$$f(x) = x^3 + 2x^2 + 5x + 4$$

Replace x with $-x$.

$$f(-x) = (-x)^3 + 2(-x)^2 + 5(-x) + 4$$
$$= -x^3 + 2x^2 - 5x + 4$$

Now count the sign changes.

$$f(-x) = -x^3 + 2x^2 - 5x + 4$$

sign change sign change sign change

There are three variations in sign. The number of negative real zeros of f is either equal to the number of sign changes, 3, or is less than this number by an even integer. This means that either there are 3 negative real zeros or there is $3 - 2 = 1$ negative real zero.

What do the results of Example 7 mean in terms of solving

$$x^3 + 2x^2 + 5x + 4 = 0?$$

Without using Descartes's Rule of Signs, we list the possible rational roots as follows:

Possible rational roots

$$= \frac{\text{Factors of the constant term, 4}}{\text{Factors of the leading coefficient, 1}} = \frac{\pm 1, \pm 2, \pm 4}{\pm 1} = \pm 1, \pm 2, \pm 4.$$

However, Descartes's Rule of Signs informed us that $f(x) = x^3 + 2x^2 + 5x + 4$ has no positive real zeros. Thus, the polynomial equation $x^3 + 2x^2 + 5x + 4 = 0$ has no positive real roots. This means that we can eliminate the positive numbers from our list of possible rational roots. Possible rational roots include only -1, -2, and -4. We can use synthetic division and test the first of these three possible rational roots of $x^3 + 2x^2 + 5x + 4 = 0$ as follows:

Test -1.

-1	1	2	5	4
		-1	-1	-4
	1	1	4	0.

The zero remainder shows that -1 is a root.

By solving the equation $x^3 + 2x^2 + 5x + 4 = 0$, you will find that this equation of degree 3 has three roots. One root is -1 and the other two roots are imaginary numbers in a conjugate pair. Verify this by completing the solution process.

Check Point 7 Determine the possible numbers of positive and negative real zeros of $f(x) = x^4 - 14x^3 + 71x^2 - 154x + 120$.

Exercise Set 3.4

Practice Exercises

In Exercises 1–8, use the Rational Zero Theorem to list all possible rational zeros for each given function.

1. $f(x) = x^3 + x^2 - 4x - 4$
2. $f(x) = x^3 + 3x^2 - 6x - 8$
3. $f(x) = 3x^4 - 11x^3 - x^2 + 19x + 6$
4. $f(x) = 2x^4 + 3x^3 - 11x^2 - 9x + 15$
5. $f(x) = 4x^4 - x^3 + 5x^2 - 2x - 6$
6. $f(x) = 3x^4 - 11x^3 - 3x^2 - 6x + 8$
7. $f(x) = x^5 - x^4 - 7x^3 + 7x^2 - 12x - 12$
8. $f(x) = 4x^5 - 8x^4 - x + 2$

In Exercises 9–16,

a. *List all possible rational zeros.*
b. *Use synthetic division to test the possible rational zeros and find an actual zero.*
c. *Use the quotient from part (b) to find the remaining zeros of the polynomial function.*

9. $f(x) = x^3 + x^2 - 4x - 4$
10. $f(x) = x^3 - 2x^2 - 11x + 12$
11. $f(x) = 2x^3 - 3x^2 - 11x + 6$
12. $f(x) = 2x^3 - 5x^2 + x + 2$
13. $f(x) = x^3 + 4x^2 - 3x - 6$
14. $f(x) = 2x^3 + x^2 - 3x + 1$
15. $f(x) = 2x^3 + 6x^2 + 5x + 2$
16. $f(x) = x^3 - 4x^2 + 8x - 5$

In Exercises 17–24,

a. *List all possible rational roots.*
b. *Use synthetic division to test the possible rational roots and find an actual root.*
c. *Use the quotient from part (b) to find the remaining roots and solve the equation.*

17. $x^3 - 2x^2 - 11x + 12 = 0$
18. $x^3 - 2x^2 - 7x - 4 = 0$
19. $x^3 - 10x - 12 = 0$
20. $x^3 - 5x^2 + 17x - 13 = 0$
21. $6x^3 + 25x^2 - 24x + 5 = 0$
22. $2x^3 - 5x^2 - 6x + 4 = 0$
23. $x^4 - 2x^3 - 5x^2 + 8x + 4 = 0$
24. $x^4 - 2x^2 - 16x - 15 = 0$

In Exercises 25–32, find an nth-degree polynomial function with real coefficients satisfying the given conditions. If you are using a graphing utility, use it to graph the function and verify the real zeros and the given function value.

25. $n = 3$; 1 and $5i$ are zeros; $f(-1) = -104$
26. $n = 3$; 4 and $2i$ are zeros; $f(-1) = -50$
27. $n = 3$; -5 and $4 + 3i$ are zeros; $f(2) = 91$
28. $n = 3$; 6 and $-5 + 2i$ are zeros; $f(2) = -636$
29. $n = 4$; i and $3i$ are zeros; $f(-1) = 20$
30. $n = 4$; -2, $-\frac{1}{2}$, and i are zeros; $f(1) = 18$
31. $n = 4$; -2, 5, and $3 + 2i$ are zeros; $f(1) = -96$
32. $n = 4$; -4, $\frac{1}{3}$, and $2 + 3i$ are zeros; $f(1) = 100$

In Exercises 33–38, use Descartes's Rule of Signs to determine the possible number of positive and negative real zeros for each given function.

33. $f(x) = x^3 + 2x^2 + 5x + 4$
34. $f(x) = x^3 + 7x^2 + x + 7$
35. $f(x) = 5x^3 - 3x^2 + 3x - 1$
36. $f(x) = -2x^3 + x^2 - x + 7$
37. $f(x) = 2x^4 - 5x^3 - x^2 - 6x + 4$
38. $f(x) = 4x^4 - x^3 + 5x^2 - 2x - 6$

In Exercises 39–52, find all zeros of the polynomial function or solve the given polynomial equation. Use the Rational Zero Theorem, Descartes's Rule of Signs, and possibly the graph of the polynomial function shown by a graphing utility as an aid in obtaining the first zero or the first root.

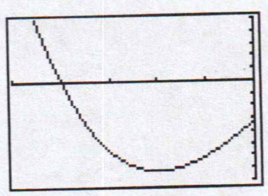
39. $f(x) = x^3 - 4x^2 - 7x + 10$
40. $f(x) = x^3 + 12x^2 + 21x + 10$
41. $2x^3 - x^2 - 9x - 4 = 0$
42. $3x^3 - 8x^2 - 8x + 8 = 0$
43. $f(x) = x^4 - 2x^3 + x^2 + 12x + 8$
44. $f(x) = x^4 - 4x^3 - x^2 + 14x + 10$
45. $x^4 - 3x^3 - 20x^2 - 24x - 8 = 0$
46. $x^4 - x^3 + 2x^2 - 4x - 8 = 0$
47. $f(x) = 3x^4 - 11x^3 - x^2 + 19x + 6$
48. $f(x) = 2x^4 + 3x^3 - 11x^2 - 9x + 15$
49. $4x^4 - x^3 + 5x^2 - 2x - 6 = 0$
50. $3x^4 - 11x^3 - 3x^2 - 6x + 8 = 0$
51. $2x^5 + 7x^4 - 18x^2 - 8x + 8 = 0$
52. $4x^5 + 12x^4 - 41x^3 - 99x^2 + 10x + 24 = 0$

Practice Plus

Exercises 53–60, show incomplete graphs of given polynomial functions.

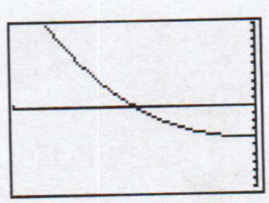
a. *Find all the zeros of each function.*
b. *Without using a graphing utility, draw a complete graph of the function.*

53. $f(x) = -x^3 + x^2 + 16x - 16$



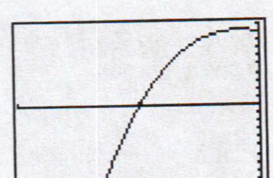
[−5, 0, 1] by [−40, 25, 5]

54. $f(x) = -x^3 + 3x^2 - 4$



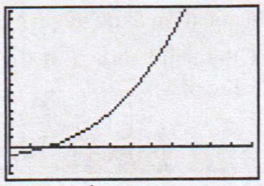
[−2, 0, 1] by [−10, 10, 1]

55. $f(x) = 4x^3 - 8x^2 - 3x + 9$



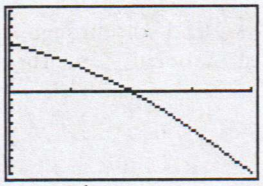
[−2, 0, 1] by [−10, 10, 1]

56. $f(x) = 3x^3 + 2x^2 + 2x - 1$



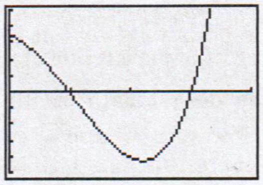
$[0, 2, \frac{1}{6}]$ by $[-3, 15, 1]$

57. $f(x) = 2x^4 - 3x^3 - 7x^2 - 8x + 6$



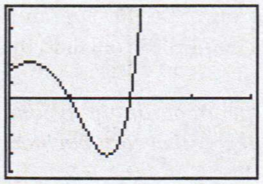
$[0, 1, \frac{1}{4}]$ by $[-10, 10, 1]$

58. $f(x) = 2x^4 + 2x^3 - 22x^2 - 18x + 36$



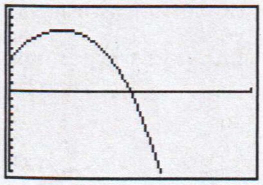
$[0, 4, 1]$ by $[-50, 50, 10]$

59. $f(x) = 3x^5 + 2x^4 - 15x^3 - 10x^2 + 12x + 8$



$[0, 4, 1]$ by $[-20, 25, 5]$

60. $f(x) = -5x^4 + 4x^3 - 19x^2 + 16x + 4$



$[0, 2, 1]$ by $[-10, 10, 1]$

Application Exercises

A popular model of carry-on luggage has a length that is 10 inches greater than its depth. Airline regulations require that the sum of the length, width, and depth cannot exceed 40 inches. These conditions, with the assumption that this sum is 40 inches, can be modeled by a function that gives the volume of the luggage, V, in cubic inches, in terms of its depth, x, in inches.

Volume	=	depth	·	length	·	width: 40 − (depth + length)
$V(x)$	=	x	·	$(x + 10)$	·	$[40 - (x + x + 10)]$

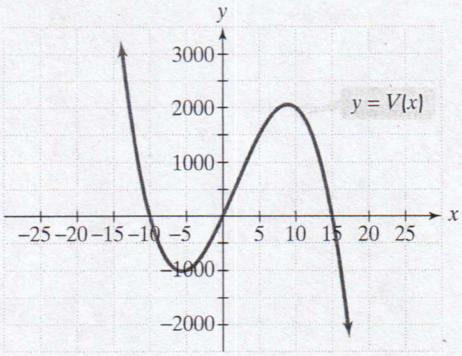
$$V(x) = x(x + 10)(30 - 2x)$$

Use function V to solve Exercises 61–62.

61. If the volume of the carry-on luggage is 2000 cubic inches, determine two possibilities for its depth. Where necessary, round to the nearest tenth of an inch.

62. If the volume of the carry-on luggage is 1500 cubic inches, determine two possibilities for its depth. Where necessary, round to the nearest tenth of an inch.

Use the graph of the function modeling the volume of the carry-on luggage to solve Exercises 63–64.



63. **a.** Identify your answers from Exercise 61 as points on the graph.
    b. Use the graph to describe a realistic domain, x, for the volume function, where x represents the depth of the carry-on luggage.

64. **a.** Identify your answers from Exercise 62 as points on the graph.
    b. Use the graph to describe a realistic domain, x, for the volume function, where x represents the depth of the carry-on luggage.

Writing in Mathematics

65. Describe how to find the possible rational zeros of a polynomial function.

66. How does the linear factorization of $f(x)$, that is,

$$f(x) = a_n(x - c_1)(x - c_2)\cdots(x - c_n),$$

show that a polynomial equation of degree n has n roots?

67. Describe how to use Descartes's Rule of Signs to determine the possible number of positive real zeros of a polynomial function.

68. Describe how to use Descartes's Rule of Signs to determine the possible number of negative roots of a polynomial equation.

69. Why must every polynomial equation with real coefficients of degree 3 have at least one real root?

70. Explain why the equation $x^4 + 6x^2 + 2 = 0$ has no rational roots.

71. Suppose $\frac{3}{4}$ is a root of a polynomial equation. What does this tell us about the leading coefficient and the constant term in the equation?