The Great Depression

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Abstract

This paper is about the explanation of the Great Depression given in Keynes' General Theory. There are two key ideas in this book that set it apart from pre-Keynesian economics: The first is that there is something distinctive about the labor market that makes the marginal disutility of labor different in general from the real wage. The second is that aggregate economic activity is determined by the 'animal spirits' of investors. This paper preserves both of these ideas, modified in a way that respects recent developments in dynamic general equilibrium theory.

Keywords Keynesian economics, Search theory, Unemployment.

1 Introduction

Although economic fluctuations in the U.S. have been relatively mild in recent decades, during the Great Depression of the 1930's the unemployment rate exceeded 20% for a protracted period of time. The Great Depression is not unique, and similar episodes have been a recurrent feature of capitalist economies since the beginning of the industrial revolution. Timothy Kehoe and Edward Prescott (2007) define a great depression to be a period of diminished economic output with at least one year where output is 20% below the trend. By this definition Argentina, Brazil, Chile, and Mexico have all experienced great depressions since 1980.

What causes big fluctuations in economic activity? For several decades after the publication of the General Theory economists thought they had an answer; the Great Depression was thought to be a failure of an unregulated capitalist economy to efficiently utilize available resources. But with the resurgence of classical ideas in the 1970's, the key premise of the General Theory, that market economies are not inherently self-stabilizing, has been called into question. Although there has been a recent resurgence of Keynesian ideas under the rubric of "new-Keynesian economics", the models studied by the new-Keynesians are hybrids that incorporate a classical core. New-Keynesian models allow for temporary deviations of unemployment from its 'natural rate' as a consequence of sticky prices but they contain a stabilizing mechanism that causes a return to the natural rate over time.

Recently, it has become fashionable to argue that episodes like the Great Depression of the 1930's are an aberration. Economists who take this view, point to post-war U.S. experience in which unemployment has been relatively low and business cycles relatively mild for long periods of time. The Depression is seen as an unusual episode that requires explanation but it is thought that the place to look for such an explanation is in the actions of the regulatory authorities. An early example is the book by Milton Friedman and Anna Schwartz (1963) who argued that the Depression was caused by incompetent monetary policy in the 1920's. More recently Cole and Ohanian (2004) have argued that Herbert Hoover's regulatory policies deepened the depression in the early 1930's. This is a remarkable turn of intellectual thought from the prevailing mood in the early post-war period when President Richard Nixon is famously quoted as saying "We are all Keynesians now".

In this paper I develop an infinite horizon model in which employment is determined by aggregate demand and I use it to tell the Keynesian story of the Great Depression. According to this story, the stock market crash of 1929 was due to a loss of confidence in the economy that caused a calamitous drop in aggregate demand. This in turn caused an increase in unemployment that was socially inefficient in the sense that the unemployed persons could and should have been profitably employed in productive activity. This story was taught to generations of undergraduates as *the* leading explanation of the Great Depression by Keynesian economists of the post war period. The competition to this explanation was not the RBC model of Kydland and Prescott (1982) but an alternative story of market failure promoted by Milton Friedman (1948). Friedman disputed the impetus to the depression, for him it was the failure of the Fed to maintain sufficient liquidity, but he did not dispute the fact that unemployment in the depression was socially inefficient.

Why do I need to revisit a story that was accepted by several generations of economists? The answer is that the theoretical foundations of this story have been discredited because Keynes did not construct a credible microfoundation to the theory of aggregate supply. I will use a search model of the labor market to provide such a foundation by modifying work of Shimer (2005) and Hall (2005) that follows earlier work in search theory: Pissarides (2000) provides an excellent summary. In this literature it is assumed that the process by which an unemployed worker finds a job requires the input of resources on the part of the firm and time on the part of the worker. When a worker and a firm meet, they determine the wage to be paid through a Nash bargain. Shimer pointed out that this assumption does not provide a good quantitative explanation of employment fluctuations and Hall proposed to replace it with an alternative wage determination mechanism; he assumed that the real wage is determined one period in advance. Shimer's criticism is commonly referred to as the 'Shimer puzzle' and it has generated a considerable amount of recent work amongst economists and graduate students who are exploring alternative wage determination mechanisms in an attempt to reconcile the volatility of vacancies and unemployment with a model in which economic fluctuations are driven by productivity shocks.

In this paper I take an alternative approach. I develop a model in which the labor market is cleared by search but instead of closing it with an explicit bargaining assumption, I assume only that all firms must offer the same wage. This leads to a theory in which there are many wages all of which are consistent with a zero profit equilibrium and it provides a microfounded analog of Keynes' idea that there are many levels of economic activity at which the macroeconomy may be in equilibrium. To select an equilibrium and close the model I introduce the idea that households form beliefs about the future value of productive capital and I show that for any sequence of self-fulfilling beliefs, less than a given bound, there exists a Keynesian equilibrium. This equilibrium will in general be inefficient in the sense that a benevolent social planner would prefer a different employment level that may be higher or lower. Hence, I am able to articulate the Keynesian story of the Great Depression in a model with well defined microfoundations in which no individual agent has an incentive to deviate from his chosen action.

At the end of the paper I will study a second question: can the model explain not only the Great Depression but also the wartime recovery? In studying this question I will have cause to revisit an important debate that arose in the post-war literature. Does fiscal policy matter? Before the rational expectations revolution of the 1970's macroeconomists attempted to extend Keynesian economics to dynamic environments by building microfoundations to each of the components of the Keynesian model. Lucas (1967) and Treadway (1971) estimated models of investment, Milton Friedman (1957) provided a permanent income theory of the theory of consumption function and Friedman (1956) breathed new life into the Quantity Theory of Money by making the case for a stable demand-for-money function. A central goal of this research was to provide a quantitative explanation of the effects of government policy on employment and prices. This goal ran into theoretical difficulties when Alan Blinder and Robert Solow (1973) pointed out that dynamic Keynesian models had no role for fiscal policy since a one dollar increase in government expenditure was predicted to *crowd out* an equal amount of private consumption expenditure if consumption and investment functions were derived from optimizing behavior by a representative family. In section 12 I will show that crowding out is a logical consequence of the representative agent model in which government cannot influence the real rate of interest. Hence, the model developed in this paper does not provide a good vehicle for explaining the wartime recovery.

2 Preferences

I will study a multi-commodity intertemporal representative family model in which there is a single capital good in fixed supply. The simplification of non-reproducible capital enables me to draw out a relationship between the value of the stock market (represented by the value of capital) and the level of economic activity.

There is a unit measure of identical representative infinitely lived families. There are n consumption goods and $K_t = 1$ units of capital. Preferences are described by the following logarithmic utility function

$$J_t = \sum_{s=t}^{\infty} \left[\beta^{s-1} \sum_{i=1}^{n} g_i \log \left(C_{i,s} \right) \right], \tag{1}$$

where

$$\sum_{i=1}^{n} g_i = 1, \tag{2}$$

and the g_i are preference weights. Each family sends a measure 1 of members to look for a job every period. All jobs last for one period and there is 100% labor market turnover. These assumptions are very strong and are made to facilitate the exposition of the model.

The household faces the sequence of budget constraints

$$p_{k,t} K_{t+1} = \left(p_{k,t} + rr_t \right) K_t + w_t L_t - \sum_{i=1}^{n} p_{i,t} C_{i,t}, \qquad t = 1, ...\infty, \tag{3}$$

$$H_t \leq 1, \tag{4}$$

$$L_t = \tilde{q}_t H_t, \tag{5}$$

$$U_t = H_t - L_t, \tag{6}$$

and the 'no-Ponzi scheme' constraint,

$$\lim_{T \to \infty} Q_t^T K_{t+1} \geq 0. \tag{7}$$

The notation is defined as follows. H_t is the measure of family members that search, L_t is the measure that find a job and U_t is the measure that remain unemployed. w_t is the money wage, $p_{i,t}$ is the money price of good i, $\tilde{q}_t$ is the probability that a searching worker will find a job, $C_{i,t}$ is consumption of good i, K_t is the family's ownership of capital, rr_t is the rental price of capital and $p_{k,t}$ is the money price of a unit of capital. All of these terms are defined for each date t. All date t prices are in different date-t units of account that I refer to as date-t money. The variable Q_t^T represents the relative price of date-T money in terms of date-t money and is given by the expression

$$Q_t^T = \prod_{k=t}^{T-1} \frac{1}{(1+i_k)}, \quad T > t, \tag{8}$$

$$Q_t^t = 1. \tag{9}$$

I assume riskless borrowing and lending at money rate of interest i_t and a no arbitrage condition then implies that

$$1 + i_t = \frac{p_{k,t+1} + rr_{t+1}}{p_{k,t}}, \tag{10}$$

where i_t is the money rate of interest between dates t and $t+1$. Since all families are identical there will be no borrowing or lending in equilibrium.

3 The Consumption Function

Since the household derives no disutility from work, it will choose to send all of its members into the labor force to look for a job. At the end of each period all workers are fired and, in the next period, the entire labor force is rehired. I make this assumption to facilitate exposition. Dropping it is an important extension that I will leave for future work.

The first order conditions for the problem are represented by an Euler equation and a set of intertemporal first-order conditions that together imply

$$\frac{1}{C_t} = \frac{\beta}{C_{t+1}} (1 + i_t), \tag{11}$$

where consumption expenditure, C_t is defined as

$$C_t \equiv \sum_{i=1}^{n} p_{i,t} C_{i,t}. \tag{12}$$

Equation (11) describes how aggregate consumption expenditure, measured in dollars, evolves over time. This equation will be central later in this paper when I discuss crowding out. For completeness, it may be helpful also to write down the solution to the household's problem by deriving an equation in which consumption expenditure is described as a function of prices, its time endowment and the hiring probability; all variables that it takes as given in equilibrium. This solution requires first that we define some alternative concepts of wealth.

Let

$$h_t = w_t L_t + \frac{h_{t+1}}{1+i_t}, \tag{13}$$

be the *human wealth* of the family.[1] By iterating this equation forwards and using the no-Ponzi scheme constraint (7) human wealth can be written in terms of prices and hiring probabilities,

$$h_t = \sum_{s=t}^{\infty} Q_t^s w_s L_s = \sum_{s=t}^{\infty} Q_t^s w_s \tilde{q}_s. \tag{14}$$

The household also has *financial wealth* in the forms of claims to capital

$$(p_{k,t} + rr_t)\, K_t.$$

The sum of financial and human wealth is *total wealth* W_t,

$$W_t = (p_{k,t} + rr_t)\, K_t + h_t. \tag{15}$$

The solution to the household problem is to spend a fixed fraction of total wealth on consumption goods and consumption expenditure is given by

$$C_t = (1-\beta)\, W_t. \tag{16}$$

4 Technology

This section mirrors the technology described in Farmer (2008) and the interested reader is referred to that paper for a more complete description. There is a unit measure of non-reproducible capital that must be allocated across industries in every period. There is also a unit measure of workers, all of whom will be allocated, in equilibrium, to the activity of labor market search. I assume that each industry is described by a Cobb-Douglas production function and that capital is rented in a competitive rental market. Labor

[1] This is a slightly non-standard definition since wealth would normally be defined as the net present value of the labor endowment. Here it is the net present value of labor income.

is hired in a search market. I assume further that labor in each firm is divided between a recruiting department and a production department. These assumptions lead to the observation that average and marginal products are equal and are equated to factor prices.

Production is competitive and output of the $i'th$ commodity, denoted $Y_{i,t}$, is produced by a Cobb-Douglas function

$$Y_{i,t} \equiv K_{i,t}^{a_i} X_{i,t}^{b_i}, \tag{17}$$

where

$$a_i + b_i = 1, \tag{18}$$

and

$$\sum_{i=1}^{n} K_{i,t} = K_t. \tag{19}$$

$K_{i,t}$ is the rental demand for capital by firm i and $X_{i,t}$ is the firm's allocation of labor to production. Market clearing in each industry implies that

$$C_{i,t} = Y_{i,t}. \tag{20}$$

Firms maximize profits taking $p_{i,t}$, w_t, rr_t and q_t as given. Each firm solves the problem

$$\max_{\{K_{i,t}, V_{i,t}, X_{i,t}, L_{i,t}\}} p_{i,t} K_{i,t}^{a_i} X_{i,t}^{b_i} - w_t L_{i,t} - rr_t K_{i,t} \tag{21}$$

$$L_{i,t} = X_{i,t} + V_{i,t}, \tag{22}$$

$$L_{i,t} = q_t V_{i,t}, \tag{23}$$

where $L_{i,t}$ is total labor hired by firm i and $V_{i,t}$ is the labor that it allocates to recruiting. Substituting Eqns (22) and (23) into (21) and defining

$$Q_t = (1 - 1/q_t), \tag{24}$$

leads to the reduced form expression for profits;

$$p_{i,t} Q_t^{b_i} K_{i,t}^{a_i} L_{i,t}^{b_i} - w_t L_{i,t} - rr_t K_{i,t} \tag{25}$$

which is maximized when

$$a_{i,} p_{i,t} Y_{i,t} = rr_t K_{i,t}, \tag{26}$$

and

$$b_i p_{i,t} Y_{i,t} = w_t L_{i,t}. \tag{27}$$

These expressions are identical to those that hold in an economy with a competitive labor market. This economy differs from the competitive model since the recruiting efficiency parameter Q_t is endogenously determined by aggregate economic activity but is taken parametrically by the firm; hence there is an externality in the labor market that is not priced. The following section combines Eqns (26) and (27) with consumer first-order conditions to obtain some simple aggregate equilibrium relationships.

5 Aggregate Supply

In this Section I will introduce the variable Z_t to denote nominal gdp. Recall that C_t is the nominal value of aggregate consumption and since there is no investment or government expenditure these two variables will be identical as a consequence of accounting identities. My goal here is to find a relationship between Z_t and L_t that I refer to as aggregate supply.

From the solution to the household's problem it follows that the consumer allocates a fraction g_i of total consumption expenditure to good i; that is,

$$p_{i,t} C_{i,t} = g_i C_t. \tag{28}$$

Since all production of good i is consumed, this also implies that

$$p_{i,t} Y_{i,t} = g_i C_t, \tag{29}$$

and, defining

$$\sum_{i=1}^{n} p_{i,t} Y_{i,t} \equiv Z_t, \tag{30}$$

it follows that,

$$p_{i,t} Y_{i,t} = g_i Z_t. \tag{31}$$

Substituting (31) into the first order condition for the choice of labor in industry i leads to the expression

$$b_i g_i Z_t = w_t L_{i,t}, \tag{32}$$

which can be summed over all industries to give the following expression,

$$\chi Z_t = w_t L_t, \tag{33}$$

where

$$\chi \equiv \sum_{i=1}^{n} g_i b_i. \tag{34}$$

Since money in each period is simply an accounting device there is a degree of freedom in choosing a price normalization in each period. I will choose the date t numeraire to be labor by setting

$$w_t = 1, \quad t = 1, ...\infty. \tag{35}$$

This normalization implies that $p_{i,t}$ is the inverse of the real product wage for each commodity and it allows me to write Eq (33) as

$$Z_t = \frac{1}{\chi} L_t, \tag{36}$$

an equation that I refer to as the Keynesian aggregate supply curve.

A similar exercise using the first order condition for rental capital yields the expression

$$\psi Z_t = r r_t K_t, \tag{37}$$

where

$$\psi \equiv \sum_{i=1}^{n} g_i a_i. \tag{38}$$

In Section 8 I will use equations (36) and (37) to describe how the properties of aggregates behave in a demand constrained equilibrium.

6 Search and the Labor Market

As in Farmer (2008) I assume there is an aggregate match technology that results in the following expression for aggregate employment,

$$L_t = (1)^{1/2} V_t^{1/2}, \tag{39}$$

where L_t is the measure of workers that find jobs when a measure 1 of workers search and V_t workers are allocated to recruiting in aggregate by all firms. Each firm faces an individual hiring equation

$$L_{i,t} = q_t V_{i,t}, \tag{40}$$

which, when aggregated over all firms, yields the expression

$$L_t = q_t V_t. \tag{41}$$

These equations can be rearranged to find an expression relating the measure of workers that can be hired by a single recruiter, q_t, to aggregate employment, L_t,

$$q_t = \frac{1}{L_t}. \tag{42}$$

7 The Social Planning Problem

Before discussing the properties of a Keynesian equilibrium I will provide a benchmark for what full employment means in this economy, by solving a social planning problem. As in Farmer (2008), one can show that the Keynesian equilibrium mimics the decisions of a Social Planner by allocating resources across industries in an efficient fashion; but the Keynesian equilibrium may fail to maintain full employment in a well defined sense. The purpose of this section is to define what this means.

Consider the problem,

$$\max_{\{C_{i,s},X_{i,s},V_{i,s},L_{i,s},H_s,L_s,V_s\}} J_t = \sum_{s=t}^{\infty}\left[\beta^{s-1}\sum_{i=1}^{n} g_i \log\left(C_{i,s}\right)\right] \tag{43}$$

such that

$$C_{i,s} \leq K_{i,s}^{a_i} X_{is}^{b_i} \quad i = 1, ...n \tag{44}$$

$$X_{i,s} + V_{i,s} = L_{i,s}, \tag{45}$$

$$\sum_{i=1}^{n} L_{i,s} = L_s, \quad s = t, \ldots \infty. \tag{46}$$

$$\sum_{i=1}^{n} K_{i,s} = 1, \ \sum_{i=1}^{n} V_{i,s} = V_s, \quad s = t. \ldots \infty \tag{47}$$

$$L_s = V_s^{1/2} H_s^{1/2}, \quad s = t, \ldots \infty \tag{48}$$

$$H_s \leq 1, \quad s = t, \ldots \infty \tag{49}$$

$$L_{i,s} = \frac{V_{i,s}}{V_s} L_s \quad s = t, \ldots \infty. \tag{50}$$

Equation (43) is the objective function of the social planner (identical to that of the representative agent) and Eqns (44)-(50) define the constraint set. This problem can be simplified by using the match technology to eliminate $V_{i,s}$, X_{is} and V_s from the problem. Using Eqns (44), (45) and (50) one can write the production function for good i as

$$C_{i,s} \leq K_{i,s}^{a_i} L_{i,s}^{b_i} \left(1 - L_s\right)^{b_i}. \tag{51}$$

Using this simplification the social planning problem can be restated as

$$\max_{\{C_{i,s}, L_{i,s}, L_s\}} J_t = \sum_{s=t}^{\infty} \left[\beta^{s-1} \sum_{i=1}^{n} g_i \log \left(C_{i,s}\right) \right] \tag{52}$$

subject to Eqn (51) for each commodity at each date and the set of labor constraints represented by Eqn (46).

Proposition 1 (SP) *The solution to the social planning problem has the following properties. Aggregate employment each period is given by the expression,*

$$L_s = \frac{1}{2}, \quad s = t \ldots \infty \tag{53}$$

and labor and capital are allocated across industries according to the equations,

$$L_{i,s} = \frac{g_i b_i}{\chi}, \quad K_{i,s} = \frac{g_i a_i}{\psi}, \quad s = t, \ldots \infty. \tag{54}$$

where $\sum_{i=1}^{n} g_i b_i = \chi$ *and* $\sum_{i=1}^{n} g_i a_i = \psi$.

The proof of this is in the Appendix. Note that there is an optimal unemployment rate of 50% which arises from assumptions about the match technology. By modifying the match efficiency through adding a constant or changing the match elasticity this number can be varied between 0 and 100%. I have kept the existing structure because it makes the algebra more transparent although any empirical implementation of this model would need to make these modifications.

8 Demand Constrained Equilibrium

I begin this section by defining a *demand constrained equilibrium,* which I refer to interchangeably as a *Keynesian equilibrium*, for the dynamic economy with multiple commodities. I will proceed to show that a Keynesian equilibrium exists and that it is represented by a set of aggregate equations that determine employment and gdp and a separate set of equations that describe how labor and capital are allocated across industries.

The absence of markets for the search time of workers and recruiters leads to an equilibrium model with one less equation than unknown. If firms and workers take all wages and prices as given then there is an equilibrium for every value of the sequence of hiring effectiveness parameters $\{Q_s\}_{s=t}^{\infty}$. There are many possible ways of resolving this indeterminacy each of which corresponds to a different possible belief about the future. In the General Theory, Keynes argued that the level of economic activity is pinned down by the *state of long term expectations.* In my model, this concept is represented by a self-fulfilling sequence of values for the capital good, that is, a sequence $\{p_{k,t}\}$.

Definition 2 *A (bounded) state of (long-term) expectations is a non-negative sequence* $\{p_{k,s}\}_{s=t}^{\infty}$ *with a bound* B *such that*

$$p_{k,s} < B$$

for all s*.*

I define the state of expectations to be a sequence of beliefs about the value of capital in all future periods. In a more general model, there will be a different sequence of beliefs for every type of reproducible capital and discrepancies between expectations and the interest rate will cause changes in investment expenditures. In this model I am abstracting from investment spending by assuming that there is a unique non-reproducible capital good. Even in this simple environment changes in beliefs about the value of capital will have an effect on expenditure since long-term expectations influence wealth which, in turn, influences consumption expenditure.

The following definition is of a demand constrained equilibrium in the infinite horizon economy. Following this definition, I show that aggregate variables in a DCE follow a relatively simple equation.

Definition 3 *(Demand Constrained Equilibrium) For any state of expectations a demand constrained equilibrium (DCE) is an $n-$tuple of price sequences $\{p_{i,s}\}_{s=t}^{\infty}$ $i=1,\ldots n$, a sequence of rental rates $\{rr_s\}_{s=t}^{\infty}$ a set of quantity sequences $\{Y_{i,s},X_{i,s},V_{i,s},L_{i,s},C_{i,s},H_s\}_{s=t}^{\infty}$ and a pair of sequences of numbers $\{\tilde{q}_s,q_s\}_{s=t}^{\infty}$, such that the following equations hold for all $s=t,\ldots\infty$:*
1) Feasibility and Market Clearing.

$$Y_{i,s}=K_{i,s}^{a_i}X_{i,s}^{b_i}, \tag{55}$$

$$C_{i,s}=Y_{i,s}, \tag{56}$$

$$X_{i,s}+V_{i,s}=L_{i,s}, \tag{57}$$

$$V_s=\sum_{i=1}^{n}V_{i,s}, \qquad L_s=\sum_{i=1}^{n}L_{i,s}, \qquad K_s=\sum_{i=1}^{n}K_{i,s}, \tag{58}$$

$$L_s=H_s^{\frac{1}{2}}V_s^{\frac{1}{2}}, \tag{59}$$

$$H_s=1, \qquad K_s=1. \tag{60}$$

2) Consistency with optimal choices by firms and households.

$$1=b_{i,s}\frac{p_{i,s}Y_{i,s}}{L_{i,s}}, \qquad rr_s=a_{i,s}\frac{p_{i,s}Y_{i,s}}{K_{i,s}}, \tag{61}$$

$$C_{i,s}=g_i\left(1-\beta\right)W_s, \tag{62}$$

$$W_s=\left(p_{k,s}+rr_s\right)K_s+h_s, \qquad h_s=L_s+\frac{h_s}{1+i_s}, \tag{63}$$

$$1+i_s=\frac{p_{k,s+1}+rr_{s+1}}{p_{k,s}}. \tag{64}$$

3) Search market equilibrium:

$$\tilde{q}_s=L_s, \tag{65}$$

$$q_s=\frac{L_s}{V_s}. \tag{66}$$

Equations (55)-(60)) define technologies, adding up constraints and market clearing conditions. Eqns (61)-(64) are first order conditions that define solutions to individual optimizing problems and (65) and (66) define search market equilibrium.

Proposition 4 (DCE) *There exists a unique Demand Constrained Equilibrium for every state of expectations with bound*

$$B \leq \frac{\psi\beta}{\chi(1-\beta)}.$$

In a DCE, for $s = t, \ldots,$ aggregate expenditure, aggregate employment and the rental rate are described by Equations (67)-(69),

$$Z_s = \frac{1}{\psi}\frac{p_{k,s}(1-\beta)}{\beta}, \tag{67}$$

$$L_s = \chi Z_s, \tag{68}$$

$$rr_s = \frac{p_{k,s}(1-\beta)}{\beta}. \tag{69}$$

Equations (70)-(71), which hold for all $i = 1, \ldots n$ and all $s = t, \ldots\infty$, determine the allocation of factors across industries,

$$K_{i,s} = \frac{a_{i,}g_i}{\psi}, \tag{70}$$

$$L_{i,s} = b_{i,}g_i Z_s. \tag{71}$$

The price in wage units of each commodity is given by the expression

$$p_{i,s} = \left(\frac{\psi Z_s}{a_i}\right)^{a_i}\left(\frac{1}{b_i}\right)^{b_i}\left(\frac{1}{1-\chi Z_s}\right)^{b_i}, \tag{72}$$

and the physical quantity of each good produced is given by the equation

$$Y_{i,s} = \left(\frac{a_i}{\psi}\right)^{a_i}(b_i Z_s)^{b_i} g_i (1-\chi Z_s)^{b_i}. \tag{73}$$

The proof of Proposition 4 is in the appendix.

9 Keynes and the Great Depression

According to Keynes, the Great Depression was caused by a failure of aggregate demand. The model developed in this paper provides a simplified framework for understanding his explanation. In 1929 the stock market fell

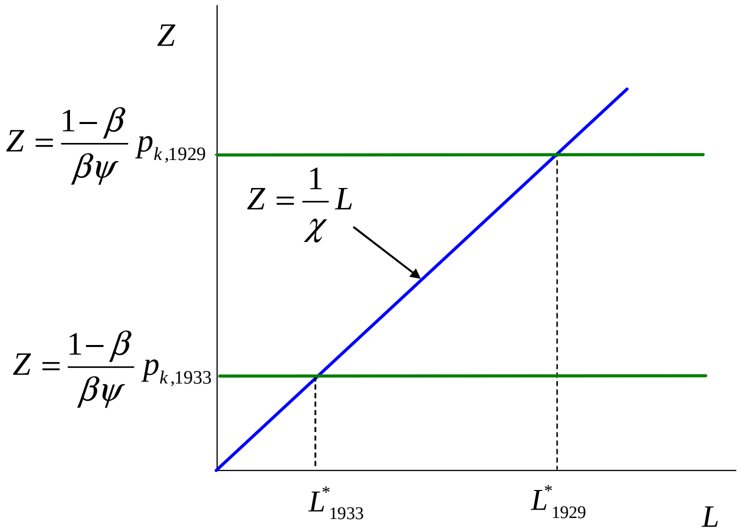

Figure 1: The Keynesian Explanation for the Great Depression

13% in one day. The drop in stock market value was followed by drop in expenditure on new capital goods from 16% of gdp in 1929 to 6% in 1933 and a corresponding dramatic increase in unemployment from 8% to 25% of the labor force. The economy did not recover until 1942 when the United States entered World War II.

Figure (1) illustrates the Keynesian explanation for these events. In 1929 investors lost confidence in the economy causing a self-fulfilling drop in stock market prices and a subsequent fall in investment purchases. This in turn triggered a drop in consumption expenditure through a multiplier effect.

In the model p_k is an exogenous driving variable and a fall in p_k causes an increase in unemployment. On the figure, Z falls from $\frac{1-\beta}{\beta\psi} p_{k,1929}$ to $\frac{1-\beta}{\beta\psi} p_{k,1933}$ and as the economy moves down the aggregate supply curve employment falls from L^*_{1929} to L^*_{1933}. Is this explanation consistent with the data?

Figure (2) plots the value of the Standard and Poors Stock Market index in constant dollars against an investment series and the unemployment rate plotted on an inverted scale. This figure shows that although the model does not explain the investment data, since capital is fixed, it does capture the increase in unemployment that accompanies the crash. Notice, however, that the recovery in the unemployment rate that occurred in the 1940's is

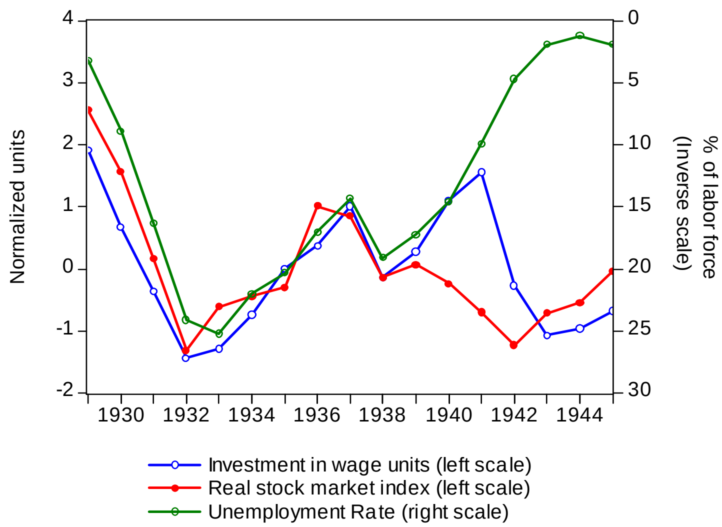

Figure 2: Investment, Stocks and Unemployment in the Depression

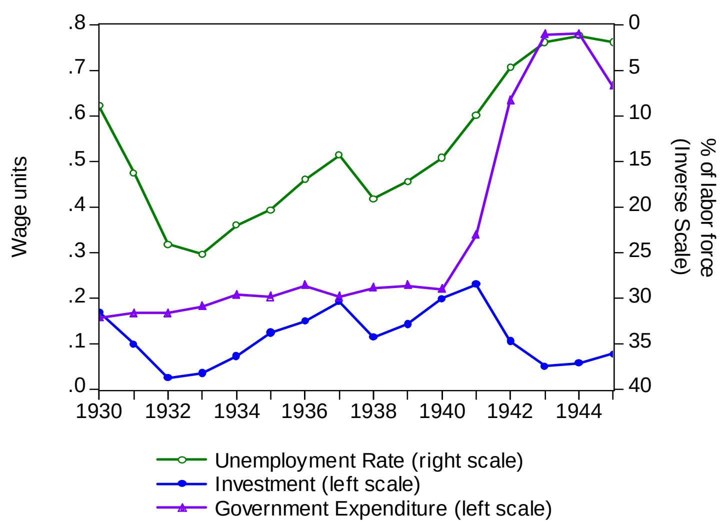

Figure 3: Unemployment Investment and Government Purchases in the Recovery

not accompanied by an increase in the value of the S&P index - neither is it accompanied by an increase in private investment expenditure.

Figure (3) plots the investment and unemployment series from Figure (2) and, in addition it plots the data for government purchases. The investment and government purchases data on this figure are measured in wage units and are comparable to each other. Notice that although investment falls, government purchases shoot up as the United States enters World War II in 1942. The Keynesian model explains the recovery with this fact since, in the textbook static version of the model, equilibrium gdp and employment are functions of autonomous expenditure which consists of the sum of investment and government purchases. I now turn to the question: Can the Dynamic Keynesian model developed in this paper explain the wartime recovery?

10 Efficiency of Equilibrium

Before discussing fiscal policy one would like to understand why it might be necessary. Keynes argued that unemployment was a waste of resources and that full employment could be restored by government expenditure financed either by taxes or by borrowing. In this section I will give the first of these assertions a theoretical foundation by comparing the efficiency of a Keynesian equilibrium with that of the social planning optimum.

Although the model I have described is inspired by the General Theory, it is not identical to it and overemployment as well as underemployment is a possibility. Recall that the model-economy has a stationary efficient employment level of 50% in every period. Although a unit mass of workers searches for employment it is not efficient for all of them to be employed. Any employment level greater than 50% would require that each firm allocate too many of its workers to the recruiting department and would result in a fall in the physical output produced in each industry and a corresponding drop in social welfare.

In a Keynesian equilibrium the value of GDP is proportional to the value of physical capital and is given by the expression

$$Z_t = \alpha p_{k,t}. \tag{74}$$

Employment, proportional to GDP, is equal to

$$L_t = \chi Z_t, \tag{75}$$

where $\alpha = (1-\beta)/(\beta\psi)$. In an economy with a stock market, $p_{k,t}$ represents the value of equity. The value of $p_{k,t}$ is determined by the 'state of long term expectations', which was also famously described in the General Theory as the 'animal spirits' of investors. In the model there is no underlying uncertainty in the physical environment but in reality technology, preferences and endowments as well as political and social variables are themselves changing in unknown ways. Keynes argued that the future cannot be quantified in a way that has become common in modern macroeconomics as agents are assumed to know the probability distributions of all uncertain future events. As a consequence, the belief of agents in the form of the animal spirits of investors becomes an independent driving force of the business cycle.

I have argued elsewhere (1999) that animal spirits should be modeled by building general equilibrium models in which there is an indeterminate continuum of equilibria, indexed by beliefs. In my earlier work these equilibria represented different non-stationary paths each of which converged to the same steady state. The model I have developed is an extension of this idea to allow beliefs to influence the steady state itself.

How does the Keynesian equilibrium compare with the social planning solution? The answer is that there is a continuum of Keynesian equilibria indexed by $\{p_{k,t}\}$. In any given period there will be a unique value

$$p^*_{k,t} = \frac{1}{2\alpha\chi}, \tag{76}$$

such that when the stock market price $p_{k,t}$ is equal to $p^*_{k,t}$, the Keynesian Equilibrium implements the social planning solution and $L_t = 1/2$. If $p_{k,t} < p^*_{k,t}$, there will be inefficiently high unemployment and if $p_{k,t} > p^*_{k,t}$ gdp and employment will be too high. In this case gdp is high because prices are high and welfare and physical output could be increased by an increase in the unemployment rate: in common parlance, the economy is *overheating.* In either case, if $p_{k,t}$ is too high or too low, the Keynesian equilibrium will be inefficient in the sense that a different belief by investors would result in an unambiguous increase in social welfare. In a calibrated model one might expect the socially efficient unemployment rate to be considerably less than 50%; perhaps 5% would be a good guess. If unemployment rose to 20% (or higher) as it did in the early 1930's the additional 15% represents workers that could have been gainfully employed in producing consumption and investment goods. Even if only half of this additional unemployment was due to an inefficiency of the kind I have modeled in this paper, the implied welfare loss would be substantial.

During the 1930's government spending was widely discussed as a possible remedy to the Great Depression but the remedy was not effectively put into practice until 1942 when the United States entered World War II. In the textbook Keynesian model consumption is a function of income that is itself the sum of investment, consumption and government spending. In this simple model an increase in government spending causes an increase in equilibrium income that in effect, pays for itself. The following section explores the possibility of telling a similar story in the context of the intertemporal representative agent model of this paper.

11 Household and Government

Consider the following variation on the model developed so far. Let there be a government that purchases commodities $G_{i,t}$ in each period. To keep the model simple I will assume that

$$G_{i,t} = g_i G_t \tag{77}$$

where the weights g_i for $i = 1, \dots n$ are the same as the preference weights of the consumer. This assumption allows me to abstract from distribution effects associated with changes in the composition of aggregate demand between consumption and government purchases. To pay for its purchases, the government levies an income tax at rate τ_t on labor income, or it may borrow money from households by issuing debt B_t. The assumption that there is no capital tax is not inconsequential since one might wish to use capital taxes or subsidies to influence intertemporal prices. Since tax-subsidy schemes of this kind are not the focus of the expansionary fiscal policies that I am interested in I will abstract from capital taxation in this section.

The government faces the following sequence of constraints,

$$B_{s+1} = B_s \left(1 + i_{s-1}\right) + G_s - \tau_s L_s, \quad s = t, ... \tag{78}$$

with the no-Ponzi scheme condition

$$\lim_{T \to \infty} Q_t^T B_T \leq 0. \tag{79}$$

Here, $\tau_t L_t$ is the tax revenue from the labor income tax and I have used the normalization $w_s = 1$ to remove w_s from the flow budget constraint. The sequence of constraints (78) together with Eqn (79) , is equivalent to the single infinite-horizon constraint

$$\sum_{s=t}^{\infty} Q_t^s G_s + B_t \left(1 + i_{t-1}\right) \leq \sum_{s=t}^{\infty} Q_t^s \chi \tau_s Z_s, \tag{80}$$

where I have replaced L_t by χZ_t from the aggregate supply curve.

How does the introduction of a government that spends, taxes and issues debt, influence the solution to the consumer's problem? Recall that the household solves the problem,

$$\max_{\{C_{i,s}\}} J_t = \sum_{s=t}^{\infty} \left[\beta^{s-1} \sum_{i=1}^{n} g_i \log \left(C_{i,s}\right) \right]. \tag{81}$$

When we introduce taxes and government debt into the model the budget constraint faced by the household becomes

$$\sum_{s=t}^{\infty} Q_t^s \left(\sum_{i=1}^{n} p_{i,s} C_{i,s} \right) \leq \sum_{s=t}^{\infty} Q_t^s \tau_s L_s + A_t. \tag{82}$$

where

$$A_t = p_{k,t} + rr_t + B_t \left(1 + i_{t-1}\right), \tag{83}$$

represents its initial wealth. Aggregating first-order conditions for this problem leads to the consumption Euler equation,

$$\frac{1}{C_t} = \frac{\beta}{C_{t+1}} \left(1 + i_t\right), \tag{84}$$

and riskless arbitrage implies,

$$1 + i_t = \frac{p_{k,t+1} + rr_{t+1}}{p_{k,t}}. \tag{85}$$

It follows from this analysis that the introduction of government purchases does not alter the household's consumption-Euler equation. This fact has an important implication for the usefulness of the representative agent model in telling the story of the 1940's recovery.

12 Crowding Out

In this section I define a government expenditure plan and a fiscal policy and using these definitions I describe the characteristics of an equilibrium in an economy with government. I define a class of fiscal policies that restricts spending by government to have the same distributional pattern as spending by households. Although this restriction is not strictly necessary in the sense that one could define an equilibrium without it, the assumption simplifies the characterization of an equilibrium. It would not be surprising if expenditure by government on a particular sector of the economy has distributional consequences by changing relative prices but that is not what one normally means by the effectiveness of fiscal policy.

A fiscal policy as I define it has two components. First, it is a decision by government to purchase a given quantity of goods and services in every period. Second; it is a decision on whether those purchases should be financed by raising taxes or by borrowing.

Definition 5 (Expenditure Policy) *A (distributionally neutral) expenditure policy is a set of non-negative sequences* $\{G_{i,s}\}_{s=t}^{\infty}$ *and an initial debt level* $B_t(1+i_{t-1})$ *such that*

$$G_{i,s} = g_i G_s \tag{86}$$

for all s*. An expenditure policy together with a pair of sequences* $\{\tau_s, B_s\}_{s=t}^{\infty}$ *is called a fiscal policy. If there exists a pair of sequences* $\{\tau_s, B_s\}_{s=t}^{\infty}$ *such that the budget equation*

$$\sum_{s=t}^{\infty} Q_t^s G_s + B_t(1+i_{t-1}) \leq \sum_{s=t}^{\infty} Q_t^s \chi \tau_s Z_s, \tag{87}$$

is satisfied then the expenditure policy $\{G_{i,s}\}_{s=t}^{\infty}$ *is said to be feasible for price sequence* $\{Q_t^s\}$*.*

Given this definition, a particular class of stationary policies is of particular interest.

Definition 6 (Stationary Fiscal Policy) *A feasible (distributionally neutral) fiscal policy is stationary if the sequences* $\{G_{i,s}, \tau_s, B_s\}_{s=t}^{\infty}$ *do not depend on* s*.*

Given these definitions I now provide a relatively straightforward extension of Definition 3 to show how a Keynesian equilibrium is modified in the presence of government expenditure.

Definition 7 (DCEG) *For any state of expectations* $\{p_{k,s}\}_{s=t}^{\infty}$ *and any distributionally neutral expenditure policy* $\{G_{i,s}\}_{s=t}^{\infty}$ *a demand constrained equilibrium with Government (DCEG) is an* $n-$*tuple of price sequences* $\{p_{i,s}\}_{s=t}^{\infty}$ $i = 1, \ldots n$, *a sequence of rental rates* $\{rr_s\}_{s=t}^{\infty}$ *and implied present value prices* $\{Q_t^s\}_{s=t}^{\infty}$, *a set of quantity sequences* $\{Y_{i,s}, X_{i,s}, V_{i,s}, L_{i,s}, C_{i,s}, H_s\}_{s=t}^{\infty}$, *a set of tax and debt sequences* $\{\tau_t, B_s\}_{s=\tau}^{\infty}$ *such the the policy is feasible for the present value prices* $\{Q_t^s\}$, *and a pair of sequences of numbers* $\{\tilde{q}_s, q_s\}_{s=t}^{\infty}$, *such that the following equations hold for all* $s = t, \ldots \infty$:
1) Feasibility and Market Clearing.

$$Y_{i,s} = K_{i,s}^{a_i} X_{i,s}^{b_i}, \tag{88}$$

$$C_{i,s} + G_{i,s} = Y_{i,s}, \tag{89}$$

$$X_{i,s} + V_{i,s} = L_{i,s}, \tag{90}$$

$$V_s = \sum_{i=1}^{n} V_{i,s}, \qquad L_s = \sum_{i=1}^{n} L_{i,s}, \qquad K_s = \sum_{i=1}^{n} K_{i,s}, \tag{91}$$

$$L_s = H_s^{\frac{1}{2}} V_s^{\frac{1}{2}}, \tag{92}$$

$$H_s = 1, \qquad K_s = 1. \tag{93}$$

2) Consistency with optimal choices by firms and households.

$$1 = b_{i,s} \frac{p_{i,s} Y_{i,s}}{L_{i,s}}, \qquad rr_s = a_{i,s} \frac{p_{i,s} Y_{i,s}}{K_{i,s}}, \tag{94}$$

$$C_{i,s} = g_i (1 - \beta) W_s, \tag{95}$$

$$h_s = L_s + \frac{h_s (1 - \tau_s)}{1 + i_s}, \tag{96}$$

$$A_s = (p_{k,s} + rr_s) K_s + B_s (1 + i_{s-1}) \tag{97}$$

$$W_s = A_s + h_s, \tag{98}$$

$$1 + i_s = \frac{p_{k,s+1} + rr_{s+1}}{p_{k,s}}. \tag{99}$$

3) Search market equilibrium:

$$\tilde{q}_s = L_s, \tag{100}$$

$$q_s = \frac{L_s}{V_s}. \tag{101}$$

This definition differs in three ways from (3). First, Eqn (89) is modified to recognize the allocation of resources between household and government sectors. Second, the definition of human wealth in Eqn (96) is modified to include only the after tax value of labor income. Third, financial wealth of the household sector, defined in Eqn (97) includes government debt.

Definition 8 (Stationary DCEG) *A DCEG is stationary if all variables are independent of calendar time.*

I now have enough machinery to define the main idea of this section. I will deal with the case in which households have stationary pessimistic expectations in the sense that p_k is constant and permanently less than p_k^*. Stationarity is a strong assumption but a useful one since it is the case that Keynes believed was characteristic of the Great Depression. In the General Theory he argued that unemployment may be an equilibrium phenomenon (in the sense of a stationary state). To capture this feature I assume that agents' expectations are unchanging and that the economy is in a stationary Keynesian equilibrium with an unemployment rate that is inefficiently high.

I would like to be able to model Keynes' prescription of increased government expenditure as a way out of the Great Depression. The following proposition demonstrates that the representative agent environment is not a good vehicle with which to make this case because one dollar of government expenditure is predicted to crowd out an equal amount of private consumption expenditure.

Proposition 9 (Crowding Out) *Let $\{p_{k,s}\}$ be a bounded stationary state of expectations such that*

$$p_{k,s} = p_k < \frac{\psi\beta}{\chi(1-\beta)}, \quad s = t \ldots \infty.$$

Let $\{G_s\}$ be a stationary sequence of expenditures such that

$$G_s = G \leq \frac{1}{\psi}\frac{p_{k,s}(1-\beta)}{\beta}, \quad s = t.\ldots\infty.$$

There exists a unique stationary Demand Constrained Equilibrium with government. This equilibrium has the following characteristics. Aggregate expenditure, aggregate employment and the rental rate are described by Equations (102)-(104),

$$Z_s = Z = \frac{1}{\psi}\frac{p_k(1-\beta)}{\beta}, \tag{102}$$

$$L_s = L = \chi Z, \tag{103}$$

$$rr_s = rr = \frac{p_k(1-\beta)}{\beta}. \tag{104}$$

Equations (105)-(106), which hold for all $i = 1, \ldots n$ and all $s = t, \ldots\infty$, determine the allocation of factors across industries,

$$K_{i,s} = K_i = \frac{a_{i,}g_i}{\psi}, \tag{105}$$

$$L_{i,s} = L_i = b_{i,}g_i Z. \tag{106}$$

The price in wage units of each commodity is given by the expression

$$p_{i,s} = p_i = \left(\frac{\psi Z}{a_i}\right)^{a_i}\left(\frac{1}{b_i}\right)^{b_i}\left(\frac{1}{1-\chi Z}\right)^{b_i}, \tag{107}$$

and the physical quantity of each good produced is given by the equation

$$Y_{i,s} = Y_i = \left(\frac{a_i}{\psi}\right)^{a_i}(b_i Z)^{b_i} g_i (1-\chi Z)^{b_i}. \tag{108}$$

Consumption and government purchases of each commodity are allocated as follows

$$C_{i,s} = \left(\frac{Z-G}{Z}\right)Y_{i,s}, \tag{109}$$

$$G_{i,s} = \left(\frac{G}{Z}\right)Y_{i,s}. \tag{110}$$

The proof of this proposition mirrors that of Proposition 4 and it hinges on the fact that the household Euler equation is unchanged by the introduction of government. In aggregate, it can be written as

$$\frac{1}{C_s} = \frac{\beta}{C_{s+1}} \left(\frac{p_{k,s+1} + rr_{s+1}}{p_{k,s}} \right), \tag{111}$$

or, using the first order conditions from production,

$$\frac{1}{C_s} = \frac{\beta}{C_{s+1}} \left(\frac{p_{k,s+1} + \psi Z_s}{p_{k,s}} \right). \tag{112}$$

In a stationary equilibrium this implies

$$Z_s = \frac{1}{\psi} p_k \left(\frac{1-\beta}{\beta} \right). \tag{113}$$

Equation (113) implies that gdp in a stationary equilibrium is independent of government expenditure and is a function only of the state of expectations. Since

$$Z = C + G, \tag{114}$$

it follows that a one dollar increase in government expenditure must crowd out one dollar of private consumption expenditure. This is exactly the point made by Blinder and Solow in 1973 and since government spending and private spending is allocated in the same proportion across industries the allocation of each commodity to households and government is in proportion to aggregate spending as in Eqns (109) and (110).

13 Concluding Comments

What have we learned from this exercise? When economists of the post-war period began to provide microfoundations to Keynesian economics they turned to the Ramsey model of a representative agent as the simplest formal framework within which to model the evolution of dynamic equilibrium models. The static consumption function modeled consumption as a function of income but it was soon realized that a forward looking agent should be concerned not just with current income but with wealth or in Friedman's terms, 'permanent income'.

The Keynesian remedy to unemployment was to replace investment by government expenditure and the Hicks-Hansen framework illustrated in a relatively simple model why this ought to work. The rational expectations revolution of the 1970's threw out the Hicks-Hansen apparatus because it did not cope well with the simultaneous appearance of high inflation and high unemployment in the 1970's. But the model could not have been rejected so quickly on empirical grounds if it was not already on weak theoretical foundations. In this paper I have attempted to shore up the theoretical foundations of aggregate supply by providing a sound microfoundation to the idea that there may be a continuum of stationary equilibrium unemployment rates indexed by beliefs. This has led us down a road and towards a conclusion that was trod in 1973 by Blinder and Solow and it begs for the development of a richer household structure that can restore the possibility of telling the second part of the Keynesian story in a consistent way. The representative agent structure leaves us with an unanswered puzzle: Why does fiscal policy matter for aggregate economic activity?

14 Appendix

Proof of Proposition 1. Let $\lambda_{i,s}$ be the multiplier on the $i'th$ constraint (51) at date s, and let μ be the multiplier on (46). The following three first order conditions follow from the choice of $C_{i,s}, L_{i,s}$ and L_s

$$\frac{\beta^{s-1}g_i}{C_{i,s}} = \lambda_{i,s}, \tag{115}$$

$$\frac{\lambda_{i,s}b_iC_{i,s}}{L_{i,s}} = \mu, \tag{116}$$

$$\sum_{\iota=1}^{n} \frac{\lambda_{i,s}b_iC_{i,s}}{1-L_s} = \mu. \tag{117}$$

Combining (115) with (116) and summing over i gives,

$$\beta^{s-1}\chi = \mu L_s. \tag{118}$$

Combining (115) with (117) yields,

$$\beta^{s-1}\chi = \mu\left(1-L_s\right). \tag{119}$$

Together, these equations imply

$$L_s = 1/2. \tag{120}$$

To obtain the allocations of labor across industries combine Eqns (115), (116) and (118). The allocation of capital follows from a similar analysis using the first-order condition for capital. ■

Proof of Proposition 4. The proof of existence is constructive. Since labor supply is bounded above by 1, and since, in a DCE, $L_s = \chi Z_s$, from Eq (36), Z_s is bounded above by χ^{-1}. By aggregating the consumer Euler equations one obtains Eq (11) which can be combined with (10) and the market clearing conditions to give

$$\frac{1}{Z_s} = \frac{\beta}{Z_{s+1}}\left(\frac{p_{k,s+1} + rr_{k,s+1}}{p_{k,s}}\right). \tag{121}$$

Using Eq (37), (60) and rearranging terms,

$$\frac{1}{Z_s} = \frac{\beta}{Z_{s+1}}\left(\frac{p_{k,s+1}}{p_{k,s}}\right) + \frac{\beta\psi}{p_{k,s}}, \tag{122}$$

which can be iterated forward to obtain the expression

$$\frac{1}{Z_s} = \frac{\beta\psi}{p_{k,s}} \left(1 + \beta + \beta^2 ...\right). \tag{123}$$

Since $\beta \in (0,1)$ the infinite sum on the RHS of (123) converges to $(1-\beta)$ and rearranging this expression then leads to Eq (67). Since Z_s is bounded above by χ^{-1}, it follows that a valid equilibrium requires

$$p_{k,s} \leq \frac{\psi\beta}{\chi(1-\beta)}. \tag{124}$$

Eq (68) follows from (36) and (69) follows from combining (67) with (121). Equations (70) and (71) follow directly from rearranging the first order conditions for the firm. To obtain Equation (72), note that the production function for good i can be written, in reduced form, as

$$Y_{i,s} = Q_s^{b_i} K_{i,s}^{a_i} L_{i,s}^{b_i}, \tag{125}$$

where it follows from (24) (36), and (42) that

$$Q_s = (1 - \chi Z_s). \tag{126}$$

Using the fact that the consumer spends a fraction g_i on good i leads to the equation

$$p_{i,s} = \frac{g_i Z_s}{Y_{i,s}}. \tag{127}$$

Combing Equations (125) and (127), substituting for $K_{i,s}$ and $L_{i,s}$ from (70) and (71) leads to Eq (72). Equation (73) is derived similarly. ■

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