Business Cycles with Heterogeneous Agents

Roger E. A. Farmer[1]
UCLA and CEPR
Department of Economics: UCLA
8283 Bunche Hall
Box 951477
Los Angeles CA 90095-1477
rfarmer@econ.ucla.edu

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Abstract

I show how to construct a stochastic long-lived overlapping generations model, based on a non-stochastic model developed by Olivier Blanchard [2] and Philippe Weil [14], that nests the RBC model as a special case. My innovation over previous work is to add an aggregate stochastic shock. I provide three different calibrations of the model. One mimics the RBC model and the other two are heterogeneous agent economies (HA and HATAX) with and without corporate income taxes. I show that the HA and HATAX models can explain the low safe rate of interest that has been observed for long periods in U.S. data. The HATAX model can also explain the fact that the investment to GDP ratio in US data is lower than the profit share. All three models are almost identical in their predictions for the comovements and volatilities of consumption, investment, employment and GDP at business cycle frequencies.

1 Introduction

This paper is part of a research agenda in which I a develop a stochastic version of an overlapping generations model that can be calibrated and used in applied work. The focus of the current paper is on dynamic efficiency in a model with capital. In a related paper [7] I study asset pricing and the equity premium in an endowment economy. In both papers I adapt a model first discussed by Olivier Blanchard [2] and later developed by Philippe Weil [14] in which one thinks of the economy as populated by a growing number of infinite horizon agents. The Blanchard-Weil model is non-stochastic at the aggregate level. The innovation of my work is to add an aggregate stochastic shock.

In the current paper I construct a model that nests the RBC model as a special case; but in which the safe rate of return is below the growth rate. In spite of this fact, the economy passes the test of Abel-Mankiw-Zeckhauser and Summers [1], who point out that dynamic inefficiency in a competitive economy implies that the investment rate should exceed the profit share. Since the profit share has been roughly 30% and the investment rate only 18%, Abel-et-al conclude that the U.S. economy cannot be dynamically inefficient. This argument is correct in an economy with no distortions but in the U.S. there is a tax on corporate profits that distorts the capital allocation decision. I show that when one accounts for a corporate profits tax of 46%, the historical average in post-war data, my calibrated economy is consistent with the historical observation of a low safe rate of return and with an investment rate that is lower than the profit share.

2 Historical Data on the Safe Rate of Return

In the period from 1934 to 1979 there were only six years when the interest rate on three month treasury bills exceeded the growth rate of nominal GDP. These were 1938, 1946, 1949, 1954, 1957 and 1970. The average treasury bill rate for this period was 2.9% with a standard deviation of 2.5 and the average rate of nominal income growth was equal to 8.23% with a standard deviation of 5.5. The data for this period is illustrated in Figure 1. It is difficult to view this Figure without forming the impression that the U.S. government did not face a binding present value budget constraint before 1979. Since the growth rate of tax revenues consistently exceeded the interest rate on government debt, the government could have lowered taxes without planning to raise them again in the future. This conceptual experiment is in contrast to the Ricardian Equivalence proposition which asserts that any reduction in current taxes must be met by a planned increase in future taxes of equal net present value.[1]

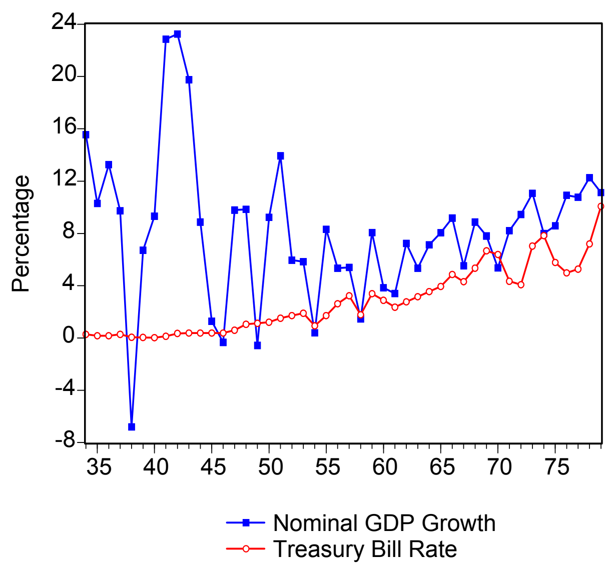

Figure 1: The Interest Rate and the Growth Rate in U.S. Data.

It is difficult to explain the existence of a low safe rate of interest with the real business cycle model since the wealth of the representative agent is unbounded in such economies when the riskless interest rate is lower than the growth rate.[2] Most recent work on business cycles recognizes the low

[1]The argument in this paragraph was first made by Michael Darby [5]. Miller and Sargent [9] responded to Darby's argument by pointing out that one should not expect the interest rate to be independent of policy. It is likely that the attempt to run a large deficit would lead to an increase in the interest rate and change the nature of the equilibrium. The model developed in this paper can be used to develop the Miller-Sargent argument formally in a computable dynamic general equilibrium model.

[2]This result relies on the assumption that there exists a lower bound on the value of the future endowment stream of at least one agent. By relaxing this assumption it is possible for an RBC economy to display a low rate of return if the degree of risk aversion of the representative agent is very high. This is the route followed by Abel et al [1], who give an example of a representative agent economy in which endowment *growth* is lognormally distributed (and hence the endowment of the representative agent is unbounded below).

safe rate of return as a puzzle to be explained. Typically, papers in this literature choose the rate of time preference of the representative agent to equal approximately 3%. This choice implies that the mean return to capital in the model will be equal to 7% which was the historical mean return to equity in the U.S. over the last century. Models calibrated in this way have difficulty explaining why the riskless rate over the same period was only 2%. In this paper I take an alternative approach. I calibrate a business cycle model by choosing the rate of time preference of a typical agent to be 1%. I assume that all population growth consists of new agents entering the economy and I show that my model economy has a safe interest rate of approximately 2%. The government, in my model, does not face a binding present value budget constraint, at least for low rates of lump-sum taxation since future agents can be made to pay for current expenditures.

3 The Model

3.1 Households

I will describe a stochastic version of an economy that was first studied by Philippe Weil [14]. It is populated by infinitely lived families that have logarithmic preferences over consumption and discount factor $\beta \in [0, 1)$. These agents maximize the discounted present value of a function,

$$U^i = \log\left(C_t^i\right) - \lambda l_t^i, \tag{1}$$

where l_t^i is hours supplied to the market by household i. I have chosen a specification for which preferences are linear over leisure to give the model the best possible chance of capturing the observed volatility in hours. This specification is widely used in the real business cycle literature following the arguments of Gary Hansen [8] and Richard Rogerson [11] that indivisible leisure can be modeled “as if” agents had linear preferences over hours worked.

The families in my economy trade a complete set of Arrow securities. Each period, the agents of household i supply labor hours l_t^i to the market, they purchase consumption commodities C_t^i and they accumulate securities $A_{t+1}^i\left(S'\right)$ for each value of S'. The budget constraint of a representative family is given by

$$\sum_{S'} Q_{t+1}\left(S'\right) A_{t+1}^{i}\left(S'\right) = A_{t}^{i}\left(S\right) + \omega_t l_t^i + T_t^i - C_t^i.$$

$A_{t+1}^{i}\left(S'\right)$ is the quantity of security S' purchased for price $Q_{t+1}\left(S'\right)$ at date t. The term $A_t^i\left(S\right)$ on the right side of this constraint is the security that pays off at date t if state S is realized. ω_t is the wage, l_t^i is hours worked, C_t^i is the household's purchase of consumption commodities and T_t^i represents lump-sum transfers. I define the human wealth of the household as follows,[3]

$$H_t^i = \omega_t l_t^i + T_t^i + \sum_{S'} Q_{t+1} H_{t+1}^i.$$

Using this definition one can represent the solution to the household's problem as follows:

$$C_t^i = (1-\beta)\left(A_t^i + H_t^i\right), \tag{2}$$

$$\lambda C_t^i = \omega_t. \tag{3}$$

Equation (2) says that the household consumes a fixed fraction of wealth each period. Equation (3) equates the marginal disutility of working divided by the marginal utility of consumption to the real wage. The assumption that utility is linear in leisure implies that consumption is independent of wealth and hence all generations have the same consumption profile.[4] This assumption is important in enabling me to obtain a simple expression for the price of an Arrow security and I will return to it in Section (4).

3.2 Demographics

The population in my economy grows at rate n so that if N_t is the number of agents alive at date t then

$$N_{t+1} = (1+n) N_t.$$

[3] This is a non-standard definition of human wealth which is typically used to refer to the value of the time-endowment of a representative agent. In my economy, human wealth is an endogenous variable that is different, in equilibrium, for agents of different generations.

[4] It also possible to solve explicitly for a model in which utility is logarithmic in leisure, although this model delivers a more complicated expression for the price of an Arrow security. I am indebted to Leo Kaas for pointing out to me that linear leisure leads to a particularly simple expression for security prices.

Following Weil [14] I want to think of the agents in my model as "dynasties". Each period, households have some number of children that inherit the family fortune. But there are also "unloved children" that are cut off from inheritance and who must form their own new dynasties. The parameter n represents the growth rate of these new agents.

Define the terms

$$\begin{aligned} A_t &= \sum_i A_t^i, \quad H_t = \sum_i H_t^i, \quad L_t = \sum_i l_t^i, \\ l_t &= \frac{L_t}{N_t}, \qquad C_t = \sum C_t^i, \quad T_t = \sum T_t^i, \end{aligned}$$

to be aggregate versions of the individual variables and consider the accumulation of aggregate physical assets, A_t. Summing the budget equation over all households gives

$$\sum_{S'} Q_{t+1}(S') A_{t+1}(S') = A_t(S) + \omega_t L_t + T_t - C_t, \tag{4}$$

and since capital is the only store of wealth,

$$A_t(S) = (1 + r_t) K_t(S). \tag{5}$$

Equation (5) defines the total value of Arrow securities in state S at date t to equal the value of capital plus the net after-tax return-to-capital, $r_t K_t$.

3.3 Technology

The resource constraint is standard,

$$K_{t+1} = K_t(1 - \delta) + Y_t - C_t.$$

K_{t+1} is the capital stock in period $t+1$, δ is the depreciation rate and Y_t is aggregate output. I assume that output is produced with a Cobb-Douglas production function,

$$Y_t = S_t K_t^{\alpha} (M_t N_t l_t)^{1-\alpha},$$

where S_t is a stationary stochastic productivity shock, α is a parameter that measures the elasticity of output with respect to capital and $l_t N_t$ is aggregate labor input. Deterministic labor augmenting technical progress, represented by M_t, grows at rate m;

$$M_t = (1+m)\, M_{t-1},$$

and productivity is governed by the process

$$S_{t+1} = S_t^{\rho} \exp\left(e_{t+1}\right),$$

where e_{t+1} is an error with zero mean that represents the innovation to the productivity shock. I assume that the support of e_{t+1} is finite and that it corresponds to the set of Arrow securities in the sense that for each element of the support of e_{t+1} there is an associated security.

It will also be useful to define the aggregate growth factor g such that

$$(1+g) \equiv (1+n)\,(1+m)\,.$$

There are two terms on the right side of this identity because growth arises from two sources; growth of new dynasties, represented by $(1+n)$ and growth of labor productivity, represented by $(1+m)$.

I assume competitive factor markets so that the real wage ω_t and the rental rate rr_t are given by the expressions

$$\omega_t = \frac{(1-\alpha)\,Y_t}{L_t}, \quad rr_t = \frac{\alpha Y_t}{K_t},$$

where I have exploited the Cobb-Douglas functional form to write the marginal products of labor and capital in this way. I assume that corporate profits are subject to tax at the rate τ^c so that the net after tax return to capital, r_t, is given by the equation:

$$r_t = (rr_t - \delta)\,(1-\tau^c)\,.$$

A household that holds a unit of capital loses δ units to depreciation and earns rental rate rr_t. It pays taxes on rents minus depreciation at the corporate tax rate, τ^c.

Throughout this paper I will analyze economies for which the government budget is balanced,

$$(\alpha Y_t - \delta K_t)\,\tau^c = T_t.$$

For policies in this class all tax revenues from corporate taxes are redistributed to households in the form of lump sum transfers.

The parameter τ^c is important because Abel et al [1] have argued that the U.S. economy is dynamically efficient even though the safe rate of return has been lower than the growth rate of GDP for long periods of U.S. history. Their argument hinges on the fact that, in a dynamically efficient economy, the share of national income paid to capital will exceed the ratio of investment to GDP. In the U.S., although the safe rate of interest has been lower than the growth rate, the share of income paid to capital has historically exceeded the investment ratio. I will show in Section 5 that a long-lived overlapping generations economy can capture both of these facts when the capital income tax is set at a value consistent with much of post-war policy. Since the U.S. tax code allows for the wage bill to be deducted as an expense against revenues, the corporate income tax acts as a distortionary tax on fixed capital owned by corporations and it lowers the equilibrium capital stock.

The final equation of the model is the pricing equation for rental capital. Absence of arbitrage opportunities and the existence of a complete set of contingent securities markets implies that the rental rate is related to the price of an Arrow security by the following equation,

$$\sum_{S'} Q_{t+1}(S')(1+r_t) = 1. \tag{6}$$

4 A Stationary Representation of the Model

4.1 Defining New Variables

My next task is to find a stationary representation of the equations of the model. For this purpose I will normalize each of the variables, deflating it by the two variables M_t and N_t that grow exogenously. Define each of the following transformed variables:

$$\begin{aligned} k_t &= \frac{K_t}{M_t N_t}, \quad c_t = \frac{C_t}{M_t N_t}, \quad y_t = \frac{Y_t}{M_t N_t}, \\ h_t &= \frac{H_t}{M_t N_t}, \quad a_t = \frac{A_t}{M_t N_t}. \end{aligned}$$

Using these definitions, the equations of the model can be expressed in the following way, beginning with two static equations.

$$\begin{aligned} y_t &= S_t k_t^{\alpha} l_t^{1-\alpha}, && (7) \\ \lambda c_t &= \frac{(1-\alpha)\, y_t}{l_t}. && (8) \end{aligned}$$

Equation (7) is the production function in per-capita terms and Equation (8) is the labor market clearing equation. The model is completed by the following two dynamic equations.

$$(1+g)\, k_{t+1} = k_t (1-\delta) + y_t - c_t, \tag{9}$$

$$\frac{1}{c_t} = \frac{\beta}{(1+m)} E_t \left[\frac{1}{c_{t+1}} RR_{t+1} \right] \tag{10}$$

where

$$RR_{t+1} \equiv \left(1 + \left(\alpha \frac{Y_{t+1}}{K_{t+1}} - \delta \right) (1-\tau^c) \right).$$

Equation (9), is the resource constraint and (10) is an aggregate consumption Euler equation. Notice that the right side of this expression is divided by $(1+m)$ and not by $(1+g)$ as it would be in a representative agent economy. The following paragraph explains why.

Each dynasty at date t will choose consumption to obey the equation

$$\frac{1}{C_t^i} = \frac{\beta \pi (S')}{Q_{t+1}(S')\, C_{t+1}^i (S')},$$

where $\pi(S')$ is the probability that state S' occurs, β is the discount factor and $Q_{t+1}(S')$ is the price for delivery of a unit of the commodity in state S' at date $t+1$. When this expression is summed over all generations alive at two consecutive dates, and normalized by the growth factor $M_t N_t$ it leads to equation (11).

$$\frac{1}{C_t} = \frac{\beta \pi (S') (1+n)}{Q_{t+1}(S')\, C_{t+1}(S')}. \tag{11}$$

In this equation C_t is aggregate consumption. It is important to notice that the term $(1+n)$ appears in the numerator of Equation (11) because the consumption at date $t+1$ of those who were alive at date t is equal to $1/(1+n)$ of aggregate consumption at date $t+1$. The additional fraction $n/(1+n)$ of aggregate $t+1$ consumption is accounted for by the new generations born at date $t+1$. After normalizing Equation (11), multiplying both sides by $M_t N_t$, and taking the conditional expectation at date t,one obtains equation (10).

4.2 Steady State Equations

My next task is to find a balanced growth path for this economy, represented as a non-stochastic steady state of the transformed equations. I begin by defining $\bar{k}, \bar{c}, \bar{l}$ and $\bar{y}$ to be the steady state values of k_t, c_t, l_t and y_t in a version of the model for which $S_t = 1$ for all t. This assumption shuts down all of the stochastic shocks. Now define

$$c_y = \frac{\bar{c}}{\bar{y}}, \quad k_y = \frac{\bar{k}}{\bar{y}},$$

and write the steady state version of Equations (9) and (10) in terms of these transformed variables.

$$(g+\delta) k_y = 1 - c_y, \tag{12}$$

$$\frac{1}{c_y} = \frac{\beta}{(1+m)} \left(\frac{1}{c_y}\right) \left(1 + \left(\frac{\alpha}{k_y} - \delta\right)(1-\tau^c)\right). \tag{13}$$

By substituting Equation (12) into (13) one arrives at a linear equation in c_y. In the following analysis I calibrate the model and use Maple (the computational engine in Scientific Workplace) to solve for the steady state value of c_y. Given this value I compute k_y from Equation (12) and $\bar{l}$ from the the steady state version of labor market clearing, Equation (8);

$$\bar{l} = \frac{(1-\alpha)}{\lambda c_y}. \tag{14}$$

One can also recover an expression for $\bar{y}$ from the production function and expressions for $\bar{c}$ and $\bar{k}$ from the definitions of c_y and k_y.

5 Calibration

In this section I add the assumption that uncertainty arises from a highly autocorrelated productivity shock and I calibrate three versions of the model. One version is a standard RBC economy in which there is no population growth. In the other two versions I allow for the entry of new dynasties and I calibrate the discount parameter and the population growth rate to match the implied safe rate of return to U.S. data. The equations of the model are as follows:

$$
\begin{aligned}
(1+g)\,k_{t+1} &= k_t\,(1-\delta) + y_t - c_t, && (15)\\
\frac{1}{c_t} &= \frac{\beta}{(1+m)} E_t\left[\frac{1}{c_{t+1}} RR_{t+1}\right], && (16)\\
y_t &= S_t\,(k_t)^{\alpha}\,(l_t)^{1-\alpha}, && (17)\\
\lambda c_t &= (1-a)\,\frac{y_t}{l_t}, && (18)\\
S_t &= S_{t-1}^{\rho}\exp(e_t), && (19)\\
RR_{t+1} &\equiv \left(1+\left(\frac{\alpha y_{t+1}}{k_{t+1}}-\delta\right)(1-\tau^c)\right). && (20)
\end{aligned}
$$

The evolution of the productivity shock is modeled in Equation (19). The parameter ρ represents the degree of autocorrelation of this shock and e_t is its innovation. It is typical in the literature to argue that S_t is directly observable and is equal to the Solow residual. With the exception of the term $(1+m)$ that enters Equation (16) this model is *identical* to a standard real business cycle economy. In the case when no new generations are allowed to enter the economy, the parameter m is equal to g and in this case the heterogeneous agent model and the real business cycle model coincide.

In the period from 1950 to 1979 the average growth rate of real GDP was 3.8%. I have chosen this period because it is the longest span in post-war history in which the nominal rate of growth was consistently above the treasury bill rate. I started the sample in 1950 to exclude war-time fluctuations in debt and deficits. I ended it in 1979 because the data following 1980 looks very different and in the 1980's the interest rate exceeded the growth rate. Growth slowed dramatically in the 1980's and the interest rate rose.[5]

[5] Miller and Sargent [9] point out that one would not expect the interest rate to be independent of policy. It seems likely that the steep rise in the real interest rate in the

DATA	RBC	HA	HATAX	Concept
$g = 0.038$	0.038	same	same	GDP growth rate
$m = 0.014$	0.038	0.014	0.014	Productivity growth rate
β (unmeasured)	0.97	0.99	0.99	Discount factor
$\alpha = 0.36$	0.36	same	same	Capital's share
$\rho = 0.93$	0.93	same	same	Autocorrelation of Shock
$\delta = 0.06$	0.06	same	same	Depreciation rate
$\sigma = .016$	0.016	same	same	Stnd. dev. of Shock
$\tau^c = 0.46$	0	0	0.46	Corporate tax rate
$r \in \{0.001, 0.07\}$	0.07	0.024	0.024	Interest rate
$k/y = 4.3$	2.8	4.3	3.4	Capital/output ratio
$i/y = 0.29$	0.27	0.42	0.34	Investment ratio
$1 - LSH = 0.36$	0.36	0.36	0.36	Profit ratio

Table1: Calibrating three models

In Table 1 I have calibrated my model using three different sets of assumptions. The first set, listed in column 2 of Table 1, turns the model into a relatively standard RBC economy by picking the growth rate of new dynasties to equal zero.[6] The second two sets, in columns 3 and 4, represent heterogeneous agent economies (HA) in which there are new agents appearing each period. In all three calibrations I have chosen the same value for the growth rate, g. I have also chosen depreciation and the properties of the Solow residual to take values that are relatively standard in the calibration literature. There are three parameters that are different across the three calibrations.

Table 1, column 1 reports the growth rate of real chain weighted GDP from the US NIPA accounts. This was equal to 3.8% on average from 1950 through 1979. The rate of growth of the Solow residual (computed as $\log\left(Y/K^{0.36}L^{0.64}\right)$ was equal to 1.4%.[7] For the HA and HATAX models I

early 1980's was directly connected to monetary and fiscal policy changes that occurred during the Reagan administration. In the heterogeneous agent model, a reduction in taxes will raise the equilibrium interest rate. If the reduction is large enough, the model may switch from one in which the riskless interest rate is lower than the growth rate, to one in which it exceeds the growth rate.

[6]Since $(1+g) = (1+m)(1+n)$, the proposition that $n = 0$ is implied by setting $g = m$.

[7]The capital stock series used for this construction was computed from a series on

set $m = 1.4\%$ and $g = 3.8\%$ to match these numbers. For the RBC model I set both g and m equal to 3.8%. In the RBC model, population growth and productivity growth have the same effect on the equilibrium values of all other variables. Hence, this difference in the calibrations across the two models has no important effects on any other parts of the models.

The numbers for the capital/output ratio, labor's share and the investment rate, reported in the table, are taken from Prescott and McGrattan [10] page 39 Tables C1 and C2 by adding up the shares of corporate and non-corporate sectors. The capital stock number is high, but it includes government capital, housing and a measure of intangible capital. Since these different stocks depreciate at different rates, it may be misleading to put too much weight on the discrepancy between this number and the capital stocks in the three single sector models in which I have imposed a single 6% depreciation rate.

In the RBC calibration I set the discount factor to 0.97. In combination with the assumption that $g = 3.8\%$, this choice of parameters delivers an equilibrium interest rate of approximately 7%, equal to the rate of time preference plus the growth rate. In the two heterogeneous agent economies, HA and HATAX, I have set the agent's discount rate to 0.99. In combination with the assumption that $m = 1.4\%$ and $g = 3.8\%$ this choice of parameters delivers an equilibrium interest rate of 2.4%. In the data the ex post real T-bill rate had a mean of 0.15% with a standard deviation of 1.2% and the real stock return as computed by Shiller [13] had a mean of 7.1% with a standard deviation of 16%. By choosing a rate of time preference of 0.99 I was able to bring down the rate of return to 2.4%, less than the growth rate, but not as low as the observed mean of the ex post T-bill rate of 0.15%. It would have been possible to lower the safe rate of return still further by raising the discount rate at the cost of generating unrealistically high values for the investment ratio in the HATAX economy.

Column 3 (the HA economy) represents a heterogeneous agent economy in which the corporate income tax rate is zero and in this economy the steady state interest rate is equal to 2.4%. Compare this with the steady state interest rate in the RBC economy of 7%. As Abel et al [1] point out, a low real interest rate economy will be associated with an unrealistically high investment/income ratio (equal to 42% as opposed to 29% in the data) that

private plus government investment using a perpetual inventory method and assuming a depreciation rate of 6%.

exceeds the share of income going to capital (equal to 36%).

Column 4 adjusts the model to account for a corporate profit tax of 46%, which is representative of the rates in force in the post-war period.[8] Explicitly allowing for a corporate profit tax has no noticeable effect on the after-tax real interest rate in equilibrium but is has a substantial effect on the equilibrium capital-output and investment-output ratios. The capital-output ratio falls to 3.4, and the investment-output ratio to 34%. The HATAX model can account for the low safe rate of interest and it gives a figure for the investment to GDP ratio that is not too far from the historically observed ratio of private plus government investment to GDP. The figure in U.S data was equal to 29% and in the HATAX model it is 34% when the corporate income tax is set at 46%.

6 A Linearized Version of the Model

In this section I show how to linearize the model and in Section 7 I use the linearized model to simulate artificial time series from three different calibrations.

First define the vectors X_t and Y_t as follows;

$$Y_t = \begin{bmatrix} \hat{c}_t \\ \hat{k}_t \\ \hat{s}_t \end{bmatrix}, \quad X_t = \begin{bmatrix} \hat{l}_t \\ \hat{y}_t \end{bmatrix}.$$

where hats indicate proportional deviations from the non-stochastic steady state. To derive a linearized model I first separate out those equations that contain values of X_t and Y_t at a point in time (I call these static equations) from equations that contain values at different points in time; (I call these dynamic equations). The static equations are Equations (17) and (18) and the dynamic equations are (15), (16) and (19). Now linearize the static equations as follows;

$$B_1 X_t = B_2 Y_t, \tag{21}$$

[8] John Seater [12] estimates that the average marginal coporate tax rate was 1% from 1909-1915. At the end of WWI it jumped to 10% and increased sporadically during the interwar years to reach a new high of 19% in 1938-39. At the end of WWII it jumped to 70% and in the period from 1949-1975 it never fell below 38% and was equal to 48% for much of the 1970's.

where the matrices B_1 and B_2 are derived in the appendix. A key insight in understanding the properties of the dynamics that I report below is that the elements of the matrices B_1 and B_2 are the same for all three parameterizations of the model. This implies that the comovements of X_t and Y_t are the same for the RBC model and for the HA and HATAX models.

Now write the dynamic equations as follows,

$$A_1 Y_{t+1} + A_2 X_{t+1} + A_3 Y_t + A_4 X_t + A_5 V_{t+1} = 0, \tag{22}$$

where

$$V_{t+1} = \begin{bmatrix} \hat{e}_{t+1} \\ \hat{w}_{t+1} \end{bmatrix},$$

and

$$\hat{w}_{t+1} = E_t \left[a_4 \hat{c}_{t+1} + a_5 \hat{k}_{t+1} + a_6 \hat{y}_{t+1} \right] - \left[a_4 \hat{c}_{t+1} + a_5 \hat{k}_{t+1} + a_6 \hat{y}_{t+1} \right],$$

is the Euler equation error. The terms A_1, A_2, A_3, A_4 and A_5 are parameter matrices that contain functions of the deep parameters of the model, and the parameters a_4, a_5, and a_6 are linearization coefficients defined in the appendix.

Substituting Equation (21) into Equation (22) gives the expression

$$Y_t = J_1 Y_{t+1} + J_2 V_{t+1}, \tag{23}$$

where the matrices J_1 and J_2 are functions of $A_1, A_2, A_3, A_4, A_5, B_1$ and B_2. Equation (23) expresses the current value of Y_t as a function of future values and future expected values of itself and of future realizations of e_{t+1}. The behavior of Y_t (and hence of X_t) will depend in part on the values of the eigenvalues of J_1.

Since the vector Y_t contains one free variable (a variable not pinned down by an initial condition) a necessary and sufficient condition for the rational expectations equilibrium to be unique is that one of the eigenvalues of J_1 is inside the unit circle.[9] When this condition holds, as it does in all three of my calibrated examples, one can find a linear function

$$\hat{c}_t = q_1 \hat{k}_t + q_2 \hat{s}_t, \tag{24}$$

[9] The solution technique for this class of models is, by now, well known and there are many good references that explain the procedure in depth. One of the first articles on the topic is by Blanchard and Kahn [3]. There is a detailed exposition in Chapters 2 and 3 of Farmer [6].

that describes the behavior of $\hat{c}_t$ as a function of the state variables $\hat{k}_t$ and $\hat{s}_t$ in a rational expectations equilibrium.[10]

In the simulations reported below I calculated the values of q_1 and q_2 for each of the three models, RBC, HA and HATAX. I then defined vectors

$$X_t^* = \begin{bmatrix} \hat{c}_t \\ \hat{l}_t \\ \hat{y}_t \end{bmatrix}, \quad Y_t^* = \begin{bmatrix} \hat{k}_t \\ \hat{s}_t \end{bmatrix}$$

and used Equations (23) and (24) to write the dynamics of the system in state space form

$$\begin{aligned} Y_{t+1}^* &= J_3 Y_t^* + J_4 e_{t+1}, \\ X_t^* &= M Y_t^*. \end{aligned}$$

The matrices J_3, J_4 and M are functions of the linearization coefficients of the model and of the derived parameters q_1 and q_2. Details of these computations are reported in the appendix.

In Table 1 I showed that the three models differ in their first moments. The following section illustrates the behavior of their second moments by graphing artificial data from a single simulation of each of the models.

7 Simulating Data

In Figures 2, 3 and 4 I compare simulated data for the three artificial economies with business cycles from US data. In each figure, the top left panel represents US data, and the other three panels report simulations of each of the three models for a single drawing of the errors $\{e_t\}_{t=1}^{30}$. Each model uses the same random drawing and each panel of these three figures compares the deviations of GDP from trend with one additional series: Figure 2 plots consumption and GDP, Figure 3 plots employment and GDP and Figure 4 plots investment and GDP. In every case the data and the model simulations have been detrended with the Hodrick-Prescott filter.[11]

[10] Let $J_1 = Q \Lambda Q^{-1}$ where Λ is a diagonal matrix of eigenvalues and Q is a matrix of eigenvectors. The linear function described in the text is found by setting the row of $Q^{-1} Y_t$ associated with the eigenvalue of J_1 that lies inside the unit circle, equal to zero.

[11] Artifical data was generated in Gauss using code available at http//:www.farmer.sscnet.ucla.edu. Actual and artifical data was detrended (using Eviews) with the Hodrick Prescott filter using a smoothing parameter of 100.

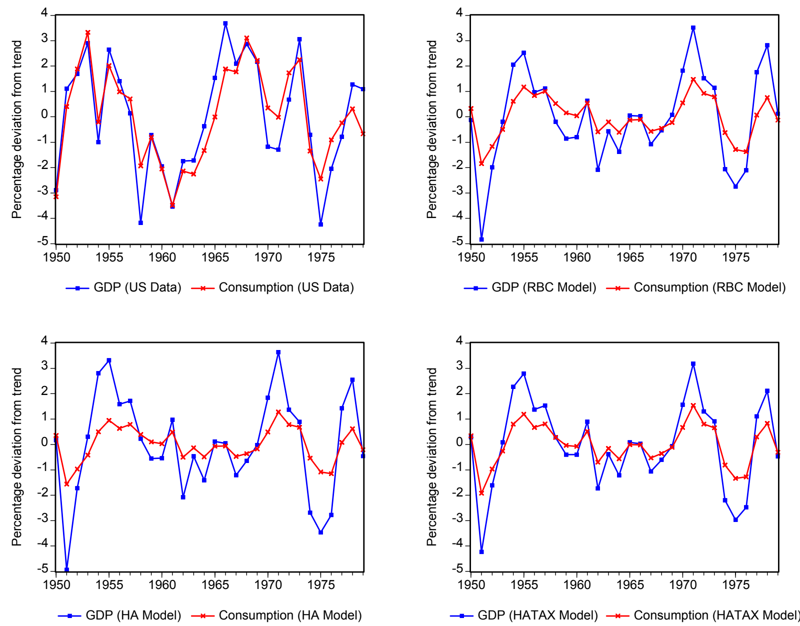

Figure 2: Consumption and GDP

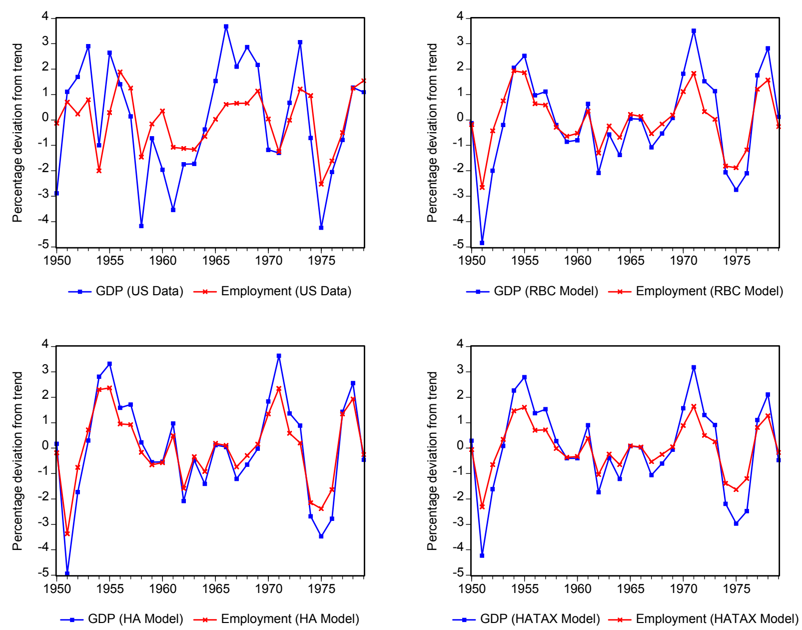

Figure 3: Employment and GDP

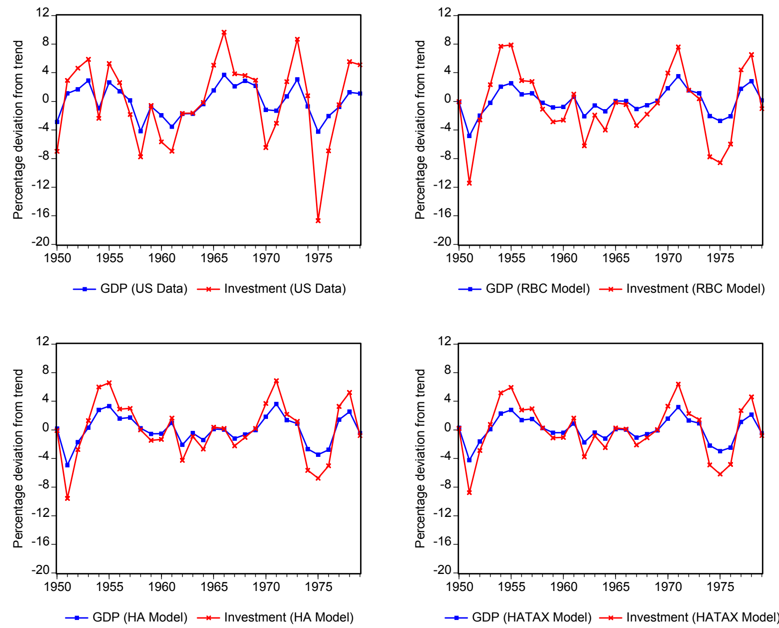

Figure 4: Investment and GDP

Notice the similarity of the three artificial economies with each other. Although the HA and HA tax models predict very different values for the mean rate of return from the predictions of the RBC economy, all three models generate remarkably similar fluctuations. I have already drawn attention to the fact that for all three models the matrix M that links X_t and Y_t is the same. This implies that, if the dynamic behavior of the sequences $\{Y_t\}$ is similar across models then so is the behavior of $\{X_t\}$. The similarity across models of the artificial data illustrated in Figures 2, 3 and 4 shows that the variables $\{Y_t\}$ are also similar.

To understand why the dynamics of the models are so close, consider the matrix J_3 that governs the dynamics of capital accumulation. This matrix has two roots λ_1 and λ_2 that are compared across models in Table 2.

Concept	**RBC**	**HA**	**HATAX**
λ_1	0.797	0.884	0.843
λ_2	0.930	0.930	0.930

Table 2: Eigenvalues of J_3

The second root of J_3 is equal to ρ, the parameter that governs the autocorrelation properties of the technology shock; this was set equal to 0.93

for all three models. Notice also that the first root, λ_1 is similar; it takes the values 0.797, 0.884 and 0.843 in the three different economies. It is this fact that explains why the autocorrelation properties of the series appear similar in all three simulations.

8 The Equity Premium Puzzle

The HATAX model does a reasonably good job of explaining business cycle fluctuations. It can also explain why the safe rate of return is less than the growth rate. But it performs no better than the RBC model at explaining why the return to equity has been historically much higher than the return to treasury bills. In the RBC model the safe rate of return is too high. In the HA and HATAX models the return to capital is too low.

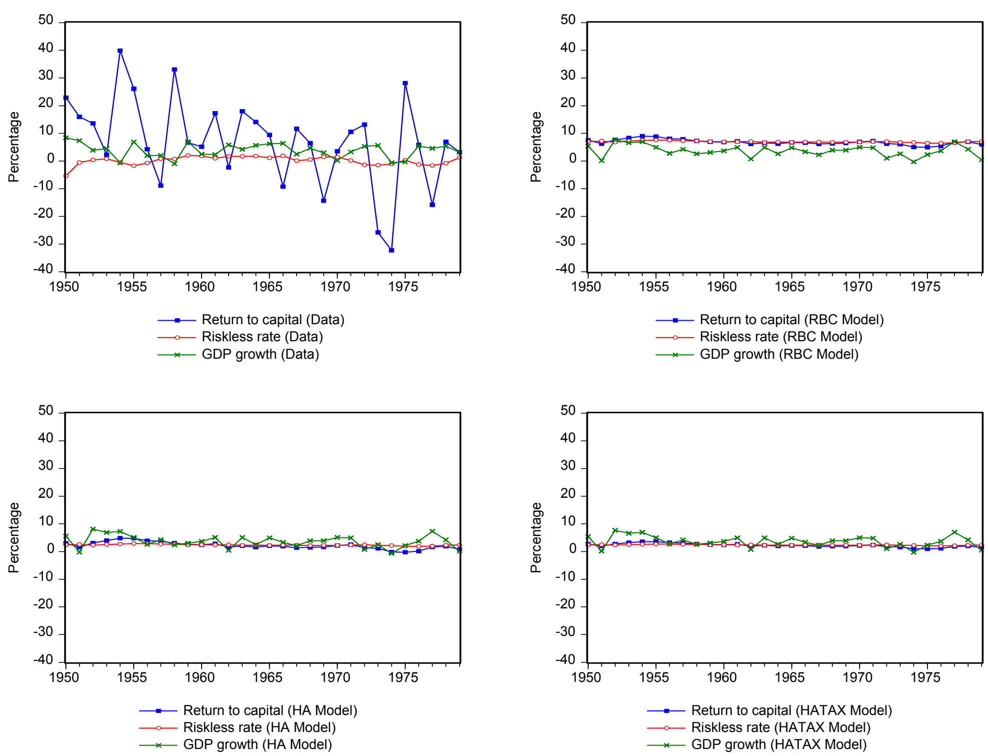

Figure 5: The Riskless Interest Rate and the Return to Capital

Figure 5 compares the GDP growth rate, the riskless rate of return and the return to capital in the data with simulated data for the three models.

In the data the "riskless rate" is the ex-post real rate computed as the T-bill rate minus realized inflation and the return to capital is Robert Shiller's series on the S&P 500 as reported in [13]. In the models the riskless rate and the return to capital are computed as linear approximations to the expressions;

$$\begin{aligned} \text{Riskless rate} &= \quad E_t\left[\frac{\pi\left(S'\right)}{Q_{t+1}\left(S'\right)}\right] = E_t\left[\frac{c_{t+1}}{c_t}\frac{(1+m)}{\beta}\right], \\ \text{Return to capital} &= \quad \left(\frac{\alpha}{k_y} - \delta\right)(1-\tau^c). \end{aligned}$$

None of these series has been filtered and, in the case of the simulated data, the mean of the safe rate has been added back into the series. In all three models the variance of the return to capital is much lower than the variance of the return to the stock market from the U.S. data.

	DATA		RBC		HA		HATAX	
	Risky	Safe	Risky	Safe	Risky	Safe	Risky	Safe
Mean	6.97	0.14	7.05	7.05	2.16	2.35	2.42	2.42
Std. Dev.	16	1.57	1.06	0.31	1.26	0.24	0.68	0.17

Table 3: Rates of Return

Table 3 reports the means and standard deviations of the return to capital (labeled "risky") and the risk-free rate (labeled "safe"). For the data the means and standard deviations refer to 30 years of annual data from 1950 through 1979. For the models the table reports the average 30 year mean and the average 30 year standard deviation for 10,000 different 30 year simulations. Since the model statistics that I have reported are returns from the linearized model (ignoring second moments) the equity premia in these computed series are zero by construction. The risky and safe returns in the model differ only because of sampling error.

The most striking feature of Figure 5 and of Table 3 is the tremendous difference between volatilities of returns. In the data the return to the S&P 500 is approximately ten times greater than the mean value of the safe rate of interest. In all three models the safe return is five to ten times less volatile than the real return to T-bills in the data and, in addition, the risky rate in the models is only three to four times greater than the safe return. Although the HATAX model can capture the relative ranking growth rates and interest rates; along with the RBC model, it fails spectacularly at explaining volatilities.[12]

9 Conclusion

I have developed a small macroeconomic model that may prove to be a useful alternative to the representative agent model. It has one more parameter than the standard real business cycle model; but otherwise it is identical in all respects. In spite of this superficial similarity, the heterogeneous agent economy has very different implications. First, and the main focus of the current paper, the model can explain why the return to government debt has been historically low. One might think that this issue was already understood since the overlapping generations model can deliver equilibria for which the interest rate is lower than the growth rate. But in overlapping generations economies with no distortions, low interest rate economies are known to imply that the investment rate should be counter-factually high. Specifically, Abel et. al. showed that in dynamically inefficient economies with no distortions, the investment rate exceeds the profit rate. I have shown that one can construct a tractable overlapping generations model in which the interest rate is below the growth rate and yet the model passes the test of Abel et. al. due to the presence of distortionary taxation. Further, my calibration is consistent with cyclical fluctuations in which the relative volatilities of consumption, investment and labor supply match those of the U.S. data with about the same degree of accuracy as a standard RBC model.

[12] To address the issue of excess volatility, in a related paper [7], I study an economy that differs from the current model in two respects. First, preferences are logarithmic, rather than linear, over leisure. This innovation is important since it changes the expression for the pricing kernel in a significant way. Second, the model is an endowment economy in which there is an endowed stock of capital that is priced by a sequence of agents. This alternative structure *is* able to explain excess volatility and the equity premium but it requires a departure from the one-sector model. In future work I hope to combine both sets of ideas in a single model.

10 Appendix

The model has the following equations.

$$\begin{aligned}
k_{t+1}(1+g) &= k_t(1-\delta) + y_t - c_t, \\
\frac{1}{c_t} &= \frac{\beta}{(1+m)} E_t\left[\frac{1}{c_{t+1}}\left(1+\left(\alpha\frac{y_{t+1}}{k_{t+1}}-\delta\right)(1-\tau^c)\right)\right], \\
y_t &= S_t k_t^{\alpha} l_t^{1-\alpha}, \\
\lambda c_t &= \frac{(1-\alpha)\,y_t}{l_t}, \\
S_t &= S_{t-1}^{\rho} e_t
\end{aligned}$$

In the steady state,

$$\begin{aligned}
\bar{k}(1+g) &= \bar{k}(1-\delta) + \bar{y} - \bar{c}, \\
\frac{1}{\bar{c}} &= \frac{\beta}{(1+m)} E_t\left[\frac{1}{\bar{c}}\left(1+\left(\alpha\frac{\bar{y}}{\bar{k}}-\delta\right)(1-\tau^c)\right)\right], \qquad (25) \\
\bar{y} &= \bar{k}^{\alpha}\bar{l}^{1-\alpha}, \\
\lambda\bar{c} &= \frac{(1-\alpha)\,\bar{y}}{\bar{l}}, \\
S &= 1. \qquad (26)
\end{aligned}$$

Define the variables

$$k_y = \frac{k}{y}, \quad c_y = \frac{c}{y}$$

and solve the following equations for their steady state values.

$$\begin{aligned}
(g+\delta)\,k_y &= 1 - c_y \qquad (27) \\
\left[c_y - k_y(1-\beta)\,n\right](1+m) &= \beta c_y\left[1+\left(\frac{\alpha}{k_y}-\delta\right)(1-\tau^c)\right]
\end{aligned}$$

For the calibration in Table 1, this leads to the solutions:

$$\begin{aligned}
\text{RBC} &: \quad \{c_y = 0.728\,58, \quad k_y = 2.758\,3\}, \\
\text{HA} &: \quad \{c_y = 0.579\,50, \quad k_y = 4.273\,4, \}, \\
\text{HATAX} &: \quad \{c_y = 0.662\,29, \quad k_y = 3.432\,1\}.
\end{aligned}$$

The linearized model is given by the equations,

$$\begin{aligned}
\hat{k}_{t+1} &= a_1\hat{k}_t + a_2\hat{y}_t + a_3\hat{c}_t \\
-\hat{c}_t &= a_4\hat{c}_{t+1} + a_5\hat{k}_{t+1} + a_6\hat{y}_{t+1} + \hat{w}_{t+1}, \\
\hat{s}_{t+1} &= \rho\hat{s}_t + e_{t+1}, \\
\hat{y}_t &= \hat{s}_t + \alpha\hat{k}_t + (1-\alpha)\,\hat{l}_t, \\
\hat{c}_t &= \hat{y}_t - \hat{l}_t, \\
\hat{w}_{t+1} &= E_t\left[a_4\hat{c}_{t+1} + a_5\hat{k}_{t+1} + a_6\hat{y}_{t+1}\right] - \left[a_4\hat{c}_{t+1} + a_5\hat{k}_{t+1} + a_6\hat{y}_{t+1}\right],
\end{aligned} \tag{28}$$

And the linearized value of investment is

$$i_{t+1} = \frac{1}{1-c_y}y_{t+1} + \frac{-c_y}{1-c_y}c_{t+1}.$$

The linearization coefficients are defined as follows:

$$\begin{aligned}
a_1 &= \frac{(1-\delta)}{1+g}, \qquad a_2 = \frac{1}{(1+g)}\frac{1}{k_y}, \\
a_3 &= -\frac{1}{(1+g)}\frac{c_y}{k_y}, \qquad a_4 = -1 \\
a_5 &= -\frac{\alpha\,(1-t)}{k_y\,(1-\delta\,(1-t)) + \alpha\,(1-t)}, \\
a_6 &= \frac{\alpha\,(1-t)}{k_y\,(1-\delta\,(1-t)) + \alpha\,(1-t)}.
\end{aligned} \tag{29}$$

In matrix form the static equations can be written as:

$$B_1X_t + B_2Y_t = 0. \tag{30}$$

where

$$B_1 = \begin{bmatrix} 1-\alpha & -1 \\ -1 & 1 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 0 & \alpha & 1 \\ -1 & 0 & 0 \end{bmatrix},$$

$$X_t = \begin{bmatrix} \hat{l}_t \\ \hat{y}_t \end{bmatrix}, \quad Y_t = \begin{bmatrix} \hat{c}_t \\ \hat{k}_t \\ \hat{s}_t \end{bmatrix},$$

and the dynamic equations as

$$A_1 Y_{t+1} + A_2 X_{t+1} + A_3 Y_t + A_4 X_t + A_5 V_{t+1} = 0. \tag{32}$$

where

$$\begin{aligned} A_1 &= \begin{bmatrix} 0 & 1 & 0 \\ a_4 & a_5 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \qquad A_2 = \begin{bmatrix} 0 & 0 \\ 0 & a_6 \\ 0 & 0 \end{bmatrix}, \\ A_3 &= \begin{bmatrix} -a_3 & -a_1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & -\rho \end{bmatrix}, \qquad A_4 = \begin{bmatrix} 0 & -a_2 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, \\ A_5 &= \begin{bmatrix} 0 & 0 \\ 0 & 1 \\ -1 & 0 \end{bmatrix}. \end{aligned} \tag{33}$$

Using (30) and (32) this leads to

$$Y_t = J_1 Y_{t+1} + J_2 V_{t+1}$$

where

$$\begin{aligned} J_1 &= -\left(A_3 - A_4 B_1^{-1} B_2\right)^{-1} \left(A_1 - A_2 B_1^{-1} B_2\right), \\ J_2 &= -\left(A_3 - A_4 B_1^{-1} B_2\right)^{-1} A_5. \end{aligned}$$

The eigenvalues of J_1 for the three parameterizations are given by:

$$\begin{aligned} \text{Eigenvalues in RBC case} &: 0.797\,21, 1.\,216\,7, 1.\,075\,3 \\ \text{Eigenvalues in HA case} &: 0.884\,49, 1.\,042\,1, 1.\,075\,3 \\ \text{Eigenvalues in HATAX case} &: 0.84329, 1.\,0983, 1.\,075\,3 \end{aligned}$$

And the matrix Q^{-1} where Q is the matrix of eigenvectors is given by,[13]

$$\text{RBC case:} \quad Q^{-1} = \begin{bmatrix} -2.\,023\,2 & 1 & 1.\,375\,2 \\ 1 & 0 & -2.\,393\,8 \\ 0 & 0 & 1 \end{bmatrix},$$

[13] For the RBC case the matrix Q^{-1} was computed by inverting the matrix of eigenvectors using the eigenvector routine in Scientific Workplace. For the HA and HATAX cases I used the Jordan form routine which leads to a different normalization of the eigenvectors. In all three cases I checked the procedures by comparing them to the eigenvalue decomposition in Gauss.

$$\text{HA case:}\quad Q^{-1}=\begin{bmatrix} 1.4039 & -0.68224 & -0.8317 \\ -2.0861\times10^{-9} & 1.228\times10^{-7} & 1.00000 \\ -0.5221 & 1.3522\times10^{-7} & 1.6812 \end{bmatrix},$$

$$\text{HATAX case:}\quad Q^{-1}=\begin{bmatrix} 1.8602 & -0.74809 & -1.3334 \\ -1.0756\times10^{-6} & 2.3191\times10^{-7} & 1.0 \\ -0.18479 & 2.2898\times10^{-7} & 1.2318 \end{bmatrix}$$

The next step is to pick the row of Q^{-1} associated with the smallest eigenvalue of J_1 and set the product of this row with the vector of variables equal to zero to obtain a linear function relating $\hat{c}_t$ to $\hat{k}_t$ and $\hat{s}_t$.

$$\text{RBC case:}\qquad \begin{bmatrix} -2.0232 & 1 & 1.3752 \end{bmatrix}\begin{bmatrix} \hat{c}_t \\ \hat{k}_t \\ \hat{s}_t \end{bmatrix}=0$$

$$\begin{aligned} \hat{c}_t &= \frac{1}{2.0232}\hat{k}_t+\frac{1.3752}{2.0232}\hat{s}_t \\ q_1 &= 0.49427, \quad q_2=0.67972 \end{aligned}$$

$$\text{HA case:}\qquad \begin{bmatrix} 1.4039 & -0.68224 & -0.8317 \end{bmatrix}\begin{bmatrix} \hat{c}_t \\ \hat{k}_t \\ \hat{s}_t \end{bmatrix}=0$$

$$\begin{aligned} \hat{c}_t &= \frac{68224}{1.4039}\hat{k}_t+\frac{0.8317}{1.4039}\hat{s}_t \\ q_1 &= 0.48596, \quad q_2=0.59242 \end{aligned}$$

$$\text{HA tax case:}\qquad \begin{bmatrix} 1.8602 & -0.74809 & -1.3334 \end{bmatrix}\begin{bmatrix} \hat{c}_t \\ \hat{k}_t \\ \hat{s}_t \end{bmatrix}=0$$

$$\begin{aligned} \hat{c}_t &= \frac{0.74809}{1.8602}\hat{k}_t+\frac{1.3334}{1.8602}\hat{s}_t \\ q_1 &= 0.40216, \quad q_2=0.7168 \end{aligned}$$

We now seek a stochastic linear difference equation in the state vector $\left\{\hat{k}_t,\hat{s}_t\right\}$. To find this equation, take the expression

$$\hat{c}_t=q_1\hat{k}_t+q_2\hat{s}_t$$

and add it to the static equations to give an augmented set of static equations

$$\begin{bmatrix} 0 & 1-\alpha & -1 \\ 1 & 1 & -1 \\ -1 & 0 & 0 \end{bmatrix} \begin{bmatrix} \hat{c}_t \\ \hat{l}_t \\ \hat{y}_t \end{bmatrix} + \begin{bmatrix} \alpha & 1 \\ 0 & 0 \\ q_1 & q_2 \end{bmatrix} \begin{bmatrix} \hat{k}_t \\ \hat{s}_t \end{bmatrix} = 0.$$

or

$$B_3 X_t^* + B_4 Y_t^* = 0,$$

where

$$\begin{aligned} B_3 &= \begin{bmatrix} 0 & 1-\alpha & -1 \\ 1 & 1 & -1 \\ -1 & 0 & 0 \end{bmatrix}, \quad B_4 = \begin{bmatrix} \alpha & 1 \\ 0 & 0 \\ q_1 & q_2 \end{bmatrix}, \\ X_t^* &= \begin{bmatrix} \hat{c}_t \\ \hat{l}_t \\ \hat{y}_t \end{bmatrix}, \quad Y_t^* = \begin{bmatrix} \hat{k}_t \\ \hat{s}_t \end{bmatrix}. \end{aligned}$$

In terms of the vectors X^* and Y^* the dynamic equations take the form:

$$A_6 Y_{t+1} + A_7 Y_t + A_8 X_t + A_9 e_{t+1} = 0,$$

where

$$\begin{aligned} A_6 &= \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}, \quad A_7 = \begin{bmatrix} a_1 & 0 \\ 0 & \rho \end{bmatrix}, \\ A_8 &= \begin{bmatrix} a_3 & 0 & a_2 \\ 0 & 0 & 0 \end{bmatrix}, \quad A_9 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}. \end{aligned}$$

or

$$\begin{aligned} & A_6 Y_{t+1} + A_7 Y_t + A_8 X_t + A_9 e_{t+1} = 0 \\ & Y_{t+1} = J_3 Y_t + J_4 e_{t+1}, \end{aligned} \tag{34}$$

where

$$J_3 = -A_6^{-1} \left(A_7 - A_8 B_3^{-1} B_4 \right), \quad J_4 = -A_6^{-1} A_9.$$

To simulate data for Figures 3, 4 and 5 I generated a series of normal shocks with standard deviation 0.016 and I simulated data using Equation (34) where the parameter matrices J_3 and J_4 were chosen according to the three different model specifications given in Table 1.

References

[1] Abel, Andrew, N. Gregory Mankiw, Lawrence H. Summers and Richard J. Zeckhauser. (1989). "Assessing Dynamic Efficiency: Theory and Evidence". *Review of Economic Studies* 56, 1–20.

[2] Blanchard, Olivier J., (1985). "Debt Deficits and Finite Horizons," *Journal of Political Economy* 93, 2 223-247.

[3] _______________ and Charles M. Kahn. (1980) "The solution of linear difference models under rational expectations", *Econometrica* 48: 1305–11.

[4] Cass, David. (1984) "On capital overaccumulation in the aggregate, neoclassical model of economic growth: A complete characterization. *Journal of Economic Theory* 4: 200-23.

[5] Darby, Michael R. (1984) "Some Pleasant Monetarist Arithmetic," *Federal Reserve Bank of Minneapolis Quarterly Review*, 8(2): 15-20.

[6] Farmer, Roger (1998). *The Macroeconomics of Self-Fulfilling Prophecies,* MIT Press 1998 (2nd edition).

[7] Farmer, Roger E. A. (2002). "Fiscal Policy, Equity Premia and Heterogeneous Agents". UCLA mimeo.

[8] Gary D. Hansen, (1985). "Indivisible Labor and the Business Cycle," *Journal of Monetary Economics* 16: 309-28.

[9] Miller, Preston and Thomas J. Sargent. "A Reply to Darby" *Federal Reserve Bank of Minneapolis Quarterly Review*, 8(2).

[10] McGrattan, Ellen R. and Edward C. Prescott, (2001). "Taxes, Regulations and Asset Prices", Federal Reserve Bank of Minneapolis, Working Paper #610.

[11] Rogerson, Richard (1988). "Indivisible Labor, Lotteries and Equilibrium", *Journal of Monetary Economics* 21.

[12] Seater, John, (1982). "Marginal Federal Personal and Corporate Income Tax Rates in the U.S., 1909-1975", *Journal of Monetary Economics* 10, 361–381.

[13] Shiller, Robert. (2000). *Irrational Exuberance* Princeton University Press, Princeton N.J.

[14] Weil, Philippe, (1987). "Overlapping Families of Infinitely Lived Agents," *Journal of Public Economics*, 38: 183-198. *Cycles Research*, Tom Cooley, (ed.) Princeton University Press, Princeton New Jersey.