Chapter 4

1. For answers to question (1) see the definitions on page 75 of the text.

2. The excess demand for good 1 is

$$f_1(p) = x_1 - \omega_1 = \frac{p_1}{p_1 + p_2 + p_3}\left(\frac{p_1}{p_2} - 3\right).$$

Similarly, the excess demand for good 2 is

$$f_2(p) = x_2 - \omega_2 = \left(\frac{p_3}{p_2}\right)^2 + \frac{p_1}{p_1 + p_2}.$$

a. According to Walras' Law, the value of excess demands in all three markets should sum to zero. Therefore, the excess demand for the third good is

(1-1)
$$f_3(p) = -\frac{p_1 f_1(p) + p_2 f_2(p)}{p_3} = \frac{p_1^2}{p_3(p_1 + p_2 + p_3)}\left(3 - \frac{p_1}{p_2}\right) - \frac{p_3}{p_2} - \frac{p_1 p_2}{p_3(p_1 + p_2)}.$$

b. The price simplex is a vector of positive numbers with elements that sum to unity:

(1-2)
$$\Delta(p) = \left\{ p \in R_+^l \,|\, \sum_{j=1}^{l} p_j = 1 \right\}.$$

c. No, these demand functions could not have come from preferences that satisfy all of the assumptions imposed in the chapter. Consider what happens to the excess demands as we take a sequence of price vectors $p^n \to \bar{p} \equiv (0, \bar{p}_2, \bar{p}_3)$:

(1-3)
$$f_1(p^n) \to 0, \quad f_2(p^n) \to \left(\frac{\bar{p}_3}{\bar{p}_2}\right)^2, \quad f_3(p^n) \to -\frac{\bar{p}_3}{\bar{p}_2}.$$

Clearly, from (1-3), the norm of the excess demand vector $\| f(p^n) \| \to \| f(\bar{p}) \| < \infty$ which violates property 5 stated on page 73 of the text.

d. The Debreu-Sonnenschein-Mantel theorem states that any continuous function $f(p)$ which satisfies Walras' Law, can be an excess demand function for some economy, provided that there are *at least* as many consumers as there are commodities. For the theorem to apply in this case, we need the number of consumers *m,* to be greater than or equal to 3.

3. Figure 4.1 illustrates the constant function $x^1(p)=1$.

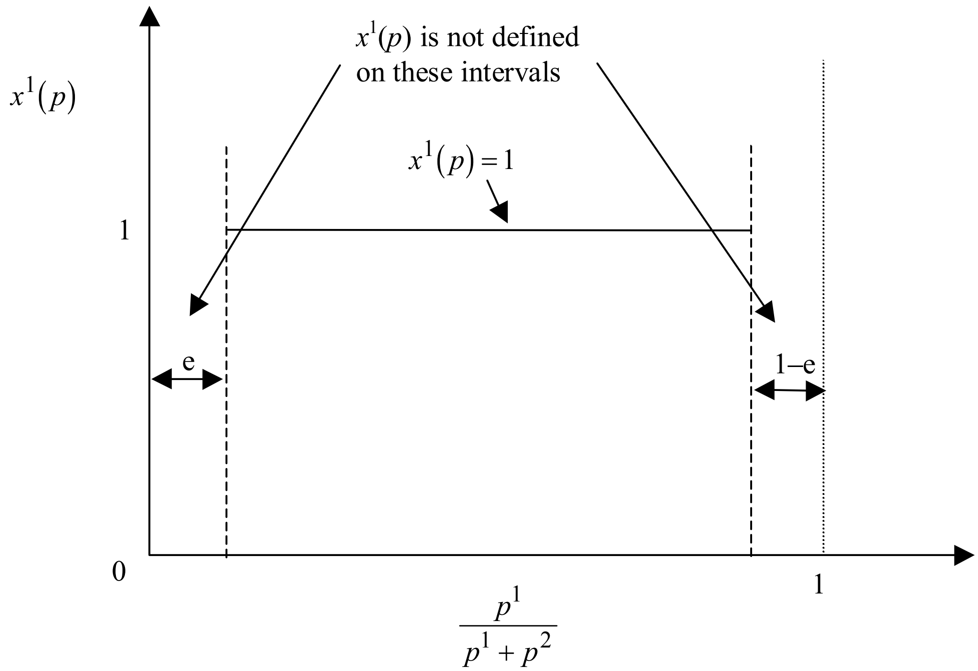

Figure 4.1

According to the DSM theorem, this function could be the first component of the excess demand function for some well-defined economy with at least two consumers. However, it does not cross the axis and there is therefore no equilibrium. The apparent paradox is resolved by the fact that the DSM theorem does not require $x(p)$ to replicate the excess demand function over the entire price simplex; only in its interior. Any economy that satisfies the assumptions of general equilibrium theory *will* possess an equilibrium. This equilibrium must therefore occur in one of the intervals $[0,e]$ or $[1-e,1]$.

4. Applying Walras' law it follows that:

$$f^3 = -\frac{p_1 f^1 + p_2 f^2}{p_3} = \frac{-p_1(p_1-p_2)}{p_3^{\ 2}} - \frac{p_2 p_1}{p_2^{\ 2}}$$

5. This problem deals with general equilibrium theory.

a. Mr. A maximizes $U^A = \log x_1^A + \log x_2^A + \log x_3^A$ subject to the budget constraint $\sum_{j=1}^{3} p_j x_j^A = p_1$. He owns one unit of good 1. Given his preferences he will choose to spend one third of his income on each good: therefore, his excess demand functions are:

(5-1) $$f_1^A(p) = -\frac{2}{3}, \quad f_2^A(p) = \frac{p_1}{3p_2}, \quad f_3^A(p) = \frac{p_1}{3p_3}.$$

Mr. B maximizes $U^B = x_1^B x_2^B x_3^B$ subject to the budget constraint $\sum_{j=1}^{3} p_j x_j^B = p_2$. He owns one unit of good 2 and since his preferences are an increasing monotonic transformation of Mr. A's preferences he also will choose to spend one third of his income on each good. Mr. B's excess demand functions are:

(5-2) $$f_1^B(p) = \frac{p_2}{3p_1}, \quad f_2^B(p) = -\frac{2}{3}, \quad f_3^B(p) = \frac{p_2}{3p_3}.$$

Mr. C maximizes $U^C = -\left(\frac{1}{x_1^C}\right) - \left(\frac{1}{x_2^C}\right) - \left(\frac{1}{x_3^C}\right)$ subject to the budget constraint $\sum_{j=1}^{3} p_j x_j^B = p_3$. He owns one unit of good 3. Taking ratios of marginal utilities and substituting into the budget constraint one can establish that his excess demand functions are:

(5-3) $$f_1^C(p) = \frac{p_3}{\left(\sqrt{p_1} + \sqrt{p_2} + \sqrt{p_3}\right)\sqrt{p_1}}, \quad f_2^C(p) = \frac{p_3}{\left(\sqrt{p_1} + \sqrt{p_2} + \sqrt{p_3}\right)\sqrt{p_2}},$$
$$f_3^C(p) = \frac{p_3}{\left(\sqrt{p_1} + \sqrt{p_2} + \sqrt{p_3}\right)\sqrt{p_3}} - 1.$$

Hence, the aggregate demand functions for each of the three goods are:

(5-4)
$$f_1(p) = -\frac{2}{3} + \frac{p_2}{3p_1} + \frac{p_3}{\sqrt{p_1}\left(\sqrt{p_1} + \sqrt{p_2} + \sqrt{p_3}\right)},$$
$$f_2(p) = \frac{p_1}{3p_2} - \frac{2}{3} + \frac{p_3}{\sqrt{p_2}\left(\sqrt{p_1} + \sqrt{p_2} + \sqrt{p_3}\right)},$$
$$f_3(p) = \frac{p_1}{3p_3} + \frac{p_2}{3p_3} + \frac{p_3}{\sqrt{p_3}\left(\sqrt{p_1} + \sqrt{p_2} + \sqrt{p_3}\right)} - 1.$$

b. The aggregate excess demand function is the vector $f(p) = \left(f_1(p), f_2(p), f_3(p)\right)$, where the components $f_i(p)$ are defined in (5-4).

c. Zero degree homogeneity in *p*, means that;

$f(\lambda p) = \left(f_1(\lambda p) f_2(\lambda p) f_3(\lambda p)\right) = \left(f_1(p), f_2(p), f_3(p)\right) = f(p)$,

this can be directly verified from (5-4):

$$f_1(\lambda p) = -\frac{2}{3} + \frac{\lambda p_2}{3\lambda p_1} + \frac{\lambda p_3}{\sqrt{\lambda p_1}\left(\sqrt{\lambda p_1} + \sqrt{\lambda p_2} + \sqrt{\lambda p_3}\right)} = -\frac{2}{3} + \frac{p_2}{3p_1} + \frac{p_3}{\sqrt{p_1}\left(\sqrt{p_1} + \sqrt{p_2} + \sqrt{p_3}\right)}$$

$$f_2(\lambda p) = \frac{\lambda p_1}{3\lambda p_2} - \frac{2}{3} + \frac{\lambda p_3}{\sqrt{\lambda p_2}\left(\sqrt{\lambda p_1} + \sqrt{\lambda p_2} + \sqrt{\lambda p_3}\right)} = \frac{p_1}{3p_2} - \frac{2}{3} + \frac{p_3}{\sqrt{p_2}\left(\sqrt{p_1} + \sqrt{p_2} + \sqrt{p_3}\right)}$$

$$f_3(\lambda p) = \frac{\lambda p_1}{3\lambda p_3} + \frac{\lambda p_2}{3\lambda p_3} + \frac{\lambda p_3}{\sqrt{\lambda p_3}\left(\sqrt{\lambda p_1} + \sqrt{\lambda p_2} + \sqrt{\lambda p_3}\right)} - 1 = \frac{p_1}{3p_3} + \frac{p_2}{3p_3} + \frac{p_3}{\sqrt{p_3}\left(\sqrt{p_1} + \sqrt{p_2} + \sqrt{p_3}\right)} - 1$$

To show that this solution satisfies Walras' Law:

$$p_1 f_1(p) = -\frac{2p_1}{3} + \frac{p_2}{3} + \frac{\sqrt{p_1}\, p_3}{\left(\sqrt{p_1} + \sqrt{p_2} + \sqrt{p_3}\right)},$$

$$p_2 f_2(p) = \frac{p_1}{3} - \frac{2p_2}{3} + \frac{\sqrt{p_2}\, p_3}{\left(\sqrt{p_1} + \sqrt{p_2} + \sqrt{p_3}\right)},$$

$$p_3 f_3(p) = \frac{p_1}{3} + \frac{p_2}{3} + \frac{\sqrt{p_3}\, p_3}{\left(\sqrt{p_1} + \sqrt{p_2} + \sqrt{p_3}\right)} - p_3.$$

Summing these terms gives:

$$\sum_{i=1}^{3} p_i f^i(p) = \left(-\frac{2p_1}{3} + \frac{p_1}{3} + \frac{p_1}{3}\right) + \left(\frac{p_2}{3} - 2\frac{p_2}{3} + \frac{p_2}{3}\right) + \frac{p_3\left(\sqrt{p_1} + \sqrt{p_2} + \sqrt{p_3}\right)}{\left(\sqrt{p_1} + \sqrt{p_2} + \sqrt{p_3}\right)} - p_3 = 0.$$