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Animal spirits in a monetary model[☆]

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ABSTRACT

We integrate Keynesian economics with general equilibrium theory in a new way. We develop a simple graphical apparatus, the IS-LM-NAC framework, that can be used by policy makers to understand how policy affects the economy. A new element, the No-Arbitrage-Condition (NAC) curve, connects the interest rate to current and expected future values of the stock market and it explains how 'animal spirits' influence economic activity. Our framework provides a rich new approach to policy analysis that explains the short-run and long-run effects of policy.

1. Introduction

In the lead-up to the 2008 financial crisis, a consensus developed among academic macroeconomists that the problem of macroeconomic stability had been solved. According to that consensus, the New-Keynesian dynamic stochastic general equilibrium (DSGE) model provides a good first approximation to the way that monetary policy influences output, inflation and unemployment. In its simplest form, the NK model has three equations; a dynamic IS curve, a policy equation that describes how the central bank sets the interest rate, and a New-Keynesian Phillips curve. In its more elaborate form, the New-Keynesian DSGE model is reflected in work that builds on the medium scale DSGE model of Smets and Wouters (2007).

The NK model evolved from post-war economic theory in which the Keynesian economics of the *General Theory*, (Keynes, 1936), was grafted onto the microeconomics of Walrasian general equilibrium theory (Walras, 1899). Paul Samuelson, in the third edition of his undergraduate textbook, (Samuelson, 1955), referred to this hybrid theory as the 'neoclassical synthesis'. According to the neoclassical synthesis, the economy is Keynesian in the short-run, when not all wages and prices have adjusted to clear markets; it is classical in the long-run, when all wages and prices have adjusted to clear markets and the demands and supplies for all goods and for labor are equal.[1]

The neoclassical synthesis is still the main framework taught in economics textbooks, and, in the form of 'dynamic IS-LM analysis', it is used by policy makers to frame the way they think about the influence of changes in fiscal and monetary policy on economic activity.[2] This paper proposes an alternative framework. Building on work by Farmer (2010a) we integrate

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[1] This characterization of the history of thought is drawn from Farmer (2010b) and elaborated on in Farmer (2016b).

[2] See, for example, Mankiw (2015).

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Keynesian economics with general equilibrium theory in a new way to demonstrate that low-income high-unemployment inefficient equilibria may be sustained in the long run. Our work displays two main differences from the New Keynesian model.

First, the steady state equilibria of our model display dynamic indeterminacy. For every steady state equilibrium, there are multiple dynamic paths, all of which converge to the same steady state. We use that property to explain how changes in the money supply may be associated with immediate changes in real economic activity without invoking artificial barriers to price change. Prices in our model are set one period in advance, but there are no explicit costs of price adjustment.[3]

Second, our model displays steady state indeterminacy. We adopt a labor search model in which the presence of externalities generates multiple steady state equilibria. Unlike classical search models we do not close the model by assuming that firms and workers bargain over the wage.[4] Instead, as in Farmer (2010a, 2012b, 2016b), firms and workers take wages and prices as given and employment is determined by aggregate demand. We use that feature to explain why unemployment is highly persistent in the data. Persistent unemployment, in our model, represents potentially permanent deviations of the market equilibrium from the social optimum.[5]

To close our model, we assume that equilibrium is selected by 'animal spirits' and we model that idea with a *belief function* as in Farmer (1999, 2002, 2012a). We treat the belief function as a fundamental with the same methodological status as preferences and endowments and we study the implications of that assumption for the ability of monetary policy to influence inflation, output and unemployment. Although we use a stylized calibration of our model to generate impulse response graphs, this paper is not a serious piece of data analysis: Our contribution is to introduce a new pedagogical tool, the IS-LM-NAC model, and to illustrate the use of that tool with a series of policy exercises. We refer the reader to Farmer and Nicolò (2018), "Keynesian economics without the Phillips curve", for an empirical application of our framework.

There have been many attempts to build micro-foundations to the IS-LM model. The most popular is the dominant New-Keynesian model that appears in modern graduate textbooks (Galí, 2008; Woodford, 2003). Bilbiie (2008, 2019) and Dong et al. (2016) use credit market imperfections to generate Keynesian results from micro foundations and a micro-founded approach that stresses financial market imperfections has found its way into the undergraduate curriculum in the UK with the influential textbook by Carlin and Soskice (2014). Our main difference from these approaches is the ability of our model to generate deviations of the unemployment rate from the social optimum that can persist forever in the absence of monetary or fiscal policy intervention.

The supply side of our model was developed in, Farmer (2012b, 2013) and in Farmer (2016b) where Farmer refers to a model closed by beliefs, as a "Keynesian search model" to distinguish it from the classical approach to search theory (Diamond, 1982; Mortensen, 1970; Pissarides, 1984). Keynesian search theory replaces the classical assumption that the bargaining weight is a parameter with the alternative assumption that the unemployment rate is demand determined.

The current paper builds on the Keynesian search approach by including the real value of money balances in the utility function to capture the function of money as a means of exchange. The addition of money leads to genuinely new results from the real model in Farmer's previous work. For example, we show that, under some specifications of beliefs, money may be non-neutral. An unanticipated shock to the money supply may have a permanent effect on the unemployment rate through its influence on beliefs about the real value of future wealth.

2. The model

We construct a two-period overlapping generations model. In every period there are two generations of representative households; the young and the old.[6] The young inelastically supply one unit of labor, but, due to search frictions, a fraction of young individuals remain unemployed in any given period. We assume that there is perfect insurance within the household and that labor income is split between current consumption, interest bearing assets, and money balances.

Households hold money, physical capital and financial assets in the form of government bonds. Money is dominated in rate-of-return and is held for transaction purposes. We model this by assuming that real money balances yield utility as in Patinkin (1956). The old generation receives interest on capital and bonds and they sell assets to the young generation. We close the markets for physical capital and labor by assuming that there is one unit of non-reproducible capital and that the labor-force participation rate is constant and equal to one. We also assume that government bonds are in zero net supply.

[3] For earlier papers that invoke that idea see Farmer and Woodford (1997), Farmer (1991, 1999, 2000, 2002), Matheny (1998), and Benhabib and Farmer (2000). Although we do not explicitly adopt the assumptions of menu costs (Mankiw, 1985) or price rigidity (Christiano et al., 2005; Smets and Wouters, 2007), these are both possible explanations for agents in our model to select an equilibrium in which prices are predetermined.

[4] By classical search models, we mean the literature that builds on work by Diamond (1982), Mortensen (1970), and Pissarides (1976).

[5] King et al. (1991), Beyer and Farmer (2007), and Farmer (2012c, 2015) find evidence of a unit root in the U.S. unemployment rate. Sticky-price models with a unique determinate steady-state equilibrium have difficulty generating enough persistence to understand this fact, as do unique-equilibrium models of the monetary transmission mechanism that assume sticky information (Mankiw and Reis, 2007) or rational inattention, (Sims, 2003). Our approach generates *permanent* equilibrium movements in the unemployment rate that are consistent with a unit root, or near unit root, in U.S. unemployment data and is complimentary to theories that explicitly model small costs of price change. Blanchard and Summers (1986, 1987) attribute persistent unemployment to models that display hysteresis. Our model has that feature, but for different reasons than the explanation given by Blanchard and Summers. For a recent survey that explains the evolution of models of dynamic and steady state indeterminacy, see Farmer (2016a).

[6] The restriction to two-period lives is made for expository purposes only. In Appendix A we develop a long-lived version of our approach using the Blanchard (1985) perpetual youth model.

There is a single good produced by a continuum of competitive firms. Firms rent capital from old generation individuals and hire young generation individuals. Hiring labor is subject to search frictions. Firms take prices and wages as given and they allocate a fraction of labor to recruiting. We assume that every worker allocated to recruiting can hire q new workers, where q is taken as given by firms but determined in equilibrium by the search technology. Every worker allocated to recruiting is one less worker allocated to production.

Search in the labor market generates multiple equilibria. To select equilibrium, we assume that economic agents form beliefs about the real value of their financial wealth using a belief function that is a primitive of our model. Our Keynesian search approach differs from the more usual assumption in the classical labor search literature where the equilibrium is pinned down by Nash bargaining over the real wage.[7]

To make our model transparent, we consider only permanent unanticipated shocks to beliefs. There is no uncertainty regarding other fundamentals of the economy.

Our model provides a microfoundation for the textbook Keynesian cross, in which the equilibrium level of output is determined by aggregate demand. Our labor market structure explains why firms are willing to produce any quantity of goods demanded, and our assumption that beliefs are fundamental determines aggregate demand. In our model, beliefs select an equilibrium and in that equilibrium, the unemployment rate may differ permanently from the social planning optimum.

3. Aggregate supply

There is a unit continuum of competitive firms. We represent the capital and labor employed and output produced by each individual firm with the symbols K_t, L_t, and Y_t.[8] To refer to aggregate labor and aggregate output we use the symbols $\bar{L}_t$ and $\bar{Y}_t$. The variables K_t, L_t, and Y_t are indexed by $j \in [0, 1]$ where

$$\bar{K}_t = \int_j K_t(j)dj, \quad \bar{L}_t = \int_j L_t(j)dj, \quad \bar{Y}_t = \int_j Y_t(j)dj.$$

Since all firms face the same prices and will make the same decisions, it will always be true that $K_t(j) = K_t,\ L_t(j) = L_t$ and $Y_t(j) = Y_t,$ hence, we will dispense with the subscript j in the remainder of our exposition.

All workers work only in the first period of their life. A firm puts forward a production plan in which it proposes to allocate X_t workers to production and V_t workers to recruiting where

$$L_t = X_t + V_t.$$

Output is given by the expression

$$Y_t = K_t^{\alpha} X_t^{1-\alpha},$$

and the total number of workers employed at the firm is equal to

$$L_t = q_t V_t, \tag{1}$$

where the firm takes q_t as given. Putting these pieces together, we may express the output of the firm as

$$Y_t = K_t^{\alpha}\left[\left(1 - \frac{1}{q_t}\right)L_t\right]^{1-\alpha}. \tag{2}$$

Firms maximize profit,

$$P_t Y_t - R_t K_t - W_t L_t,$$

by choosing how much capital and labor to hire. Here P_t is the money price of goods, R_t is the money rental rate of capital and W_t is the money wage. Perfect competition implies that factors earn their marginal products and profit is equal to zero.

$$(1-\alpha)\frac{Y_t}{L_t} = \frac{W_t}{P_t} \quad \text{and} \quad \alpha\frac{Y_t}{K_t} = \frac{R_t}{P_t}. \tag{3}$$

Notice that Eq. (2) looks like a classical production function with one exception. The variable, q_t, which represents labor market tightness, influences total factor productivity. One may show that q_t is greater than 1 in equilibrium. A low value of q_t corresponds to a tight labor market in which firms must devote a large amount of resources to recruiting and in which productivity is low. A high value of q_t corresponds to a loose labor market in which firms may devote a small amount of resources to recruiting and in which productivity is high.

At the aggregate level, we assume the existence of a matching technology that determines aggregate employment $\bar{L}_t$ as a function of aggregate resources devoted to recruiting, $\bar{V}_t$, and the aggregate number of unemployed searching workers, $\bar{U}_t$. This function is given by,

$$\bar{L}_t = m\left(\bar{V}_t, \bar{U}_t\right) \equiv \left(\Gamma \bar{V}_t\right)^{1/2}, \tag{4}$$

[7] Farmer (2016b, Chapter 7) distinguishes Keynesian search models, where employment is determined by aggregate demand, from classical search models, where employment is determined by Nash bargaining.

[8] The model developed in this section is drawn from Farmer (2012b).

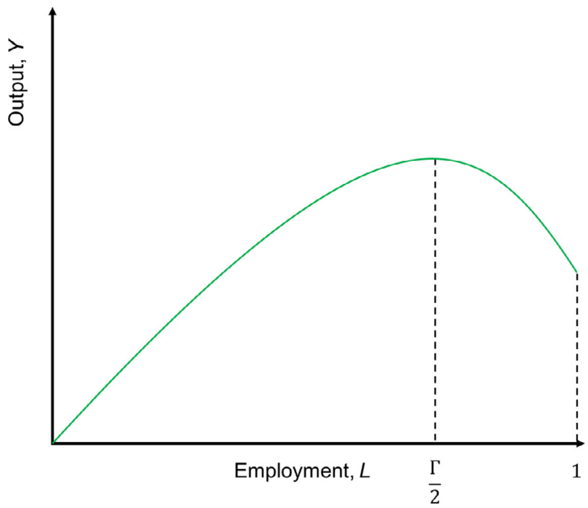

Fig. 1. The social production function.

where $\bar{U}_t = 1$ because workers are fired every period and the number of searching workers is equal to 1 at the beginning of every period.[9] Because the economy is endowed with one unit of labor, the end-of-period unemployment rate can be defined as

$$U_t = 1 - \bar{L}_t.$$

The parameter Γ in the matching function determines the efficiency of the matching technology. In a symmetric equilibrium where $L_t = \bar{L}_t$, we may combine Eqs. (1), (2) and (4) to find an expression for Y_t in terms of L_t and $\bar{L}_t$

$$Y_t = K_t^{\alpha}\left[L_t\left(1 - \frac{\bar{L}_t}{\Gamma}\right)\right]^{1-\alpha}, \tag{5}$$

where $\bar{L}_t/\Gamma = 1/q_t$.

Eq. (5) is the private production function. This function represents the connection between the output of an individual firm, Y_t, the capital and labor inputs at the level of the firm, K_t and L_t, and the labor input of all other firms, $\bar{L}_t$. The private production function is distinct from the social production function, Eq. (6),

$$\bar{Y}_t = \bar{K}_t^{\alpha}\left[\bar{L}_t\left(1 - \frac{\bar{L}_t}{\Gamma}\right)\right]^{1-\alpha}, \tag{6}$$

which represents the connection between aggregate output $\bar{Y}_t$ and aggregate capital and labor inputs, $\bar{K}_t$ and $\bar{L}_t$. We illustrate the properties of the social production function on Fig. 1. On this figure, we see that output is increasing in employment up to a maximum that occurs at $\Gamma/2$.

The social production function exhibits search externalities. For large values of aggregate employment, $\bar{L}_t$, the labor market becomes very tight and further reduction of unemployment is costly. As firms allocate more workers to the recruiting activity, those workers are withdrawn from production. If employment increases beyond $\Gamma/2$, additional increases in aggregate employment become counter-productive.[10] The value of unemployment at the social optimum,

$$U = 1 - \frac{\Gamma}{2}, \tag{7}$$

is our definition of the *natural rate of unemployment.*[11]

[9] This simplification requires the assumption that workers can, in effect, recruit themselves. Farmer (2012b) discusses the assumption further and Farmer (2013) drops the assumption and treats employment as an additional state variable. In the complete dynamic model, the fraction of workers assigned to recruiting is a small fraction of the workforce, as opposed to the current formulation where, at the social optimum, 50% of the firm's workers are engaged in recruiting. Because nothing of substance is added to the model by studying the full dynamics of the labor market we have chosen, in this paper, to use the simpler model, where labor is not a state variable, for expositional purposes.

[10] In the special case when $\Gamma = 1$, output is maximized when $\bar{L} = 1/2$ and, when $\bar{L} = 1$, aggregate output falls to zero.

[11] Friedman (1968) defined the natural rate of unemployment to be the *equilibrium* rate. That definition only makes sense when equilibrium is unique. In our model, there is a continuum of steady state equilibria and in this framework it makes more sense to define the natural rate of unemployment to be the unemployment rate at the social planning optimum.

4. Aggregate demand

There is a continuum of households. Each household lives for two periods and derives utility from consumption when young C_t^y, consumption when old C_{t+1}^o, and real money balances accumulated in the first period of their life M_{t+1}/P_t. Labor does not deliver disutility, and therefore the participation rate is always equal to 1.[12]

Preferences are given by a logarithmic utility function. Households maximize expected utility,

$$u_t = \log\left(C_t^y\right) + \beta \mathbb{E}_t\left[\log\left(C_{t+1}^o\right)\right] + \delta \log\left(\frac{M_{t+1}}{P_t}\right) \tag{8}$$

where the mathematical expectation is taken with respect to the future realization of the stock market. We describe determination of the value of the stock market in the next section.

In the first period of their life, households earn labor income W_tL_t. They use their income to purchase current consumption $P_tC_t^y$, capital goods $P_{K,t}K_{t+1}$ and government bonds B_{t+1}. All prices are in terms of money.

In the second period of life, households rent capital to firms and earn the rental payment $R_{t+1}K_{t+1}$ and interest accrued on their loan to the government $(1+i_t)B_{t+1}$. In addition, at the end of the period they sell capital and money to the new young generation. The first and second period budget constraints are given by the following equations:

$$P_tC_t^y + M_{t+1} + B_{t+1} + P_{K,t}K_{t+1} = W_tL_t, \tag{9}$$

$$P_{t+1}C_{t+1}^o = (R_{t+1} + P_{K,t+1})K_{t+1} + (1+i_t)B_{t+1} + M_{t+1}. \tag{10}$$

The no-arbitrage condition (NAC) implies that the return to government bonds must be equal to the return on physical capital, when evaluated in terms of utility from consumption in the second period,

$$\mathbb{E}_t\left[\frac{\beta}{C_{t+1}^o}\left(\frac{1+i_t}{P_{t+1}/P_t} - \frac{(P_{K,t+1}+R_{t+1})/P_{t+1}}{P_{K,t}/P_t}\right)\right] = 0. \tag{11}$$

Here the first term in round parentheses is the expected real interest rate payed on government bonds. The second term in the round parentheses is the real return to physical capital. In words, this equation states that the young are indifferent between investing in bonds and capital. Using this condition, and defining real savings of the young in interest-bearing non-monetary assets as follows,

$$S_t^y = (B_{t+1} + P_{K,t}K_{t+1})/P_t, \tag{12}$$

we can write the young's consumption function C_t^y, the demand for real money balances M_{t+1}/P_t and the young's real savings function S_t^y that solve the utility maximization problem:

$$C_t^y = \frac{1}{1+\beta+\delta}\frac{W_tL_t}{P_t}, \tag{13}$$

$$\frac{M_{t+1}}{P_t} = \frac{\delta}{1+\beta+\delta}\left(\frac{1+i_t}{i_t}\right)\frac{W_tL_t}{P_t}. \tag{14}$$

$$S_t^y = \frac{W_tL_t}{P_t} - C_t^y - \frac{M_{t+1}}{P_t}. \tag{15}$$

Substituting for consumption and money balances in (15) gives the following alternative expression for saving

$$S_t^y = \frac{1}{1+\beta+\delta}\left(\beta - \frac{\delta}{i_t}\right)\frac{W_tL_t}{P_t}. \tag{16}$$

The saving of the young is an increasing function of the money interest rate because money and consumption are substitutes in utility and the money interest rate is the opportunity cost of holding money. In the traditional IS-LM model, saving is sometimes written as an increasing function of the real interest rate. That channel for the interest rate to influence saving is missing from our model because of our simplifying assumptions that utility is logarithmic and that labor supply occurs only in youth.[13]

To simplify the exposition of our model, we assume that government bonds are in zero net supply and we concentrate on the role of monetary policy. We study a policy in which the central bank keeps the money supply M_t^* constant, and where that policy is expected to continue forever. In that environment we study the effect of an unanticipated change in M_t^* that we implement through an unanticipated cash transfer to the old generation. In future work we plan to study the role of fiscal interventions.

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[12] Allowing for disutility from participation in the labor market would make the participation rate endogenous. We do not pursue that modification here because, in the U.S. data, the participation rate appears to be driven mostly by demography and does not exhibit a pronounced co-movement with unemployment at business cycle frequencies.

[13] Relaxing the unitary elasticity of intertemporal substitution by considering a utility function of the form $U(C^y, C^o, M/P) = \log(C^y) + \beta\log\left(C^o + \bar{C}\right) + \delta\log(M/P)$ would add the real interest rate as an argument of the savings function. When $\bar{C} > 0$, the intertemporal substitution effect dominates the income effect, making the savings function increasing in both money interest rate as the price of money and the real interest rate as the relative price of consumption when old. In this model, we adopt $\bar{C} = 0$ for expository purposes.

5. The role of beliefs

Although our work is superficially similar to the IS-LM model and its modern New Keynesian variants; there are significant differences. By grounding the aggregate supply function in the theory of search and, more importantly, by dropping the Nash bargaining assumption, we arrive at a theory where preferences, technology and endowments are not sufficient to uniquely select an equilibrium.

Following Farmer (2012b) we close our model by making beliefs fundamental. Farmer studies that assumption in the context of a purely real representative agent model. In the current paper we explore the implications of multiple steady state equilibria in a model where money is used as a means of exchange and where the representative agent assumption is replaced by a model of overlapping generations.[14]

The assumption that beliefs are fundamental is not sufficient to explain *how* they are fundamental and the belief function could take different forms. In our view, beliefs are most likely learned and we see the work of Evans and Honkapohja (2001) as a promising avenue in describing how a particular belief function may arise. In this respect beliefs are similar to preferences.[15]

Economists assume that a human being is described by a preference ordering and that by the time a person achieves adulthood he or she is able to make choices over any given commodity bundle. But those choices are learned during childhood; they are not inherited. At the age of twenty one, an Italian is likely to choose a glass of wine with a meal; a German is more likely to choose a beer. But a German child, adopted into an Italian family at birth, will grow up with the preferences of his adoptive parents, not with those of his biological parents. Beliefs, in our view, are similar.

During a period of stable economic activity, people learn to make forecasts about future variables by projecting observations of variables of interest on their information from the recent past. When there is a change in the environment, caused by a policy shift or a large shock to fundamentals, they continue to use the beliefs that they learned from the past. That argument suggests that we should treat the parameters of the belief function in the same way that we treat the parameters of the utility function. They are objects that we would expect to remain stable over the medium term and that should be estimated using econometric methods.

In this paper we investigate one plausible assumption about the belief function and we study its role as a way of closing our model. We assume that beliefs are determined by the equation

$$\mathbb{E}_t^*\left[\frac{P_{K,t+1}}{P_{t+1}}\right] = \Theta_t, \tag{17}$$

where the expectations operator in Eq. (17) is subjective and reflects the beliefs of a representative person of the probabilities of future events. To impose discipline on our analysis we assume that expectations are rational; that is,

$$\mathbb{E}_t^*\left[\frac{P_{K,t+1}}{P_{t+1}}\right] = \mathbb{E}_t\left[\frac{P_{K,t+1}}{P_{t+1}}\right] = \Theta_t, \tag{18}$$

where the expectation $\mathbb{E}$ is taken with respect to the true probabilities in a rational expectations equilibrium.

Because there is no aggregate investment in our model, capital represents an input in fixed supply. We interpret P_K to be the the average price of assets traded in the stock market and changes in P_K represent self-fulfilling shifts in perceptions of financial wealth.

6. The equations of the model

The equilibrium of our model is described by the following seven equations. To obtain these equations, we used two facts. First, factor incomes are proportional to GDP,

$$\frac{R_t K_t}{P_t} = \alpha Y_t \quad \text{and} \quad \frac{W_t L_t}{P_t} = (1-\alpha) Y_t \tag{19}$$

and second, in a symmetric equilibrium, total employment equals individual employment and each firm employs one unit of capital,

$$L_t = \bar{L}_t \quad \text{and} \quad K_t = 1. \tag{20}$$

Now we turn to a description of each of the seven equations that comprise our model.

$$\frac{1-\alpha}{1+\beta+\delta}\left(\beta - \frac{\delta}{i_t}\right) Y_t = \frac{P_{K,t}}{P_t}. \tag{21}$$

[14] Plotnikov (2013, 2019) explores a similar idea in a version of a real business cycle model, closed with Farmer's (2012b) Keynesian search model of the labor market. Plotnikov closes his model with the assumption that beliefs about future human wealth are adaptive and he shows that a model, closed in this way, generates jobless recoveries.

[15] The discussion in this section closely follows the presentation in Farmer (2016b).

Eq. (21) describes equilibrium in the asset markets. It equates the demand for interest bearing assets by the young (the young's real savings function S_t^y) to the real value of the single unit of capital ($P_{K,t}/P_t$) available in the economy. Since government bonds are in zero supply, the young's savings must be equal to the purchases of capital sold by the old generation. Eq. (21) is our analog of the IS curve.

$$\frac{M_{t+1}^*}{P_t} = \frac{(1-\alpha)\delta}{1+\beta+\delta}\left(\frac{1+i_t}{i_t}\right)Y_t. \tag{22}$$

Eq. (22) is the money market clearing condition and it is our equivalent of the LM curve. Here M_{t+1}^* is the stock of money exogenously determined by the central bank and available for the young generation to hold as part of their optimal portfolio.

$$\mathbb{E}_t\left[\frac{\beta}{C_{t+1}^o}\left(\frac{1+i_t}{P_{t+1}/P_t} - \frac{(P_{K,t+1}+\alpha P_{t+1}Y_{t+1})/P_{t+1}}{P_{K,t}/P_t}\right)\right] = 0. \tag{23}$$

Eq. (23) is the no-arbitrage condition (NAC) between the money interest rate and the return to capital. This equation represents the assumption that physical capital and government bonds pay the same rate of return and it has no analog in the simplest version of the IS-LM model.

$$P_t C_t^o = \alpha P_t Y_t + P_{K,t} + M_t^*. \tag{24}$$

Eq. (24) is the expenditure function of the old. It says that the old's expenditure on consumption must be equal to the income plus principal from selling capital plus the value of the money held by the old.

$$Y_t = \left[\left(1 - \frac{L_t}{\Gamma}\right)L_t\right]^{1-\alpha}. \tag{25}$$

Eq. (25) is the social production function. This equation serves only to determine employment and it plays the role of the 45 degree line in the Keynesian Cross model.

Next, real GDP is the sum of the consumption of the two generations

$$Y_t = C_t^y + C_t^o. \tag{26}$$

Finally, we add a seventh equation, the belief function (27).

$$\mathbb{E}_t\left[\frac{P_{K,t+1}}{P_{t+1}}\right] = \Theta_t. \tag{27}$$

The belief function distinguishes our model from the New Keynesian approach and it replaces the New Keynesian Phillips curve. In the absence of this new element, the other six equations would not uniquely determine the seven endogenous variables $\{Y_t, P_t, i_t, P_{K,t}, L_t, C_t^y, C_t^o\}$. The belief function is an equation that determines how much households are willing to pay for claims on the economy's capital stock. It represents the aggregate state of confidence or 'animal spirits' and, in combination with the other six equations of the model, the belief function selects an equilibrium.

In our comparative statics exercises in Section 8, we compare two alternative specifications for the belief function. In one specification we assume that,

$$\Theta_t = \Theta \quad \text{for all } t. \tag{28}$$

We call this assumption *fixed beliefs* and it amounts to the assumption that, in the collective view of asset market participants, the stock market has some fixed real value measured in terms of the CPI.

In a second specification we assume that

$$\Theta_t = \frac{P_{K,t}}{P_t}. \tag{29}$$

We call this assumption *adaptive beliefs* and it amounts to the assumption that, in the collective view of market participants, the real value of the stock market is a random walk. This second assumption, which is a better description of the actual behavior of stock market prices, has a non-standard implication that we draw attention to in Section 10.3. It implies that unanticipated shocks to the money supply can have permanent effects on the steady state unemployment rate.[16]

Eqs. (21), (22), (23), (24), (26) and (27) determine aggregate demand. Given beliefs $\{\Theta_t\}$ and monetary policy M_t^*, these equations select an equilibrium sequence for $\{Y_t, P_t, i_t, P_{K,t}, C_t^y, C_t^o\}$ and Eq. (25) determines how much labor firms need to hire to satisfy aggregate demand. Since employment is determined recursively, in the subsequent parts of the paper we dispense with Eq. (25) in our discussion of equilibrium.

[16] Farmer (2012c, 2015) finds evidence that the real value of the stock market and the unemployment rate can be parsimoniously modeled as co-integrated random walks. Our work in this paper provides one possible theoretical model that can explain this finding.

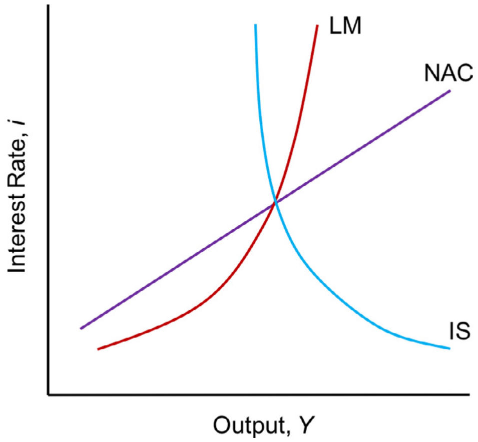

Fig. 2. The IS-LM-NAC representation of the steady state.

7. The IS-LM-NAC representation of the steady-state

In this section, we show that the steady-state equilibrium of our model admits a representation that is similar to the IS-LM representation of the *General Theory* developed by Hicks and Hansen. The IS-LM model is a static construct in which the price level is predetermined. To provide a fully dynamic model, Samuelson closed the IS-LM model by adding a price adjustment equation that later New-Keynesian economists replaced with the New-Keynesian Phillips curve.

We take a different approach. We select an equilibrium by closing the labor market with a belief function. Our model consists of the IS curve, the LM curve and the NAC curve. The NAC curve is a new element that equates the return to capital to the nominal interest rate. And unlike the interpretation of animal spirits that was popularized by Akerlof and Shiller (2009), pessimistic animal spirits are fully rational. The people in our model are rational and have rational expectations but they are, sometimes, unable to coordinate on a socially efficient outcome.

The following equations characterize the steady-state equilibrium:

$$\text{IS:}\quad \frac{1-\alpha}{1+\beta+\delta}\left(\beta-\frac{\delta}{i}\right)Y=\Theta, \tag{30}$$

$$\text{LM:}\quad \frac{M}{P}=\frac{(1-\alpha)\delta}{1+\beta+\delta}\left(\frac{1+i}{i}\right)Y, \tag{31}$$

$$\text{NAC:}\quad i=\frac{\alpha Y}{\Theta}. \tag{32}$$

Eqs. (30)–(32) determine the three unknowns: Y, i and P, for given values of M and Θ. We treat $\Theta=\mathbb{E}[P_K/P]$ as a new exogenous variable that reflects investor confidence about the real value of their financial assets and by making Θ exogenous we provide a new interpretation of Keynes' idea that equilibrium is selected by 'animal spirits'.

In (Y, i) space, the IS and NAC curves determine Y and i and the price level adjusts to ensure that the LM curve intersects the IS and NAC curves at the steady state. We illustrate the determination of a steady state equilibrium in Fig. 2.

The IS curve, Eq. (30), is downward sloping and its position is determined by animal spirits, Θ.

In a steady state equilibrium, beliefs about future wealth are self-fulfilling. When people feel wealthy, they *are* wealthy. Beliefs about wealth determine consumption, and firms hire as much labor as necessary to satisfy demand. The value of capital in a rational expectations equilibrium adjusts to match the beliefs.

8. Two comparative static exercises

In this section we ask how shifts in exogenous driving variables affect the equilibrium values of Y, i and P. We conduct two comparative static exercises. In the first exercise we increase Θ from a low value to a higher value at some date, $t=1$, and we assume that it remains constant thereafter. In the second exercise, we hold Θ fixed forever and we increase the stock of money.[17]

Consider first, the experiment of an increase in the belief about the value of financial wealth. A greater value of Θ influences output through two channels. Firstly, since consumers believe, correctly, that they are wealthier, real consumption of goods and services increases. The IS curve shifts to the right. Moreover, higher asset prices reduce the interest rate and the NAC curve becomes flatter. These effects are illustrated in Fig. 3.

[17] In Section 10 we consider an alternative model of expectation formation in which the belief about the future value of capital is equal to its current realized value.

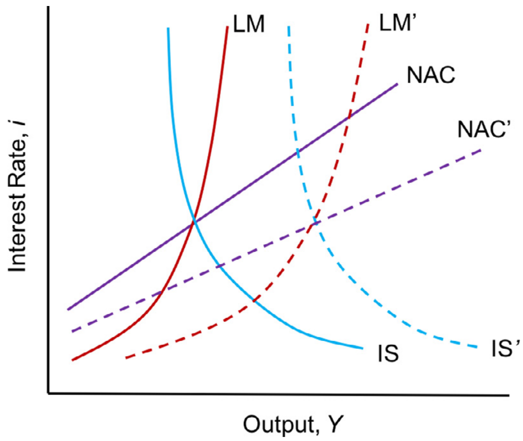

Fig. 3. An increase in confidence.

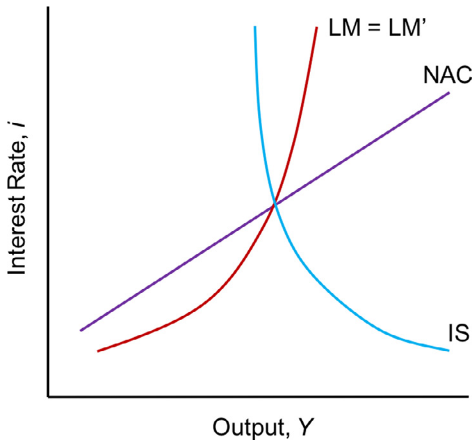

Fig. 4. An increase in the money supply.

As people become more confident, the IS curves shifts to the right beginning at the solid IS curve and ending at the dashed IS curve. At the same time, the NAC curve shifts down and to the right, from the solid NAC to the dashed NAC curve. Because output increases, the demand for real money balances increases, and the price level must be lower in the new steady state equilibrium. This is reflected on Fig. 3 by a rightward shift in the LM curve. Because the class of Cobb–Douglas utility functions implies a unitary elasticity of intertemporal substitution, the intertemporal substitution effect and the income effect cancel each other out and, at the new equilibrium, the interest rate remains unchanged.

Consider next, the effect of an increase in the stock of money, which we illustrate on Fig. 4.

Eqs. (30) and (32) determine the equilibrium values of output and the interest rate independently of the stock of money. The demand for real balances depends only on Y and i and, once these variables have been determined, the price level, P, adjusts to equate the real value of the money supply to the real value of money demand. It follows that changes in the supply of money will cause proportional changes in the price level and the nominal value of wealth, leaving output and the interest rate unchanged.

Fig. 4 illustrates the effects of a change in M on a graph. The LM curve after the increase in the money supply is identical with the LM curve before the change, illustrating the concept that money, in our model, is neutral. However, as we will show in Section 10, this result depends on the form of the belief function. If beliefs about the future value of financial wealth depend on the current realized value of wealth, an increase in the money supply may have a permanent real effect on output through its effect on business and consumer confidence.

9. Dynamic equilibria

In this section we shift from a comparison of steady states to a description of complete dynamic equilibria. To study the equilibria of the complete model, we use the algorithm, GENSYS, developed by Sims (2001). First, we choose a constant

Table 1
Calibration.

Parameter	Definition	Value
α	Share of capital in output	0.33
β	Subjective discount rate	0.50
δ	Coefficient on real money balances in utility	0.05

sequence $\{M, \Theta\}$ to describe policy and we log-linearize the dynamic equations around a steady state. Let

$$x_t \equiv \left[y_t, \tilde{i}_t, p_t, p_{K,t}, \mathbb{E}_t[y_{t+1}], \mathbb{E}_t[p_{t+1}], \mathbb{E}_t[p_{K,t+1}]\right]' \tag{33}$$

be log deviations of the endogenous variables from their steady state values. Let

$$\varepsilon_t \equiv [m_t, \theta_t]' \tag{34}$$

log deviations of the exogenous variables from their initial values and define three new variables,

$$\eta_t^1 \equiv p_t - \mathbb{E}_{t-1}[p_t], \tag{35}$$

$$\eta_t^2 \equiv p_{K,t} - \mathbb{E}_{t-1}[p_{K,t}], \tag{36}$$

$$\eta_t^3 \equiv y_t - \mathbb{E}_{t-1}[y_t]. \tag{37}$$

These new variables represent endogenous forecast errors. Next, we log-linearize Eqs. (21)–(23) and (27) and we append them to Eqs. (35)–(37). That leads to the following linear system of seven equations in seven unknowns,

$$\Gamma_0 x_t = \Gamma_1 x_{t-1} + \Psi\varepsilon_t + \Pi\eta_t, \tag{38}$$

The matrix Ψ is derived from the linearized equations and it explains how shocks to M and shocks to Θ influence each of the equations of the model.

Once we have provided a model of beliefs, the steady state of our system is determinate. For every specification of the belief function, Eq. (27), there is a unique steady state. In this sense, our animal spirits model is similar to any dynamic stochastic general equilibrium model. For a given specification of fundamentals, there is a unique predicted outcome.

But the fact that the model, augmented by a belief function, has a unique steady state, is not enough to uniquely determine a dynamic equilibrium. To establish uniqueness of a dynamic equilibrium, we must show that for every representation of fundamentals, where fundamentals now include beliefs, there is a unique dynamic path converging to the steady state. The uniqueness or non-uniqueness of dynamic equilibria is determined by the properties of the matrices Γ_0 and Γ_1, in Eq. (38).

To establish the properties of a dynamic equilibrium, we must provide a calibrated version of the model since determinacy of equilibrium is, in general, a numerical question. To study determinacy, we used the calibration from Table 1.

For this calibration, we found that our model has one degree of indeterminacy. In words, that implies that for any set of initial conditions there is a one dimensional continuum of dynamic paths, all of which converge to a given steady state. In practice, it means that the rational expectations assumption is not sufficient to uniquely determine all three of the forecast errors, η_t, as functions of the fundamental shocks, ε_t. When the model displays dynamic indeterminacy, there are many ways that people may use to forecast the future, all of which are consistent with a rational expectations equilibrium (Farmer, 1991; 1999).

Following Farmer (2000), we resolve this indeterminacy by selecting a particular equilibrium for which

$$\eta_t^1 \equiv p_t - \mathbb{E}_{t-1}[p_t] = 0. \tag{39}$$

This equation is a special case of Eq. (35). In words, this assumption means that money prices are set one period in advance. It is important to note that price stickiness does not violate the property of rational expectations. The equilibrium with sticky prices is one of many possible equilibria of the economy where agents form self-fulfilling beliefs about wealth and it is an equilibrium that explains an important property of the data: Unanticipated monetary shocks have real short run effects and they feed only slowly into prices.

In our model, the equilibrium is selected by the way that people form beliefs. How should we view the choice of an equilibrium with predetermined prices? Farmer has argued elsewhere (Farmer, 1999) that when there are multiple equilibria, we should allow the data to determine how people form beliefs in the real world. It may be, for example, that a small but unmodeled cost of changing prices leads market participants to an equilibrium where prices are predetermined. Here, we choose to display the properties of the predetermined price equilibrium and we refer the reader to the paper by Farmer and Nicolò (2018) which provides evidence that a predetermined price equilibrium of this kind is a good fit to US data.

10. Three dynamic experiments

In this section, using the parameter values from Table 1, we analyze three dynamic experiments. In the first experiment, we begin from a steady state, and we ask how a permanent unanticipated increase in confidence affects the endogenous variables of the model. In the second and third experiments, we ask how a permanent unanticipated increase in the stock of money affects the economy.

In our second experiment, the belief of households about the future real value of the stock market is invariant to its current value. In our third experiment, households expect the future real value of the stock market to be equal to its current value. We refer to the alternative assumptions in experiments two and three as fixed and adaptive beliefs.

In the case of fixed beliefs, the experiment of increasing the money supply, has the same long-run effects that it would have in a classical model in which output is supply determined: Money is neutral. In contrast, if households form their beliefs adaptively, a permanent increase in the money supply has a permanent effect on output. Money is non-neutral because it increases the real value of the stock market in the short run and that increase is translated, through a confidence effect, into a permanent increase in beliefs about the value of the stock market.

Is it reasonable to think that a change in a nominal variable may have *permanent* real effects? We think so. Farmer (2012a) and Farmer and Nicolò (2018) have estimated a model of the US economy in which beliefs about future income growth are equal to current income growth and they have shown that a belief function of this kind outperforms standard New-Keynesian models closed by a Phillips curve. In their model, the central bank sets the money interest rate and changes in the interest rate have a permanent effect on the unemployment rate by shifting the economy from one equilibrium to another. In our model a similar shift from one steady state equilibrium to another is achieved by an increase in the money supply which raises share prices and has a permanent effect on animal spirits.

10.1. Experiment 1: A shock to confidence

Fig. 5 displays the dynamic paths of eight variables in response to a one time increase in beliefs about the future value of capital. We call this a shock to confidence.

Panel (a) depicts the value of beliefs about the future real value of the stock market, $\mathbb{E}_t\left[P_{K,t+1}/P_{t+1}\right]$. This is the variable we refer to as Θ. In our first experiment, Θ increases by one percent and it remains one percent higher for ever after. Panel (b) shows the value of the money supply, which we hold fixed for this experiment.

Panel (c) shows that, in period 2, output increases and remains permanently higher by one percent. This occurs because rational forward-looking consumers increase their spending on goods and services and firms respond by hiring additional workers to produce these goods. Panel (d) shows that the price level falls and stays permanently lower. Greater output increases the demand for real money balances and the price level must fall to equate the demand and supply of money.

Panel (e) shows that, in period 2, the realized value of real stock-market wealth increases by one percent. That follows from the rational expectations assumption; people expected the value of share prices to increase and, in a rational expectations equilibrium, that belief is supported by the way that people form their beliefs in period 2 and in all subsequent periods. From panel (g), we see that the real interest rate jumps up in period 1 and reverts to its steady-state value thereafter. Because the price level and the money interest rate do not adjust in the first period, the real interest rate adjustment is achieved by a self-fulfilling adjustment to the expected future price level.

We want to draw attention to several features of these impulse responses. First, although adjustment to a confidence shock is delayed, the delay lasts for only one period. That follows from the stylized nature of a model in which there are no endogenous propagation mechanisms. Second, prices do not respond at all in the first period. In the New-Keynesian model, prices are sticky because of adjustment costs or restrictions on choice. Although we are not averse to the possibility that restrictions of this kind may be important in the real world, they are not an essential element of our theory. In our model, prices are fixed because people rationally anticipate that output, not prices, will respond to unanticipated shocks.

If models in this class are to be taken seriously as descriptions of data, they must be tied down by an assumption about how beliefs are formed. To give the model empirical content, one must assume that the belief function remains time invariant at least over the medium term. If that assumption holds, the parameters of the belief function can be estimated in the same way that econometricians estimate preference parameters. See Farmer (2012a) and Farmer and Nicolò (2018) for examples of empirical exercises that estimate a version of this model on U.S. data. These papers tie down the equilibrium of the theoretical model by treating the covariance of prices with contemporaneous variables as a parameter of the belief function. In the empirical work of Farmer (2012a) and Farmer and Nicolò (2018), it is the sticky price equilibrium that best explains data.

10.2. Experiment 2: A shock to the money supply with fixed beliefs

In Sections 10.2 and 10.3 we show that the way economic agents form beliefs about the future, matter for the long-term effect of monetary shocks.

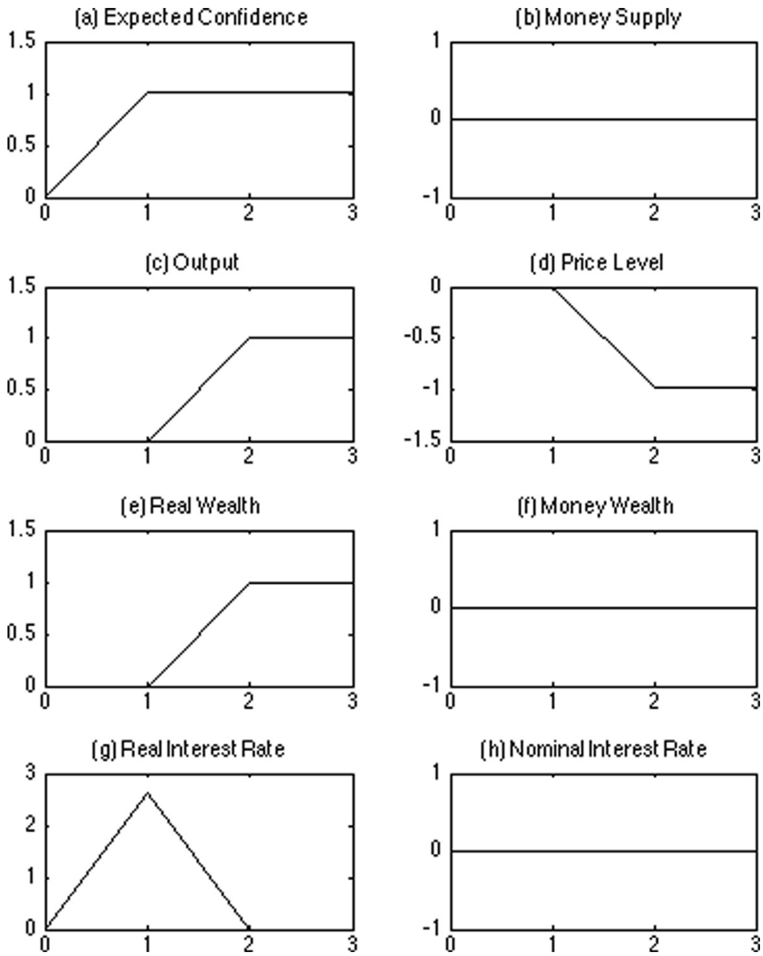

Fig. 5. A permanent shock to confidence.

In Section 10.2, we consider the case of fixed beliefs, which we model with Eq. (40),

$$\mathbb{E}_t\left[\frac{P_{K,t+1}}{P_{t+1}}\right] = \Theta. \tag{40}$$

Fig. 6 displays the dynamic paths for the variables of this economy in response to a shock to the money supply when beliefs are modeled in this way. This shock is reflected in Panel (b) which depicts the time path for M. We assume that at date 1, M increases by one percent and that it remains one percent higher forever after. Panel (a) reflects our assumption that beliefs are fixed.

Panel (c) shows that output increases temporarily in the first period by one percent. This happens because prices are predetermined and are unable to adjust until period 2. Instead, the increase in the money supply causes an increase in aggregate demand that is met by a corresponding temporary increase in output and employment. Firms hire more workers to satisfy the increased aggregate demand.

Panel (d) shows that prices respond in period 2 and remain 1 percent higher. This increase neutralizes the increase in the money supply and is consistent with the return to steady state of output reflected in panel (c). Panels (e) and (f) show that the real value of shares in the stock market increases and then returns to its original value. In contrast, the money value of shares in the stock market goes up by one percent and remains permanently higher. From panel (g) we see that the real interest rate falls in period 1 and panel (h) shows that the money interest rate remains constant during the entire exercise.

10.3. Experiment 3: A shock to the money supply with adaptive beliefs

To model adaptive beliefs, we replace Eq. (40), with Eq. (41),

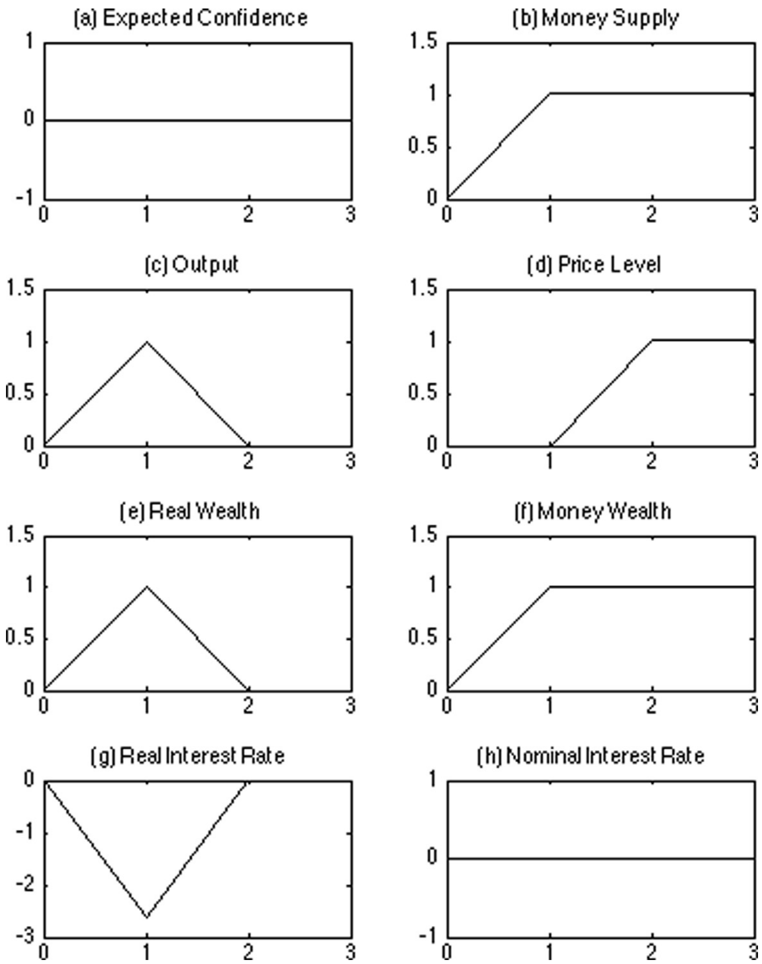

Fig. 6. A permanent shock to the supply of money under fixed beliefs.

$$\mathbb{E}_t\left[\frac{P_{K,t+1}}{P_{t+1}}\right] = \frac{P_{K,t}}{P_t}. \tag{41}$$

When beliefs are adaptive, households expect the real value of the stock market to be a random walk. This is a special case of a more general model in which beliefs are formed by the following adaptive expectations equation,[18]

$$\mathbb{E}_t\left[\frac{P_{K,t+1}}{P_{t+1}}\right] = \lambda\left(\frac{P_{K,t}}{P_t}\right) + (1-\lambda)\mathbb{E}_{t-1}\left[\frac{P_{K,t}}{P_t}\right], \quad \lambda \in [0,1].$$

Fig. 7 displays the dynamic paths for the variables of this economy in response to a shock to the money supply when beliefs about the real value of shares in the stock market are determined by a random walk. We assume that M increases by one percent and that it remains one percent higher for ever after. The shock to the money supply is reflected in Panel (a).

The increase in the stock of money causes an increase in the money price of financial assets; this is shown in Panel (f). Because the price of goods is predetermined, the increase in the nominal share price is also an increase in its real price as shown in Panel (e). Panel (b) shows that beliefs about the future real value of shares respond to this monetary shock and they remain permanently one percent higher in all subsequent periods. Panel (c) shows that the increase in the real value of the stock market triggers an increase in output that is sustained because of the effect of the increase in the money supply on beliefs, as reflected in Panel (b). Panels (d), (g) and (h) show that the price level, and the real and nominal interest rates

[18] We have restricted ourselves to the special case of $\lambda = 1$ because Farmer (2012a) estimated a model that allows λ to lie in the interval [0,1] and found that empirically, the data favors a model where $\lambda = 1$.

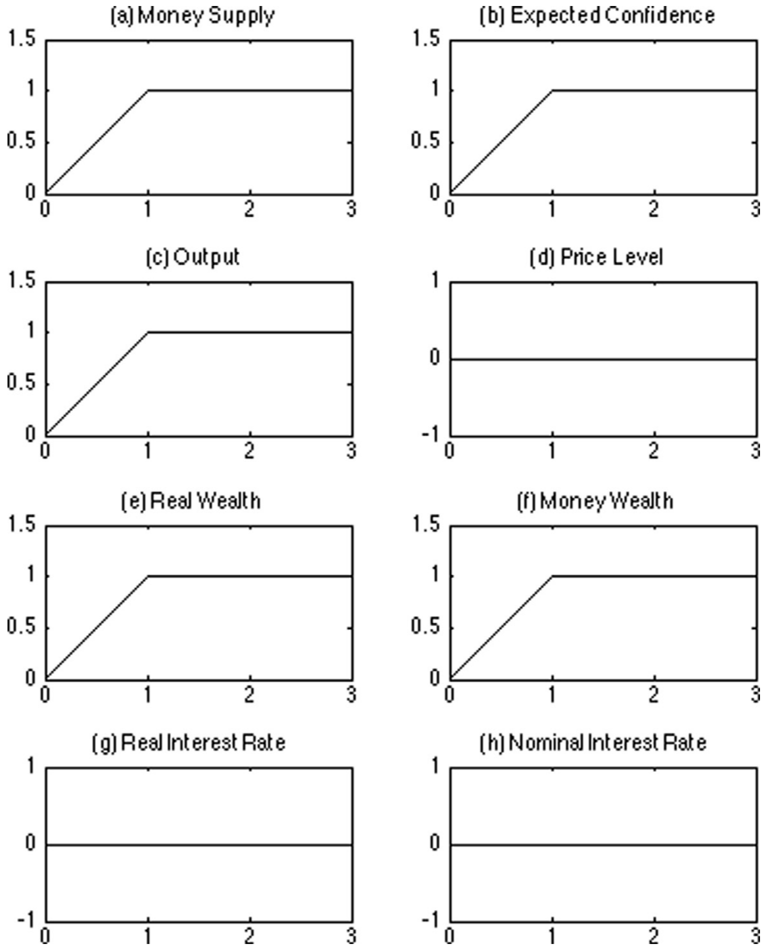

Fig. 7. A permanent shock to the supply of money under adaptive beliefs.

do not respond at all to a one off permanent increase in the money supply which is reflected entirely in changes to output and in the real value of financial assets.

11. Conclusion

We have proposed a fresh way of thinking about the monetary transmission mechanism. By integrating Keynesian economics with general equilibrium theory in a new way, we have provided an alternative to the IS-LM framework that we call the IS-LM-NAC model. Our new model provides an alternative narrative to New-Keynesian economics to explain how macroeconomic policy influences prices and employment.

Our approach differs from New Keynesian economics in two fundamental ways. First, our model displays dynamic indeterminacy. We focus on a dynamic path with predetermined prices to show that changes in the money supply may affect real economic activity even if all nominal prices are perfectly flexible. Second, our model displays steady state indeterminacy that arises as a consequence of search frictions in the labor market. We replace the classical search assumption that firms and workers bargain over the wage, with the Keynesian search assumption that beliefs about the future value of the stock market select a steady-state equilibrium. In our view, beliefs should be treated as a new fundamental of the model. The belief function advances our understanding of why the unemployment rate is so persistent in real world data.

Finally, we have presented a simple graphical apparatus that can be used by policy makers to understand how policy affects the economy. A new element, the NAC curve, connects the interest rate to current and expected future values of the stock market and it explains how ‘animal spirits’ influence economic activity. The IS-LM-NAC framework provides a rich new approach to policy analysis that explains the short-run and long-run effects of policy, without the assumption that prices are prevented from moving by artificial barriers to price adjustment.

Appendix A. Generalizing our approach

A referee has pointed out that our graphical method relies on the assumption that only current income enters decision rules and asks if our model generalizes to models of permanent income or models with long-lived agents. The answer is yes.[19]

A.1. The demand side of the economy

Here, we adapt the Blanchard perpetual youth model (Blanchard, 1985) to deliver a version of the IS-LM-NAC model in steady state. To keep the presentation simple, we assume that there is no aggregate uncertainty. The case of aggregate uncertainty can be handled by modeling the pricing kernel using the work of Farmer et al. (2008) and adapting the methods described in Farmer (2018).

Time is discrete and indexed by $t = 0, 1, 2, \ldots$. There is a total mass of population equal to 1. Every agent faces a constant probability λ of surviving into the next period. At the beginning of each period, a fraction $1 - \lambda$ of agents die, and an equal mass of individuals is born. Generations are indexed by the date of birth s. As in Blanchard (1985), we assume the existence of annuities and a competitive set of life insurance companies that pay every agent an additional return to financial wealth in return for a claim on the agent's wealth upon death. Free entry guarantees zero profit of the insurance companies.

Conditional on surviving, agents discount the future at rate $0 < \beta < 1$. Each generation's preferences are defined over sequences of consumption $\{C_t^s\}_{t=s}^{\infty}$ and real money balances $\{M_{t+1}^s / P_t\}_{t=s}^{\infty}$,

$$\sum_{\tau=0}^{\infty} (\beta\lambda)^{\tau} \left((1-\delta) \log C_{t+\tau}^s + \delta \log \left(\frac{M_{t+1+\tau}^s}{P_{t+\tau}} \right) \right),$$

where consumption and money holdings are indexed by the generation s for $s \le t$.

Households hold three assets: money balances, M_{t+1}^s, that provide liquidity services and enter the utility function, government bonds B_{t+1}^s that pay the rate of return i_t, and shares S_{t+1}^s of a representative firm that has a value of F_t and that pays dividends D_t. The flow budget constraint takes the form:

$$P_t C_t^s + M_{t+1}^s + B_{t+1}^s + S_{t+1}^s F_t = W_t L_t^s + \frac{1}{\lambda} [S_t^s (D_t + F_t) + M_t^s + (1 + i_{t-1}) B_t^s].$$

The term $1/\lambda$ in this expression represents the annuity premium paid by the life insurance company in return for a claim on the agent's wealth in the event of death. We also assume that people are born with zero wealth; that is, $S_t^t = 0$, $M_t^t = 0$, and $B_t^t = 0$.

Under the no arbitrage condition and assuming no uncertainty, the dollar return to holding a government bond equals the dollar return to holding stock:

$$1 + i_t = \frac{D_{t+1} + F_{t+1}}{F_t}.$$

Because preferences are given by a Cobb-Douglas utility function, spending on consumption and money holding are each equal to a constant fraction of each agent's wealth:

$$P_t C_t^s = (1-\delta)(1-\beta\lambda) \left[\frac{1}{\lambda} (S_t^s (D_t + F_t) + M_t^s + (1 + i_{t-1}) B_t^s) + H_t^s \right],$$

$$\frac{i_t}{1+i_t} M_{t+1}^s = \delta (1-\beta\lambda) \left[\frac{1}{\lambda} (S_t^s (D_t + F_t) + M_t^s + (1 + i_{t-1}) B_t^s) + H_t^s) \right],$$

where H_t^s is the nominal value of human wealth defined recursively as

$$H_t^s = W_t L_t^s + \frac{\lambda}{1+i_t} H_{t+1}^s.$$

Define the following aggregate variables:

[19] A second issue that was raised by a referee is whether our model would survive the introduction of produced capital. We suspect that the answer is yes. But the model would need to be more elaborate in other dimensions. In a one-sector model with reproducible capital, the relative price of capital is pinned down by technology and economic fluctuations cannot be driven simply by the relative price of capital as they are in the model we present in this paper. To give full justice to a model with reproducible capital it would seem to us, that either one should build a two-sector model, or one would need to drop the assumption of a static labor market by allowing for labor as a state variable. Either of those variations would permit the value of the stock-market to diverge from the value of capital and allow the introduction of self-fulfilling beliefs about the relative price of an asset to drive business cycle fluctuations.

$$C_t = \sum_{s=-\infty}^{t} (1-\lambda)\lambda^{t-s} C_t^s, \quad H_t = \sum_{s=-\infty}^{t} (1-\lambda)\lambda^{t-s} H_t^s,$$

$$B_t = \sum_{s=-\infty}^{t} (1-\lambda)\lambda^{t-s} B_t^s, \quad S_t = \sum_{s=-\infty}^{t} (1-\lambda)\lambda^{t-s} S_t^s,$$

$$M_t = \sum_{s=-\infty}^{t} (1-\lambda)\lambda^{t-s} M_t^s.$$

where recall that, in period t, the newborns have zero assets.

The demand side of the economy is completely described by the following three equations:

$$P_t C_t = (1-\delta)(1-\beta\lambda)[S_t(D_t + F_t) + M_t + (1+i_{t-1})B_t + H_t],$$

$$\frac{i_t}{1+i_t} M_{t+1} = \delta(1-\beta\lambda)[S_t(D_t + F_t) + M_t + (1+i_{t-1})B_t + H_t],$$

$$H_t = W_t L_t + \frac{\lambda}{1+i_t} H_{t+1},$$

plus the no-arbitrage condition

$$1 + i_t = \frac{D_{t+1} + F_{t+1}}{F_t}.$$

A.2. The supply side of the economy

The production sector is comprised of a continuum of identical perfectly competitive firms. Because all firms will make the same decisions, we consider the problem of a representative producer.

The firm owns a unit of non-reproducible capital and hires labor to produce goods. The production function is given by

$$Y_t = X_t^{1-\alpha},$$

where X_t is labor used in production.

In period t, the firm hires L_t workers that must be assigned to production, X_t, and recruiting, V_t,

$$L_t = X_t + V_t.$$

Each worker assigned to recruiting hires q_t workers,

$$L_t = q_t V_t.$$

The firm maximizes its value which equals the discounted present value of its cash flow, all of which is paid out as dividends,

$$D_t = P_t Y_t - W_t L_t.$$

The value maximization problem is constrained by the following two equations,

$$L_t = X_t + V_t,$$

and

$$L_t = q_t V_t.$$

Because we assume that workers are fired and rehired every period, the firm's problem reduces to a sequence of static maximization problems with the following first order condition which holds in every period,

$$(1-\alpha) P_t Y_t = W_t L_t.$$

Dividends, in the optimal solution, are given by

$$D_t = \alpha P_t Y_t.$$

At the aggregate level, there exists a matching technology,

$$q_t \bar{V}_t = \left(\Gamma \bar{V}_t\right)^{\frac{1}{2}},$$

and, in equilibrium, $\bar{L}_t = L_t$, $\bar{V}_t = V_t$, and $L_t = (\Gamma V_t)^{1/2}$.

A.3. Equilibrium

An equilibrium is a sequence of consumption, production, employment, asset holdings and prices such that the above equations hold, the goods market clears, $C_t = Y_t$, the demand for money equals the fixed supply, $M_{t+1} = M^*_{t+1}$, debt is in zero net supply, $B_{t+1} = 0$, and the stock of capital is fixed, $S_{t+1} = 1$. We allow unemployment to be different from the social optimum and we close the model by assuming that agents form beliefs about the real value of the stock market,

$$\frac{F_{t+1}}{P_{t+1}} = \Theta_t.$$

A.4. Steady state

To derive the model equations in the steady state we first find expressions for Q, V, X and Y as functions of steady-state employment, L. These functions are derived as follows. Using the labor recruiting equation,

$$L = qV,$$

the labor allocation identity,

$$X + V = L,$$

plus the aggregate matching function and the production function we attain the following steady-state relationships

$$q = \frac{\Gamma}{L}, \quad V = \frac{L^2}{\Gamma}, \quad X = L - \frac{L^2}{\Gamma}, \quad Y = \left(L - \frac{L^2}{\Gamma}\right)^{1-\alpha}. \tag{A1}$$

Because the production function is non-monotonic in L we assume that in any search equilibrium, the economy is on the rising part of the production function; that is, $L < \frac{\Gamma}{2}$. Given this assumption, for any

$$Y < \left(\frac{\Gamma}{2}\left(1 - \frac{\Gamma}{2}\right)\right)^{1-\alpha},$$

we can find a unique value of L and, from Eqs. (A1) we can find expressions for steady-state q, V and X. Further, from the first-order conditions of individual firms we have,

$$\frac{WL}{P} = (1-\alpha)Y,$$

and

$$\frac{D}{P} = \alpha Y.$$

By substituting these expressions into the demand equations for consumption and real balances and using goods market clearing, $C = Y$, the assumption that debt is in zero net supply, $B = 0$, and the fact that there is a single non-reproducible unit of capital, $S = 1$, we arrive at the following four equations in the five variables Y, $\frac{F}{P}$, $\frac{M}{P}$, $\frac{H}{P}$, and i,

$$Y = (1-\delta)(1-\beta\lambda)\left[\alpha Y + \frac{F}{P} + \frac{M}{P} + \frac{H}{P}\right], \tag{A2}$$

$$\frac{i}{1+i}\frac{M}{P} = \delta(1-\beta\lambda)\left[\alpha Y + \frac{F}{P} + \frac{M}{P} + \frac{H}{P}\right], \tag{A3}$$

$$\frac{H}{P} = (1-\alpha)Y + \frac{\lambda}{1+i}\frac{H}{P}, \tag{A4}$$

$$1 + i = \frac{\alpha Y + \frac{F}{P}}{\frac{F}{P}}. \tag{A5}$$

To close the system we assume that beliefs about the stock market (animal spirits) are given by the expression $F/P = \Theta$. These four equations determine the four real variables: Y, i, M/P, and H/P. The steady state price level, P, is determined by the nominal money supply M.

Summarizing this discussion, we can characterize steady-state equilibrium in a long-lived agent model with three equations. Two of these equations, the LM curve and the no-arbitrage equation, are similar to the two-period-lived model. These are given by the expressions,

$$\frac{M}{P} = \frac{\delta}{1-\delta}\frac{1+i}{i}Y. \tag{LM Curve}$$

and,

$$i = \frac{\alpha Y}{\Theta}. \tag{NAC Curve}$$

Note that the LM curve has a positive slope in space (Y, i) space.

To obtain the IS curve, substitute for M/P and H/P in Eq. (A2) and divide both sides by Y to give:

$$1 = (1-\delta)(1-\beta\lambda)\left[\alpha + \frac{\Theta}{Y} + \frac{\delta}{1-\delta}\frac{1+i}{i} + \frac{1-\alpha}{1-\frac{\lambda}{1+i}}\right]. \tag{IS Curve}$$

By totally differentiating this equation and evaluating the partial derivatives at the steady state one readily verifies that the IS curve defines a downward sloping relationship in (Y, i) space. Further, one verifies that increases in confidence, which we attribute to Θ, shifts the IS curve to the right.

Supplementary material

Supplementary material associated with this article can be found, in the online version, at doi:10.1016/j.euroecorev.2019.02.005.

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