

## DEFINITIONS AND CONCEPTS

## EXAMPLES

## 8.3 Partial Fractions

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|---|---------------------------------|
| a. Partial fraction decomposition is used on rational expressions in which the numerator and denominator have no common factors and the highest power in the numerator is less than the highest power in the denominator. The steps in partial fraction decomposition are given in the box on page 775. |                                 |
| b. Include one partial fraction with a constant numerator for each distinct linear factor in the denominator. Include one partial fraction with a constant numerator for each power of a repeated linear factor in the denominator.   | Ex. 1, p. 774;<br>Ex. 2, p. 776 |
| c. Include one partial fraction with a linear numerator for each distinct prime quadratic factor in the denominator. Include one partial fraction with a linear numerator for each power of a prime, repeated quadratic factor in the denominator.  | Ex. 3, p. 778;<br>Ex. 4, p. 780 |

## 8.4 Systems of Nonlinear Equations in Two Variables

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|--|---|
| a. A system of two nonlinear equations in two variables contains at least one equation that cannot be expressed as $Ax + By = C$ .   |   |
| b. Systems of nonlinear equations in two variables can be solved algebraically by eliminating all occurrences of one of the variables by the substitution or addition methods. | Ex. 1, p. 784;<br>Ex. 2, p. 785;<br>Ex. 3, p. 786;<br>Ex. 4, p. 788 |

## 8.5 Systems of Inequalities

- |  |   |
|--|---|
| a. A linear inequality in two variables can be written in the form $Ax + By > C$ , $Ax + By \geq C$ , $Ax + By < C$ , or $Ax + By \leq C$ .  |   |
| b. The procedure for graphing a linear inequality in two variables is given in the box on page 795. A nonlinear inequality in two variables is graphed using the same procedure.   | Ex. 1, p. 795;<br>Ex. 2, p. 796;<br>Ex. 3, p. 797;<br>Ex. 4, p. 798 |
| c. To graph the solution set of a system of inequalities, graph each inequality in the system in the same rectangular coordinate system. Then find the region, if there is one, that is common to every graph in the system. | Ex. 6, p. 800;<br>Ex. 7, p. 801;<br>Ex. 8, p. 802                   |

## 8.6 Linear Programming

- |   |                                 |
|---|---------------------------------|
| a. An objective function is an algebraic expression in three variables describing a quantity that must be maximized or minimized. | Ex. 1, p. 807                   |
| b. Constraints are restrictions, expressed as linear inequalities.  | Ex. 2, p. 808;<br>Ex. 3, p. 808 |
| c. Steps for solving a linear programming problem are given in the box on page 809.   | Ex. 4, p. 809;<br>Ex. 5, p. 810 |

## Review Exercises

## 8.1

In Exercises 1–5, solve by the method of your choice. Identify systems with no solution and systems with infinitely many solutions, using set notation to express their solution sets.

1. 
$$\begin{cases} y = 4x + 1 \\ 3x + 2y = 13 \end{cases}$$

2. 
$$\begin{cases} x + 4y = 14 \\ 2x - y = 1 \end{cases}$$

3. 
$$\begin{cases} 5x + 3y = 1 \\ 3x + 4y = -6 \end{cases}$$

4. 
$$\begin{cases} 2y - 6x = 7 \\ 3x - y = 9 \end{cases}$$

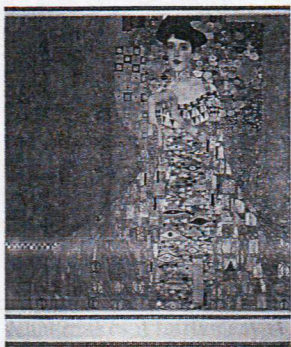
5. 
$$\begin{cases} 4x - 8y = 16 \\ 3x - 6y = 12 \end{cases}$$

6. A company is planning to manufacture computer desks. The fixed cost will be \$60,000 and it will cost \$200 to produce each desk. Each desk will be sold for \$450.

- Write the cost function,  $C$ , of producing  $x$  desks.
- Write the revenue function,  $R$ , from the sale of  $x$  desks.
- Determine the break-even point. Describe what this means.



7.



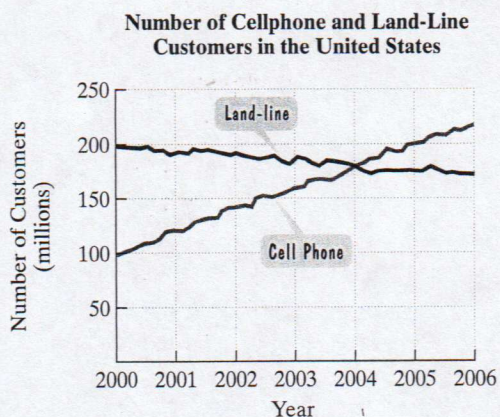
Gustav Klimt (1826–1918)  
“Mrs. Adele Bloch-Bauer, I”,  
1907. Oil on canvas,  
138 × 138 cm. Private  
Collection. Photo: Erich  
Lessing/Art Resource, NY



Pablo Picasso (1881–1973)  
“Boy with a Pipe”, 1905 (oil  
on canvas). Collection of Mr.  
and Mrs. John Hay Whitney,  
New York, USA.  
© DACS/The Bridgeman Art  
Library. © 2008 Estate of  
Pablo Picasso/Artists Rights  
Society (ARS), New York

Talk about paintings by numbers: In 2007, this glittering Klimt set the record for the most ever paid for a painting, a title that had been held by Picasso's *Boy with a Pipe*. Combined, the two paintings sold for \$239 million. The difference between the selling price for Klimt's work and the selling price for Picasso's work was \$31 million. Find the amount paid for each painting.

8. The graph shows that from 2000 through 2006, Americans unplugged land lines and switched to cellphones.



Source: Federal Communications Commission

- Use the graphs to estimate the point of intersection. In what year was the number of cellphone and land-line customers the same? How many millions of customers were there for each?
- In 2000, there were 98 million cellphone customers. For the period shown by the graph, this has increased at an average rate of 19.8 million customers per year. Write a function that models the number of cellphone customers,  $y$ , in millions,  $x$  years after 2000.
- The function  $4.3x + y = 198$  models the number of land-line customers,  $y$ , in millions,  $x$  years after 2000. Use this model and the model you obtained in part (b) to determine the year, rounded to the nearest year, when the number of cellphone and land-line customers was the

same. According to the models, how many millions of customers, rounded to the nearest ten million, were there for each?

- How well do the models in parts (b) and (c) describe the point of intersection of the graphs that you estimated in part (a)?
- The perimeter of a rectangular table top is 34 feet. The difference between 4 times the length and 3 times the width is 33 feet. Find the dimensions.
  - A travel agent offers two package vacation plans. The first plan costs \$360 and includes 3 days at a hotel and a rental car for 2 days. The second plan costs \$500 and includes 4 days at a hotel and a rental car for 3 days. The daily charge for the hotel is the same under each plan, as is the daily charge for the car. Find the cost per day for the hotel and for the car.
  - The calorie-nutrient information for an apple and an avocado is given in the table. How many of each should be eaten to get exactly 1000 calories and 100 grams of carbohydrates?

	One Apple	One Avocado
Calories	100	350
Carbohydrates (grams)	24	14

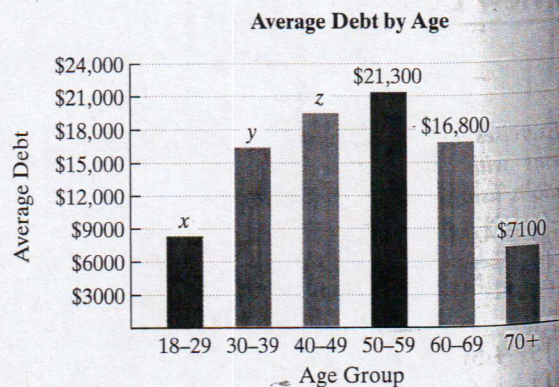
## 8.2

Solve each system in Exercises 12–13.

$$12. \begin{cases} 2x - y + z = 1 \\ 3x - 3y + 4z = 5 \\ 4x - 2y + 3z = 4 \end{cases}$$

$$13. \begin{cases} x + 2y - z = 5 \\ 2x - y + 3z = 0 \\ 2y + z = 1 \end{cases}$$

- Find the quadratic function  $y = ax^2 + bx + c$  whose graph passes through the points (1, 4), (3, 20), and (−2, 25).
- The bar graph shows the average debt in the United States, not including real estate mortgages, by age group. The difference between the average debt for the 30–39 age group and the 18–29 age group is \$8100. The difference between the average debt for the 40–49 age group and the 30–39 age group is \$3100. The combined average debt for these three age groups is \$44,200. Find the average debt for each of these age groups.



Source: Experian



## 8.3

In Exercises 16–24, write the partial fraction decomposition of each rational expression.

16.  $\frac{x}{(x-3)(x+2)}$

17.  $\frac{11x-2}{x^2-x-12}$

18.  $\frac{4x^2-3x-4}{x(x+2)(x-1)}$

19.  $\frac{2x+1}{(x-2)^2}$

20.  $\frac{2x-6}{(x-1)(x-2)^2}$

21.  $\frac{3x}{(x-2)(x^2+1)}$

22.  $\frac{7x^2-7x+23}{(x-3)(x^2+4)}$

23.  $\frac{x^3}{(x^2+4)^2}$

24.  $\frac{4x^3+5x^2+7x-1}{(x^2+x+1)^2}$

## 8.4

In Exercises 25–35, solve each system by the method of your choice.

25.  $\begin{cases} 5y = x^2 - 1 \\ x - y = 1 \end{cases}$

26.  $\begin{cases} y = x^2 + 2x + 1 \\ x + y = 1 \end{cases}$

27.  $\begin{cases} x^2 + y^2 = 2 \\ x + y = 0 \end{cases}$

28.  $\begin{cases} 2x^2 + y^2 = 24 \\ x^2 + y^2 = 15 \end{cases}$

29.  $\begin{cases} xy - 4 = 0 \\ y - x = 0 \end{cases}$

30.  $\begin{cases} y^2 = 4x \\ x - 2y + 3 = 0 \end{cases}$

31.  $\begin{cases} x^2 + y^2 = 10 \\ y = x + 2 \end{cases}$

32.  $\begin{cases} xy = 1 \\ y = 2x + 1 \end{cases}$

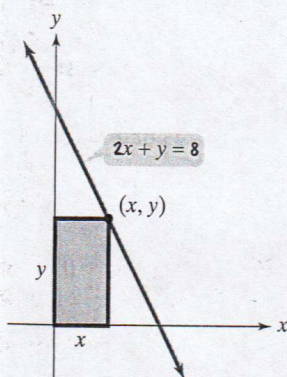
33.  $\begin{cases} x + y + 1 = 0 \\ x^2 + y^2 + 6y - x = -5 \end{cases}$

34.  $\begin{cases} x^2 + y^2 = 13 \\ x^2 - y = 7 \end{cases}$

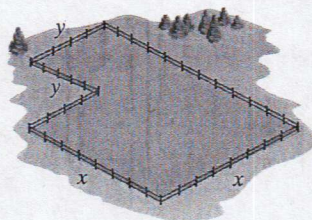
35.  $\begin{cases} 2x^2 + 3y^2 = 21 \\ 3x^2 - 4y^2 = 23 \end{cases}$

36. The perimeter of a rectangle is 26 meters and its area is 40 square meters. Find its dimensions.

37. Find the coordinates of all points  $(x, y)$  that lie on the line whose equation is  $2x + y = 8$ , so that the area of the rectangle shown in the figure is 6 square units.



38. Two adjoining square fields with an area of 2900 square feet are to be enclosed with 240 feet of fencing. The situation is represented in the figure. Find the length of each side where a variable appears.



## 8.5

In Exercises 39–45, graph each inequality.

39.  $3x - 4y > 12$

40.  $y \leq -\frac{1}{2}x + 2$

41.  $x < -2$

42.  $y \geq 3$

43.  $x^2 + y^2 > 4$

44.  $y \leq x^2 - 1$

45.  $y \leq 2^x$

In Exercises 46–55, graph the solution set of each system of inequalities or indicate that the system has no solution.

46.  $\begin{cases} 3x + 2y \geq 6 \\ 2x + y \geq 6 \end{cases}$

47.  $\begin{cases} 2x - y \geq 4 \\ x + 2y < 2 \end{cases}$

48.  $\begin{cases} y < x \\ y \leq 2 \end{cases}$

49.  $\begin{cases} x + y \leq 6 \\ y \geq 2x - 3 \end{cases}$

50.  $\begin{cases} 0 \leq x \leq 3 \\ y > 2 \end{cases}$

51.  $\begin{cases} 2x + y < 4 \\ 2x + y > 6 \end{cases}$

52.  $\begin{cases} x^2 + y^2 \leq 16 \\ x + y < 2 \end{cases}$

53.  $\begin{cases} x^2 + y^2 \leq 9 \\ y < -3x + 1 \end{cases}$

54.  $\begin{cases} y > x^2 \\ x + y < 6 \\ y < x + 6 \end{cases}$

55.  $\begin{cases} y \geq 0 \\ 3x + 2y \geq 4 \\ x - y \leq 3 \end{cases}$

## 8.6

56. Find the value of the objective function  $z = 2x + 3y$  at each corner of the graphed region shown. What is the maximum value of the objective function? What is the minimum value of the objective function?

