Chapter 1

1. In economics the word equilibrium refers to a situation in which no economic agent has an incentive to change his behavior. Different branches of economic theory use different but related concepts. In general equilibrium theory the prevalent concept is that of a competitive equilibrium. This refers to an allocation of commodities to every agent and a set of prices at which agents may freely trade these commodities. A competitive equilibrium is a set of prices and an allocation with the property that no individual would prefer to exchange his allocation for some other allocation that is attainable through trade at equilibrium prices. A competitive equilibrium will often be summarized by its associated prices. In economies in which agents live for ever, these prices will consist of an infinite sequence of prices that may be changing through time.

In the physical sciences, equilibrium refers to a situation in which some measurable quantity is constant through time. The behavior of a physical system is entirely determined by its state at t = 0, i.e. the boundary condition, $y_0 = \bar{y}_0$. If the variables in a physical system grow at constant rates the system is said to be in a stationary equilibrium. On the other hand, the behavior of economies depends on expectations of human beings. The boundary condition is often a boundedness condition, $\lim_{t\to\infty}|y_t| < \infty$. In a stationary equilibrium there is an invariant probability distribution of the variables in the system. If an economic equilibrium is constant through time we say that it is a stationary equilibrium.

2. Figure 1.1 represents the graph of the equation (2-2)

(2-1) $$y_t = 0.5 y_{t+1}$$

which can also be expressed as

(2-2) $$y_{t+1} = 2 y_t :$$

Sequences that obey equation (2-2) are of the form:

(2-3) $$y_t = 2^{t-1} y_1 .$$

All sequences of this kind grow without bound except for the steady state solution represented by the initial condition

(2-4) $$y_1 = 0 .$$

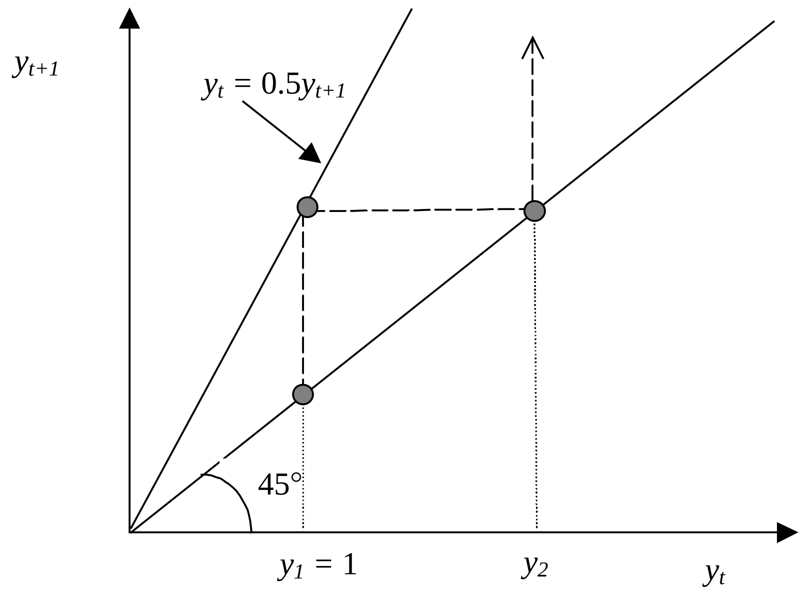

Figure 1.1

Sequences that obey equation (2-2) are of the form:

(2-5) $$y_t = 2^{t-1} y_1.$$

All sequences of this kind grow without bound except for the steady state solution represented by the initial condition

(2-6) $$y_1 = 0.$$

If the data of the problem give the initial condition $y_1 = 1$, then the unique solution to this difference equation grows without bound. If there is no initial condition as part of the problem then the unique solution that remains bounded is the steady state solution

(2-7) $$y_t = 0, \quad \textit{for all } t.$$

3. We seek a first order difference equation in the single state variable p_t^E. The equations of the model are

(3-1) $$p_t = a p_{t+1}^E,$$

(3-2) $$p_{t+1}^E = \lambda p_t^E + (1-\lambda) p_t, \quad p_1^E = 1.$$

Equation (3-1) should be interpreted as the reduced form of an economic model in which the current value of the price level depends on expectations of its future value. Equation (3-2) is a rule used by agents to form expectations. Before the advent of rational expectations *adaptive expectations* rules of this kind were common ways of specifying how agents learn. Substituting for p_t in (3-2) from (3-1) leads to the expression

(3-3) $$p_{t+1}^E = \frac{\lambda}{\left(1-a\left(1-\lambda\right)\right)} p_t^E,$$

which has a bounded solution for arbitrary initial beliefs whenever

(3-4) $$\left|\frac{\lambda}{1-a\left(1-\lambda\right)}\right| \le 1.$$

The parameter λ in equation (3-3) represents the speed with which agents learn. If $\lambda = 1$ then $p_{t+1}^E = p_t^E$ for all t and in this case agents never learn to adjust their expectations in the face of contradiction by experience. If, instead $\lambda = 0$ then agents always believe that the previous period's actual price will be repeated in the current period. In both these cases condition (3-4) is satisfied for any value of a.

4. An economic model is summarized by a difference equation in a set of state variables together with a number of initial conditions. It is usually the case that the state variables can be chosen to remain bounded in equilibrium. If, given the initial conditions, a unique bounded sequence satisfies the difference equation we say the model is regular. If a continuum of bounded sequences satisfies the equation, we say the model is irregular.