Solution

a. We find $f(x + h)$ by replacing x with $x + h$ each time that x appears in the equation.

$$f(x) = 2x^2 \quad - \quad x \quad + \quad 3$$

Replace x with $x + h$. | Replace x with $x + h$. | Replace x with $x + h$. | Copy the 3. There is no x in this term.

$$\begin{aligned} f(x + h) &= 2(x + h)^2 - (x + h) + 3 \\ &= 2(x^2 + 2xh + h^2) - x - h + 3 \\ &= 2x^2 + 4xh + 2h^2 - x - h + 3 \end{aligned}$$

b. Using our result from part (a), we obtain the following:

This is $f(x + h)$ from part (a).

This is $f(x)$ from the given equation.

$$\frac{f(x+h) - f(x)}{h} = \frac{\boxed{2x^2 + 4xh + 2h^2 - x - h + 3} - (2x^2 - x + 3)}{h}$$

$$= \frac{2x^2 + 4xh + 2h^2 - x - h + 3 - 2x^2 + x - 3}{h}$$
Remove parentheses and change the sign of each term in the parentheses.

$$= \frac{(2x^2 - 2x^2) + (-x + x) + (3 - 3) + 4xh + 2h^2 - h}{h}$$
Group like terms.

$$= \frac{4xh + 2h^2 - 1h}{h}$$
Simplify.

We wrote $-h$ as $-1h$ to avoid possible errors in the next factoring step.

$$= \frac{h(4x + 2h - 1)}{h}$$
Factor h from the numerator.

$$= 4x + 2h - 1$$
Divide out identical factors of h in the numerator and denominator.

Check Point 5 If $f(x) = -2x^2 + x + 5$, find and simplify each expression:

a. $f(x + h)$ **b.** $\dfrac{f(x+h) - f(x)}{h}, h \neq 0.$

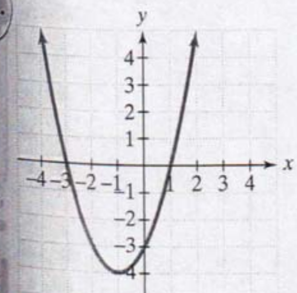
Exercise Set 2.2

Practice Exercises

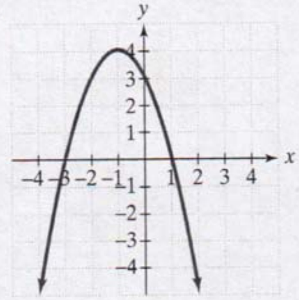
In Exercises 1–12, use the graph to determine

a. *intervals on which the function is increasing, if any.*
b. *intervals on which the function is decreasing, if any.*
c. *intervals on which the function is constant, if any.*

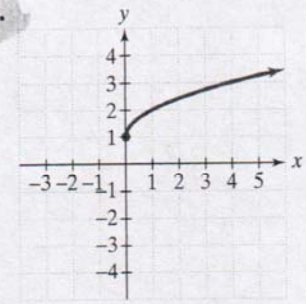
1.



2.



3.



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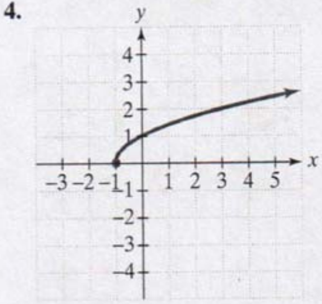

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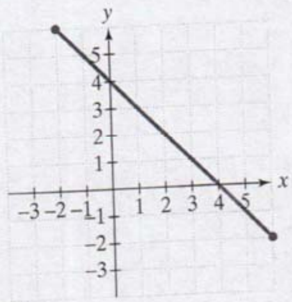

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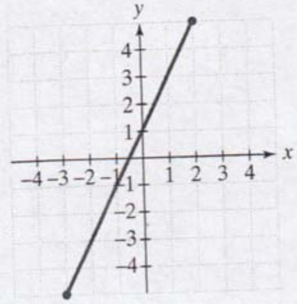

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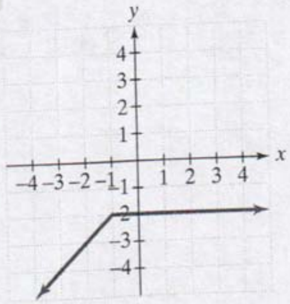

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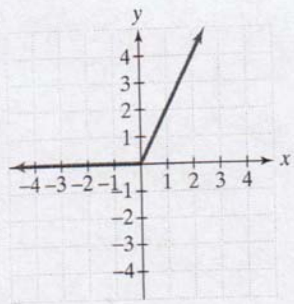

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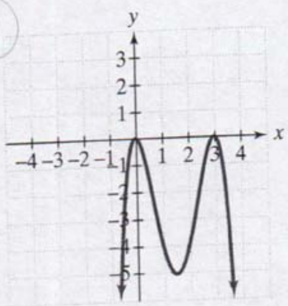

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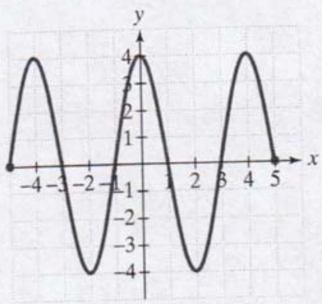

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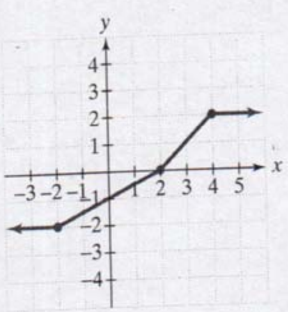

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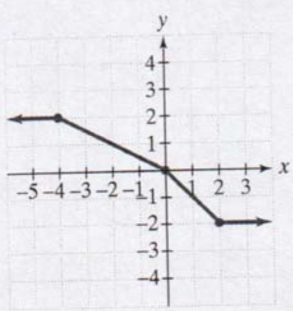
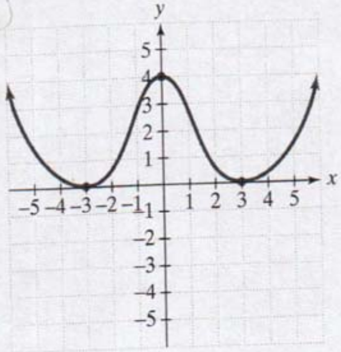


In Exercises 13–16, the graph of a function f is given. Use the graph to find each of the following:

a. *The numbers, if any, at which f has a relative maximum. What are these relative maxima?*

b. *The numbers, if any, at which f has a relative minimum. What are these relative minima?*

13.



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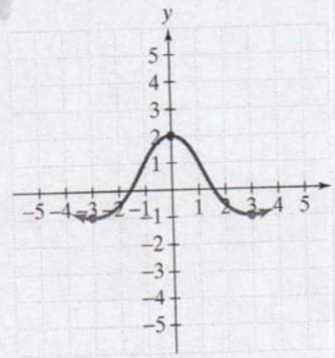

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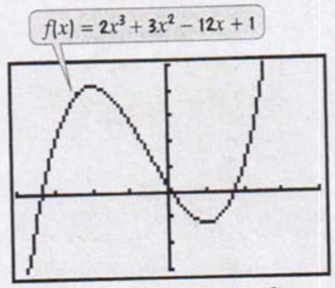
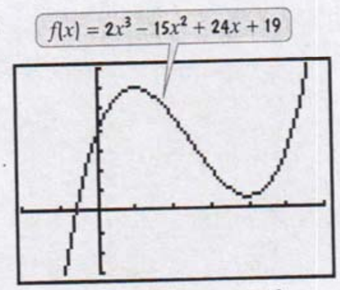


[−4, 4, 1] by [−15, 25, 5]

16.



[−2, 6, 1] by [−15, 35, 5]

In Exercises 17–28, determine whether each function is even, odd, or neither.

17. $f(x) = x^3 + x$

18. $f(x) = x^3 - x$

19. $g(x) = x^2 + x$

20. $g(x) = x^2 - x$

21. $h(x) = x^2 - x^4$

22. $h(x) = 2x^2 + x^4$

23. $f(x) = x^2 - x^4 + 1$

24. $f(x) = 2x^2 + x^4 + 1$

25. $f(x) = \frac{1}{5}x^6 - 3x^2$

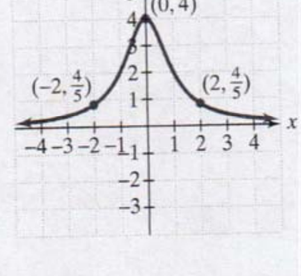
26. $f(x) = 2x^3 - 6x^5$

27. $f(x) = x\sqrt{1 - x^2}$

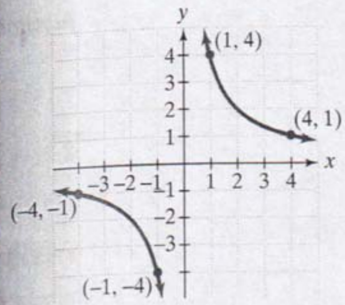
28. $f(x) = x^2\sqrt{1 - x^2}$

In Exercises 29–32, use possible symmetry to determine whether each graph is the graph of an even function, an odd function, or a function that is neither even nor odd.

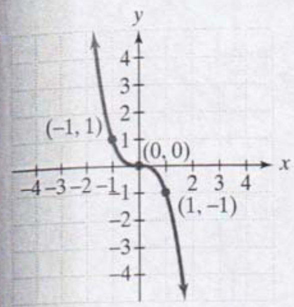
29.



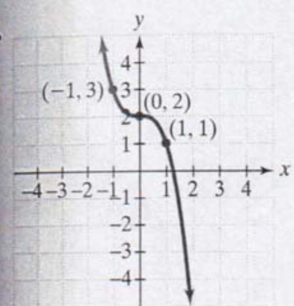
30.



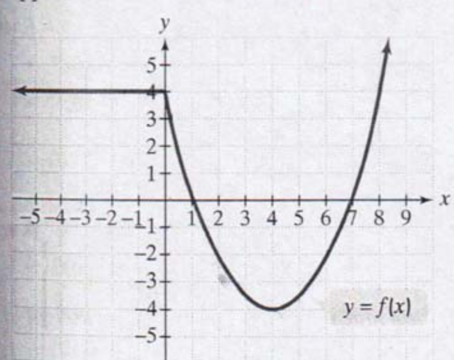
31.



32.



33. Use the graph of f to determine each of the following. Where applicable, use interval notation.



a. the domain of f

b. the range of f

c. the x-intercepts

d. the y-intercept

e. intervals on which f is increasing

f. intervals on which f is decreasing

g. intervals on which f is constant

h. the number at which f has a relative minimum

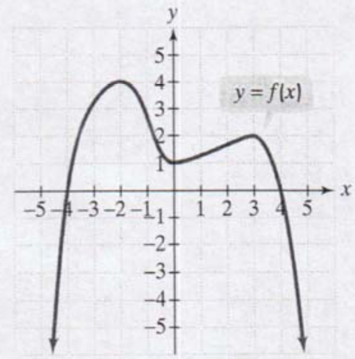
i. the relative minimum of f

j. $f(-3)$

k. the values of x for which $f(x) = -2$

l. Is f even, odd, or neither?

34. Use the graph of f to determine each of the following. Where applicable, use interval notation.



a. the domain of f

b. the range of f

c. the x-intercepts

d. the y-intercept

e. intervals on which f is increasing

f. intervals on which f is decreasing

g. values of x for which $f(x) \le 0$

h. the numbers at which f has a relative maximum

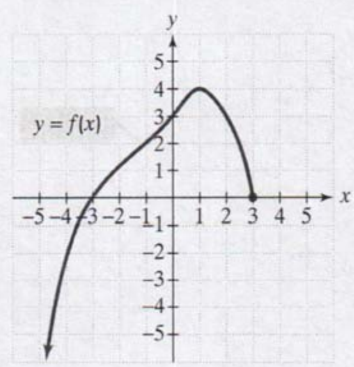
i. the relative maxima of f

j. $f(-2)$

k. the values of x for which $f(x) = 0$

l. Is f even, odd, or neither?

35. Use the graph of f to determine each of the following. Where applicable, use interval notation.



a. the domain of f

b. the range of f

c. the zeros of f

d. $f(0)$

e. intervals on which f is increasing

f. intervals on which f is decreasing

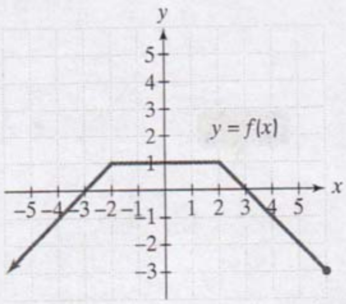
g. values of x for which $f(x) \le 0$

h. any relative maxima and the numbers at which they occur

i. the value of x for which $f(x) = 4$

j. Is $f(-1)$ positive or negative?

36. Use the graph of f to determine each of the following. Where applicable, use interval notation.



a. the domain of f

b. the range of f

c. the zeros of f

d. $f(0)$

e. intervals on which f is increasing

f. intervals on which f is decreasing

g. intervals on which f is constant

h. values of x for which $f(x) > 0$

i. values of x for which $f(x) = -2$

j. Is $f(4)$ positive or negative?

k. Is f even, odd, or neither?

l. Is $f(2)$ a relative maximum?

In Exercises 37–42, evaluate each piecewise function at the given values of the independent variable.

37. $f(x) = \begin{cases} 3x + 5 & \text{if } x < 0 \\ 4x + 7 & \text{if } x \geq 0 \end{cases}$

a. $f(-2)$ **b.** $f(0)$ **c.** $f(3)$

38. $f(x) = \begin{cases} 6x - 1 & \text{if } x < 0 \\ 7x + 3 & \text{if } x \geq 0 \end{cases}$

a. $f(-3)$ **b.** $f(0)$ **c.** $f(4)$

39. $g(x) = \begin{cases} x + 3 & \text{if } x \geq -3 \\ -(x + 3) & \text{if } x < -3 \end{cases}$

a. $g(0)$ **b.** $g(-6)$ **c.** $g(-3)$

40. $g(x) = \begin{cases} x + 5 & \text{if } x \geq -5 \\ -(x + 5) & \text{if } x < -5 \end{cases}$

a. $g(0)$ **b.** $g(-6)$ **c.** $g(-5)$

41. $h(x) = \begin{cases} \dfrac{x^2 - 9}{x - 3} & \text{if } x \neq 3 \\ 6 & \text{if } x = 3 \end{cases}$

a. $h(5)$ **b.** $h(0)$ **c.** $h(3)$

42. $h(x) = \begin{cases} \dfrac{x^2 - 25}{x - 5} & \text{if } x \neq 5 \\ 10 & \text{if } x = 5 \end{cases}$

a. $h(7)$ **b.** $h(0)$ **c.** $h(5)$

In Exercises 43–54, the domain of each piecewise function is $(-\infty, \infty)$.

a. *Graph each function.*

b. *Use your graph to determine the function's range.*

43. $f(x) = \begin{cases} -x & \text{if } x < 0 \\ x & \text{if } x \geq 0 \end{cases}$

44. $f(x) = \begin{cases} x & \text{if } x < 0 \\ -x & \text{if } x \geq 0 \end{cases}$

45. $f(x) = \begin{cases} 2x & \text{if } x \leq 0 \\ 2 & \text{if } x > 0 \end{cases}$

46. $f(x) = \begin{cases} \frac{1}{2}x & \text{if } x \leq 0 \\ 3 & \text{if } x > 0 \end{cases}$

47. $f(x) = \begin{cases} x + 3 & \text{if } x < -2 \\ x - 3 & \text{if } x \geq -2 \end{cases}$

48. $f(x) = \begin{cases} x + 2 & \text{if } x < -3 \\ x - 2 & \text{if } x \geq -3 \end{cases}$

49. $f(x) = \begin{cases} 3 & \text{if } x \leq -1 \\ -3 & \text{if } x > -1 \end{cases}$

50. $f(x) = \begin{cases} 4 & \text{if } x \leq -1 \\ -4 & \text{if } x > -1 \end{cases}$

51. $f(x) = \begin{cases} \frac{1}{2}x^2 & \text{if } x < 1 \\ 2x - 1 & \text{if } x \geq 1 \end{cases}$

52. $f(x) = \begin{cases} -\frac{1}{2}x^2 & \text{if } x < 1 \\ 2x + 1 & \text{if } x \geq 1 \end{cases}$

53. $f(x) = \begin{cases} 0 & \text{if } x < -4 \\ -x & \text{if } -4 \leq x < 0 \\ x^2 & \text{if } x \geq 0 \end{cases}$

54. $f(x) = \begin{cases} 0 & \text{if } x < -3 \\ -x & \text{if } -3 \leq x < 0 \\ x^2 - 1 & \text{if } x \geq 0 \end{cases}$

In Exercises 55–76, find and simplify the difference quotient

$$\frac{f(x + h) - f(x)}{h}, h \neq 0$$

for the given function.

55. $f(x) = 4x$

56. $f(x) = 7x$

57. $f(x) = 3x + 7$

58. $f(x) = 6x + 1$

59. $f(x) = x^2$

60. $f(x) = 2x^2$

61. $f(x) = x^2 - 4x + 3$

62. $f(x) = x^2 - 5x + 8$

63. $f(x) = 2x^2 + x - 1$

64. $f(x) = 3x^2 + x + 5$

65. $f(x) = -x^2 + 2x + 4$

66. $f(x) = -x^2 - 3x + 1$

67. $f(x) = -2x^2 + 5x + 7$

68. $f(x) = -3x^2 + 2x - 1$

69. $f(x) = -2x^2 - x + 3$

70. $f(x) = -3x^2 + x - 1$