# THE ECONOMIC JOURNAL

*NOVEMBER* 1991

*The Economic Journal*, **101** (*November* 1991), 1369–1379
*Printed in Great Britain*


## STICKY PRICES*


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This paper is about sticky prices. It is typical to use this term to refer to a class of theoretical models in which there is some kind of artificial barrier that prevents markets from clearing. I shall mean something different.

There is a sense in which it should be uncontroversial to assert that prices are sticky. By this I mean that one can arrive at a working definition of 'sticky prices' and one can demonstrate that this definition characterises US time series data. One approach is suggested by a recent paper of Chris Sims (1989) in which he compares the impulse response functions in US time series data with a set of impulse response functions that are generated by an artificial economy populated by a representative agent:

> In the simulated data, price responds sharply to every kind of innovation; in the actual data it responds much more slowly and weakly to interest rates and money, and not at all to real output.

When I refer to 'sticky prices' in a theoretical model I will mean that the price level does not respond to a contemporaneous disturbance to one of the other variables. I will demonstrate, by means of an example, that this definition of sticky prices does not require that one should abandon the market clearing assumption. The model that I describe in this paper is a fully articulated rational expectations market clearing model. In this sense it is 'classical'. However the model is capable of replicating the price responses that one observes in the data. In this sense it is 'Keynesian'. I hope to persuade the reader that more elaborate models of this kind will provide an explanation of business fluctuations that represents a viable middle ground between real business cycle theory and the 'neo-Keynesian' agenda.

### I. THE MAIN IDEA

Rational expectations models with market clearing are characterised by functional difference equations. I shall take a simple case in which the stock of real balances $m_t$ is the only state variable. Models in this class will typically lead one to study an equation of the form:

$$m_t = b\mathrm{E}_t m_{t+1} + x, \tag{1}$$


* The author wishes to thank the National Science Foundation for supporting this research under grant number SES 87 2243.

where $b$ is a parameter, $E_t$ is the expectation operator conditional on time $t$ information and in general $x$ will represent a set of exogenous variables that follows a known stochastic process. I shall assume that equation (1) is a good approximation to some nonlinear equation, derived from economic theory, and I will take $x$ to be constant.

A rational expectations equilibrium is a sequence of probability distributions $\{F_t(m_t)\}_{t=1}^{\infty}$ that satisfies equation (1). It is useful to distinguish two possible cases one of which I shall refer to as *regular* and the other as *irregular*. The regular case has the parameter restriction $|b| < 1$ derived from economic theory. In this case equation (1) can be iterated into the future to arrive at the equilibrium,

$$m_t = \frac{x}{1-b}. \tag{2}$$

The regular case is associated with a unique rational expectations equilibrium in which, in the absence of disturbances to the exogenous variable $x$, the stock of real balances is constant.[1]

The idea behind this paper is to exploit the fact that economic theory does not necessarily lead to the restriction $|b| < 1$. It is possible to construct well specified models in which $|b| > 1$ and in this case any stochastic process in the class:

$$m_t = \frac{1}{b}(m_{t-1} - x) + \epsilon_t, \tag{3}$$

generates an equilibrium sequence of probability distributions, where $\epsilon_t$ is an arbitrary random variable with conditional mean zero that is believed to affect economic activity. I refer to this situation as the *irregular* case and I will show that irregular models are associated with equilibria in which prices may be predetermined one period in advance.

In the irregular case equilibria are indeterminate in the sense that arbitrarily close to one equilibrium there is another one. If one restricts oneself to non-stochastic sequences then any sequence of variables $\{m_t\}_{t=1}^{\infty}$ generated by picking an initial value of $m_1$ close to the steady state and iterating the difference equation:

$$m_t = \frac{1}{b}(m_{t-1} - x), \tag{4}$$

is an equilibrium. These equilibria are indeterminate because the choice of initial condition is arbitrary. If one allows for the possibility that agents may condition their beliefs on non-fundamental randomness, 'sunspots', then there exist multiple *stationary* rational expectations equilibria each of which is characterised by a stationary stochastic process generated by equation (3) for an arbitrary choice of the process generating $\epsilon$.

Although the existence of the irregular case has been known for some time much of the literature has been directed at finding conditions under which it

[1] Throughout this paper I will use the word equilibrium in the sense of general equilibrium theory to mean a sequence of variables that is consistent with the assumptions of market clearing and rational choice. An equilibrium that is invariant through time will be referred to as *stationary*.

can be eliminated. I shall argue that a more fruitful approach is to exploit the possibility that a model with indeterminate equilibria may be used to explain the comovements of prices and quantities that characterise the data. Rather than think of these equilibria as 'indeterminate' I prefer to think of them as equilibria in which beliefs may independently influence allocations. Once beliefs are parameterised, each equilibrium has a unique implication for the time series properties of the endogenous variables.

## II. A SIMPLE EXAMPLE

The economy has a single nonstorable commodity $y_t$ that is produced using the following technology:

$$y_t = \alpha n_{t-1} + \phi n_t, \quad \alpha > 0, \quad \phi > 0, \tag{5}$$

where $n_t$ represents labour input. Notice that outputs in periods $t-1$ and $t$ are joint products. Other technologies will deliver similar results to those reported below although this is the simplest that I have found which generates an equilibrium with the properties that I will describe. The time delay in production is needed to make the demand for labour sensitive to intertemporal prices. The inclusion of current output in the technology is important since it allows quantities to respond contemporaneously to demand shocks.[2]

Agents in this economy live for two periods and have perfect foresight of future prices. They work when young but consume in both periods of life. The opportunity set of an individual in the generation that is born in period $t$ is represented by a pair of lifecycle budget constraints:[3]

$$\omega_t(n_t^s - n_t^d) + \phi n_t^d - c_t^{yd} - m_t^d \geqslant 0, \tag{6}$$

$$\alpha n_t^d + m_t/\pi_{t+1} - c_{t+1}^{od} \geqslant 0. \tag{7}$$

Putting these two inequalities together, one arrives at the lifecycle constraint of a representative individual:

$$\omega_t(n_t^s - n_t^d) + \phi n_t^d + \alpha n_t^d \pi_{t+1} \geqslant c_t^{yd} + \pi_{t+1} c_{t+1}^{od}. \tag{8}$$

In the above inequalities, $\omega_t$ is the real wage, $n_t^s$ is labour supply, $n_t^d$ is labour demand, $c_t^{yd}(c_t^{od})$ is consumption demand of the young (old), $m_t^d$ is the real demand for money, $p_t$ is the price of goods in terms of money and $\pi_t \equiv p_t/p_{t-1}$ is the inflation factor between periods $t-1$ and $t$. Agents must choose both a demand and a supply of labour because they must act in the capacity of producer as well as that of consumer.

[2] In an equilibrium in which prices do not respond immediately there must be some way for *quantities* to increase contemporaneously to meet an increase in demand. I have found examples of economies where agents store inventories of goods and where the simpler technology: $y_t = \alpha n_{t-1}$ can be used, that display equilibria with similar properties to the example that I describe here. However, these examples lead to equilibria that are characterised by difference equations of order two or more.

[3] I have suppressed a symbol that would differentiate the individual agent from the aggregate in order to cut down on notation.

Rather than model individual utility functions directly I assume that aggregate demand functions in period $t$ are given by the following expressions:

$$n_t^s = n^s(\omega_t), \tag{9}$$

$$c_t^{yd} = c^d(\omega_t n_t^s), \tag{10}$$

$$m_t^d = \omega_t n_t^s - \omega_t n_t^d - c_t^{yd} + \phi n_t^d, \tag{11}$$

$$c_t^{od} = m_{t-1}/\pi_t + \alpha n_{t-1}^d, \tag{12}$$

where the functions $n^s(.)$ and $c^d(.)$ are continuous, increasing and differentiable, and the first derivative of $c^d(.)$ is between zero and one. These functions are continuous and they obey Walras law. It follows from the Debreu Sonnenschein theorem that there exists an economy with at most four consumers in each generation which generates these demands as the outcome of maximising behaviour on the part of rational individuals.[4]

Young agents in this economy must choose how much labour to supply, how much to consume when young and how much to consume when old. The aggregate demands and supplies that are generated by their rational choices are described above in equations (9), (10), (11) and (12). They must also choose whether to store their wealth between periods in the form of money or by setting up a firm and holding stocks of goods in process. If money is held as a store of wealth in equilibrium then holding money must appear equally attractive as demanding labour and turning it into inventories. Since the lifecycle budget constraint is linear in labour demand it follows that in a monetary equilibrium:

$$\omega_t = \phi + \alpha\pi_{t+1}. \tag{13}$$

Notice from equation (13) that expected inflation is positively related to the real wage and from equation (9) that the real wage is positively related to labour supply. These features allow a stationary state to be found in which increases in government spending lead to increases in inflation, real wages and employment. In more familiar versions of this model real wages and inflation are inversely related and as a result many comparative static results are of the opposite sign from those that hold in my example.

To close the model I assume that real government spending is financed by printing money and that there is no alternative outside asset. The government budget identity then implies that:

$$p_t/p_{t+1} = \frac{m_{t+1} - g}{m_t}, \tag{14}$$

where $g$ represents government spending and $m_t$ is the supply of real balances. Substituting equations (9), (10) and (11) into equation (14) to write $m_t$ in terms of $\omega_t$ and using equation (13) to write $p_t/p_{t+1}$ in terms of $\omega_t$ one arrives at the following difference equation which characterises equilibria for this economy:

$$\frac{\alpha}{\omega_t - \phi} = \frac{m(\omega_{t+1}) - g}{m(\omega_t)}. \tag{15}$$

[4] The preferences $U = \log(c_t - n_t^2) + \log(c_{t+1})$ will generate demand functions which satisfy all of these conditions for an economy with a representative agent in each generation. I have avoided this example however because I will need an additional restriction which it violates. This restriction is discussed in footnote (5).

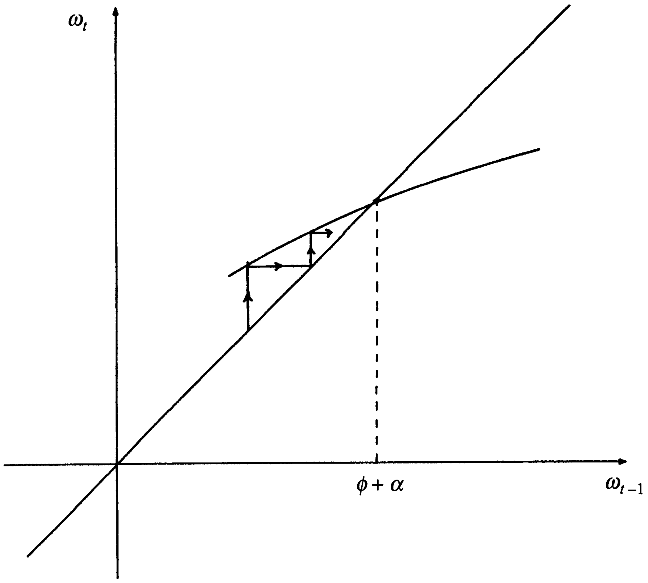


Fig. 1. The dynamics of the example.

When $g = 0$ the model has a steady state in which $\omega = \phi + \alpha$. The local behaviour of this difference equation, which is governed by the slope of the implicit function (15) around the steady state $\phi + \alpha$, is represented in Fig. 1. This steady state will have positive valued fiat money if:

$$\phi n^s(\phi+\alpha) > c^{yd}[(\phi+\alpha)\, n^s(\phi+\alpha)]. \tag{16}$$

An example of an economy that satisfies this inequality is provided by the following set of demand and supply functions:

$$n_t^s = \omega_t^\gamma; \tag{17}$$

$$c_t^{yd} = \delta n_t^s \omega_t; \tag{18}$$

for $\phi > \delta(\phi+\alpha)$. These functions imply that consumption and the demand for money can be described in terms of the real wage:

$$c_t^{yd} = \delta \omega_t^{1+\gamma}; \tag{19}$$

$$m_t^d = \phi \omega_t^\gamma - \delta \omega_t^{1+\gamma}; \tag{20}$$

where equation (20) follows from (11) after substituting in the labour market equilibrium condition, $n_t^s = n_t^d$. Equation (20) is depicted in Fig. 2.

Notice from Fig. 2 that I have chosen $\alpha$ in such a way that $\phi < \phi + \alpha < [\phi\gamma/\delta(\gamma+1)]$.[5] Since the model has a monetary steady state at $\omega = \phi + \alpha$, and since the demand for money achieves a maximum at $\omega = [\gamma\phi/(1+\gamma)\delta]$, it follows for the above parameter configuration that the economy has a steady

[5] This construction is not possible for the preferences discussed in footnote (4). There may well be simple examples with one agent per generation that allow one to construct a stationary state in with both $m$ and $m'$ are positive but I have not found one.

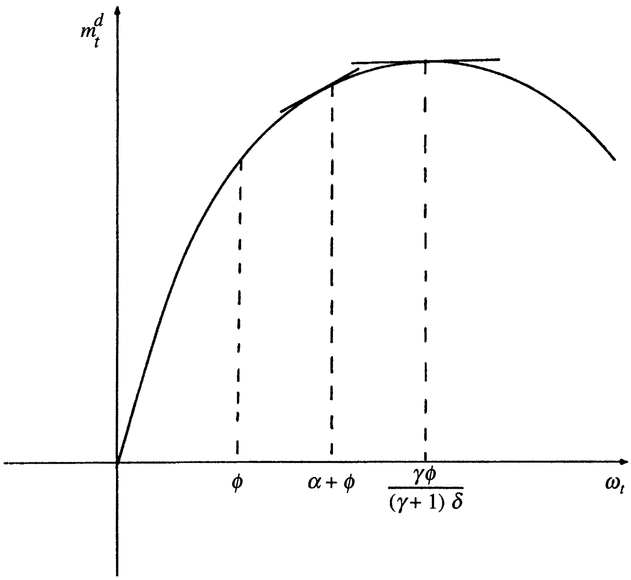


Fig. 2. The demand for money as a function of the real wage.

state at which the demand for money is both positive and increasing. These properties are important for the reasons described below.

To find out how the model behaves close to the steady state one may linearise equation (15). Appealing to the implicit function theorem the dynamic equilibria around this study state can be described by the solutions to a first order difference equation of the form:

$$\omega_t = f(\omega_{t-1}), \tag{21}$$

where the first derivative of the function $f$ is given by:[6]

$$f' = \frac{\alpha}{\omega-\phi} - \frac{m}{m'(\omega-\phi)} = 1 - \frac{m}{am'}. \tag{24}$$

When $|f'| < 1$ sequences of real balances that begin close to the steady state will converge back towards it. For this to occur the economy must be parameterised in such a way that both $m$ and $m'$ are positive at the steady state. It must also be true that $(m/\alpha m') < 1$. One example that satisfies these conditions is provided by the parameterisation of equation (20) for values of $\gamma = 7$, $\phi = \frac{3}{4}$, $\alpha = \frac{1}{4}$ and $\delta = \frac{1}{2}$. In this case there is a monetary steady state at $\omega = 1$. At this steady state the demand for money is positive and equal to $\frac{1}{4}$, the

[6] In this expression, $\omega$ and $m$ are evaluated at their steady state values and $m'$ is the derivative of $m$ also evaluated at the steady state. The slope of the demand for money function is given by:

$$m' \equiv \phi n' - c'n - c'\omega n'; \tag{22}$$

which for the example given in equation (20) generates the expression:

$$m' = \gamma\phi\omega^{\gamma-1} - \delta(\gamma+1)\,\omega^{\gamma}. \tag{23}$$

slope of the demand for money function is equal to $\frac{5}{4}$ and slope of the equation linking $\omega_t$ to $\omega_{t-1}$ is given by the expression:

$$f' = 1 - \frac{m}{\alpha m'} = \frac{1}{5}.$$

The importance of the existence of a stationary monetary equilibrium in which $|f'| < 1$ is that under these conditions there exists a non-stationary equilibrium for any level of initial real balances in the neighbourhood of $m_0 = \frac{1}{4}$. Furthermore, members of the set of non-stationary equilibria all converge back to the monetary steady state. In more familiar examples of the overlapping generations model it is the steady state in which money has no value which is locally indeterminate. Consequently not much interest has been directed towards the set of non-stationary monetary equilibria since they all converge to a demonetised economy.

Some interest has been shown in versions of the standard overlapping generations model in which the government finances expenditure by printing money.[7] This model is potentially interesting since one may show that the set of non-stationary equilibria converges to a *monetary* steady state. However, this steady state is on 'the wrong side of the inflationary Laffer curve' in the sense that increases in government expenditure lead to *lower* inflation. The above example does not suffer from this counter factual implication.[8]

## III. THE ECONOMICS OF THE EXAMPLE

The above example was designed to show that simple equilibrium models may contain a continuum of equilibria. To isolate a specific equilibrium one might recognise that the path of prices and quantities depends not only on 'fundamentals' but also on what people believe. In this part of the paper I will try to give some intuition about the economics that underlies this situation by comparing two possible equilibria which are distinguished by different beliefs about the future. Both sets of beliefs can be self-fulfilling but they generate very different time paths for the endogenous variables of the model.

I will compare two possible responses of the economy to a conceptual experiment which is equivalent, in a stochastic model, to an unanticipated switch in policy regime. In this experiment, one assumes that for all recorded history the level of government purchases has remained constant at zero. At date $T$ there is an unanticipated increase in the stock of money which is used to purchase commodities but for all dates later than $T$ the money stock is

[7] This is the model studied in Farmer and Woodford (1984). Azariadis and Cooper (1985) have also examined the idea that predetermined price equilibria can lead to a model with Keynesian properties.

[8] Totally differentiating (15) at the monetary stationary state it follows that:

$$\left.\frac{\partial \omega}{\partial g}\right|_{g=0} = \frac{(\omega-\phi)/\alpha}{(m/\alpha)+m'\{[(\omega-\phi)/\alpha]-1\}} = \frac{\alpha}{m} > 0, \qquad (25)$$

that is, increases in the level of government purchases cause output to expand, people hold more money, and the inflation (tax) rate increases.

(rationally) anticipated to remain at its new higher level. Since the monetary stationary state is indeterminate there is a range of possible values of the initial level of prices all of which are consistent with a perfect foresight equilibrium. As there are many possible equilibrium paths for prices, there are also many possible ways in which the economy could respond to a regime change.

At one extreme, it is possible that prices might jump in such a way that the economy remains at the stationary state $\omega_t = \alpha + \phi$. In this scenario the jump in prices would cause the holders of money balances to be taxed exactly enough to pay for the government purchases in period $T$. This inflation tax would have been unforeseen in the sense that the agents who had chosen to hold money did so in the belief that the distribution of prices had point mass at its historically determined level. The unforeseen shock that occurs in this scenario is inconsistent with a single rational expectations equilibrium, that characterises the economy both before and after date $T$, because at the date of the policy switch agents act on beliefs about the *distribution* of prices which later turns out to have been incorrect. However, the behaviour that I have just described is not the only possible way in which this economy might react to an unforeseen event.

Suppose instead that the price level at date $T$ does not respond to a contemporaneous increase in the stock of money. In this alternative equilibrium, firms respond to an increase in demand by increasing output and hiring more labour. Since part of the output that is produced will be sold in period $T+1$ it is possible for this expansion in economic activity to be a rational response if firms (correctly) anticipate that prices will increase. This situation is depicted in Fig. 3. Firms expect that $p_{t+1}/p_t$ will be positive and this expectation implies that, at the level of real wages that has existed historically,

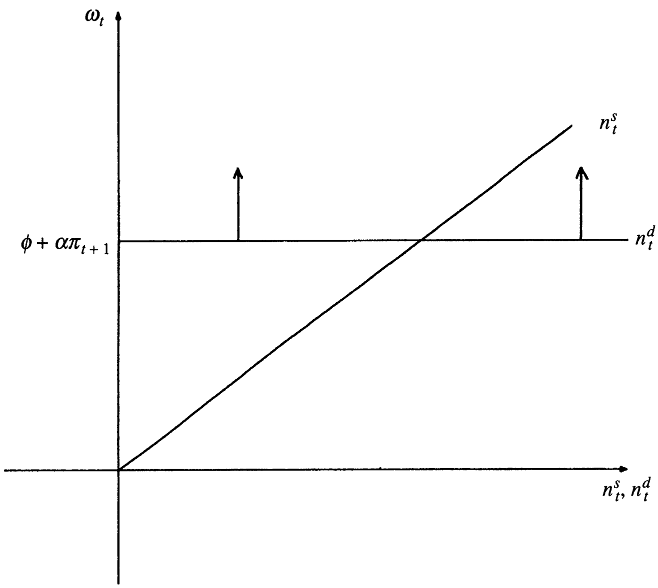


Fig. 3. The labour market.

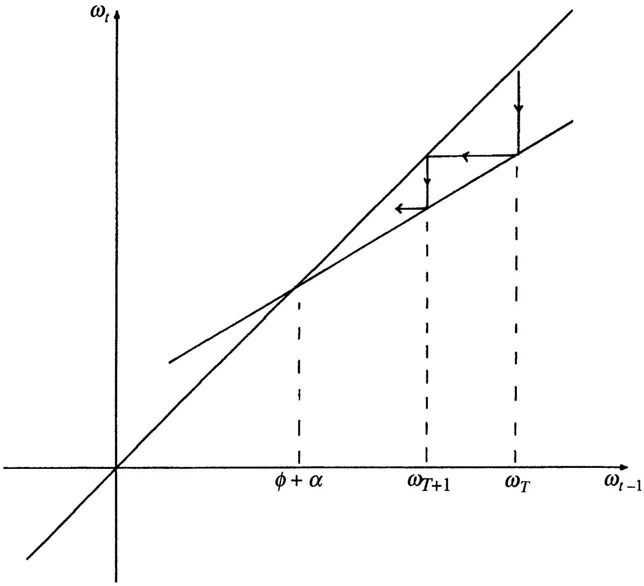


Fig. 4. The response to a monetary shock.

they should switch from holding money as a store of wealth into holding goods in process. Acting on this belief, firms hire more labour and drive up wages to the point at which $\omega_t = \phi + \alpha\pi_{t+1}$.

In period $T+1$ the nominal money stock remains constant at its new higher level but output, employment and real wages contract back towards the stationary state in which $\omega = \phi + \alpha$; this scenario is depicted in Fig. 4. Since the demand for (real) money is locally increasing in real wages, the real demand for money in period $T+1$ will be *lower* than in period $T$. To equate demand and supply, the price level in period $T+1$ must exceed the price level in period $T$. This is the rationally anticipated increase in the price level that triggered the expansion in economic activity in the first place.

## IV. WHICH IS THE APPROPRIATE EQUILIBRIUM?

The economy that I have described is one in which agents have perfect foresight of future prices. However, one may use the techniques described in Farmer and Woodford (1984) to construct stochastic equilibria in this economy which are characterised by first order stochastic difference equations. There are analogies between the equilibria of the perfect foresight economy and the stochastic equilibria of the more complicated example. I will concentrate, however, on the perfect foresight case.[9]

[9] In my discussion I shall use the terms perfect foresight and rational expectations interchangeably in recognition of the fact that a perfect foresight equilibrium is a special case of a rational expectations equilibrium in which the distributions of the variables of the model are degenerate with mass points at particular values.

The two different perfect foresight equilibria that I have described represent alternative ways of modelling rational expectations responses to a regime switch. They differ in the implicit assumptions that are made about the mechanism that agents use to forecast the future. In any rational expectations model with multiple equilibria there are many ways in which agents could form beliefs, each of which may be self fulfilling. This observation suggests that one should ask why one method of forecasting might become adopted over another.

My own answer to this question is explained in more depth in Farmer (1991). It comes down to a comparison of the properties of alternative forecasting rules in the face of an unforeseeable switch in policy regime of the kind that I discussed above for the perfect foresight example. In this economy there is one very important difference between an equilibrium in which prices jump and an equilibrium in which they do not. This difference is most forcefully illustrated by comparing the forecast rules that agents use to support equilibrium in the two alternative situations.

Consider first, an economy in which agents have learnt that the perfect foresight price level is given by the expression;

$$p_{t+1} = p^*(M_{t+1}); \tag{26}$$

where $p^*$ is the rational expectations pricing function associated with a stationary perfect foresight equilibrium in which money has value and $M_{t+1}$ is the nominal quantity of money. In this economy agents in period $T-1$ will forecast $p_T$ on the mistaken assumption that the distribution of $M_T$ has point mass at its historically given level. Ex post, they will turn out to have been mistaken and they must update their beliefs about the distribution of the forcing variables to learn about the new monetary regime. A forecast rule of the kind described in equation (26) has the disadvantage that it requires knowledge of the future distribution of policy variables. But this is not true of all forecast rules!

Suppose instead that agents forecast future prices using the function:

$$\pi_{t+1} = f(\omega_{t-1}; g)\,\alpha + \phi. \tag{27}$$

This forecasting rule coincides with the equation that describes the properties of a non-stationary perfect foresight equilibrium. If $\omega_{T-1}$ is equal to its stationary value then the price level at date $T$ that is forecast by this equation will be the same as that which is forecast using (26). The difference in these two rules occurs in the way that they behave in the face of a regime switch. If the government conducts an unanticipated regime change at date $T$ and if agents continue to forecast prices with equation (27) at the point of the switch, then the effect of the regime change will be felt on quantities rather than prices.

Although both of the forecasting rules that I have described will support a rational expectations equilibrium for any given policy regime, they are not alike in the way in which they perform in the face of a *change* in regime. Whereas agents who use equation (26) must know the distribution of future values of monetary policy, no such information is required in the case of an equilibrium with predetermined prices since equation (27) does not depend upon the values of $M_t$. If all agents in all generations continue to forecast using equation (27), then nobody will be fooled by a change in regime. Another way of saying this is that, in some economies in which there are multiple equilibria, one can sometimes find an equilibrium which is immune to the 'Lucas critique'. The economy that I have presented above is an example of such an economy. In my model the 'Lucas proof' equilibrium is one in which prices are predetermined one period in advance and in which the impulse response functions of the economy are 'Keynesian' in the sense that quantities rather than prices respond contemporaneously to clear markets in the face of demand shocks.

## V. CONCLUSION

A predetermined price equilibrium has very different time series implications from those of the equilibria that arise in standard rational expectations models. It is my contention that these implications are likely to allow us to construct model economies with equilibria which are in much closer accord with the data than those which follow from representative agent economies. In a world where the future is continually changing in unknown (and unknowable) ways, there are clear advantages to forecasting schemes with the properties that I describe. Whether agents do indeed use such schemes is an empirically falsifiable proposition. My current research agenda involves the construction of more realistic economies that exhibit the kinds of features of the model described in this paper. By comparing the impulse response functions of these economies with those of time series data one can rank the performance of alternative models. It is too early to tell if this research agenda will prove successful but my preliminary investigations look promising.

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*Date of receipt of final typescript: February 1991*

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