Implicit Contracts with Asymmetric Information and Bankruptcy: The Effect of Interest Rates on Layoffs

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This paper develops a model in which a firm writes labour contracts with workers and debt contracts with creditors. Firms have more information than do the owners of the factors of production and they are also subject to limited liability. We show that if the limited liability constraint is binding then the employment level is inefficient relative to a situation of symmetric information. The firm is then embedded into a partial equilibrium model in which the real rate of interest is exogenously determined. We show that increases in the real rate of interest increase the inefficiency of the optimal employment contract and lead to layoffs in more states of nature than would occur at lower real interest rates.

1. INTRODUCTION

Recent literature in the area of implicit contracts, has shown considerable interest in the assumption of asymmetric information (henceforth A.I.). Papers by Azariadis (1982), Green and Kahn (1982) and Grossman and Hart (1981) have all shown that A.I. will interfere with optimal risk sharing, but although these models produce non-Walrasian outcomes it is not clear that they are capable of explaining the observed variations in employment over the business cycle. In the absence of a link between the level of employment and other macroeconomic variables, these new versions of contract theory are open to the charge that they are presenting a complicated explanation of frictional unemployment.

One potential link between variations in output, and variations in intertemporal prices, is suggested by models which explicitly allow for the possibility of bankruptcy. Sappington (1983) has shown that bankruptcy will generate inefficient outcomes even between risk neutral agents and Farmer (1984) has shown that if Sappington type contracts are incorporated into a macroeconomic structure then the resulting model predicts that bankruptcies will vary countercyclically over the business cycle.

Although bankruptcy is important, however, it is unlikely that bankruptcies alone will account for all of the output loss during a recession. The purpose of this paper is to show that the assumptions of A.I. and limited liability will generate a model in which the employment level is inefficient and in which the magnitude of this inefficiency varies systematically with the expected real rate of interest. This model predicts that the observed increase in the incidence of layoffs during recessions is directly attributable to the effects of high interest rates on aggregate supply.

Some evidence of the phenomenon to be explained is presented below.¹
Figure 1 is a scatter diagram of the ex post real rate of return, realized in period \( t \), against a measure of deviations from potential output.\(^2\) A high G.N.P. gap is associated with a low level of G.N.P. and vice versa. The relationship in Figure 1 is plotted using U.S. annual data although similar relationships exist in Canadian quarterly data, in Canadian annual data, and between unemployment and a real interest rate in U.S. annual data.\(^3\) The argument which will be developed in this paper suggests that Figure 1 should be interpreted as an aggregate supply relationship and that high interest rates cause firms to write contracts in which workers are laid off in more states of nature than would otherwise be the case.\(^4\)

The paper is structured in the following way. In Sections 2 and 3 we outline a model of a firm, which hires factors in competitive contract markets. We assume that firms have superior information and that they are subject to limited liability. Section 4 shows that the design of a contract may be set up as an optimal control problem and Section 5 derives the properties of the optimal contract under the constraints imposed by asymmetric information. In Section 6 we embed this firm into a competitive equilibrium model in which the real rate of interest is taken as exogenous and we consider the comparative static exercise of varying the interest rate. We show that if the interest rate increases, then the firm will write factor contracts which exhibit lower levels of employment than would otherwise be the case. Finally, Section 7 presents a short conclusion.

2. STRUCTURE OF THE PRODUCTION TECHNOLOGY

Consider a firm which produces a single commodity \( y \), using the services of \( N \) workers and one machine. For the purposes of Sections 2–5 \( N \) will be taken as given, although this assumption will be relaxed in Section 6. The technology is assumed to take the form

\[
y = f(n)S,
\]

\[
f' > 0, \quad f'' < 0,
\]
\[ S \in [\bar{S}, \bar{S}] \subset \mathbb{R}_+ , \]  

\[ p(S \leq x) = H(x) = \int_{s-S}^x h(S) dS. \]

(2.1) is a stochastic production function which is assumed to be concave and twice continuously differentiable. Workers are assumed to be identical and, ex post, each worker supplies \( n/N \) hours of labour. \( n \) therefore, represents the total hours of labour input. \( S \) is a non-negative random variable, contained in the interval \([\bar{S}, \bar{S}]\) which represents productivity shocks which are unobservable to the factors of production, but observed by the firm. \( h(S) \) and \( H(S) \) represent the probability density function and the cumulative density function of \( S \). For some purposes it will be convenient to define the hazard rate \( r(S) \).

\[ r(S) = \frac{h(S)}{1 - H(S)}. \]  

It will be assumed that firms assemble the factors of production, ex ante, in a production location. We impose a sufficiently high ex post mobility cost so that no movement may take place in the production period—firms must bid for the factors of production, ex ante, by offering them contracts. An employment contract is defined as a wage payment \( W(S) \) and an employment level \( n(S) \) for each value of the state of nature, \( S \). Similarly a debt contract is defined to be a payment \( B(S) \).

### 3. CONSTRAINTS UPON THE CHOICE OF A CONTRACT

#### 3.A. Incentive compatibility

The assumption that only the firm may observe the state of nature imposes a constraint on the set of feasible contracts. It will be convenient to define the cost to the firm which is borne in any state of nature—this will be denoted as \( C(S) \) where

\[ C(S) = NW(S) + B(S). \]  

Since the factors of production are unable to observe the state of nature, \( S \), they must rely on the firm to reveal this state ex post and it may be shown that there will be no loss to restricting attention to that class of contracts for which telling the truth is in the firm’s own best interest.\(^5\) If we let \( m \) denote the state of nature which is revealed by the firm then an incentive compatible contract will be one for which the solution to

\[ \max_m Sf(n(m)) - C(m) \quad \text{s.t. } 0 \leq n(m) \leq N \]  

occurs at \( m = S \).

Assume initially that \( n(S) \) and \( C(S) \) are continuous functions. Then the first-order conditions to this problem imply that a factor supply contract \( \{C(S), n(S)\} \) must obey the constraint

\[ Sf_n n_S - C_S = 0 \]  

at all values of \( S \) for which the inequality constraints are non-binding where \( n_S, C_S \) is the derivative of \( n(m) \), \( C(m) \) evaluated at \( S = m \). From the second-order conditions it may be shown that for any contract \( \{C(S), n(S)\} \) to represent a maximum it is both necessary and sufficient that (3.3) holds and

\[ n_S \geq 0 \quad \text{where } n_S = \frac{dn}{dS} . \]
The inequality (3.4) follows from comparing the total differential of (3.3) with the differential of the second-order condition to (3.2), evaluated at \( S = m \).

One method that one might use to find an optimal contract would be to set up an optimal control problem in which \( C(S) \) and \( n(S) \) appear as state variables. However, this approach is incorrect, since one cannot guarantee \textit{a priori} that \( C(S) \) and \( n(S) \) are continuous functions. One way around this problem is to define a new function \( \Pi(S) = Sf(n(S)) - C(S) \) which must be continuous in \( S \) if the contract is to be incentive compatible.\(^6\) \( \Pi(S) \) is the profit of the firm as a function of the true state of nature, if it follows a truth telling strategy. We use \( \Pi(S) \) as the only state variable and reformulate the incentive compatibility constraint, (3.3), in terms of \( \Pi(S) \):

\[
\Pi_S = f(n(S)). \tag{3.3'}
\]

Similarly, the second-order condition (inequality (3.4)) may be restated in the following form:

\[
n(S) \succeq n(S') \quad \text{for all } S \succeq S'. \tag{3.4'}
\]

3.B. \textit{The factor supply constraint}

The owners of the factors of production consist of two groups, workers and creditors, both of which will be characterized by a representative agent. In order to attract either labour or capital to the production location the firm must bid for factors, ex ante, in a competitive contract market. This will impose a further constraint on the set of employment contracts and on the set of debt contracts which we shall aggregate in order to describe a single “factor supply constraint”.

Turning first to the representative worker, it will be assumed that utility may be represented by the twice differentiable von-Neuman–Morgenstern utility function:

\[
U^W = W(S) - V\left(\frac{n(S)}{N}\right) \tag{3.5}
\]

where

\[
V_n > 0, \quad V_{nn} > 0. \tag{3.6}
\]

All workers are assumed to be treated identically and so variations in employment take the form of variations in hours—we shall therefore be unable to distinguish between unemployment and underemployment. \( n(S) \) represents the total hours supplied to the firm of which each worker supplies \( n(S)/N \); \( W(S) \) represents the associated wage which he or she receives.

Notice that (3.5) is a special case of the utility function \( H(W(S) - V(n(S)/N)) \) which has been frequently used in the literature.\(^7\) We assume that \( U^W \) is linear in \( W(S) \), which imposes risk neutrality with respect to income risk.

The owners of capital will also be assumed to be risk neutral, with a collective utility function of the form

\[
U^b = B(S). \tag{3.7}
\]

If workers have alternative ex ante opportunities which offer at least \( \bar{u} \) in expected utility terms, then the expected value of taking employment with the firm must be at least as great as this amount. Similarly, if the owners of capital can earn a gross expected return of \( R \) from other sources then the expected value of a debt contract must be at least as great as \( R \). Collectively, these requirements impose the following constraint on
the firm:

$$E \left[ C(S) - NV \left( \frac{n(S)}{N} \right) \right] \geq N\bar{u} + R$$

(3.8)

where \( E(\cdot) \) is the expectation operator associated with the distribution of \( S \). (3.8) will be referred to as the factor supply constraint.

Notice that, by assumption, workers and creditors are risk neutral and so they are indifferent to the distribution of payments across states of nature. For any factor supply contract \( \{C(S), n(S)\} \) there will, therefore, be a continuum of feasible wage and employment contracts.

3.C. Limited liability

Much of the existing contract literature focuses on the role of A.I. in disrupting optimal risk sharing, and it is by now well known that incentive compatible contracts lead to underemployment if firms are risk averse. In this paper we assume that firms are risk neutral, but that payments in some states of nature are bounded—this assumption introduces an artificial concavity into the utility function of the firm which leads it to act like a risk averse firm. This assumption proves to be interesting because the "degree of concavity" of the firm's utility function and hence the efficiency of its employment contracts can be shown to vary as this constraint becomes more or less binding.

Following Sappington (1983) we refer to a situation of bounded state dependent payments as one of limited liability. Limited liability will be modelled by assuming that the firm has collateral, \( \delta \), which may represent the scrap value of the capital stock, and it may also include the financial wealth or the personal guarantees of the owners of the firm. In any state of nature the payments of the firm to the factors of production can be no greater than the total amount which is produced, plus the collateral of the firm

$$\Pi(S) + \delta \geq 0.$$  

(3.9)

The limited liability constraint (3.9), together with the incentive compatibility constraint and the factor supply constraint serve to restrict the set of feasible contracts. In the following section we examine the choice of a Pareto Optimal contract from amongst this limited set.

4. SETTING UP THE PROBLEM

If firms bid for capital and for workers in an ex ante contract market, then each firm is forced to offer a competitive contract to the representative factor supplier. Each firm is assumed to design a contract \( \{C(S), n(S)\} \) which maximizes its own expected profits subject to the constraints which were outlined in the previous section—this problem may be formally stated in the following way:

Let \( b = \{\Pi_0, \Pi_1\} \) be control parameters, let \( n(S) \) be a control variable and let \( \Pi(S) \) be a state variable.

**Problem P**

$$\max_{n, n(S)} \int_{s=S}^{S} \Pi(S) h(S) dS$$

(4.1)
such that

$$\int_{S}^{S} \left[ Sf(n(S)) - \Pi(S) - NV \left( \frac{n(S)}{N} \right) \right] h(S) dS \equiv N\bar{u} + R,$$

(4.2)

$$\frac{d\Pi}{dS} = f(n(S)),$$

(4.3)

$$\Pi(S) = \Pi_0,$$

(4.4i)

$$\Pi(S) = \Pi_1,$$

(4.4ii)

$$\delta + \Pi_0 \geq 0,$$

(4.5)

$$n(S) \geq 0,$$

(4.6)

$$n(S) \geq n(S') \text{ for all } S \geq S',$$

(4.7)

*Remark 4.1.* The constraint (4.7) is a necessary condition for a solution to $P$ to be incentive compatible. It is very difficult to impose this constraint directly; however, we are able to impose a condition on the density function $h(S)$ which guarantees that the solution to $P$, $\hat{n}(S)$, obeys the inequality (4.7) for all values of $S$. This condition is introduced below as regularity condition $R1$.

*Remark 4.2.* The bankruptcy constraint

$$\Pi(S) + \delta \geq 0$$

must hold at all values of $S$. But since $\Pi(S)$ is increasing in $S$, if the contract is incentive compatible, the constraint on the functional $\Pi(S)$ may be replaced by a constraint on the control parameter $\Pi_0$ which is described by equation (4.5).

Problem $P$ takes the same form as the optimal control problem of Bolza, to which a set of necessary conditions were provided by Hestenes (1966) and are reproduced in Takayama (1974). Our assumptions about the properties of the technology also guarantee that these conditions will be sufficient—and a theorem to this effect is provided in Takayama (1974 p. 660).

5. CHARACTERIZING THE OPTIMAL CONTRACT

Before proceeding to characterize the solution to $P$ it is useful to have a point of reference with which the Pareto optimal contract under asymmetric information and limited liability can be compared. A natural standard of comparison is the solution to the problem in the absence of these constraints, which we shall refer to as a First Best Optimal solution ($F.B.O.$).

*Definition 1.* A First Best Optimal contract, $\{n^*(S), C^*(S)\}$, ($F.B.O.$) is characterized by the conditions:

$$\left[ -Sf_n(n^*(S)) + V_n\left( \frac{n^*(S)}{N} \right) \right] n^*(S) = 0$$

(5.1)

where $n^*(S) \geq 0$ and $-Sf_n(n^*(S)) + V_n(n^*(S)/N) \geq 0$

$$E \left[ C^*(S) - NV \left( \frac{n^*(S)}{N} \right) \right] = N\bar{u} + R.$$

(5.2)
Condition (5.1) is the employment rule for an optimal contract which is defined by the familiar requirement the marginal rates of transformation should be equal in the activities of production and consumption in all states for which non-negativity of employment is satisfied. Equation (5.2) serves to define a set of optimal cost schedules. Since both parties are risk neutral, any schedule which satisfies the factor supply constraint is optimal because the agents are indifferent as to the allocation of payoffs across states.

We characterize the solution to P in Theorem 1. In order to keep the presentation simple we impose the following regularity conditions on the properties of the density function of $S$ and on the utility function $V(n(S)/N)$.

R1. $r_S(S) \geq 0$.

R2. $V_n(n/N) \to \infty$ as $n \to N$.

R1 stipulates that the hazard rate of $S$ should be non-decreasing. This is a property shared by many density functions, for example, "the Exponential, the Gamma and Weilbull with degrees of freedom parameter larger than 1, the Normal Distribution, Laplace, and the Uniform".9 R1 guarantees that the second order incentive compatibility constraint will not bind which simplifies the optimal employment rule by excluding a set of potential corner solutions (see Remark (4.1), above).

R2 guarantees that it will never be optimal for $n(S)$ to approach $N$ which ensures that the constraint $N - n(S) \geq 0$ will not bind.

\textit{Definition 2.} Define $\theta$ by:

$$\theta = S - \frac{\hat{\psi}}{r(S)}$$

(5.3)

where

$$\hat{\psi} = (\hat{\lambda} - 1)/\hat{\lambda}$$

(5.4)

and $\hat{\lambda}$ is the optimal value of the multiplier associated with constraint (4.2).

\textbf{Theorem 1.} Assume R1 and R2. Let $\{\hat{n}(s), \hat{N}(S)\}$ be the solution to P, and let $\hat{C}(S)$ be the cost schedule which is derived from this solution. Then:

1. If the bankruptcy constraint is non-binding, that is, if

$$Sf(n_0^*) + \delta \geq Nu + R + NV\left(\frac{n_0^*}{N}\right)$$

where $n_0^* = n^*(S)$

it follows that the optimal contract is F.B.O. and it satisfies:

$$\hat{n}(S) = n^*(S),$$

(5.5i)

$$\hat{C}(S) = Nu + R + NV\left(\frac{\hat{n}(S)}{N}\right) \in C^*(S).$$

(5.5ii)

2. If the bankruptcy constraint is binding, that is, if

$$Sf(n_0^*) + \delta < Nu + R + NV\left(\frac{n_0^*}{N}\right)$$
then the optimal contract is inefficient; it satisfies:

\[ \hat{n}(S) = n^*(\theta), \]  
\[ \hat{C}(S) = \delta + f(S)\hat{n}_0 + NV\left(\frac{\hat{n}(S)}{N}\right) - NV\left(\frac{\hat{n}_0}{N}\right) + \int_{x=T}^{S} \frac{\hat{\psi}f_n\hat{n}_n}{r(x)}\,dx \in C^*(S), \]  
where \( \theta < S \) for all \( S < \bar{S} \)

\[ \theta = S \quad \text{if } S = \bar{S} \quad \text{and} \quad \hat{n}_0 = \hat{n}(\bar{S}) \]

further; \( T \) is defined by:

\[ T = S \quad \text{if } \hat{n}(S) > 0 \quad \text{for all } S \in [S, \bar{S}] \]
\[ T = \max \{S|\hat{n}(S) = 0\} \quad \text{otherwise.} \]

For the proof of Theorem 1 see Appendix A.

Remark 5.1. Efficiency of the cost schedule is not difficult to achieve because of the assumption of risk neutrality. Incentive compatibility will pick out one of the set of cost schedules which satisfy (5.2) by requiring that the slope of the optimal cost schedule should be tied to the slope of the employment schedule at all points of differentiability, that is, \( C_S = Sf_n\hat{n}_S \).

Remark 5.2. There may be a state \( T^* \leq T \) with the property that \( n^*(S) = 0 \) for all \( S \leq T^* \). For states below \( T^* \) unemployment is efficient because the marginal product of labor falls short of the marginal disutility of work and the non-negativity constraint, \( \hat{n}(S) \leq 0 \), is binding. Part (2) of Theorem 1 implies that for all \( S > T^* \) employment will be lower, if the bankruptcy constraint is binding, than it would be in a F.B.O. contract.

Remark 5.3. The regularity conditions R1 and R2 simplify the form of the optimal employment rule but they are not essential for any of our results.

It is a well known result from the moral hazard literature that A.I. contracts between risk neutral agents can be made first best by giving the individual with superior information all residual risk. This is the contract which is described by (5.5); these conditions guarantee a constant utility level to the factors of production in every state. In order for a contract of this type to be feasible, however, the firm must have sufficient collateral to be able to meet this guarantee even in the worst possible state. If this condition is violated then in order to make up for the possibility of loss in bad states of nature the factors of production must receive a bonus in favourable states. However, this bonus payment interferes with the firm’s ex post employment decision by raising the marginal cost of employing an additional unit of labour above the disutility of employment. The profit maximizing firm makes its ex post employment decision by equating the marginal cost of employment to the marginal revenue product—but since the marginal cost schedule is steeper in an inefficient contract than in a F.B.O. contract, the firm will hire less labour. The multiplier \( \hat{\psi} \) measures how far the optimal contract deviates from the F.B.O. solution. (The firm pursues the same employment policy as would occur in a first best contract if the state of nature was not \( S \) but \( (S - \hat{\psi}/r(S)). \) The value of \( \hat{\psi} \) will depend upon the parameters of the problem and variations in these parameters will cause variations in the efficiency of the contract. Since the inefficiency of an A.I. contract arises from the inability of the
firm to guarantee factor payments, it seems plausible that an increase in the value of outside opportunities will cause the firm to write less efficient contracts because it will become less able to offer guarantees. This conjecture is proved in Theorem 2. Before stating this theorem, it is useful to introduce some notation to enable us to describe explicitly how employment in each state depends on the number of workers, \( N \), and on the efficiency parameter, \( \hat{\psi} \).

**Definition 3.** Let \( g(S, \hat{\psi}, N) = \hat{n} \) be the function which is implicitly defined by:

\[
\left( S - \frac{\hat{\psi}}{r(S)} \right) f_n(\hat{n}) = V_n \left( \frac{\hat{n}}{N} \right).
\]

**Theorem 2.** Let \( \hat{V} = N\hat{u} + R \). Assume (1) a feasible contract exists, (2) \( \hat{V} > Sf(n_0^*) + \delta - NV(n_0^*/N) \). Then \( d\hat{n}(S)/d\hat{V} < 0 \) for all \( S \in (T, \bar{S}) \).

For the proof of Theorem 2 see Appendix B.

In Theorem 2 we have shown that an increase in the firm's factor bill, holding constant \( \bar{u} \) and \( N \), will cause it to write a contract which exhibits lower employment levels than would otherwise be the case. However, this result does not tell us anything about the impact of the rate of interest on employment in an equilibrium model of the economy in which \( \bar{u} \) and \( N \) are determined endogenously. In order to establish a link between the rate of interest and the level of employment we must model the equilibrium response of an economy which is comprised of a number of firms each of which displays the behaviour described above.

6. A PARTIAL EQUILIBRIUM MODEL

The paper so far has dealt with a single firm which takes the number of workers in the labour pool as given. In this section, we embed this firm in an equilibrium model of the economy, and we allow the number of workers to be chosen endogenously.\(^{10}\) The model is partial since we take the real rate of interest to be exogenously determined. It could be closed by embedding it in an overlapping generations model similar to that of Diamond (1965), or alternatively it could be treated as a small open economy which takes the real rate of interest from the rest of the world.

We assume that the technology of the firm exhibits constant returns to scale. Since it is well known that the size of the firm is indeterminate in such models, we choose the size of a unit of capital to equal unity. The collateral of the firm, \( \delta \), may then be interpreted as the fraction of this unit of capital which remains after depreciation. We then enquire into the optimal choice of \( N \) for the firm in perfect competition. If the number of workers in the economy is determined exogenously, then the choice of \( N \) for the individual firm will implicitly determine the number of firms. We treat \( N \) as a continuous variable.

The model we consider has two periods. Firms bid for labour and capital in period 1 taking \( R \) and \( \bar{u} \) as given. Since \( S \) is observed only by firms, they must write contracts with the factors of production of the type discussed in Section 5. In period 2 \( S \) is revealed to the firm and the employment level is determined subject to the provisions of the contract.

We now turn to examining how \( \bar{u} \) and \( N \) are determined in perfect competition. Our method is to derive two equilibrium conditions, and to show that these two equations may be solved simultaneously for \( \bar{u} \) and \( N \). First we examine how the profits of the firm will depend on the size of its work force, \( N \), and on the total factor bill, \( N\bar{u} + R \).
We may define the value function of the firm to be the maximum expected net present value as a function of the parameters \( N \), and \( N\bar{u} + R \)—i.e., for any given values of \( N \), \( \bar{u} \) and \( R \) a firm which solves the maximization problem described in Section 5 will have a value described by the function

\[
J = J(N, N\bar{u} + R).
\]  

(6.1)

By applying the envelope theorem to the value function it is possible to derive the partial derivatives which take the form\(^{11}\)

\[
J_1 = \lambda \int_S^\hat{S} \left[ \frac{\hat{n}}{N} V_a \left( \frac{\hat{n}}{N} \right) - V \left( \frac{\hat{n}}{N} \right) \right] h(S) dS > 0,
\]  

(6.2)

\[
J_2 = -\lambda < 0.
\]  

(6.3)

\( \lambda \) is the optimal value of the multiplier associated with the factor supply constraint and it is related to the “inefficiency” parameter by the identity \( \psi = (1 - 1/\lambda) \). Equation (6.3) is an analogue of the static result that a multiplier represents the value of relaxing a constraint. Equation (6.2) demonstrates that the comparative static exercise of increasing the number of workers in a firm, holding constant its fixed factor costs, will increase the value of the firm—this is because, ex post, it creates less disutility to maintain any given employment level if the workload is shared amongst a larger number of workers.

If each firm chooses its number of workers optimally then the following condition will hold in equilibrium

\[
J_1(N, N\bar{u} + R) + J_2(N, N\bar{u} + R)\bar{u} = 0.
\]  

(6.4)

Equation (6.4) states that each firm takes \( \bar{u} \) and \( R \) as given and it hires workers up until the point at which the marginal expected benefit of an additional worker is zero. If there is free entry and exit of firms then we have a second equilibrium condition which is given by the requirement that profits will be competed away to zero—this condition takes the form

\[
J(N, N\bar{u} + R) = 0.
\]  

(6.5)

Equations (6.4) and (6.5) determine the two endogenous variables \( \bar{u} \) and \( N \) as functions of the real rate of return in an analogous way to the determination of the wage and of the capital labour ratio in a simple perfect information problem. We assume that a feasible equilibrium contract exists and proceed to examine the comparative static responses of the system around such an equilibrium. Totally differentiating (6.4) and (6.5) yields

\[
\begin{bmatrix}
J_1 + J_2 \bar{u} \\
J_{11} + 2J_{12} \bar{u} + J_{22} \bar{u}^2
\end{bmatrix}
\begin{bmatrix}
J_2 N \\
J_{12} N + J_2 + J_{22} \bar{u} \bar{N}
\end{bmatrix}
\begin{bmatrix}
dN \\
d\bar{u}
\end{bmatrix}
= 
\begin{bmatrix}
-J_2 dR \\
-(J_{12} + J_{22} \bar{u}) dR
\end{bmatrix}.
\]  

(6.6)

By inverting (6.6), noting that \( J_1 + J_2 \bar{u} = 0 \), it is possible to derive the following comparative static results

\[
\frac{dN}{dR} = \frac{-(J_2)^2}{|H|} = \frac{-\lambda^2}{|H|} > 0,
\]  

(6.7)

\[
\frac{d\bar{u}}{dR} = \frac{1}{N} < 0.
\]  

(6.8)
\(|H|\) is the determinant of the matrix of coefficients on the L.H.S. of (6.6) which is given by

\[ |H| = -J_2N(J_{11} + 2\bar{u}J_{21} + J_{22}\bar{u}^2). \] (6.9)

By arguments similar to those used in the proof of Theorem 2 it may be shown that \(|H| < 0\), which implies that the firm will respond to an increase in the rate of interest by hiring more workers. This is a response which would also occur in a world of symmetric information although the magnitude of the response will be different if agents are asymmetrically informed. Since the size of the unit of capital at each firm is exogenously fixed the system attains an adjustment in the capital labour ratio by allocating more workers to each firm—if the economy-wide number of workers is fixed then this adjustment will drive some firms out of business.

The increase in the total number of workers, in response to a higher real interest rate, will have two related effects on the level of employment in any particular state. These effects are summarized below in Theorem 3.

**Theorem 3.** Assume (i) a feasible contract exists (ii) \(Sf(n^*_S) + \delta < N\bar{u} + R + NV(n^*_S/N)\). Then for \(S \in (T, \bar{S})\)

\[
\frac{d\hat{n}}{N} \frac{dN}{dR} = \left( g_N - \frac{\hat{n}}{N} \right) \frac{1}{N} \frac{dN}{dR} + \frac{1}{N} g_{\psi} \frac{d\hat{\psi}}{dR},
\] (6.10)

where

\[
\left( g_N - \frac{\hat{n}}{N} \right) \frac{1}{N} \frac{dN}{dR} < 0,
\] (6.11i)

\[
\frac{1}{N} g_{\psi} \frac{d\hat{\psi}}{dR} < 0.
\] (6.11ii)

**Proof.** (6.10) follows directly from totally differentiating \(g(S, \psi, N)/N\) w.r.t. \(R\). (6.11i) follows from Definition 3

\[
g_N = \frac{\hat{n}}{N} \left( \frac{V_{nn}f_n}{N} \left( \frac{V_{nn}f_n}{N} - V_{nn}f_{nn} \right) \right) < \frac{\hat{n}}{N}
\] (6.12)

since \(f_{nn} < 0\).

To establish (6.11ii) note that we can write the value of the firm in the form (see Appendix B)

\[
J = \int_{S-\bar{S}}^{\bar{S}} f(\hat{n})(1-H)\,dS - \delta.
\] (6.13)

But \(J = dJ = 0\), hence

\[
0 = \int_{S-\bar{S}}^{\bar{S}} f_n \left( g_{\psi} \frac{d\hat{\psi}}{dR} + g_N \frac{dN}{dR} \right)(1-H)\,dS.
\] (6.14)

But \(g_N > 0, dN/dR > 0\) and \(g_{\psi} < 0\) which implies that \(d\hat{\psi}/dR > 0\). ||

Theorem 3 establishes that the fraction of workers unemployed will rise (in states of positive employment) in response to an increase in the rate of interest. There are two sources of this effect one of which would be observed in a world of symmetrically informed agents. The first effect arises because there are more workers associated with each machine
and so the marginal unit of employment is less productive in any given state of nature. The second effect arises only in a contract between asymmetrically informed agents. As a result of an increase in the interest rate, the equilibrium ex ante wage $\bar{u}$ will fall. But the total value of ex ante factor opportunities $N\bar{u} + R$ will rise as the fall in the wage is offset by the increase in interest costs and by an increase in the number of workers

$$\frac{d}{dR}(N\bar{u} + R) = \left(N\frac{d\bar{u}}{dR} + 1 + \bar{u}\frac{dN}{dR}\right) = \bar{u}\frac{dN}{dR} > 0 \quad (6.15)$$

the second equality and the inequality follow from (6.8) and (6.7) respectively. The firm is forced to pay a higher expected return to its factors of production but it has limited collateral and so it must increase the bonus payment in good states of nature i.e. $\psi$ rises. But a secondary effect of this increase is to drive a bigger wedge between marginal rates of transformation thus making the contract less efficient.

7. CONCLUSION

This paper has attempted to explain the observed incidence of layoffs during recessions using a model where all agents are rational and prices are ex ante competitive. Our results show that, in a world of asymmetric information, variations in the rate of interest may manifest themselves as variations in the incidence of layoffs. Further, the incidence of layoffs is excessive, in the sense that, there is more unemployment than one would observe in a world in which agents were symmetrically informed. One potential criticism of these results is that the existence of a perfect equity market for capital will remove the link between interest rates and employment. Implicit in the analysis, however, is the assumption that A.I. precludes the existence of such markets and so at least some portion of the firm must be financed by debt contracts which are not state dependent. There are a variety of ways of closing a model of this kind in order to make the interest rate endogenous. But, whichever method is used, the resulting structure will be one with incomplete markets in which there is potentially a role for government intervention.

APPENDIX A

Proof of Theorem 1. From Hestenes theorem (Takayama (1974), p. 651)—if $[\hat{\beta}, \hat{\Pi}(S)]$ is a solution to $P$, then there exist multipliers $\lambda$, $p(S)$, $q_1(S)$, $q_2(S)$ not vanishing simultaneously and a function:

$$L = H + q_1\Pi_0 + q_2n, \quad (A.1)$$

$$H = \left(\Pi + \lambda \left[Sf(n) - \Pi - NV\left(\frac{n}{N}\right)\right]\right)h + pf(n). \quad (A.2)$$

Note that a multiplier $p_0$ should be associated with the first term of the Hamiltonian in equation (A.2). We implicitly assume that a normality condition holds (see Takayama (1974), p. 612–613) which allows us to drop this additional variable.

First-order conditions for the maximization of $L$ w.r.t. $n$ imply:

$$\lambda(Sf_n - V_n)h + \hat{p}f_n + \hat{q}_2 = 0. \quad (A.3)$$

From the Kuhn–Tucker conditions it follows that $\hat{q}_2\hat{n} = 0$ and $\hat{q}_2 \geq 0$, $\hat{n} \geq 0$. Hence (A.3) may be written in the form

$$\left(-\left(S + \frac{\hat{p}}{\lambda h}\right)f_n + V_n\right)\hat{n} = 0 \quad (A.4)$$
where:
\[ \hat{n} \geq 0, \quad -\left( S + \frac{\hat{p}}{\lambda h} \right) f_n + V_n \geq 0. \]

The costate variable \( \hat{p} \) is obtainable by integrating the Euler–Lagrange equation (A.5) and using the transversality conditions (A.6) (Takayama, p. 660):
\[ \frac{d\hat{p}}{dS} = h(\hat{\lambda} - 1), \quad (A.5) \]
\[ \hat{p}(\bar{S}) = 0, \quad \hat{p}(S) = -\int_{S-\bar{S}}^{\bar{S}} \hat{q}_1(S) dS = -\hat{Q}_1 \leq 0. \quad (A.6) \]

Integrating (A.5) it follows that
\[ \hat{p}(S) = \hat{p}(\bar{S}) + (\hat{\lambda} - 1) H(S) \quad (A.7) \]
and using (A.6) and (A.7) one may establish that \( \hat{\lambda} - 1 = \hat{Q}_1 \geq 0 \). It then follows that
\[ \frac{\hat{p}(S)}{\lambda h(S)} = -\frac{(\hat{\lambda} - 1)}{\hat{\lambda}} \frac{(1 - H(S))}{h(S)} = \frac{\hat{\psi}}{r(S)} \quad \text{(A.8)} \]

where \( \hat{\psi} = (\hat{\lambda} - 1)/\hat{\lambda} \) and \( r(S) = h(S)/[1 - h(S)] \).

Substituting (A.8) into (A.4) we have:
\[ \left( -\left( S - \frac{\hat{\psi}}{r(S)} \right) f_n + V_n \right) \hat{n} = 0. \quad (A.9) \]

It follows by Definitions 1 and 2 that
\[ \hat{n}(S) = n^*(\theta). \quad (A.10) \]

Notice that R1 is sufficient to guarantee that \( \hat{n}(S) \) is non-decreasing and so the second-order condition for incentive compatibility, inequality (4.7), will not bind.

Define \( T \) by:
\[ T = S \quad \text{if } \hat{n}(S) > 0 \quad \text{for all } S \in [\bar{S}, \bar{S}] \]
\[ T = \max \{ S | \hat{n}(S) = 0 \} \quad \text{otherwise}. \]

From (A.9), \( \hat{n}(S) \) is defined by:
\[ V_n\left( \frac{\hat{n}}{N} \right) = \left( S - \frac{\hat{\psi}}{r(S)} \right) f_n(\hat{n}) \quad (A.11) \]
for \( S > T \) (\( \hat{n}(S) = 0 \) otherwise). Integrating (4.3), and using the definition of \( \Pi(S) \) we have
\[ \hat{C}(S) = Sf(\hat{n}) - \hat{N}_0 - \int_{x=T}^{\bar{S}} f(\hat{n})dx. \quad (A.12) \]

For \( S > T \) \( \hat{n} \) is differentiable and so we may integrate (A.11) by parts and combine the result with (A.12) to give:
\[ \hat{C}(S) = \hat{C}_0 + \int_{x=T}^{\bar{S}} \frac{\hat{\psi} f_n \hat{n}_x dx}{r(x)} + NV\left( \frac{\hat{n}(S)}{N} \right) - NV\left( \frac{\hat{n}_0}{N} \right) \quad (A.13) \]

where \( \hat{C}_0 = \hat{C}(\bar{S}) \), \( \hat{n}_0 = \hat{n}(\bar{S}) \). But from (A.6), (A.7) and the definition of \( \hat{\psi} \) if follows that \( \hat{\psi} > 0 \) if and only if \( \hat{Q}_1 > 0 \) hence from the Kuhn–Tucker conditions \( \hat{\psi} > 0 \) if and only if the bankruptcy constraint (equation (4.5)) is binding.
The constraint (4.2) always holds with equality since $\hat{\lambda} = 1 + \hat{Q} > 0$. If bankruptcy is not binding then $\hat{\psi} = 0$ and the constraint (4.2) determines $\hat{C}_0$. In this case the cost schedule is as described in Theorem 1, equation (5.5ii). If bankruptcy is binding then $\hat{C}_0$ is determined from the constraint (4.5) and the constraint (4.2) determines $\hat{\psi}$. In this case the optimal cost schedule is as described in Theorem 1, equation (5.6ii).

Finally, from the definition of $\theta$ it follows that $\theta < S$, if $\hat{\psi} > 0$, for all $S < \bar{S}$; $\theta = S$ if $S = \bar{S}$ since $1/r(\bar{S}) = 0$. 

APPENDIX B

Proof of Theorem 2. Let $J(\hat{\psi})$ be the value of an optimal contract to the firm

$$J(\hat{\psi}) = \int_{S=\bar{S}}^{S} [Sf(\hat{n}) - \hat{C}]hdS.$$

Integrating (B.1) by parts yields

$$J(\hat{\psi}) = [Sf(\hat{n}) - \hat{C}]H|_{S=\bar{S}}^{S} - \int_{S=\bar{S}}^{S} [f(\hat{n})]HdS$$

The second term of (B.2) follows from invoking the first-order incentive compatibility condition $C_S = Sf_n n_S$. (Note that the derivatives $C_S(S)$ and $n_S(S)$ are discontinuous at $S = T$. However the derivative of the profit flow $d/dS (Sf(n) - C)$ is continuous along an incentive compatible contract since $n$ is continuous in $S$.)

Now since $H(\bar{S}) = 0$, $H(\bar{S}) = 1$ and $\hat{C}(\bar{S}) = Sf(\hat{n}(\bar{S}))+\delta$ (from Theorem 2, Assumption 2) we can write (B.2) in the form

$$J(\hat{\psi}) = \int_{S=\bar{S}}^{S} (1 - H)f(\hat{n})dS - \delta.$$

Notice that

$$\frac{dJ}{d\psi} = \int_{S=T}^{S} (1 - H)f_n g_\psi dS$$

and by the envelope theorem:

$$\frac{dJ}{dV} = -\lambda < 0.$$

For $S \in (T, \bar{S})$:

$$\frac{d\hat{n}}{dV} = \frac{d\hat{n}}{d\psi} \frac{dJ}{d\psi} \frac{dJ}{dV}.$$

But from differentiating $g(\cdot)$ w.r.t. $\hat{\psi}$ if follows that $d\hat{n}/d\hat{\psi} = g_\psi < 0$ (see Definition 3) and hence (B.4), (B.5) and (B.6) imply $d\hat{n}/dV < 0$. 

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NOTES
1. $R_{i-1}$ is defined by

$$R_{i-1} = (1 + r_{i-1}) - \frac{P_i}{P_{i-1}}$$

where $r_{i-1}$ is the annual rate on three month treasury bills, and $P_i$ is the G.N.P. deflator. G.N.P. gap is a measure of deviations from potential output. All data is from Data Resources Inc. (1981).

2. Note that the two significant deviations from the relationship in Figure 1 occur in 1974, and 1975 just after the oil price shock. This is consistent with the interpretation that over most of the sample period aggregate supply was stable and the observed scatter of points represents a supply relationship which was traced out by movements in aggregate demand. The deviations in 1974 and 1975 may be explained by a temporary shift in supply as the economy adjusted during the transition to higher energy prices.

3. All of these relationships hold between an ex post interest rate variable and an output or employment variable after correcting for autocorrelation. The G.N.P. gap variable was chosen for Figure 1, rather than an unemployment variable, because it required no correction for autocorrelation.

4. Note that this argument is not necessarily inconsistent with the intertemporal substitution mechanism (see Barro (1981)) which suggests that aggregate supply is an increasing function of the expected current real interest rate as opposed to the realized lagged real interest rate. The relationship between current interest rates and output or employment is much less clearly defined in the data sets which we have examined. This is consistent with macroeconomic theory which predicts that both demand and supply should depend on current interest rates and hence a simple scatter diagram is likely to be uninformative.


6. This result, in a different context, is discussed in Moore (1985). Our example is simpler than Moore's since $n(S)$ is bounded by the workers' endowment of leisure; this simplifies the proof of continuity of the profit function. I am grateful to John Moore for pointing out to me the method used in this paper which has been known for some time in the literature on optimal taxation. This method does not impose continuity of $n(S)$ and $C(S)$ a priori although in our problem it turns out that the optimal contract is continuous.

7. This form is less restrictive than it first appears for the reasons discussed in Hart (1983). It may be shown that the introduction of a risk neutral third party, e.g. a social insurance scheme, will remove the implication (discussed in Cooper (1983)) that the exact form of the utility function of workers is critical in determining whether overemployment or underemployment results from A.I. contracts.

10. The restriction which we impose on ex post mobility avoids many of the problems which are addressed in other papers with a general equilibrium setting. See for example Geanakoplos and Ito (1981).
11. The proof of the appropriate envelope theorem involves writing the objective function in the form:

$$J = \int_{S}^{S} [L(\beta, \alpha, \hat{\alpha}, \hat{\rho}, \hat{q}) - \hat{\rho} \hat{S}] dS$$

where $L$ is the Lagrangian described in Appendix A and hats over variables indicate that they are evaluated at the optimal values.

I am indebted to Larry Epstein for outlining the method of proving this result.
12. The proof of this result is available from the author on request.

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