Chapter 11

1. There are two main issues in monetary theory:

a) Why does an unbacked paper asset have value.

b) Why do government bonds pay interest i.e. why is there "rate of return dominance" of one paper asset over another with apparently identical risk characteristics.

Legal restrictions theory was put forward by Bryant and Neil Wallace to explain rate of return dominance. The theory asserts that in the absence of legal restrictions, for example, a prohibition on the issue of private bank notes, government debt would not bear interest. Some might argue that it is a "better" theory of money than money in the utility function since it explains what is and what is not money. Money is the object that is declared money by government fiat.

2. Money is

a) A store of value

b) A standard of deferred payment

c) A unit of account

d) A medium of exchange

In the overlapping generations model money satisfies properties a) through c) but it is not in any meaningful sense a medium of exchange.

3.

3.a If the cash-in-advance constraint is binding then

(3.a-1) $$c_t = \frac{m_{t-1}}{p_t}.$$

And we can write the budget constraint as:

(3.a-2) $$c_{t+1} = \frac{m_t}{p_{t+1}} = \left[\frac{b_{t-1}}{p_{t+1}}\left(1+r_{t-1}\right) + \frac{w_t}{p_{t+1}} l_t - \frac{b_t}{p_{t+1}}\right].$$

Substituting this expression into the objective function leads to the problem:

(3.a-3) $$\max_{c_1,\{l_t,b_t\}_{t=1}^{\infty}} \frac{c_1^{1-\rho}}{1-\rho} + \sum_{t=1}^{\infty} \beta^t \frac{1}{1-\rho}\left[\frac{b_{t-1}}{p_{t+1}}\left(1+r_{t-1}\right) + \frac{w_t}{p_{t+1}} l_t - \frac{b_t}{p_{t+1}}\right]^{1-\rho} - \sum_{t=1}^{\infty} \beta^{t-1} \frac{l_t^{1+\gamma}}{1+\gamma} \quad .$$

The first order conditions for this problem are:

(3.a-4) $$l_t^{\gamma} = c_{t+1}^{-\rho}\beta\frac{w_t}{p_{t+1}} = c_{t+1}^{-\rho}\beta\frac{p_t}{p_{t+1}}$$

where the second equality follows from the assumption that labor can be transformed into output one for one.

(3.a-5) $$\frac{1}{p_t}c_t^{-\rho} = \beta\frac{(1+r_t)}{p_{t+1}}c_{t+1}^{-\rho}.$$

Equation (3.a-5) serves only to define the sequence of interest rates. Using (3.a-4), the production function (which implies $l_t = c_t$) and the cash in advance constraint and defining $\mu_t = \frac{m_t}{m_{t-1}}$ we can write the required difference equation as:

(3.a-6) $$\left(\frac{m_{t-1}}{p_t}\right)^{\gamma} = \left(\frac{m_t}{p_{t+1}}\right)^{-\rho}\beta\frac{m_t}{p_{t+1}}\frac{p_t}{m_{t-1}}\frac{m_{t-1}}{m_t} \quad \Rightarrow (z_t)^{\gamma} = \frac{(z_{t+1})^{1-\rho}}{z_t}\frac{\beta}{\mu_t}.$$

Rearranging terms gives the first order difference equation:

(3.a-7) $$z_{t+1} = \left(\frac{\mu_t}{\beta}\right)^{\frac{1}{1-\rho}}(z_t)^{\frac{1+\gamma}{1-\rho}}.$$

3.b Since the right-hand-side is a monotonic function of z_t this equation has one non-zero steady state.

3.c Indeterminacy requires that the slope of the difference equation:

(3.c-1) $$\log(z_{t+1}) = \frac{1+\gamma}{1-\rho}\log(z_t) + \frac{1}{1-\rho}\log(\mu_t) - \frac{1}{1-\rho}\log(\beta)$$

be less than one in absolute value: i.e.

(3.c-2) $$\left|\frac{1+\gamma}{1-\rho}\right| < 1.$$

4.

4.a One needs to assume that

(4.a-1) $$\underset{T\to\infty}{Lim}\, Q_1^T\left(M_T + B_T\right) \geq 0, \qquad Q_1^T = \prod_{s=1}^{T-1}\frac{1}{1+r_s}.$$

This is sometimes known as a no Ponzi scheme constraint.

4.b The transversality condition requires that:

(4.b-1) $$\lim_{T\to\infty} \beta^{T-1} U_c(C_T)(M_{T+1} + B_{T+1}) \le 0.$$

Since the first order condition for the choice of consumption requires that

(4.b-2) $$\beta^{T-1} U_c(C_T) = Q_1^T P_t,$$

the transversality condition and the no-Ponzi scheme constraint are reverse inequalities of each other. One says that the family cannot be outside its budget constraint; the other says that it would like to be if possible.

4.c Using the production function and the budget constraint we can write the optimization problem as follows:

(4.c-1) $$\max U = \sum_{t=1}^{\infty} \beta^{t-1} \left\{ \frac{\left[\frac{M_{t-1}}{P_t} + (1+r_{t-1}) \frac{B_{t-1}}{P_t} + \frac{X_t}{P_t} + \left(\frac{M_{t-1}}{P_t} \right)^{\gamma} - \frac{M_t}{P_t} - \frac{B_t}{P_t} \right]^{1-\theta}}{1-\theta} \right\}$$

The required first order conditions are:

(4.c-2) $$\frac{1}{P_t} C_t^{-\theta} = \beta \frac{C_{t+1}^{-\theta}}{P_{t+1}} \left[1 + \gamma \left(\frac{M_t}{P_{t+1}} \right)^{\gamma-1} \right].$$

(4.c-3) $$\frac{1}{P_t} C_t^{-\theta} = \beta \frac{C_{t+1}^{-\theta}}{P_{t+1}} [1 + r_t].$$

4.d If the government picks the rate of interest then comparing (4.c-2) with (4.c-3) implies

(4.d-1) $$\gamma \left(\frac{M_t}{P_{t+1}} \right)^{\gamma-1} = r, \quad \Rightarrow \quad m = \left(\frac{r}{\gamma} \right)^{\frac{1}{\gamma-1}}, \quad Y = \left(\frac{r}{\gamma} \right)^{\frac{\gamma}{\gamma-1}}.$$

4.e When the government picks $M_t = \mu M_{t-1}$ we can use the production function together with (4.c-2) to give:

(4.e-1) $$\frac{1}{P_t} m_t^{-\gamma\theta} = \beta \frac{1}{P_{t+1}} m_{t+1}^{-\gamma\theta} \left[1 + \gamma (m_{t+1})^{\gamma-1} \right].$$

Using the definition of monetary policy:

(4.e-2) $$m_t^{1-\gamma\theta} = \frac{\beta}{\mu} m_{t+1}^{1-\gamma\theta}\left[1+\gamma\left(m_{t+1}\right)^{\gamma-1}\right].$$

Which is (11.27) from page 257, (lagged one period).

4.f In the steady state:

(4.f-1) $$m = \left\{\frac{1}{\gamma}\left[\frac{\mu}{\beta}-1\right]\right\}^{\frac{1}{\gamma-1}}.$$

4.g Log linearizing (4.c-2) gives:

(4.g-1) $$z_t = \left[1-\frac{1-\gamma}{1-\gamma\theta}\frac{r}{1+r}\right]z_{t+1}+k$$

where k is a constant, r is the steady state interest rate and

$$\frac{r}{1+r} = \frac{\gamma m^{\gamma-1}}{1+\gamma m^{\gamma-1}}$$

is the log linearization of the term in square brackets in equation (4.e-1).

4.h We restrict attention to non-stationary equilibria that converge to the stationary monetary steady state. These will exist in the constant money growth regime whenever:

(4.h-1) $$\left\|\left[1-\frac{1-\gamma}{1-\gamma\theta}\frac{r}{1+r}\right]\right\| \; > \; 1 \quad \Rightarrow \quad \left\|\left[1-\frac{1-\gamma}{1-\gamma\theta}\left(1-\frac{\beta}{\mu}\right)\right]\right\| \; > \; 1,$$

where the second equality follows from (4.c-3) evaluated at the steady state.