DEFINITIONS AND CONCEPTS | **EXAMPLES**

b. A rational inequality can be expressed as $f(x) < 0$, $f(x) > 0$, $f(x) \leq 0$, or $f(x) \geq 0$, where f is a rational function. The procedure for solving such inequalities begins with expressing them so that one side is zero and the other side is a single quotient. Find boundary points by setting the numerator and denominator equal to zero. Then follow a procedure similar to that for solving polynomial inequalities. — Ex. 3, p. 388

3.7 Modeling Using Variation

a. A procedure for solving variation problems is given in the box on page 395.

b.

English Statement	Equation	Examples
y varies directly as x. y is directly proportional to x.	$y = kx$	Ex. 1, p. 395
y varies directly as x^n. y is directly proportional to x^n.	$y = kx^n$	Ex. 2, p. 396
y varies inversely as x. y is inversely proportional to x.	$y = \frac{k}{x}$	Ex. 3, p. 398; Ex. 4, p. 399
y varies inversely as x^n. y is inversely proportional to x^n.	$y = \frac{k}{x^n}$	
y varies jointly as x and z.	$y = kxz$	Ex. 5, p. 401

Review Exercises

3.1

In Exercises 1–4, use the vertex and intercepts to sketch the graph of each quadratic function. Give the equation for the parabola's axis of symmetry. Use the graph to determine the function's domain and range.

1. $f(x) = -(x + 1)^2 + 4$
2. $f(x) = (x + 4)^2 - 2$
3. $f(x) = -x^2 + 2x + 3$
4. $f(x) = 2x^2 - 4x - 6$

In Exercises 5–6, use the function's equation, and not its graph, to find

a. *the minimum or maximum value and where it occurs.*
b. *the function's domain and its range.*

5. $f(x) = -x^2 + 14x - 106$
6. $f(x) = 2x^2 + 12x + 703$

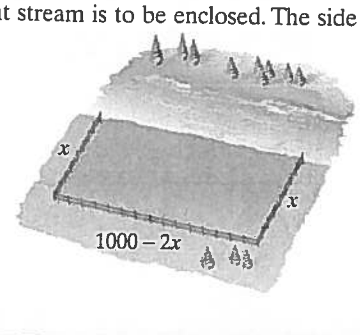
7. A quarterback tosses a football to a receiver 40 yards downfield. The height of the football, $f(x)$, in feet, can be modeled by

$$f(x) = -0.025x^2 + x + 6,$$

where x is the ball's horizontal distance, in yards, from the quarterback.

a. What is the ball's maximum height and how far from the quarterback does this occur?
b. From what height did the quarterback toss the football?
c. If the football is not blocked by a defensive player nor caught by the receiver, how far down the field will it go before hitting the ground? Round to the nearest tenth of a yard.
d. Graph the function that models the football's parabolic path.

8. A field bordering a straight stream is to be enclosed. The side bordering the stream is not to be fenced. If 1000 yards of fencing material is to be used, what are the dimensions of the largest rectangular field that can be fenced? What is the maximum area?



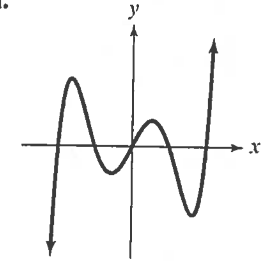
9. Among all pairs of numbers whose difference is 14, find a pair whose product is as small as possible. What is the minimum product?

3.2

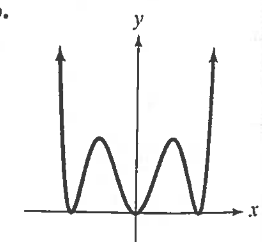
In Exercises 10–13, use the Leading Coefficient Test to determine the end behavior of the graph of the given polynomial function. Then use this end behavior to match the polynomial function with its graph. [The graphs are labeled (a) through (d).]

10. $f(x) = -x^3 + x^2 + 2x$
11. $f(x) = x^6 - 6x^4 + 9x^2$
12. $f(x) = x^5 - 5x^3 + 4x$
13. $f(x) = -x^4 + 1$

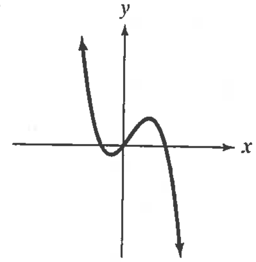
a.



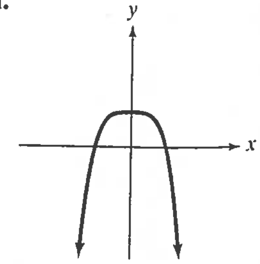
b.



c.



d.



14. The polynomial function

$$f(x) = -0.87x^3 + 0.35x^2 + 81.62x + 7684.94$$

models the number of thefts, $f(x)$, in thousands, in the United States x years after 1987. Will this function be useful in modeling the number of thefts over an extended period of time? Explain your answer.

15. A herd of 100 elk is introduced to a small island. The number of elk, $f(x)$, after x years is modeled by the polynomial function
$$f(x) = -x^4 + 21x^2 + 100.$$
Use the Leading Coefficient Test to determine the graph's end behavior to the right. What does this mean about what will eventually happen to the elk population?

In Exercises 16–17, find the zeros for each polynomial function and give the multiplicity of each zero. State whether the graph crosses the x-axis, or touches the x-axis and turns around, at each zero.

16. $f(x) = -2(x - 1)(x + 2)^2(x + 5)^3$
17. $f(x) = x^3 - 5x^2 - 25x + 125$
18. Show that $f(x) = x^3 - 2x - 1$ has a real zero between 1 and 2.

In Exercises 19–24,

a. *Use the Leading Coefficient Test to determine the graph's end behavior.*
b. *Determine whether the graph has y-axis symmetry, origin symmetry, or neither.*
c. *Graph the function.*

19. $f(x) = x^3 - x^2 - 9x + 9$
20. $f(x) = 4x - x^3$
21. $f(x) = 2x^3 + 3x^2 - 8x - 12$
22. $f(x) = -x^4 + 25x^2$
23. $f(x) = -x^4 + 6x^3 - 9x^2$
24. $f(x) = 3x^4 - 15x^3$

In Exercises 25–26, graph each polynomial function.

25. $f(x) = 2x^2(x - 1)^3(x + 2)$
26. $f(x) = -x^3(x + 4)^2(x - 1)$

3.3

In Exercises 27–29, divide using long division.

27. $(4x^3 - 3x^2 - 2x + 1) \div (x + 1)$
28. $(10x^3 - 26x^2 + 17x - 13) \div (5x - 3)$
29. $(4x^4 + 6x^3 + 3x - 1) \div (2x^2 + 1)$

In Exercises 30–31, divide using synthetic division.

30. $(3x^4 + 11x^3 - 20x^2 + 7x + 35) \div (x + 5)$
31. $(3x^4 - 2x^2 - 10x) \div (x - 2)$
32. Given $f(x) = 2x^3 - 7x^2 + 9x - 3$, use the Remainder Theorem to find $f(-13)$.
33. Use synthetic division to divide $f(x) = 2x^3 + x^2 - 13x + 6$ by $x - 2$. Use the result to find all zeros of f.
34. Solve the equation $x^3 - 17x + 4 = 0$ given that 4 is a root.

3.4

In Exercises 35–36, use the Rational Zero Theorem to list all possible rational zeros for each given function.

35. $f(x) = x^4 - 6x^3 + 14x^2 - 14x + 5$
36. $f(x) = 3x^5 - 2x^4 - 15x^3 + 10x^2 + 12x - 8$

In Exercises 37–38, use Descartes's Rule of Signs to determine the possible number of positive and negative real zeros for each given function.

37. $f(x) = 3x^4 - 2x^3 - 8x + 5$
38. $f(x) = 2x^5 - 3x^3 - 5x^2 + 3x - 1$
39. Use Descartes's Rule of Signs to explain why $2x^4 + 6x^2 + 8 = 0$ has no real roots.

For Exercises 40–46,

a. *List all possible rational roots or rational zeros.*
b. *Use Descartes's Rule of Signs to determine the possible number of positive and negative real roots or real zeros.*
c. *Use synthetic division to test the possible rational roots or zeros and find an actual root or zero.*
d. *Use the quotient from part (c) to find all the remaining roots or zeros.*

40. $f(x) = x^3 + 3x^2 - 4$
41. $f(x) = 6x^3 + x^2 - 4x + 1$
42. $8x^3 - 36x^2 + 46x - 15 = 0$
43. $2x^3 + 9x^2 - 7x + 1 = 0$
44. $x^4 - x^3 - 7x^2 + x + 6 = 0$
45. $4x^4 + 7x^2 - 2 = 0$
46. $f(x) = 2x^4 + x^3 - 9x^2 - 4x + 4$

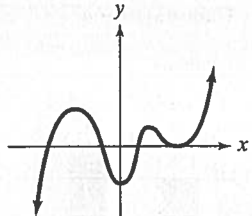
In Exercises 47–48, find an nth-degree polynomial function with real coefficients satisfying the given conditions. If you are using a graphing utility, graph the function and verify the real zeros and the given function value.

47. $n = 3$; 2 and $2 - 3i$ are zeros; $f(1) = -10$
48. $n = 4$; i is a zero; -3 is a zero of multiplicity 2; $f(-1) = 16$

In Exercises 49–50, find all the zeros of each polynomial function and write the polynomial as a product of linear factors.

49. $f(x) = 2x^4 + 3x^3 + 3x - 2$
50. $g(x) = x^4 - 6x^3 + x^2 + 24x + 16$

In Exercises 51–54, graphs of fifth-degree polynomial functions are shown. In each case, specify the number of real zeros and the number of imaginary zeros. Indicate whether there are any real zeros with multiplicity other than 1.

51.


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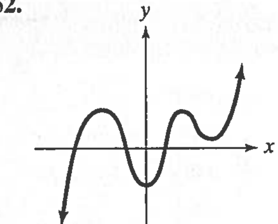

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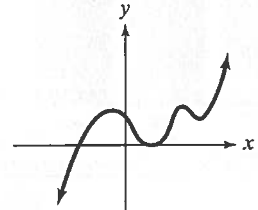

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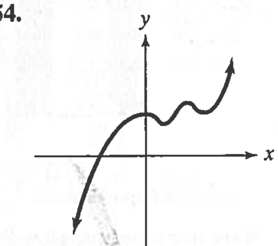


3.5

In Exercises 55–56, use transformations of $f(x) = \frac{1}{x}$ *or* $f(x) = \frac{1}{x^2}$ *to graph each rational function.*

55. $g(x) = \dfrac{1}{(x + 2)^2} - 1$
56. $h(x) = \dfrac{1}{x - 1} + 3$

In Exercises 57–64, find the vertical asymptotes, if any, the horizontal asymptote, if one exists, and the slant asymptote, if there is one, of the graph of each rational function. Then graph the rational function.

57. $f(x) = \dfrac{2x}{x^2 - 9}$
58. $g(x) = \dfrac{2x - 4}{x + 3}$

59. $h(x) = \dfrac{x^2 - 3x - 4}{x^2 - x - 6}$

60. $r(x) = \dfrac{x^2 + 4x + 3}{(x + 2)^2}$

61. $y = \dfrac{x^2}{x + 1}$

62. $y = \dfrac{x^2 + 2x - 3}{x - 3}$

63. $f(x) = \dfrac{-2x^3}{x^2 + 1}$

64. $g(x) = \dfrac{4x^2 - 16x + 16}{2x - 3}$

65. A company is planning to manufacture affordable graphing calculators. The fixed monthly cost will be \$50,000 and it will cost \$25 to produce each calculator.

a. Write the cost function, C, of producing x graphing calculators.

b. Write the average cost function, $\overline{C}$, of producing x graphing calculators.

c. Find and interpret $\overline{C}(50)$, $\overline{C}(100)$, $\overline{C}(1000)$, and $\overline{C}(100{,}000)$.

d. What is the horizontal asymptote for the graph of this function and what does it represent?

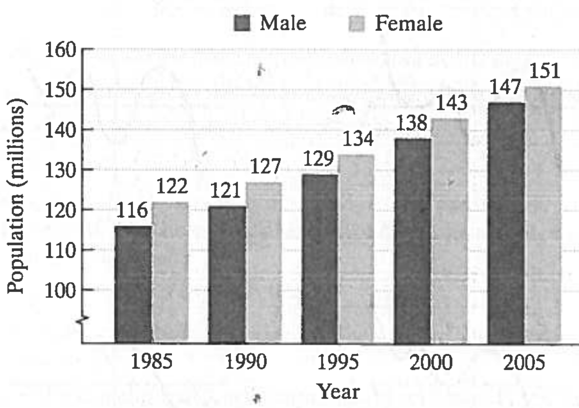
Exercises 66–67 involve rational functions that model the given situations. In each case, find the horizontal asymptote as $x \to \infty$ and then describe what this means in practical terms.

66. $f(x) = \dfrac{150x + 120}{0.05x + 1}$; the number of bass, $f(x)$, after x months in a lake that was stocked with 120 bass

67. $P(x) = \dfrac{72{,}900}{100x^2 + 729}$; the percentage, $P(x)$, of people in the United States with x years of education who are unemployed

68. The bar graph shows the population of the United States, in millions, for five selected years.

Population of the United States



Source: U.S. Census Bureau

Here are two functions that model the data:

$M(x) = 1.58x + 114.4$ — Male U.S. population, $M(x)$, in millions, x years after 1985

$F(x) = 1.48x + 120.6$. — Female U.S. population, $F(x)$, in millions, x years after 1985

a. Write a function that models the total U.S. population, $P(x)$, in millions, x years after 1985.

b. Write a rational function that models the fraction of men in the U.S. population, $R(x)$, x years after 1985.

c. What is the equation of the horizontal asymptote associated with the function in part (b)? Round to two decimal places. What does this mean about the percentage of men in the U.S. population over time?

3.6

In Exercises 69–74, solve each inequality and graph the solution set on a real number line.

69. $2x^2 + 5x - 3 < 0$

70. $2x^2 + 9x + 4 \geq 0$

71. $x^3 + 2x^2 > 3x$

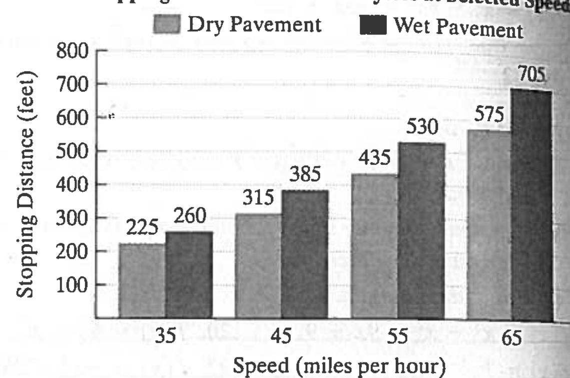
72. $\dfrac{x - 6}{x + 2} > 0$

73. $\dfrac{(x + 1)(x - 2)}{x - 1} \geq 0$

74. $\dfrac{x + 3}{x - 4} \leq 5$

75. The graph shows stopping distances for motorcycles at various speeds on dry roads and on wet roads.

Stopping Distances for Motorcycles at Selected Speeds



Source: National Highway Traffic Safety Administration

The functions

$$f(x) = 0.125x^2 - 0.8x + 99$$

Dry pavement

and

Wet pavement

$$g(x) = 0.125x^2 + 2.3x + 27$$

model a motorcycle's stopping distance, $f(x)$ or $g(x)$, in feet, traveling at x miles per hour. Function f models stopping distance on dry pavement and function g models stopping distance on wet pavement.

a. Use function g to find the stopping distance on wet pavement for a motorcycle traveling at 35 miles per hour. Round to the nearest foot. Does your rounded answer overestimate or underestimate the stopping distance shown by the graph? By how many feet?

b. Use function f to determine speeds on dry pavement requiring stopping distances that exceed 267 feet.

76. Use the position function

$$s(t) = -16t^2 + v_0t + s_0$$

to solve this problem. A projectile is fired vertically upward from ground level with an initial velocity of 48 feet per second. During which time period will the projectile's height exceed 32 feet?

3.7

Solve the variation problems in Exercises 77–82.

77. Many areas of Northern California depend on the snowpack of the Sierra Nevada mountain range for their water supply. The volume of water produced from melting snow varies directly as the volume of snow. Meteorologists have determined that 250 cubic centimeters of snow will melt to 28 cubic centimeters of water. How much water does 1200 cubic centimeters of melting snow produce?