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The Role of Options in the Resolution of Agency Problems: A Comment

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THE AIM OF THE contracting literature in economics is to explain observed contractual arrangements, or organizations, as minimizing the costs of incentive conflicts. In the agency cost literature of financial economics, the contractual arrangement at issue is the mix of securities held by inside management of a firm and outside suppliers of capital—the ownership structure of the firm, in Jensen and Meckling's [8] terminology. In explaining the ownership structure of a corporation, Jensen and Meckling focus on two sources of agency or incentive-conflict costs: (a) the manager's tendency to consume excessive perquisites, and (b) the manager's tendency to invest in prospects of high risk so as to effect an ex post transfer from bondholders to stockholders. A recent article in this journal by Haugen and Senbet [6] examines the role of stock options in resolving these agency problems and claims to show, in particular, that a call option in the insider portfolio can eliminate the first type of agency costs.

This article re-examines the role of options in the resolution of agency problems. We argue, in Section I, that options cannot eliminate the agency problem of excessive perquisite consumption, contrary to Haugen and Senbet. The essential point is that unless outside financing can be provided entirely by riskless debt, then outsiders must have a residual claim on the net income of the firm in some states of the world. Outsiders therefore share in the costs of increased perquisite consumption, whatever the ownership structure of the firm. This negative externality means that excessive perquisite consumption is inevitable.

The question remains: Do call options nonetheless have a role in the *mitigation* of agency costs? An insider portfolio consisting only of equity leaves the manager with the same proportion of residual claim in each state of the world. We show, using a formal principal-agent model, that the efficient insider's share of profits is relatively high in favorable (high-profit) states of the world. Insider call options arise as means of transferring residual claim to the manager in favorable states of the world relative to low-profit states.

The basic agency model has been solved in substantial generality by Holmström [7] and others. We employ here a more specific model that draws on the techniques developed by Green and Kahn [4] and that is an extension to continuous states of the finite-state model of Sappington [12]. The extension allows a simple characterization of the optimal contract, a characterization that

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reveals the economic intuition behind the role of options. This formal model is presented in Section II of the paper. Section III contains the main results, and the final section offers a summary and conjectures related to extensions of the model.

I. Previous Literature: An Assessment of the Haugen-Senbet Claim

Haugen and Senbet (H-S) adopt the traditional agency setting of a single owner-manager who raises capital for a firm by issuing outside securities. Following the issuance of securities, the manager makes a decision on the amount of perquisite consumption. The stochastic income of the firm is then realized and the security holders (including the manager) are paid. As in Jensen-Meckling, the issuance of outside equity means that the manager does not bear the full costs of increased perquisite consumption and therefore has an incentive to consume excessive perquisites. By assumption, the manager cannot contract to consume only the efficient amount of perquisites. The market anticipates rationally the manager's decision, pricing the securities accordingly, and the manager therefore bears the efficiency cost of the excessive perquisite consumption.

H-S suggest that, in addition to any inside equity, the manager should hold a call option to buy back the entire firm after the firm's income is observed. They note that, *if* the value of the resulting insider portfolio is fully responsive, at the margin, to an increase in perquisite consumption, then the agency problem is solved; the manager, bearing the full marginal costs of perquisites, chooses the efficient amount of perquisites.

Unfortunately, the authors do not investigate the *feasibility* of an insider portfolio with a value that responds dollar for dollar at the margin to an increase in perquisites. As H-S note (p. 633), a necessary condition for this is that the ex ante value of the outside portfolio, consisting of a $(1 - \alpha)$ share of equity plus the writer's position in the call, must be invariant at the margin to a change in perquisites. Now, the realized value of this outside portfolio is E, the exercise price, conditional upon the firm's net income exceeding $E/(1 - \alpha)$, since in this event the call option to purchase the entire outside portfolio is exercised by the manager. The value of the outside portfolio conditional upon sufficient profits is thus constant and independent of perquisite consumption.

But the outsiders, not the manager, are the residual claimants conditional upon income not exceeding $E/(1 - \alpha)$. In this event the call option is not exercised, and the realized value of the outsiders' portfolio varies by $(1 - \alpha)$ for each dollar change in perquisite consumption. The *ex ante* value of the outsider portfolio will therefore fall, when perquisites increase by one dollar, by approximately $(1 - \alpha)$ times the probability that income is less than $E/(1 - \alpha)$. *The outside portfolio will be invariant to perquisites at the margin* (and the inside portfolio fully responsive) *only if this probability is zero*—i.e., *only if the call option is always exercised.*

The outside portfolio necessary to eliminate agency costs consists, in sum, of equity plus a writer's position in a call to sell this equity that is exercised with a probability of one—a portfolio that yields the exercise price with certainty. But this outsider portfolio could be relabeled simply as riskless debt, and the insider's position relabeled as one hundred percent of the equity.

The upshot of the model is a rediscovery of a well-known result: If (and only if) the required amount of outside financing is small enough relative to the minimum possible return on assets so that it can be provided by risk-free debt, the insider is left as the sole residual claimant. There is no separation between ownership and control in this case and no agency problem. The only "call option" that can eliminate agency costs is the allocation of all equity to insiders, in the case where risk-free debt financing is feasible. Insider call options cannot eliminate agency costs in the nontrivial case (intended by H-S) where risk-free financing is impossible.

The remainder of this article investigates whether insider call options can be explained as part of the second-best efficient insider portfolio, which minimizes but does not eliminate agency costs.

II. The Basic Model

A necessary condition for an agency problem to arise, in the conventional setting of a single manager seeking outside financing for a firm, is that a contract cannot be written and enforced costlessly to ensure efficient managerial decisions. In addition to the impossibility of complete contracting, positive agency costs require that the manager not be the sole residual claimant. This second condition may be satisfied because the manager is risk averse; in this case it is not *optimal* to leave the entire residual claim with the manager (Holmström [7], Shavell [13]). Alternatively, it may not be *feasible* to leave the manager with the entire residual claim (i.e., to finance with riskless debt) because of limited liability, e.g., limited wealth, on the part of the manager (Sappington [12]). Even if the manager is risk neutral, an agency problem arises in this case because outsiders must hold some part of the residual claim in at least some states of the world.

Where does the Jensen-Meckling (J-M) model fit into this framework? In the J-M and H-S models, the manager's preferences are defined over perquisite consumption and the *market value* of the manager's random income stream. The conditions under which an individual's random income stream can be summarized by its market value, in representing the individual's preferences, are strong: complete security markets must exist, so that any random income can be completely insured (traded) in the market, or both the individual and the market must value the income at its expected value and share the same expectations.[1]

The first of these sufficient conditions, complete markets, is inconsistent with the existence of an agency problem. The assumption of complete markets requires that economic agents agree upon which state of the world has occurred. But if the state is observed, the manager's effort conditional upon each state can be inferred from the realized profit and can therefore enter the contract. This means that the contract is complete, i.e., that agency costs are zero. We suggest,

[1] Actually, the necessary and sufficient condition is a straightforward generalization of these two sufficiency conditions.

therefore, that risk neutrality is a necessary implicit assumption in the models of J-M and H-S.

A wealth constraint is thus critical for agency costs in the Jensen-Meckling framework. The model analyzed below is the simplest complete framework in which the optimal ownership can be characterized when a wealth constraint is the source of the agency problem. We assume that economic agents are risk neutral—partly because this allows us to maintain what we have argued to be the implicit J-M and H-S assumption, but mainly in order to focus on the incentive (as opposed to the risk-sharing) role of options in the ownership structure.[2] The case of risk aversion is discussed in Section IV. In addition, within the resulting model, an assumption that the manager makes the perquisite or effort decision *after* the realization of uncertainty turns out to be critical in explaining realistic financial structures. The formal model is given by the following set of assumptions:

(A1) A project is available to an entrepreneur-manager[3] at an indivisible cost C. The project yields gross returns, x, that depend upon the manager's effort e and the realized state of the world θ, $\theta \in [\underline{\theta}, \overline{\theta}] \subset R^{+}$. ("Effort" is equivalent to a reduction in perquisites in Jensen and Meckling's terminology.)

(A2) The production function $x = F(e, \theta)$ yields stochastic constant returns to scale.

Assumption (A2) is not essential for all of our results but greatly simplifies the analysis. Normalization of units of output and, if necessary, relabeling of states of the world allow us, by virtue of (A2), to express

$$F(e, \theta) = \theta \cdot e. \qquad (1)^4$$

(A3) The manager's own initial wealth is $W_0 < C$. The manager therefore raises money by selling a claim $S(x)$ on the observable output x of the project. $M(x) \equiv x - S(x)$ accrues to the manager.

[2] B. Klein has stated,

> "One should be hesitant to accept risk aversion explanations for contractual terms, because these explanations are logically equivalent to relying on tastes to explain behaviour and because they ignore the separate insurance markets that may develop in response to such tastes." (Klein [9, p. 370].)

While this may be overstating the matter for our purpose of explaining financial contracts, isolating the possible *incentive* reasons for contractual terms, as opposed to risk-sharing explanations, is surely the place to start. Interestingly, Klein goes on to suggest (p. 370) that risk-sharing may be a necessary explanation of the separation of ownership and control. In our view, as reflected by the limited liability or wealth constraint in our formal model, it is the differential endowments of managerial ability and wealth across individuals that is at the heart of the separation of ownership and control.

[3] It is not essential that entrepreneurial ability, or the property rights to surplus from the project, be tied to managerial ability. If instead there is a competitive market for managers, the entrepreneur bears the agency costs. Following the principal-agent and the agency cost literature generally, we consider a one-period model. Repetitions would mitigate the agency problem. For an analysis of the role of reputations in alleviating agency costs, see Fama [2].

[4] Given a set of states $[\underline{\theta}, \overline{\theta}]$, relabel any state $\hat{\theta} = 1$. Define units of output (or effort) such that $F(1, 1) = 1$. Then relabel any state θ by $\tilde{\theta}(\theta) = F(e, \theta)/F(e, \hat{\theta})$ for any e. This procedure is well defined by assumption (A2). Then $F[e, \theta] = \tilde{\theta}(\theta)F(e, \hat{\theta}) = \tilde{\theta}(\theta)eF(1, 1) = \tilde{\theta}e$.

(A4) (Risk-neutral manager) The manager's utility over realized wealth is given by $V = W - v(e)$, where $v(\cdot)$ is the disutility of effort.
(A5) (Risk-neutral security market) The security market prices the security $S(x)$ at the price $ES(x)$. (All monetary values are in present value terms.)
(A6) Neither the manager's effort e nor the state of the world θ can enter a contract with the security market. The effort e is decided upon by the manager after the state θ is known.
(A7) The security market has rational expectations on the choice $e(\theta)$.

The final assumption that we must make is on the feasible class of sharing rules from which the optimal rule must be chosen. One alternative would be to assume that the feasible set consists of those rules that are implemented by the issuance of equity, debt, and a finite number of options. The following lemma (proved in the Appendix) describes this class of rules.

LEMMA: *Any piece-wise linear, continuous sharing rule can be implemented by the issuance of debt, equity, and a finite number of call options (or put options).*

This lemma is straightforward from Ross [10, 11]. As an example, the rule illustrated in Figure 1 is implemented by the issuance of debt of face value x_0 to the outsiders, equity to both the outsiders and the manager, and three call options to the manager that come "into the money" as profit reaches x_1, x_2, and x_3, respectively. These options, written by the outside equity holders, transfer residual claim from the outsiders to the manager; when the third option is exercised (at profit levels higher than x_3), the manager is left holding all equity; i.e., $M(x)$ has a slope of one. Any piece-wise linear rule that is *convex*, such as in Figure 1, is implemented by an ownership structure that has only the manager holding call options; in general, both insiders and outsiders will hold call options.[5]

This paper characterizes the optimal sharing rule in the limiting but tractable case of an infinite number of securities. That is, we take as feasible any continuous sharing rule with slope (where differentiable) between zero and one inclusive. The important implications of this characterization are the following: (a) any rule with a derivative that is higher at x_2 than at some $x_1 < x_2$ is implemented in the limit by an ownership structure that includes call options in insider portfolios; and (b) any *convex* rule is implemented in the limit by an ownership structure that includes call options held only by the insider.

III. Analysis of the Optimal Ownership Structure

The optimal contract $S(x)$ that the manager offers the security market maximizes his or her expected utility subject to the *financing* constraint that enough capital is raised to pay the cost of the project, the *incentive compatibility* (or "credibility") constraint that the effort level promised is in the manager's self-interest ex post, and the *limited liability* constraint that the proceeds of the project cover the promised payment to outside capital at the realized profit. By the lemmas of Harris and Townsend [5], the optimal contract problem, denoted by P-A, can be

[5] We identify a financial policy as the issuance of particular amounts of debt, equity, and call options. Of course, from put-call parity there is an *equivalent relabeling* of any policy in terms of debt, equity, and put options (or any other combination of three of the four types of securities).

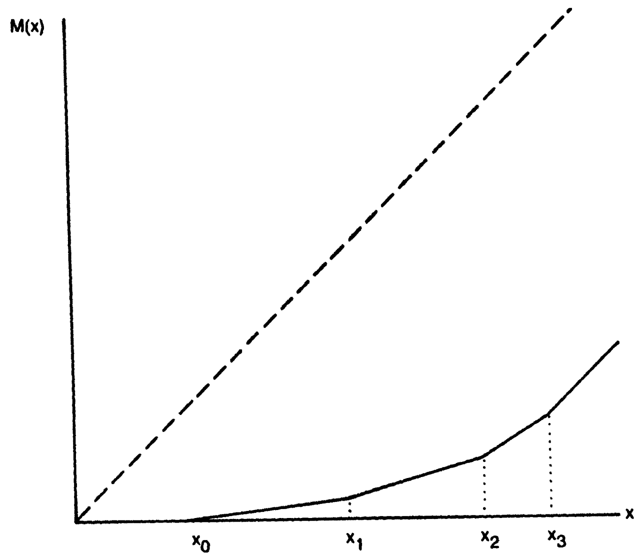

Figure 1. Piece-Wise Linear Sharing Rule for Manager

shown to be equivalent to the derivation of the optimal *direct mechanism*, a hypothetical contract in which the payment and effort are specified as functions of the state reported by the agent, subject to the constraint that the agent must have the incentive to reveal the true state. This equivalence, which allows a tractable analysis of the optimal contract, means that the optimal contract solves implicitly the following problem, which we denote by $P\text{-}A'$.[6]

$$P\text{-}A'\text{: Maximize}_{S(\theta),e(\theta)}E_\theta\{F[e(\theta),\theta]-S(\theta)-v[e(\theta)]\}, \tag{2}$$ [7]

subject to

$$\textit{Financing Constraint: } E_\theta S(\theta)\geq C-W_0, \tag{3}$$

$$\textit{Incentive Compatibility: } (\forall\theta)\ \theta\in \text{argmax}_m F[e(m),\theta]-S(m)-v[e(m)], \tag{4}$$

and

$$\textit{Limited Liability of Manager: } (\forall\theta)\ S(\theta)\leq F[e(\theta),\theta]. \tag{5}$$

Thus, the optimal financing problem can be formulated *as if* the manager sells the security $S(\theta)$ to finance the project and the market relies on the manager to

[6] Most of the derivation in this section parallels the finite-state model of Sappington [12] and is therefore kept brief. See Sappington or Farmer [3] for related details.

[7] This objective function requires some clarification. The timing of events in the model and the corresponding cash flows to management are the following: With initial wealth W_0, the manager issues the security (sharing rule) $S(\theta)$ collecting $ES(\theta)$, pays the cost C, and then ex post collects the net proceeds $F[\cdot]-V(\cdot)$ and pays out $S(\theta)$. The realized payment to the manager is therefore $W_0+E[S(\theta)]-C+F[\cdot]-V(\cdot)+S(\theta)$. Taking expectations, the manager's objective is to maximize $W_0+F-C-V$, i.e., to maximize the net value of the project. This problem is equivalent to (2) since the constraint (3) is binding at the optimum. We maintain the objective function (2) since it is standard expression in the principal-agent framework that we are applying.

announce the state. Because the optimal security satisfies the limited liability constraint, the security market knows that the project constitutes sufficient collateral for the security. The market trusts the manager to announce the correct state because the incentive compatibility constraint is met.

As a standard of comparison for the solution of $P\text{-}A'$, it is useful to characterize the solution to the *complete contracting* problem of maximizing the objective with respect to $e(\theta)$ and $S(\theta)$, in the absence of any constraint. This is easily shown to yield the (first-best) effort, denoted by $e^*(\theta)$ and given by the Kuhn-Tucker condition:

$$e^*(\theta)[F_e[e^*(\theta), \theta] - V_e[e^*(\theta)]] = 0, \tag{6}$$

whence, from (1),

$$e^*(\theta) = V'^{-1}(\theta) \quad \text{if} \quad V'^{-1}(\theta) > 0; \quad e^*(\theta) = 0 \quad \text{otherwise.} \tag{7}$$

At any state θ, define the (ex post) *agency cost* of any contract to be $F[e^*(\theta), \theta] - F[e(\theta), \theta] + v[e(\theta), \theta] - v[e^*(\theta), \theta]$. In this simple model, the agency cost is the surplus lost, ex post, due to the impossibility of writing a complete contract (the component of agency costs defined as the residual loss in Jensen and Meckling). The characterization of the optimal financial structure as the optimal direct mechanism makes it clear that the solution minimizes the expected agency costs.

To provide a tractable characterization of the solution, we must impose some conditions on the exogenous parameters of the model. Let $H(\theta)$ be the distribution of θ on $[\underline{\theta}, \overline{\theta}]$ with density $h(\theta)$. Then the hazard rate of the distribution is $r(\theta) = h(\theta)/[1 - H(\theta)]$. We shall eventually impose conditions on the first two derivatives of r. Initially, we adopt the following assumption:

(A8) $r'(\theta) \geq 0$. Equivalently, the hazard rate of the distribution of output, conditional upon any effort level, is increasing.

(A8) is imposed in order to rule out certain boundary solutions and it allows a simple characterization of the optimal contract. It is a condition that is satisfied by a large class of density functions, e.g., "the Exponential, the Gamma and Weibull with degrees of freedom parameter larger than 1, the Normal Distribution, La Place, and the uniform."[8]

For $\psi \in (0, 1)$, define $\phi(\theta, \psi) = \theta - \psi/r(\theta)$. The following proposition characterizes the (second-best) effort level that is forthcoming under the optimal contract. (Propositions are proved in the Appendix.)

PROPOSITION 1: *Under assumptions* (A1)–(A8), *the solution to $P\text{-}A'$ will specify the following effort supply*:

$$\hat{e}(\theta) = e^*[\phi(\theta, \psi)]$$

for $\psi = 1 - \gamma^{-1}$, where $\gamma \geq 0$ is the shadow price on the financing constraint.

[8] (Barlow and Proschan [1, p. 79].) Assumptions on the distribution of θ can be interpreted as assumptions on the distribution of output given an effort level, since output $= \theta e$.

COROLLARY: *For all $e^*(\theta) > 0$, the solution $\hat{e}(\theta)$ to P-A′ satisfies*

$$\hat{e}(\theta) < e^*(\theta) \quad \text{if} \quad \theta < \bar{\theta},$$

$$\hat{e}(\bar{\theta}) = e^*(\bar{\theta}) \text{ otherwise.}$$

Proposition 1 states that the manager acts in each state θ as if he or she owned the project but as if the state were $\phi(\theta, \psi)$ rather than θ. As a corollary, the effort supplied is *less* than under the complete contract in every state except the highest.

This result, that even with a sharing rule not constrained to be piece-wise linear the first-best effort is unattainable, contradicts the claim of Haugen and Senbet that a single call option can resolve the agency problem of perquisite choice.

The deficiency in effort as compared to the first-best but unattainable $e^*(\theta)$ is due to the externality imposed by outside financing. The manager is forced to share the marginal return to any extra effort input in the project with the security holders.

Note that the bias in effort is related to the constant shadow price on the financing constraint (3) and to the *hazard rate* function $r(\theta) = h(\theta)/[1 - H(\theta)]$. Clearly, the greater the need for outside financing and the larger the shadow price on the financing constraint, the greater the agency problem. To understand why the hazard rate enters as a determinant of realized agency costs, consider (heuristically) the effect of raising the residual return to the manager, $M'(\theta) \equiv 1 - S'(\theta)$, in some state of the world θ, while leaving it unchanged in other states. The benefit of doing so is to increase the manager's incentive to exert effort when θ is realized, a benefit that is proportional ex ante to $h(\theta)$, the probability that the state θ is realized (and the numerator of the hazard function). But an effect of increasing $M'(\theta)$ in the state θ is to decrease $S(\cdot)$ *in all states higher than* θ. This decrease must be offset by additional financing and therefore has a cost proportional to the shadow price on the financing constraint and to $1 - H(\theta)$, the probability of higher states (and the denominator of the hazard function).

Exactly the same reasoning explains the result, proved below, that efficiency requires a higher residual return for the manager in good (higher output) states than in bad states. The incentive benefit for greater residual return exists in both cases, but the shadow cost—through the financing constraint—of decreasing the total payment to the market in *all higher* states is greater in bad states than in good states. In the highest state $\bar{\theta}$, this shadow cost is zero, and the manager receives the full residual return. At the lower limit, $M(x) = 0$ for all output below the (endogenous) minimum possible output since the efficiency benefits of positive residual return are zero.[9]

PROPOSITION 2: *The solution to P-A satisfies*

$$S_x(F[e^*(\bar{\theta}), \bar{\theta}]) = 0,$$

[9] Note that the reason that efficiency requires a higher residual return to the manager in a higher state is not that the manager's productivity is high in that state. This incentive is exactly offset by the fact that output—and hence the shadow price of increasing the manager's total payment in that state—is also higher under stochastic constant returns to scale.

$$S(x) = x \quad for \quad x \leq F[e(\underline{\theta}), \underline{\theta}],$$

where $e(\theta)$ is the effort supply under $S(\cdot)$.

Thus, the slope of the optimal sharing rule, S_x, is unity for sufficiently low output and zero at the highest output. An immediate implication of Proposition 2, from the discussion in the last paragraph of Section I, is our main result.

COROLLARY: *The optimal contract is implementable in the limit by* (*and only by*) *an ownership structure that includes call options in insider portfolios.*

In the most general case, one might expect that S_x is not monotone in x and therefore that the efficient ownership structure includes call options in outsider portfolios as well. Warrants or convertible debt, which are observed, could equally play the role of call options held in outside portfolios and written by insiders.

Under two reasonable assumptions, however, the residual return to the security market decreases *monotonically* in output:

(A9) The manager's disutility of effort function $v(e)$ satisfies $v''' > 0$.

The distribution $H(\theta)$ must satisfy

(A10) $1/r(\theta) \equiv [1 - H(\theta)]/h(\theta)$ is concave in θ.[10]

Assumption (A9) is consistent with the idea that there is a positive level of effort, $\bar{e}$, that cannot be exceeded. Closer to $\bar{e}$, an additional unit of effort is more costly for the manager—so much so that the additional wealth necessary to compensate the manager for incurring the extra effort is a convex function of effort. Assumption (A10) is a regularity condition satisfied by a number of distributions including, for example, the normal and the uniform distributions.

PROPOSITION 3: *Under assumptions* (A1)–(A10), *the optimal solution $S(x)$ to the problem P-A is concave.*

In sum, under plausible conditions efficiency requires that the manager's residual claim on output be an increasing function of output and, therefore, that the limiting efficient ownership structure contain call options held only by the manager.

IV. Conclusions and Extensions

This paper analyzes the problem of efficient ownership structure under the simplest set of conditions giving rise to an agency problem. When a manager-entrepreneur cannot fully finance investment with riskless debt, the residual return to investment must be shared with security holders. The optimal ownership structure solves the problem of dividing residual return across states between the manager and the security market, subject to the constraint that sufficient capital be raised. We argue that the manager's proportionate share of residual return is greater in the high-output states because the shadow cost—through the

[10] An equivalent assumption could be made on the distribution of output, which is observable, conditional upon any effort level.

financing constraint—of increasing this share is lower in the high-output states. This provides an efficiency explanation of call options in managerial compensation packages.

A number of issues and extensions to the analysis are left open. The case of the manager's decision prior to θ was investigated and found to yield a discontinuous sharing role (specifically, a "bang-bang" solution to the control problem).[11] This we regard as support for our assumption of asymmetric information in explaining realistic financial structures. An open issue is the characterization within the model of the optimal capital structure in the real-world case of a finite number of securities. As one element of such a characterization, we show in an earlier version of this paper that (under suitable regularity conditions) the debt issued in the optimal ownership structure will always be risky. Debtholders—not just equityholders—are, therefore, residual claimants in some states, and the optimal financial structure involves an interior tradeoff between the agency costs of equity and those of debt.

The extension of the analysis to the case of risk-averse agents would be important, our arguments in favor of the risk-neutrality assumption (footnote 2) notwithstanding. Our conjecture is that under assumptions on preferences that generate a linear sharing rule when only risk sharing is at stake—that the manager's utility over wealth and a representative outsider's utility both belong to the class of HARA utility functions—the optimal sharing rule under our full set of assumptions would still be convex. The risk-sharing advantages of linearity would be balanced, at the margin, with the incentive advantages of increased convexity that we have documented here.[12]

A more important extension would be to introduce the additional agency problem associated with the management decision on the selection of investments. A natural assumption would be that this decision is taken immediately after the contract is struck but before the state of the world is realized. Under our risk-neutrality assumption, an all-equity insider portfolio would solve the agency problem associated with investment project selection; this portfolio would leave the manager's implicit prices or weights across states, in the valuation of projects, exactly proportional to the prices representing the collective interests of the manager and outside securityholders. But the optimal ownership structure, we conjecture, would balance at the margin this benefit of a proportional sharing rule with the advantages of convexity that are demonstrated in this article. The optimal rule would still be convex and would therefore include call options.[13]

[11] The details of this solution are in a technical appendix available from the authors.

[12] In this case of a risk-averse manager, the minimum possible share of profit to the manager may be positive. This minimum could be interpreted as a wage.

[13] The combination of the two extensions suggested may yield a proof of the efficiency of call options based on the agency problem associated with the manager's investment choice rather than the effort or perquisite choice. Jensen and Meckling (p. 353) argue essentially that a contract including wages may leave a manager's claim on residual returns higher in states of bankruptcy than in favorable states, and that this may result in a bias in the selection of investment or in the operating characteristics of the firm. Stock options mitigate this bias by giving the manager a claim on the upper tail of the income distribution.

Appendix

Proof of Lemma: We show constructively that an arbitrary piece-wise linear sharing rule can be implemented by an appropriately chosen financial policy. Let M be an arbitrary sharing rule (for the manager) with linear segments of slope between 0 and 1. M is characterized by a vector $(\lambda_0, \cdots, \lambda_n, x_0, \cdots, x_n)$, with $0 < \lambda_i < 1$, such that

$$M(x) = \lambda_0 x + \textstyle\sum_{i=1}^{n} (\lambda_i - \lambda_{i-1})\text{Max}(0, x - x_{i-1}). \tag{A1}$$

Consider the following financial policy:

1. Issue an amount $\lambda_0 x_0$ of debt to the manager and an amount $(1 - \lambda_0)x_0$ of debt to the outside market.
2. Issue $\lambda_1 N$ shares of equity to the manager and $(1 - \lambda_1)N$ shares to the outside market, where N is an arbitrary but large number.
3. For $i > 1$: if $\lambda_i > \lambda_{i-1}$, have the outside market write $(\lambda_i - \lambda_{i-1})N$ call options at an exercise price $(x_{i-1} - x_0)/N$, to be allocated to the manager. Each call option is the right to buy one share, ex post, from the writer at the exercise price. If $\lambda_i < \lambda_{i-1}$, have the manager write $(\lambda_{i-1} - \lambda_i)N$ calls to be sold to the market.

Since none of the call options is a warrant, the stock of the firm is not diluted when they are exercised. The ex post value of a share as a function of the profit x of the firm is, therefore, $EQ(x) = \max[0, (x - x_0)/N]$. The realized value of a call option with exercise price $(x_{i-1} - x_0)/N$ is

$$\max[0, EQ(x) - (x_{i-1} - x_0)N]$$
$$= \max\left[0, \max\frac{(0, x - x_0)}{N} - \frac{(x_{i-1} - x_0)}{N}\right] = \max[0, (x - x_{i-1})/N].$$

The value of the call writer's position is, of course, the negative of this. Adding the values of securities in the manager's portfolio yields the right-hand side of (A1). Q.E.D.

Proof of Proposition 1: The proof parallels the proof of Theorem 1 in Farmer [3]. The solution to $P\text{-}A'$ involves the optimization of a Hamiltonian subject to the non-negativity constraints $e(\theta) \geq 0$ and $e_\theta \geq 0$. The second of these constraints follows from the second-order conditions for incentive compatibility (see Farmer [3] for details). The Hamiltonian takes the form

$$H = [e(\theta) - S(\theta) - V(e)]h(\theta) + \gamma[S(\theta) - C + W_0] + p_1(\theta - V_e)e_\theta + p_2 e_\theta,$$

where p_1 and p_2 are co-state variables associated with $S(\theta)$ and $e(\theta)$. We treat e_θ as a control variable; $S(\underline{\theta})$, $S(\overline{\theta})$, $e(\theta)$, and $e(\overline{\theta})$ as control parameters; and $S(\theta)$ and $e(\theta)$ as state variables. The first-order incentive compatibility constraint has been used to eliminate S_θ from the Hamiltonian ($S_\theta = (\theta - V_e)e_\theta$). Optimal choice of e_θ implies

$$p_1(\theta - V_e) + p_2 = 0. \tag{A2}$$

The Euler-Lagrange equations take the form

$$\frac{dp_1}{d\theta} = (1 - \gamma)h(\theta), \tag{A3}$$

$$\frac{dp_2}{d\theta} = -h(\theta)(\theta - V_e) + p_1 V_{ee} e_\theta, \tag{A4}$$

where γ is the shadow price of the financing constraint. Differentiating (A2) using (A3) and (A4) yields

$$\theta = V_e + \frac{p_1}{\gamma h}. \tag{A5}$$

The transversality conditions to $P\text{-}A'$ yield

$$p_1(\bar{\theta}) = 0,$$

$$p_1(\underline{\theta}) = Q_1 \geq 0, \tag{A6}$$

where Q_1 is strictly positive if the wealth constraint is binding. (In this case $S(\underline{\theta})$ cannot be freely chosen and the control problem becomes one with a fixed initial condition.) Integrating (A3) yields

$$p_1(\theta) = p_1(\underline{\theta}) + (1 - \gamma)H(\theta), \tag{A7}$$

which implies, together with (A6),

$$(\gamma - 1) = Q_1 \geq 0. \tag{A8}$$

Now define $\psi \equiv (\gamma - 1)/\gamma$ and note that (A8) implies $\psi \in [0, 1]$. Then we may use (A6)–(A8) to show that

$$\frac{p_1(\theta)}{\gamma h(\theta)} = \frac{(\gamma - 1)}{\gamma} \frac{(1 - H(\theta))}{h(\theta)} = \frac{\psi}{r(\theta)}. \tag{A9}$$

But then (A5) yields

$$\left[\theta - \frac{\psi}{r(\theta)}\right] = V_e(e), \tag{A10}$$

which is the first-best optimal condition with θ replaced by $\theta - \psi/r(\theta)$. Q.E.D.

Proof of Proposition 2: Incentive compatibility requires that

$$\text{Max}_m\ \theta e(m) - S(m) - V[e(m)] \tag{A11}$$

occurs at $m = \theta$ for all θ. (m is the message that the manager conveys to the market.) First-order conditions for (A11) imply

$$\theta e_\theta - S_\theta - V_e e_\theta = 0. \tag{A12}$$

By totally differentiating (A12) and comparing the result with the second-order conditions for (A11), it follows that $e_\theta \geq 0$ if the contract is incentive compatible.

From this it follows that, along any incentive-compatible contract revenue, $x \equiv e(\theta) \cdot \theta$ will be increasing in θ, and so we can describe the state associated with any observed revenue level as a function $\theta(x)$. The sharing rule $S'(\theta)$, which solves P-A', will implicitly define the function $S(x)$, which solves P-A, and the derivative S_x will be given by the chain rule

$$S_x = S'_\theta \theta_x. \tag{A13}$$

By using (A12) and (A10), it follows that

$$S'_\theta = \frac{\psi \hat{e}_\theta}{r(\theta)}, \tag{A14}$$

where $\hat{e}_\theta(\theta)$ is the derivative of the solution to P-A' for effort supply, $\hat{e}(\theta) \equiv e^*[\phi(\theta, \gamma)]$.

θ_x is found by differentiating the function $x = \hat{e}(\theta)\cdot\theta$ and using the inverse function theorem

$$\theta_x = \frac{1}{\hat{e} + \theta \hat{e}_\theta}. \tag{A15}$$

Combining (A14) and (A15) yields

$$S_x = \frac{\psi \hat{e}_\theta}{r(\theta)} \frac{1}{(\hat{e} + \theta \hat{e}_\theta)}. \tag{A16}$$

But, since

$$\frac{1}{r(\bar{\theta})} = \frac{1 - H(\bar{\theta})}{h(\bar{\theta})} = 0,$$

it follows that $S_x[\hat{e}(\bar{\theta})\cdot\bar{\theta}] = 0$.

The second part of Proposition 2 follows directly from our assumption that limited liability is a binding constraint in at least one state. Q.E.D.

Proof of Proposition 3: $S(x)$ is concave if $S_{xx} \leq 0$. But we can write S_x in the form (from (A16))

$$S_x = \frac{\psi}{r(\theta)[\theta(x) + g(x)]}, \tag{A17}$$

where

$$g(x) \equiv \frac{\hat{e}[\theta(x)]}{\hat{e}_\theta[\theta(x)]}. \tag{A18}$$

Now

$$S_{xx} = \frac{-\psi}{D^2}[r(\theta_x + g_x) + (\theta + g)r_\theta \theta_x],$$

where D is the denominator of (A17).

Differentiating (A18) gives

$$g_x = \frac{\hat{e}_\theta^2 \theta_x - \hat{e}\hat{e}_{\theta\theta}\theta_x}{\hat{e}_\theta^2}. \tag{A19}$$

Notice that $g_x > 0$ if $\hat{e}_{\theta\theta} < 0$ and that this is a sufficient condition to guarantee $S_{xx} < 0$ since $\theta_x > 0$ (see the proof of Proposition 2) and $r_\theta > 0$ (by assumption (A8)). By definition,

$$\hat{e}(\theta) \equiv e^*[\phi(\theta, \gamma)]. \tag{A20}$$

But by assumption (A9) ($v''' > 0$) and assumption (A10) (concavity of $1/r(\theta)$), $e^*(\cdot)$ and $\phi(\cdot, \gamma)$ are concave functions; hence, $\hat{e}_{\theta\theta} < 0$. Q.E.D.

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