqs. (6^α) and (6^β) represent equality of demand and supply of money for period T in each state. This is not equivalent to goods market clearing in period T because there is an additional insurance opportunity, represented in eq. (8), that allows the supply of commodities to be different from the demand for real balances. Since seven equations are insufficient to determine eight unknowns, market clearing and rationality are insufficient to determine equilibrium. One solution to the equations occurs if one imposes the predetermined price solution given in Definition 3.

6. Properties of equilibria

It is well known that perfect foresight equilibria in the overlapping generations model may be indeterminate but, if there is uncertainty, the degree of indeterminacy is increased. Every time a random event occurs it is as if one were free to pick a new initial condition *even if agents are allowed to insure against the occurrence of the event*. The following theorem formalizes this intuition and uses it to prove the existence of a predetermined price equilibrium.[8]

Theorem 1. Assume Assumptions 1 and 2. There exists an open neighborhood of unity $N(1)$ and an $\varepsilon > 0$ such that, if $g \in G \equiv \{g | 0 \leq g \leq \varepsilon\}$ and if $x_i \in N(1)$, then there exists a predetermined equilibrium price system.

The details of the proof are in the appendix. One requires that g be small and that the monetary disturbance be small in order to ensure existence. If g becomes too large, then the family of maps,

$$p_t/p_{t+1} = F_g(p_{t-1}/p_t),$$

that describes equilibrium price sequences, may not have a fixed point. Similarly one requires that the monetary shock is small since in a predetermined price equilibrium the effect of a nominal disturbance is to shift the period T value of the argument of the map F_g. If this value is shifted too far from a fixed point, then one cannot use local arguments to establish that the sequence $\{p_t/p_{t+1}\}_{t=T}^{\infty}$ converges back to a stationary state in which money has value.

It follows from the method of proof that one can construct multiple predetermined price equilibria by picking different initial conditions in the neighborhood of the steady state $\mu^*(g)$. That is, every one of the nonstationary monetary equilibria is associated with a predetermined price equilibrium when one explicitly introduces uncertainty. The existence of multiplicity is intimately connected to the indeterminacy of the stationary state since it is this indeterminacy that allows one to construct a backward solution [in the sense of Blanchard (1979)] to the difference equation that describes rational expectations equilibria. Without indeterminacy such a solution may still exist, but one could not guaranty that the equilibria would converge to a stationary state in which money has positive value. The more familiar approach in representative agent models is to construct a forward solution that may be pinned down by a transversality condition. No such condition exists in overlapping generations economies and it is this freedom in picking boundary conditions that permits the existence of large numbers of equilibria.

The existence of predetermined price equilibria would not be particularly interesting if it were the case that quantity allocations were completely insulated from a nominal disturbance. It is therefore worthwhile establishing

[8]The proofs of Theorems 1 and 2 are related to the arguments in a paper by Peck (1988).

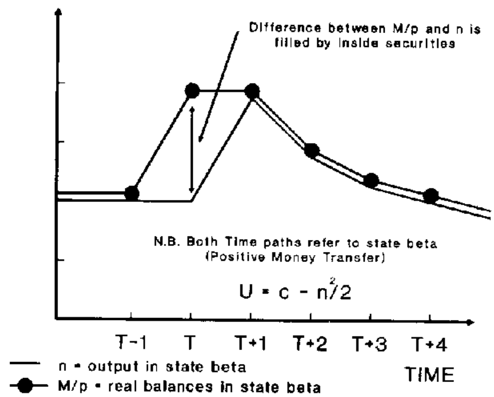

Fig. 2

some of the properties of such an equilibrium. The first case I consider is instructive since it represents a situation in which the existence of insurance markets offers a degree of insulation to quantities in the period in which the nominal disturbance occurs.

Theorem 2. Assume Assumptions 1 and 2. If $U(c, \bar{n}-n)$ is separable, i.e., $U_{12}=0$, then a predetermined equilibrium price system and the associated equilibrium allocation have the following properties:

$$p_T^\alpha = p_T^\beta, \quad c_T^\alpha = c_T^\beta, \quad n_T^\alpha = n_T^\beta, \tag{34}$$

$$q^\alpha / q^\beta = \pi^\alpha / \pi^\beta, \tag{35}$$

$$p_t^\alpha \neq p_t^\beta, \quad c_t^\alpha \neq c_t^\beta, \quad n_t^\alpha \neq n_t^\beta, \qquad t \geq T+1. \tag{36}$$

Separability implies that the real effect of a nominal shock is delayed until period $T+1$. This delay occurs because if generation T has separable preferences, then its demand functions depend only on the prices p_{T+1}^α and p_{T+1}^β to the extent that these prices affect income. Since agents face a single budget constraint, the income effect is the same in both states. The only agents who are directly affected by price variations across states in period T are generations $T-1$ and T both of whom are able to insure against such fluctuations. Thus with separability there is a sense in which participation in insurance markets is complete.

A separable example is depicted in figs. 2 and 3 which display the time series properties of output, inflation, real money balances, and money growth in state β for the utility function $U = c - n^2/2$. In this example a monetary

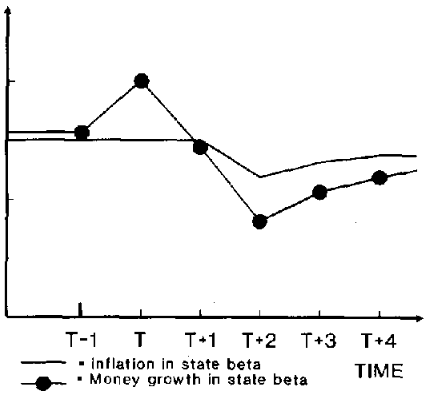

Fig. 3

disturbance has no immediate effect on output since the positive shock to 'outside' money in period T is completely neutralized by issuing 'inside' securities. Output responds in period $T+1$ when new agents, who have not been given the opportunity to trade in securities, enter the model. In period $T+1$ the rate of money growth contracts endogeneously since the base for the inflation tax has risen. As a result of this endogenous reduction in the rate of money growth, a transitory payment of interest on money is associated with a reduction in future rates of inflation.[9]

The insulation of period T quantities in the presence of insurance does not carry over to more general preferences since the actions of agents in periods $T+1$ and later may influence allocations in period T through cross-price effects in the demand functions of generation T. The properties of predetermined price equilibria, when preferences are not separable, are summarized in Theorem 3.

Theorem 3. Assume Assumptions 1 and 2. If $U_{12} \neq 0$, then a predetermined equilibrium price system and the associated equilibrium allocation have the following properties:

$$p_T^{\alpha} = p_T^{\beta}, \quad c_T^{\alpha} \neq c_T^{\beta}, \quad n_T^{\alpha} \neq n_T^{\beta}, \tag{37}$$

$$q^{\alpha}/q^{\beta} \neq \pi^{\alpha}/\pi^{\beta}, \tag{38}$$

$$p_t^{\alpha} \neq p_t^{\beta}, \quad c_t^{\alpha} \neq c_t^{\beta}, \quad n_t^{\alpha} \neq n_t^{\beta}, \qquad t \geq T+1. \tag{39}$$

[9]This example violates the strict concavity Assumption 1. The result of this departure from the assumptions of Theorem 2 is that the price level in period $T+1$ is the same in both states in contradiction to (36).

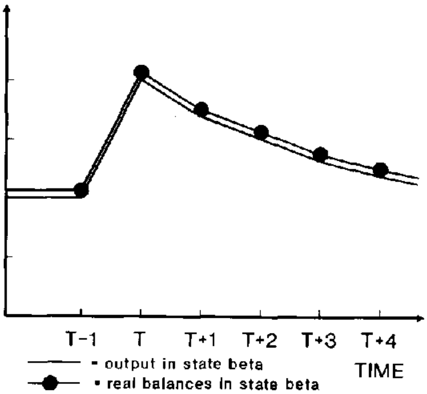

Fig. 4

Figs. 4 and 5 depict a nonseparable example which is constructed by giving generation T the preferences $U = U(c - \delta n)$, where δ is a constant. All other generations have the preferences $U = c - n^2/2$. By giving different generations different preferences one is able to simplify the description of equilibrium considerably. One may exploit the linearity across states, exhibited by the utility function of generation $T-1$, to fix the securities prices and one may simultaneously use the linearity across periods, exhibited by the utility function of generation T, to fix the period $T+1$ inflation factor. The key feature of this example is the contemporaneous response of output to the nominal monetary disturbance, which occurs in spite of a complete set of insurance markets.[10]

The structure of the model that I have investigated is a long way removed from anything that one might hope to test empirically. What then is one to make of these rather abstract results? The main point that I want to impress upon the reader is that nominal monetary disturbances may have real effects in environments in which there are no 'artificial' barriers to prevent agents

[10]This example also violates strict concavity. One implication of this violation of the conditions of the theorem is that, for this example, one may show that $p^{\alpha}_{T+1} = p^{\beta}_{T+1}$ in contradiction to (39). This follows from the linearity of utility of generation T between periods which implies that $\delta = p_T/p^{\alpha}_{T+1} = p_T/p^{\beta}_{T+1}$. For the purpose of constructing fig. 5 I have set $\delta = \mu^*(g)$, and I have depicted the values of inflation and money growth in state β. The contemporaneous response of output depicted in fig. 4 may be inferred from the budget constraint (28) which holds with equality and the government budget constraint, $M^i_T = x^i M^i_{T-1} + g p^i_T$. Together these constraints may be written in the form

$$c^i_{T+1} = \left(M^i_T/p^i_{T+1}\right) + g\left(p^i_T/p^i_{T+1}\right) = \delta\left(M^i_T/p_T + g\right),$$

which implies that in equilibrium $c^{\beta}_{T+1} \geq c^{\alpha}_{T+1}$. Finally, it follows from the insurance condition,

$$U'\left(c^{\alpha}_{T+1} - \delta n^{\alpha}_T\right) = U'\left(c^{\beta}_{T+1} - \delta n^{\beta}_T\right),$$

that output must be higher in period T in the high money state (state β).

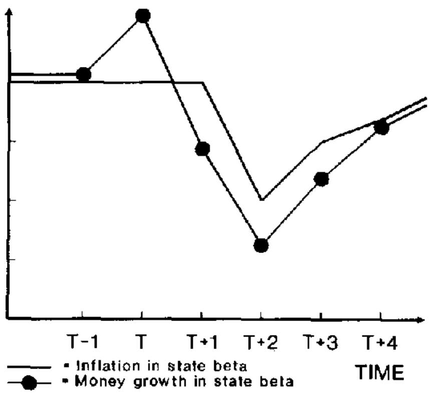

Fig. 5

from writing contingent contracts. In choosing an environment in which to make this point, I have been guided by the desire to make the mechanism, by which nominal effects are transmitted, as clear as possible.

The important feature that differentiates the model from other general equilibrium models that may be more familiar is the existence of an indeterminate steady state. My own current research is directed at investigating the properties of a range of models in this class. I am particularly interested in models that *can* be tested empirically and the direction that I am taking is towards representative agent economies in which there are important externalities. All of the models in this class contain equilibria, with complete insurance, in which purely nominal disturbances can cause contemporaneous quantity responses. The mechanism by which these effects are transmitted is essentially the same mechanism that I have described in this paper.

To sum up, even if agents can write contracts that allow them to insure against purely monetary shocks, these shocks may have real effects. The examples that I have presented show that these effects may involve an increase in employment and an increase in output in response to a proportional increase in the money supply.

7. Conclusion

It is perhaps worth closing with a few comments about the limitations of the achievements of this paper together with some puzzles that are very much still open questions. One response to the idea that I have presented might be to observe that the results of the model rest on an assumption that the economy is on the wrong side of the inflationary Laffer curve. Since we know that many 'strange' things happen in models of this class perhaps the results should be dismissed as 'unreasonable'. However, the essential feature that drives the main result of the paper is that there should exist an indeterminate steady state. This feature can exist in many models. In the overlapping generations model without government spending there will be an indeterminate steady state if there are strong income effects; this is the model exploited by Grandmont (1985) in his work on endogenous competitive business cycles. In the model of endogenous growth studied by Baxter and King (1990) an indeterminate steady state arises if production externalities are large enough, and in monetary economies in which money enters the utility function or the production function it has long been known[11] that there may be multiple convergent perfect foresight paths leading to a monetary steady state.

I *do not* want to claim that the example presented in this paper is a 'good' model of the business cycle since it has a number of undesirable features. For example, the equilibrium that I focus on makes predictions about the co-movements of inflation and government spending that are counter-factual.[12] But I *do* want to claim that we should search for models that retain the property that beliefs can influence allocations. A feature that will be shared by all such models is the ability to explain why rational agents might write contracts that permit nominal fluctuations to influence real events.

As a final comment I want to stress that sunspot equilibria are not Pareto optimal in the usual sense.[13] This does not imply, however, that one can draw strong conclusions about the possibility of Pareto improving policies. In some models in which there exist 'sticky price' equilibria it may be possible to Pareto rank alternative policy regimes. Some regimes may be characterized by the absence of 'sunspot equilibria', others may not. But before one can draw inferences about questions of this nature it will first be necessary to identify a model that *is* capable of being taken seriously as an explanation of the business cycle. My purpose in this paper has been to argue that it may be useful to conduct one's search for a model within the class of equilibrium models in which 'sunspots matter'.

[11]The work of Brock (1975) explores this idea. Most models of money permit indeterminate steady states for some parameter values but it is often claimed that such equilibria can be ruled out for 'reasonably calibrated' economies. In Benhabib and Farmer (1991) we show that, with externalities in exchange, a representative agent economy that is calibrated to fit first moments of U.S. data will generate an equilibrium with predetermined prices that matches many of the stylized facts of U.S. time series.

[12]When government spending rises, inflation is predicted to fall – this is one implication of being on the wrong side of the inflationary Laffer curve.

[13]Although one can redefine Pareto optimality in a way that restores the implication that all equilibria are Pareto optimal. Cass and Shell (1983) call the relevant concept 'dynamic Pareto optimality'.

Appendix

Proof of Theorem 1

The proof is by construction. The first step is to establish that for small positive values of government spending there exists a function $\mu^*(g)$ that is a stationary equilibrium interest factor and which coincides with the autarkic interest factor $\bar{R}$ when $g = 0$. Let $\mu^*(g)$ be the function mapping the set $G = \{g \mid 0 \leq g \leq \varepsilon\}$ into the interval $(0,1)$ that is defined by the conditions

$$m[\mu^*(g)] = \mu^*(g)m[\mu^*(g)] + g, \tag{40}$$

$$\mu^*(0) \equiv \hat{R}. \tag{41}$$

Let $F_g(x)$: $R_+ \mapsto R_+$ be the map

$$x \mapsto xm[x] + g. \tag{42}$$

The following properties of the function m and of the map F_g follow from the implicit function theorem:

$$m[\mu^*(g)] \geq 0, \tag{43}$$

$$0 \leq \left.\frac{\partial F_g(x)}{\partial x}\right|_{x=\mu^*(g)} < 1. \tag{44}$$

These properties imply that one can find an open neighborhood U of $\mu^*(g)$, a positive initial price ratio p_1/p_2, and a positive initial price p_1, such that sequences $\{p_t\}_{t=1}^{\infty}$ generated by iterating the equation $p_t/p_{t+1} = F_g(p_{t-1}/p_t)$ for initial condition $p_1/p_2 \in U$ converge to $\mu^*(g)$. Pick p_1/p_2 in U and generate $\{p_t\}_{t=1}^{T-1}$ by iterating the map F_g. From the local properties of F_g it follows that $p_{T-2}/p_{T-1} \in U$. Now define the vector

$$z \equiv \left\{\frac{p_{T-1}}{p_T}, q^\alpha, q^\beta, \frac{p_T}{p^\alpha_{T+1}}, \frac{p^\beta_{T+1}}{p^\alpha_{T+1}}\right\}, \tag{45}$$

and let $G_g(p_{T-2}/p_{T-1}, x^\alpha, x^\beta)$ be the implicit function $R^3_+ \mapsto R^5_+$ defined by eqs. (5), (6^α), (6^β), (7), and (8). Totally differentiating (5)–(8) establishes that the Jacobian of this transformation is nonsingular. When $x^\alpha = x^\beta = 1$, the map G_g is given by

$$(z,1,1) \mapsto \left(F_g(z), \pi^\alpha, \pi^\beta, F^2_g(z), 1\right),$$

where F_g^2 is the composition of F_g with itself. Since F_g maps U into itself and since F_g is continuous in (x^α, x^β), it follows that for $x^\alpha, x^\beta \in N(1)$, p_T/p^α_{T+1} and p_T/p^β_{T+1} are in U. To complete the proof observe that eqs. (3) and (4) define a pair of sequences $\{p^\alpha_t/p^\alpha_{t+1}, p^\beta_t/p^\beta_{t+1}\}^\infty_{t=T+1}$ each of which converges to $\mu^*(g)$. □

Proof of Theorem 2

Part 1

Let z represent the multiplier associated with the constraint (21) when generation $T-1$ solves the problem $\max \mathrm{E}U(c^i_T, \bar{n} - n_{T-1})$ subject to (21). Similarly let y represent the value of the multiplier associated with (29) when generation T solves $\max \mathrm{E}U(c^i_{T+1}, \bar{n} - n^i_T)$. It follows from the first-order conditions for maximum together with the separability assumption that

$$\frac{U_1(c^i_T)}{U_2(\bar{n} - n^i_T)} = \frac{z}{y}, \qquad i = \alpha, \beta. \tag{46}$$

But from goods market clearing in period T, $c^i_T + g = n^i_T$. The first part of Theorem 2 [eq. (34)] then follows from the strict concavity of U. □

Part 2

The second part of the theorem [eq. (35)] follows from part 1 since the first-order conditions also imply that

$$\frac{\pi^\alpha U_2(\bar{n} - n^\alpha_T)}{\pi^\beta U_2(\bar{n} - n^\beta_T)} = \frac{\pi^\alpha}{\pi^\beta} = \frac{p^\alpha_T q^\alpha}{p^\beta_T q^\beta} = \frac{q^\alpha}{q^\beta}. \tag{47}$$

The first equality is an implication of part 1 of the proof and the second equality follows from the definition of a predetermined price equilibrium. □

Part 3

To prove part 3 notice that

$$p^i_{T+1} c^i_{T+1} = M^{\mathrm{D}i}_T = x^i M_{T-1} + g p^i_T, \qquad i = \alpha, \beta. \tag{48}$$

The first equality is implied by (28) which holds with equality at an optimum. The second equality is implied by the government budget constraint since all money is held by generation T. Eqs. (48) imply that $p^\alpha_{T+1} c^\alpha_{T+1} \neq p^\beta_{T+1} c^\beta_{T+1}$.

Suppose that $p^{\alpha}_{T+1} = p^{\beta}_{T+1}$. Then from the first-order condition to the problem of generation T,

$$\pi^i U_1\left(c^i_{T+1}\right) = y p^i_{T+1} q^i, \qquad i = \alpha, \beta. \tag{49}$$

But then (47) together with (49) imply $p^{\alpha}_{T+1} c^{\alpha}_{T+1} = p^{\beta}_{T+1} c^{\beta}_{T+1}$, which is a contradiction, hence $p^{\alpha}_{T+1} \neq p^{\beta}_{T+1}$ and from (49) $c^{\alpha}_{T+1} \neq c^{\beta}_{T+1}$. □

Proof of Theorem 3

Part 1

To establish (37) suppose the contrary, i.e., $n^{\alpha}_T = n^{\beta}_T$ and $c^{\alpha}_T = c^{\beta}_T$. The first-order conditions for generations $T-1$ and T imply that

$$\frac{U_2\left(c^{\alpha}_{T+1}, \bar{n} - n^{\alpha}_T\right)}{U_1\left(c^{\alpha}_T, \bar{n} - n_{T-1}\right)} = \frac{U_2\left(c^{\beta}_{T+1}, \bar{n} - n^{\beta}_T\right)}{U_1\left(c^{\beta}_T, \bar{n} - n_{T-1}\right)}. \tag{50}$$

Market clearing implies that $n^i_T = c^i_T + g$. Strict concavity of U then implies $c^{\alpha}_{T+1} = c^{\beta}_{T+1}$. But since the first-order conditions also imply

$$\frac{p^{\alpha}_{T+1}}{p^{\alpha}_T} = \frac{U_1\left(c^{\alpha}_{T+1}, \bar{n} - n^{\alpha}_T\right)}{U_2\left(c^{\alpha}_{T+1}, \bar{n} - n^{\alpha}_T\right)}, \tag{51}$$

$$\frac{p^{\beta}_{T+1}}{p^{\beta}_T} = \frac{U_1\left(c^{\beta}_{T+1}, \bar{n} - n^{\beta}_T\right)}{U_2\left(c^{\beta}_{T+1}, \bar{n} - n^{\beta}_T\right)}, \tag{52}$$

it follows, if $c^{\alpha}_{T+1} = c^{\beta}_{T+1}$, that $p^{\alpha}_{T+1} = p^{\beta}_{T+1}$. But from (48) this is a contradiction, hence $n^{\alpha}_T \neq n^{\beta}_T$. □

Part 2

Inequality (38) follows from part 1 since the first-order conditions for a maximum for generation $T-1$ imply

$$\pi^i U_2\left(c^i_T, \bar{n} - n_{T-1}\right) = z q^i p^i_T, \qquad i = \alpha, \beta, \tag{53}$$

and since $p^{\alpha}_T = p^{\beta}_T$ and $c^{\alpha}_T \neq c^{\beta}_T$ it must be that $\pi^{\alpha}/\pi^{\beta} \neq q^{\alpha}/q^{\beta}$. □

Part 3

The facts that $c^{\alpha}_{T+1} \neq c^{\beta}_{T+1}$ and $n^{\alpha}_{T+1} \neq n^{\beta}_{T+1}$ follow from part 1 together with (50). Finally $p^{\alpha}_{T+1} \neq p^{\beta}_{T+1}$ follows from (48). Since the steady state equilibrium $p_t/p_{t-1} = \mu^*(g)$ is indeterminate there exist continuations of the equilibrium for $t \geq T+1$ that converge assymptotically to $\mu^*(g)$. □

References

Akerlof, G. and J. Yellen, 1985, Can small deviations from rationality make significant differences to economic equilibria?, American Economic Review 75, 708–726.

Azariadis, Costas, 1975, Implicit contracts and underemployment equilibria, Journal of Political Economy 83, 1183–1202.

Azariadis, Costas and Russell Cooper, 1985, Nominal wage–price rigidity as a rational expectations equilibrium, American Economic Review 73, 31–36.

Baily, Martin N., 1974, Wages and employment under uncertain demand, Review of Economic Studies 41, 37–50.

Ball, L. and David Romer, 1987, Sticky prices as co-ordination failure, Mimeo. (Princeton University, Princeton, NJ).

Barro, R., 1977, Long-term contracting, sticky prices, and monetary policy, Journal of Monetary Economics 3, 305–316.

Baxter, M. and R. King, 1990, Productive externalities and cyclical volatility, Mimeo. (University of Rochester, Rochester, NY).

Benhabib, J. and R.E.A. Farmer, 1991, The aggregate effects of monetary externalities, Mimeo. (University of California, Los Angeles, CA).

Blanchard, O.J., 1979, Backward and forward solutions for economies with rational expectations, American Economic Review, Papers and Proceedings 69, 114–118.

Brock, William A., 1975, A simple perfect foresight monetary model, Journal of Monetary Economics 1, 133–150.

Cass, D. and K. Shell, 1983, Do sunspots matter?, Journal of Political Economy 91, 193–227.

Cass, D. and K. Shell, 1989, Sunspot equilibria in an economy with an idealized contingent-commodities market, in: William A. Barnett, John Geweke, and Karl Shell, eds., Economic complexity: Chaos, sunspots, bubbles and non-linearity (Cambridge University Press, Cambridge).

Cooper, R., 1987, Wage and employment patterns in labor contracts: Micro-foundations and macroeconomic implications (Harwood Academic Publishers, London).

Farmer, R.E.A., 1991, Sticky prices, Economic Journal, in press.

Farmer, R.E.A. and Michael Woodford, 1984, Self-fulfilling prophecies and the business cycle, CARESS working paper no. 84-12 (University of Pennsylvania, Philadelphia, PA).

Fischer, S., 1977, Long-term contracts, rational expectations and the optimal money supply rule, Journal of Political Economy 85, 191–205.

Geanakoplos, J.D. and H.M. Polemarchakis, 1986, Walrasian indeterminacy and Keynesian macroeconomics, Review of Economic Studies 53, 755–779.

Gordon, D.F., 1974, A neo-classical theory of Keynesian unemployment, Economic Inquiry 12, 431–459.

Grandmont, J., 1985, On endogenous competitive business cycles, Econometrica 53, 995–1045.

Howitt, P. and Preston McAfee, 1988a, Animal spirits, Mimeo.

Howitt, P. and Preston McAfee, 1988b, Stability of equilibria with externalities, Quarterly Journal of Economics 103, 261–277.

Kehoe, T. and D.K. Levine, 1985, Comparative statics and perfect foresight in infinite horizon economies, Econometrica 53, 433–454.

Mankiw, G.N., 1985, Small menu costs and large business cycles, Quarterly Journal of Economics 100, 529–537.

Peck, James, 1988, On the existence of sunspot equilibria in an overlapping generations model, Journal of Economic Theory 44, 19–42.

Woodford, M., 1986, Stationary sunspot equilibria: The case of small fluctuations around a deterministic steady state, Mimeo. (University of Chicago, Chicago, IL).