Step 5 Solve the resulting system for $A, B, C,$ and D. Based on our observations in step 4, $A = 5, B = -3, C = 2,$ and $D = 0$.

Step 6 Substitute the values of $A, B, C,$ and D, and write the partial fraction decomposition.

$$\frac{5x^3 - 3x^2 + 7x - 3}{(x^2+1)^2} = \frac{Ax+B}{x^2+1} + \frac{Cx+D}{(x^2+1)^2} = \frac{5x-3}{x^2+1} + \frac{2x}{(x^2+1)^2}$$

Check Point 4 Find the partial fraction dec

Study Tip

When a rational expression contains a power of a fact
the partial fraction decomposition to allow for every n
than or equal to the power. Example:

$$\frac{2x+1}{(x-5)^2x^3}$$

$$= \frac{A}{x-5} + \frac{B}{(x-5)^2}$$

Although $(x-5)^2$ an
they are still expr
of linear factors,
Thus, the numerators are constant.

Exercise Set 8.3

Practice Exercises

In Exercises 1–8, write the form of the partial fraction decomposition of the rational expression. It is not necessary to solve for the constants.

1. $\dfrac{11x - 10}{(x-2)(x+1)}$
2. $\dfrac{5x + 7}{(x-1)(x+3)}$
3. $\dfrac{6x^2 - 14x - 27}{(x+2)(x-3)^2}$
4. $\dfrac{3x + 16}{(x+1)(x-2)^2}$
5. $\dfrac{5x^2 - 6x + 7}{(x-1)(x^2+1)}$
6. $\dfrac{5x^2 - 9x + 19}{(x-4)(x^2+5)}$
7. $\dfrac{x^3 + x^2}{(x^2+4)^2}$
8. $\dfrac{7x^2 - 9x + 3}{(x^2+7)^2}$

In Exercises 9–42, write the partial fraction decomposition of each rational expression.

9. $\dfrac{x}{(x-3)(x-2)}$
10. $\dfrac{1}{x(x-1)}$
11. $\dfrac{3x + 50}{(x-9)(x+2)}$
12. $\dfrac{5x - 1}{(x-2)(x+1)}$
13. $\dfrac{7x - 4}{x^2 - x - 12}$
14. $\dfrac{9x + 21}{x^2 + 2x - 15}$
15. $\dfrac{4}{2x^2 - 5x - 3}$
16. $\dfrac{x}{x^2 + 2x - 3}$
17. $\dfrac{4x^2 + 13x - 9}{x(x-1)(x+3)}$
18. $\dfrac{4x^2 - 5x - 15}{x(x+1)(x-5)}$
19. $\dfrac{4x^2 - 7x - 3}{x^3 - x}$
20. $\dfrac{2x^2 - 18x - 12}{x^3 - 4x}$
21. $\dfrac{6x - 11}{(x-1)^2}$
22. $\dfrac{x}{(x+1)^2}$
23. $\dfrac{x^2 - 6x + 3}{(x-2)^3}$
24. $\dfrac{2x^2 + 8x + 3}{(x+1)^3}$
25. $\dfrac{x^2 + 2x + 7}{x(x-1)^2}$
26. $\dfrac{3x^2 + 49}{x(x+7)^2}$
27. $\dfrac{x^2}{(x-1)^2(x+1)}$
28. $\dfrac{x^2}{(x-1)^2(x+1)^2}$

29. $\frac{5x^2 - 6x + 7}{(x-1)(x^2+1)}$

30. $\frac{5x^2 - 9x + 19}{(x-4)(x^2+5)}$

31. $\frac{5x^2 + 6x + 3}{(x+1)(x^2+2x+2)}$

32. $\frac{9x + 2}{(x-2)(x^2+2x+2)}$

33. $\frac{x+4}{x^2(x^2+4)}$

34. $\frac{10x^2 + 2x}{(x-1)^2(x^2+2)}$

35. $\frac{6x^2 - x + 1}{x^3 + x^2 + x + 1}$

36. $\frac{3x^2 - 2x + 8}{x^3 + 2x^2 + 4x + 8}$

37. $\frac{x^3 + x^2 + 2}{(x^2+2)^2}$

38. $\frac{x^2 + 2x + 3}{(x^2+4)^2}$

39. $\frac{x^3 - 4x^2 + 9x - 5}{(x^2 - 2x + 3)^2}$

40. $\frac{3x^3 - 6x^2 + 7x - 2}{(x^2 - 2x + 2)^2}$

41. $\frac{4x^2 + 3x + 14}{x^3 - 8}$

42. $\frac{3x - 5}{x^3 - 1}$

Practice Plus

In Exercises 43–46, perform each long division and write the partial fraction decomposition of the remainder term.

43. $\frac{x^5 + 2}{x^2 - 1}$

44. $\frac{x^5}{x^2 - 4x + 4}$

45. $\frac{x^4 - x^2 + 2}{x^3 - x^2}$

46. $\frac{x^4 + 2x^3 - 4x^2 + x - 3}{x^2 - x - 2}$

In Exercises 47–50, write the partial fraction decomposition of each rational expression.

47. $\frac{1}{x^2 - c^2} \quad (c \neq 0)$

48. $\frac{ax + b}{x^2 - c^2} \quad (c \neq 0)$

49. $\frac{ax + b}{(x - c)^2} \quad (c \neq 0)$

50. $\frac{1}{x^2 - ax - bx + ab} \quad (a \neq b)$

Application Exercises

51. Find the partial fraction decomposition for $\frac{1}{x(x+1)}$ and use the result to find the following sum:

$$\frac{1}{1\cdot 2} + \frac{1}{2\cdot 3} + \frac{1}{3\cdot 4} + \cdots + \frac{1}{99\cdot 100}.$$

52. Find the partial fraction decomposition for $\frac{2}{x(x+2)}$ and use the result to find the following sum:

$$\frac{2}{1\cdot 3} + \frac{2}{3\cdot 5} + \frac{2}{5\cdot 7} + \cdots + \frac{2}{99\cdot 101}.$$

Writing in Mathematics

53. Explain what is meant by the partial fraction decomposition of a rational expression.

54. Explain how to find the partial fraction decomposition of a rational expression with distinct linear factors in the denominator.

55. Explain how to find the partial fraction decomposition of a rational expression with a repeated linear factor in the denominator.

56. Explain how to find the partial fraction decomposition of a rational expression with a prime quadratic factor in the denominator.

57. Explain how to find the partial fraction decomposition of a rational expression with a repeated, prime quadratic factor in the denominator.

58. How can you verify your result for the partial fraction decomposition for a given rational expression without using a graphing utility?

Technology Exercise

59. Use the $\boxed{\text{TABLE}}$ feature of a graphing utility to verify any three of the decompositions that you obtained in Exercises 9–42.

Critical Thinking Exercises

Make Sense? *In Exercises 60–63, determine whether each statement makes sense or does not make sense, and explain your reasoning.*

60. Partial fraction decomposition involves finding a single rational expression for a given sum or difference of rational expressions.

61. I apply partial fraction decompositions for rational expressions of the form $\frac{P(x)}{Q(x)}$, where P and Q have no common factors and the degree of P is greater than the degree of Q.

62. Because $x + 5$ is linear and $x^2 - 3x + 2$ is quadratic, I set up the following partial fraction decomposition:

$$\frac{7x^2 + 9x + 3}{(x+5)(x^2 - 3x + 2)} = \frac{A}{x+5} + \frac{Bx + C}{x^2 - 3x + 2}.$$

63. Because $(x+3)^2$ consists of two factors of $x + 3$, I set up the following partial fraction decomposition:

$$\frac{5x + 2}{(x+3)^2} = \frac{A}{x+3} + \frac{B}{x+3}.$$

64. Use an extension of the Study Tip on page 780 to describe how to set up the partial fraction decomposition of a rational expression that contains powers of a prime cubic factor in the denominator. Give an example of such a decomposition.

65. Find the partial fraction decomposition of

$$\frac{4x^2 + 5x - 9}{x^3 - 6x - 9}.$$