EXAMPLE 11 Fair or Poor Health, by Annual Income

The bar graph in **Figure 4.21** shows people with lower incomes are more likely to report that their health is fair or poor. The function

$$f(x) = 54.8 - 12.3 \ln x$$

models the percentage of Americans reporting fair or poor health, $f(x)$, in terms of annual income, x, in thousands of dollars. According to the model, what annual income corresponds to 10% reporting fair or poor health? Round to the nearest thousand dollars.

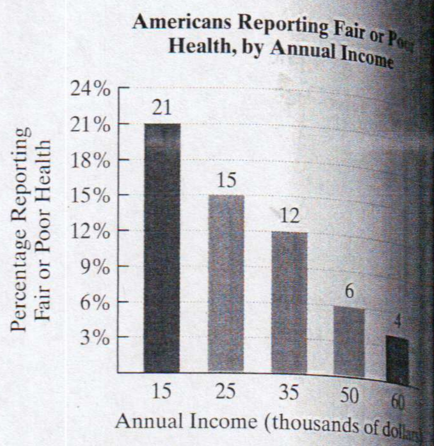


Figure 4.21
Source: William Kornblum and Joseph Julian, *Social Problems*, Twelfth Edition, Prentice Hall, 2007

Solution To find what annual income corresponds to 10% reporting fair or poor health, we substitute 10 for $f(x)$ and solve for x, the annual income.

$f(x) = 54.8 - 12.3 \ln x$ — This is the given function.

$10 = 54.8 - 12.3 \ln x$ — Substitute 10 for $f(x)$.

Our goal is to isolate $\ln x$ and then rewrite the equation in exponential form.

$-44.8 = -12.3 \ln x$ — Subtract 54.8 from both sides.

$\frac{44.8}{12.3} = \ln x$ — Divide both sides by -12.3.

$\frac{44.8}{12.3} = \log_e x$ — Rewrite the natural logarithm showing base e. This step is optional.

$e^{\frac{44.8}{12.3}} = x$ — Rewrite in exponential form.

$38 \approx x$ — Use a calculator.

An annual income of approximately \$38,000 corresponds to 10% of Americans reporting fair or poor health.

Check Point 11 According to the function in Example 11, what annual income corresponds to 25% reporting fair or poor health? Round to the nearest thousand dollars.

Exercise Set 4.4

Practice Exercises

Solve each exponential equation in Exercises 1–22 by expressing each side as a power of the same base and then equating exponents.

1. $2^x = 64$
2. $3^x = 81$
3. $5^x = 125$
4. $5^x = 625$
5. $2^{2x-1} = 32$
6. $3^{2x+1} = 27$
7. $4^{2x-1} = 64$
8. $5^{3x-1} = 125$
9. $32^x = 8$
10. $4^x = 32$
11. $9^x = 27$
12. $125^x = 625$
13. $3^{1-x} = \frac{1}{27}$
14. $5^{2-x} = \frac{1}{125}$
15. $6^{\frac{x-3}{4}} = \sqrt{6}$
16. $7^{\frac{x-2}{6}} = \sqrt{7}$
17. $4^x = \frac{1}{\sqrt{2}}$
18. $9^x = \frac{1}{\sqrt[3]{3}}$
19. $8^{x+3} = 16^{x-1}$
20. $8^{1-x} = 4^{x+2}$
21. $e^{x+1} = \frac{1}{e}$
22. $e^{x+4} = \frac{1}{e^{2x}}$

Solve each exponential equation in Exercises 23–48. Express the solution set in terms of natural logarithms. Then use a calculator to obtain a decimal approximation, correct to two decimal places, for the solution.

23. $10^x = 3.91$
24. $10^x = 8.07$
25. $e^x = 5.7$
26. $e^x = 0.83$
27. $5^x = 17$
28. $19^x = 143$
29. $5e^x = 23$
30. $9e^x = 107$
31. $3e^{5x} = 1977$
32. $4e^{7x} = 10{,}273$
33. $e^{1-5x} = 793$
34. $e^{1-8x} = 7957$
35. $e^{5x-3} - 2 = 10{,}476$
36. $e^{4x-5} - 7 = 11{,}243$
37. $7^{x+2} = 410$
38. $5^{x-3} = 137$
39. $7^{0.3x} = 813$
40. $3^{\frac{x}{7}} = 0.2$
41. $5^{2x+3} = 3^{x-1}$
42. $7^{2x+1} = 3^{x+2}$
43. $e^{2x} - 3e^x + 2 = 0$
44. $e^{2x} - 2e^x - 3 = 0$
45. $e^{4x} + 5e^{2x} - 24 = 0$
46. $e^{4x} - 3e^{2x} - 18 = 0$
47. $3^{2x} + 3^x - 2 = 0$
48. $2^{2x} + 2^x - 12 = 0$

Solve each logarithmic equation in Exercises 49–90. Be sure to reject any value of x that is not in the domain of the original logarithmic expressions. Give the exact answer. Then, where necessary, use a calculator to obtain a decimal approximation, correct to two decimal places, for the solution.

49. $\log_3 x = 4$
50. $\log_5 x = 3$
51. $\ln x = 2$
52. $\ln x = 3$
53. $\log_4(x + 5) = 3$
54. $\log_5(x - 7) = 2$
55. $\log_3(x - 4) = -3$
56. $\log_7(x + 2) = -2$
57. $\log_4(3x + 2) = 3$
58. $\log_2(4x + 1) = 5$
59. $5\ln(2x) = 20$
60. $6\ln(2x) = 30$
61. $6 + 2\ln x = 5$
62. $7 + 3\ln x = 6$
63. $\ln\sqrt{x + 3} = 1$
64. $\ln\sqrt{x + 4} = 1$
65. $\log_5 x + \log_5(4x - 1) = 1$
66. $\log_6(x + 5) + \log_6 x = 2$
67. $\log_3(x - 5) + \log_3(x + 3) = 2$
68. $\log_2(x - 1) + \log_2(x + 1) = 3$
69. $\log_2(x + 2) - \log_2(x - 5) = 3$
70. $\log_4(x + 2) - \log_4(x - 1) = 1$
71. $2\log_3(x + 4) = \log_3 9 + 2$
72. $3\log_2(x - 1) = 5 - \log_2 4$
73. $\log_2(x - 6) + \log_2(x - 4) - \log_2 x = 2$
74. $\log_2(x - 3) + \log_2 x - \log_2(x + 2) = 2$
75. $\log(x + 4) = \log x + \log 4$
76. $\log(5x + 1) = \log(2x + 3) + \log 2$
77. $\log(3x - 3) = \log(x + 1) + \log 4$
78. $\log(2x - 1) = \log(x + 3) + \log 3$
79. $2\log x = \log 25$
80. $3\log x = \log 125$
81. $\log(x + 4) - \log 2 = \log(5x + 1)$
82. $\log(x + 7) - \log 3 = \log(7x + 1)$
83. $2\log x - \log 7 = \log 112$
84. $\log(x - 2) + \log 5 = \log 100$
85. $\log x + \log(x + 3) = \log 10$
86. $\log(x + 3) + \log(x - 2) = \log 14$
87. $\ln(x - 4) + \ln(x + 1) = \ln(x - 8)$
88. $\log_2(x - 1) - \log_2(x + 3) = \log_2\left(\frac{1}{x}\right)$
89. $\ln(x - 2) - \ln(x + 3) = \ln(x - 1) - \ln(x + 7)$
90. $\ln(x - 5) - \ln(x + 4) = \ln(x - 1) - \ln(x + 2)$

Practice Plus

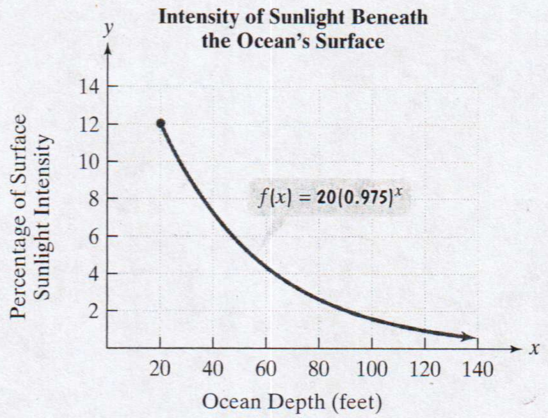
In Exercises 91–100, solve each equation.

91. $5^{2x} \cdot 5^{4x} = 125$
92. $3^{x+2} \cdot 3^x = 81$
93. $2|\ln x| - 6 = 0$
94. $3|\log x| - 6 = 0$
95. $3^{x^2} = 45$
96. $5^{x^2} = 50$
97. $\ln(2x + 1) + \ln(x - 3) - 2\ln x = 0$
98. $\ln 3 - \ln(x + 5) - \ln x = 0$
99. $5^{x^2-12} = 25^{2x}$
100. $3^{x^2-12} = 9^{2x}$

Application Exercises

101. The formula $A = 36.1e^{0.0126t}$ models the population of California, A, in millions, t years after 2005.
   a. What was the population of California in 2005?
   b. When will the population of California reach 40 million?
102. The formula $A = 22.9e^{0.0183t}$ models the population of Texas, A, in millions, t years after 2005.
   a. What was the population of Texas in 2005?
   b. When will the population of Texas reach 27 million?

The function $f(x) = 20(0.975)^x$ models the percentage of surface sunlight, $f(x)$, that reaches a depth of x feet beneath the surface of the ocean. The figure shows the graph of this function. Use this information to solve Exercises 103–104.



103. Use the function to determine at what depth, to the nearest foot, there is 1% of surface sunlight. How is this shown on the graph of f?
104. Use the function to determine at what depth, to the nearest foot, there is 3% of surface sunlight. How is this shown on the graph of f?