Chapter 12

1.

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/***********************************************************
                        Problem 12.1
                Impulse response functions
***********************************************************/

new;
library pgraph;
graphset;

let t=10;                         @number of periods@
let a={0.2 0,1 0.5};    @specification of VAR coefficients@

let e=1 0;                             @unit vector@
z=zeros(2,t);                         @initialization@

/* simulation of the system for t periods */
i=1;
b=a;
DO WHILE i<=t;
    z[.,i]=b*e;
    i=i+1;
    b=b*a;
ENDO;

/* generation of graphical output*/
/*creating a linear trend*/
tt=zeros(t,1); @initialization@
j=1;
DO WHILE j<=t;
  tt[j]=j;
  j=j+1;
ENDO;
begwind;
window(2,1,0);
setwind(1);
     _pcolor=3;
     title("Imp. resp. of y to shock in y");
     xy(tt,z[1,.]');
nextwind;
     _pcolor=5;
     title("Imp. resp. of x to shock in y");
     xy(tt,z[2,.]');
endwind;

end;
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2. This problem involves a monetary model with real balances in the utility function.

2.a

(2.a-1) $$Q_t^s = \prod_{v=t}^{s-1} \frac{1}{1+r_v}.$$

2.b The borrowing limit ensures that the value of consumption is bounded. Without this limit the family would try to consume an infinite amount in every period by borrowing.

2.c

i) $MUC = C^{-\rho}$

ii) $MDW = -m^{1-\rho}$

iii) $MURB = -L(1-\rho)m^{-\rho}$

2.d

(2.d-1)

$$= \frac{1}{1-\rho}\left[\frac{B_0}{P_1}(1+i_0)+\frac{M_0}{P_1}-\frac{B_1}{P_1}-\frac{M_1}{P_1}+L_1+\frac{T_1}{P_1}\right]^{1-\rho} - L_1\left(\frac{M_1}{P_1}\right)^{1-\rho}$$
$$+ \quad \beta\left\{\frac{1}{1-\rho}\left[\frac{B_1}{P_2}(1+i_1)+\frac{M_1}{P_2}-\frac{B_2}{P_2}-\frac{M_2}{P_2}+L_2+\frac{T_2}{P_2}\right]^{1-\rho} - L_2\left(\frac{M_2}{P_2}\right)^{1-\rho}\right\} + \; .....$$

The required first order conditions are:

(2.d-2) $$C_t^{-\rho} = \left(\frac{M_t}{P_t}\right)^{1-\rho},$$

(2.d-3) $$\frac{1}{P_t}C_t^{-\rho} = \beta\frac{1}{P_{t+1}}C_{t+1}^{-\rho}\left[1+i_t\right],$$

(2.d-4) $$\frac{1}{P_t}C_t^{-\rho}\left[1+\frac{L_t(1-\rho)\left(\frac{M_t}{P_t}\right)^{-\rho}}{C_t^{-\rho}}\right] = \beta\frac{1}{P_{t+1}}C_{t+1}^{-\rho},$$

where the term in square brackets in equation (2.d-4) is $+\frac{U_m}{U_c}$.

2.e Evaluating (2.d-3) and (2.d-4) at the steady state it follows that:

(2.e-1) $$-\frac{U_m}{U_c} = \frac{1}{1+i} \quad \Rightarrow \quad \frac{U_m}{U_c} = \frac{i}{1+i} \approx i \text{ for small } i.$$

2.f A competitive equilibrium is defined relative to a given policy. Assume that fiscal policy picks $\{B_t\}_{t=0}^{\infty}$. Assume first that monetary policy chooses the sequence of interest rates $\{i_t\}_{t=1}^{\infty}$. Then a competitive equilibrium is a sequence of money stocks $\{M_t\}_{t=1}^{\infty}$, a sequence of feasible allocations $\{L_t, Y_t\}_{t=1}^{\infty}$ and a sequence of prices $\{P_t\}_{t=1}^{\infty}$ such that all markets clear and the allocations are optimally chosen subject to the prices. If monetary policy picks the sequence $\{M_t\}_{t=1}^{\infty}$ then equilibrium is a sequence of interest rates $\{i_t\}_{t=1}^{\infty}$, a sequence of feasible allocations $\{L_t, Y_t\}_{t=1}^{\infty}$ and a sequence of prices $\{P_t\}_{t=1}^{\infty}$ such that all markets clear and the allocations are optimally chosen subject to the prices.

2.g Yes there might exist sunspot equilibria. Since this is a monetary economy, the welfare theorems do not necessarily hold and one cannot rule out sunspot equilibria a priori. If the steady state equilibrium of the perfect foresight model is indeterminate then it will be particularly easy to construct examples of sunspot equilibria.

2.h No- indeterminacy is not a necessary condition for indeterminacy. The example of a sunspot equilibrium constructed in the paper by Cass and Shell is an example of a model in which equilibrium is determinate, but there may exist sunspot equilibria.

2.i When equilibria are indeterminate, it may be possible to construct an equilibrium in which the price level is predetermined one period in advance. In an economy with a predetermined price equilibrium, a purely nominal shock will have real effects.