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BURSTING BUBBLES
On the Rationality of Hyperinflations in Optimizing Models

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It is well known that perfect foresight maximizing models frequently possess a continuum of equilibria. In some of these equilibria the price level is explosive even in models in which the money supply is constant. This paper shows that a sufficient condition to rule out these unstable paths is given by the assumption that there is a smallest non-zero unit of output.

1. Introduction

Rational expectations macroeconomic models frequently possess an embarrassingly large number of equilibria. One of these equilibria is the familiar neoclassical solution in which the price level grows at the rate of monetary expansion – but there are others in which the price level is either explosive or implosive even in models with a constant nominal supply of money. Under certain conditions it is possible to appeal to transversality conditions in order to rule out implosive price paths [Brock (1974), Brock and Scheinkman (1980)], but conditions which have previously been suggested in order to rule out hyperinflations are excessively restrictive and do not arise naturally as a result of the technological properties of a monetary economy.[1]

In contrast, this paper demonstrates that in order to rule out speculative bubbles it is sufficient that goods come in discrete units. Along an explosive price path the value of real money balances must fall over time – but the discrete nature of the economy implies that the value of money will reach zero in finite time. Since money is held only because of its future value in transactions, no agent will hold money in period t which is known to be worthless in period $t+1$ and an explosive price path cannot be an equilibrium.

2. A formal model

This argument is most easily demonstrated in a version of Samuelson's (1958) consumption loan model. Agents live for two periods and have endow-

*I would like to thank Peter Howitt for many stimulating discussions on this issue and an anonymous referee who provided the foundation for much of the discussion in section 4.

[1]See, e.g., the discussion in Obstfeld and Rogoff (1980) who point out that the conditions used by Brock (1974, 1975, 1978) and by Brock and Scheinkman (1980) amount to assuming that agents would have infinitely negative utility if money did not exist.

ments e^0 when young and e^1 when old. There is a single perishable consumption good and a durable asset, money. In the version of the model which we describe, agents have additively separable utility functions defined over consumption in the two periods. They maximize this function subject to a lifecycle budget constraint which is described below:

P.1

$$\max_{C_t^0, C_{t+1}^1} u(C_t^0) + \beta \cdot u(C_{t+1}^1),$$

$$\text{s.t.} \quad e^0 - C_t^0 = M/P_t, \quad e^1 - C_{t+1}^1 = -M/P_{t+1}.$$

We assume that there is the same number of agents in each generation and that the nominal stock of money is constant over time. These assumptions are convenient but unnecessary to the argument. Under the assumption of competitive market clearing, the economy's budget constraint will be given by

$$e^0 + e^1 = C_t^0 + C_t^1. \tag{1}$$

The first-order conditions to problem P.1 imply

$$u'(C_t^0)/u'(C_{t+1}^1) = \beta \cdot P_t/P_{t+1}. \tag{2}$$

Eqs. (1) and (2) may be combined to yield a difference equation in real money balances, which implicitly defines the set of equilibrium price sequences. This is given by

$$u'(e^0 - m_t)/u'(e^1 + m_{t+1}) = \beta \cdot m_{t+1}/m_t, \tag{3}$$

where

$$m_t \equiv M/P_t.$$

Under suitable regularity conditions on $u(\cdot)$, eq. (3) will have at least two stationary solutions. These are given by

$$m_t = 0 \quad \text{for all } t, \tag{4}$$

and

$$u'(e^0 - \bar{m}) = \beta \cdot u'(e^1 + \bar{m}). \tag{5}$$

Eq. (4) is the barter solution (which is ruled out by assumption) and eq. (5) is the neoclassical solution which possesses the neutrality properties which we usually ascribe to monetary models. In addition to these stationary solutions, there also exists a continuum of non-stationary solutions in which the value of

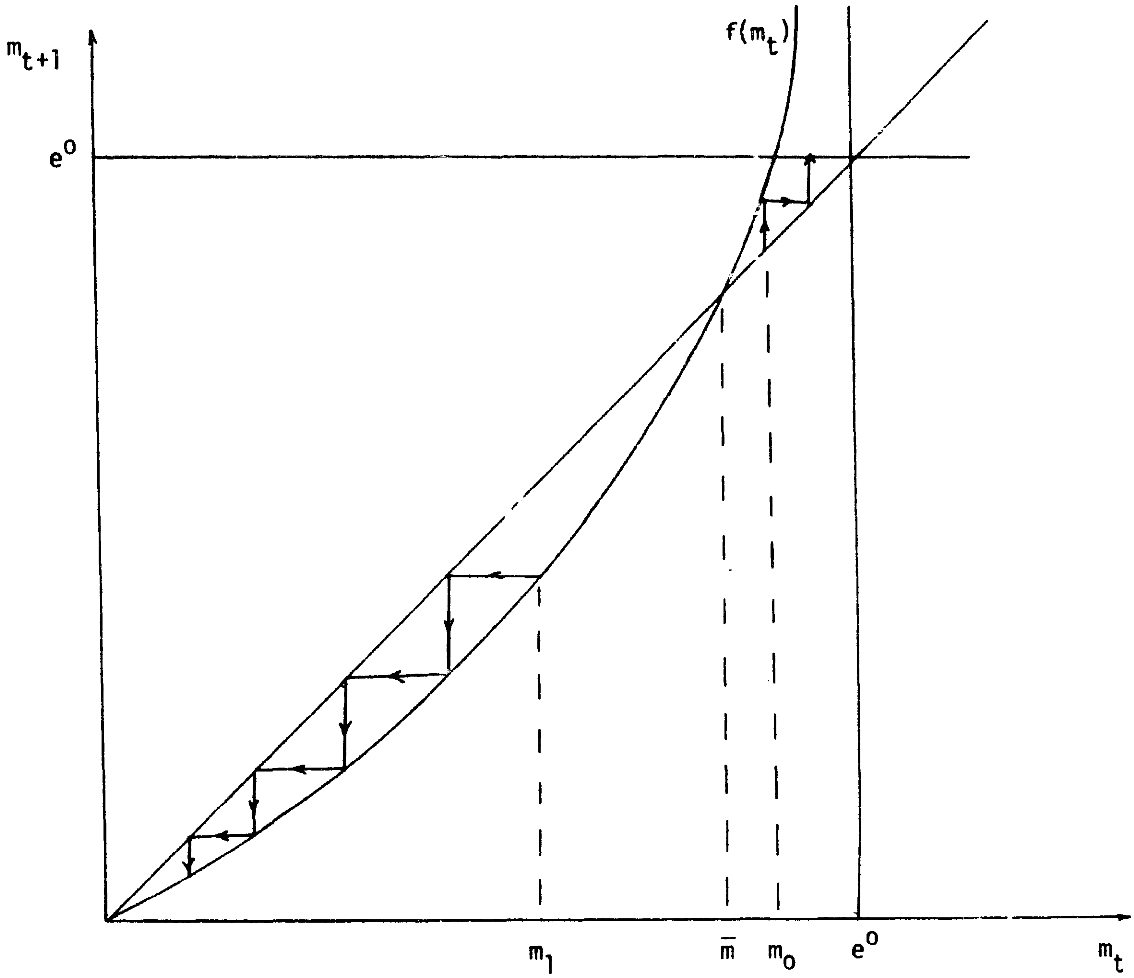

Fig. 1

money is increasing or decreasing over time. To see this, consider fig. 1 which is drawn for the case of logarithmic utility.

Solutions to the difference equation (3) are depicted by the curve $f(m_t)$. $\bar{m}$ represents the stationary neoclassical solution. Paths in which $m_0 > \bar{m}$ are characterized by ever increasing real balances (implosive prices) and paths in which $m_0 < \bar{m}$ are characterized by ever decreasing real balances (hyperinflations). It is clear that implosive paths are, in general, infeasible since eventually they imply $m_t > e^0$, and the price level becomes negative. Explosive paths do not violate any of the assumptions of our model, however, and it must be concluded that, as the model is formulated, they are suitable candidates for equilibrium.

3. Bursting bubbles

In the process of formalizing an economic argument it is common to make certain assumptions which make the analysis of the model more convenient but which are not dictated by any underlying economic rationale. One such simplification is the assumption that goods are infinitely divisible, which allows us to use the techniques of calculus in characterizing the solution to an agent's decision problem. Assumptions of this nature are justified on the grounds of tractability; however, in this instance the assumption of divisibility changes the qualitative nature of the set of feasible equilibria in a way which cannot be justified by appealing to economic intuition.

Consider the following problem which would be faced by an agent in a world in which commodities *were not* infinitely divisible:

$$\text{P.2} \qquad \max_{C_t^0 \in S} u(C_t^0) + \beta u(C_{t+1}^1),$$

$$\text{s.t.} \quad e^0 - C_t^0 = M/P, \quad e^1 - C_{t+1}^1 = -M/P_{t+1},$$

where

$$S = \{ k\Delta | k = 0, 1, \ldots, e^0/\Delta \}.$$

Δ is the smallest unit of consumption and e^0 and e^1 are the endowments of the good in the two periods both of which must be integer multiples of Δ. The consumer must pick a k^* to solve problem P.2 where k^* is the optimal number of units to consume in period t.

Suppose we let m_t^* represent the demand for real balances which arises as a solution to P.2 and we define an equilibrium price P_t^* to be a positive number such that

$$M/P_t^* = m_t^*(P_t^*/P_{t+1}^*) \equiv e^0 - c_t^0. \tag{6}$$

Then m_t^* will always be an integer multiple of Δ because the budget constraint dictates that real balances are equal to the number of units of the endowment which are not consumed. But since the endowment is finite, the set of feasible equilibrium prices will have a finite number of elements.

This set will be defined by the rule

$$P_t^* \in P \equiv \{ M/(e^0 - k^*) | k^* = 0, 1, \ldots, e^0/\Delta \}. \tag{7}$$

We may define a competitive equilibrium (C.E.) to be a sequence $\{P_t^*\}_{t=0}^{t=\infty}$ such that $P_t^* \in P$ and (6) holds for all t. But since the demand for money by the young must equal the supply of real balances by the old, a C.E. price sequence must support a sequence of real balances which satisfies the following difference equation:

$$m_t(P_t^*/P_{t+1}^*) = (P_{t-1}^*/P_t^*) \cdot m_{t-1}(P_{t-1}^*/P_t^*), \tag{8}$$

with initial condition

$$m_t(P_0^*/P_1^*) = M/P_0^*. \tag{9}$$

Eq. (8) has the same stationary solutions which were discussed for the case of a continuous state space – discreteness does nothing to preclude the existence of these equilibria. Matter are different, however, when we examine the feasibility of the sort of explosive price sequence that provided a valid solution to the economy which was described in section 2. It is immediately apparent that an explosive price sequence cannot be a candidate for equilibrium since along such a path,

$$P_{t+1}^* > P_t^* \quad \text{for all } t. \tag{10}$$

But since P has a finite number of elements there must exist a finite T such that $P_T = \infty, P_{T-1} < \infty$ and $m_T = 0, m_{T-1} > 0$. But this contradicts the assumption that $\{m_t\}_{t=0}^{t=T}$ is optimally chosen since if $m_T = 0$ then utility in period $T-1$ can be increased by setting $m_{T-1} = 0$.

In the case in which output in *divisible* we can construct an equilibrium price sequence iteratively for any initial value P_0^* by solving the difference equation (8) together with the second initial condition which is provided by eq. (9). But in the discrete case there is no guarantee that a solution to (8) will exist for an arbitrary P_0^*. Even if we could construct an explosive price sequence which satisfies (8) for a finite number of periods, that sequence will eventually attain the maximal element $P_t^* = \infty$ and money will become worthless. But if money has no value in period T then it can have no value in period $T-1$ since an individual could increase his or her utility be consuming resources rather than converting them into money which is known to have no future value. At that point the solution will violate the assumption of optimizing behaviour and so no previous element in the sequence could be supported as a perfect foresight equilibrium.

4. Alternative models of money

The overlapping generations model is by no means the only framework which will generate a demand for money by appealing to the transactions technology – the Clower cash in advance constraint, for example, has proved to be at least as useful in theoretical research.[2] Clower type models provide a

[2] I am indebted to the comments of an anonymous referee who suggested that this comparison might prove fruitful. The cash in advance constraint was first suggested by Clower (1967) and has since proved to be useful for analyzing a number of issues in monetary theory. Lucas (1980) provides a good example of such a model.

useful benchmark against which the results of this paper can be compared since in Clower models hyperinflations cannot represent feasible equilibrium price sequences. In such models the members of an infinitely lived family maximize utility which is defined over a consumption stream – they are forced to hold money because the technology is defined in such a way that money must be used for all transactions. There is usually an exogenously defined trading period and current period purchases are defined to be no greater than the previous period's accumulated money balances. In simple versions of this model there is a unique equilibrium price sequence which is given by

$$P^* = M/y \quad \text{for all } t, \tag{11}$$

M here is the stationary money stock and y is the per period endowment. It might be thought that non-stationary sequences are ruled out in this economy by the discreteness of the trading period in much the same way as they are ruled out in overlapping generations models by the discreteness of the good, i.e., a discrete good imposes a minimum holding of real balances in the Clower model – agents must always carry enough real balances to transact the income of the following period.[3] However, this analogy is misleading. As a consequence of the exogenous nature of the trading period the demand for real balances is independent of prices and depends only on the exogenous income stream. It is this independence from prices which is crucial, not the length of the trading period itself. There are versions of the overlapping generations model for which *no* non-stationary equilibria exist even if the goods space is continuous, e.g., if the utility function is logarithmic and the second period's endowment is zero then real money balances will always be a constant fraction (of the first period's endowment) which is independent of relative prices – the only equilibrium in this case in which money has value is a stationary equilibrium which is a direct analogy to the Clower case which was discussed above. The discrete state space rules out certain sequences through the transversality conditions and not through restrictions on preferences.[4]

5. Conclusion

For purposes of analytical convenience economic theorists have usually assumed that goods are perfectly divisible – this seemingly innocuous assump-

[3] Note that this minimum is always a binding constraint. Indeed as the length of the trading period goes to zero the demand for money approaches zero – this should not disturb us since one would presume that the fixed trading period is a convenient simplification which is itself based upon the sorts of issues discussed in the voluminous literature on the transactions demand for money.

[4] The discrete state space formulation is not capable of ruling out multiple stationary equilibria – nor can it rule out the sorts of cyclic non-stationary equilibria which are discussed in Azariadis (1980) along which real balances remain bounded away from zero at all times.

tion permits the use of the differential calculus in analyzing agents' decision problems and it is, therefore, an extremely useful simplification. However, this assumption admits a class of non-stationary price sequences as competitive equilibria of an overlapping generations economy even though such sequences would be inadmissable in a discrete analogue of the same model.

It has long been thought that hyperinflations should not be accepted as valid equilibria in perfect foresight monetary models in which the nominal stock of money is stationary – we simply do not observe economies which are characterized by such conditions. This paper has demonstrated that there are good grounds for ignoring these equilibria since they will eventually violate the obvious fact that output is not perfectly divisible.

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