

40. a. $\begin{bmatrix} 1 & -6 & 3 \\ 2 & -7 & 3 \\ 4 & -12 & 5 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 11 \\ 14 \\ 25 \end{bmatrix}$

b. $\{(2, -1, 1)\}$

41. a. $\begin{bmatrix} 1 & -1 & 2 & 0 \\ 0 & 1 & -1 & 1 \\ -1 & 1 & -1 & 2 \\ 0 & -1 & 1 & -2 \end{bmatrix} \begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -3 \\ 4 \\ 2 \\ -4 \end{bmatrix}$

b. $\{(2, 3, -1, 0)\}$

42. a. $\begin{bmatrix} 2 & 0 & 1 & 1 \\ 3 & 0 & 0 & 1 \\ -1 & 1 & -2 & 1 \\ 4 & -1 & 1 & 0 \end{bmatrix} \begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 9 \\ 4 \\ 6 \end{bmatrix}$

b. $\{(2, 1, -1, 3)\}$

43. $\begin{bmatrix} \frac{1}{2}e^{-x} & -\frac{1}{2}e^{-3x} \\ \frac{1}{2}e^{-3x} & \frac{1}{2}e^{-5x} \end{bmatrix}$

44. $\begin{bmatrix} \frac{1}{2}e^{-2x} & \frac{1}{2}e^{-3x} \\ -\frac{1}{2}e^{-x} & \frac{1}{2}e^{-2x} \end{bmatrix}$

45. $\begin{bmatrix} \frac{1}{8} & \frac{5}{8} \\ \frac{3}{8} & \frac{7}{8} \end{bmatrix}$

46. $\begin{bmatrix} \frac{1}{4} & \frac{5}{8} \\ \frac{1}{2} & \frac{3}{4} \end{bmatrix}$

47. $(AB)^{-1} = \begin{bmatrix} -23 & 16 \\ 13 & -9 \end{bmatrix}; A^{-1}B^{-1} = \begin{bmatrix} -3 & 11 \\ 8 & -29 \end{bmatrix}; B^{-1}A^{-1} = \begin{bmatrix} -23 & 16 \\ 13 & -9 \end{bmatrix}; (AB)^{-1} = B^{-1}A^{-1}$

48. $(AB)^{-1} = \begin{bmatrix} -11 & 26 \\ 19 & -45 \end{bmatrix}; A^{-1}B^{-1} = \begin{bmatrix} -79 & 101 \\ -18 & 23 \end{bmatrix}; B^{-1}A^{-1} = \begin{bmatrix} -11 & 26 \\ 19 & -45 \end{bmatrix}; (AB)^{-1} = B^{-1}A^{-1}$

49. $AA^{-1} = I_3$ and $A^{-1}A = I_3$

50. $\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} w & x \\ y & z \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}; \begin{bmatrix} aw + by & ax + bz \\ cw + dy & cx + dz \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}; aw + by = 1 \text{ and } ax + bz = 0$
 $cw + dy = 0 \text{ and } cx + dz = 1$
 $w = \frac{d}{ad - bc}, x = \frac{-b}{ad - bc}, y = \frac{-c}{ad - bc}, z = \frac{a}{ad - bc}; A^{-1} = \begin{bmatrix} w & x \\ y & z \end{bmatrix} = \frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

51. The encoded message is 27, -19, 32, -20.; The decoded message is 8, 5, 12, 16 or HELP.

52. The encoded message is 33, -21, 83, -61.; The decoded message is 12, 15, 22, 5 or LOVE.

53. The encoded message is 14, 85, -33, 4, 18, -7, -18, 19, -9.

54. The encoded message is -1, 59, -20, 25, 121, -48, -7, 39, -17.

65. $\begin{bmatrix} 1 & 1 \\ 2 & 3 \end{bmatrix}$

66. $\begin{bmatrix} -1 & -\frac{1}{2} \\ -3 & -2 \end{bmatrix}$

67. $\begin{bmatrix} 1 & 0 & 1 \\ 2 & 1 & 3 \\ -1 & 1 & 1 \end{bmatrix}$

68. $\begin{bmatrix} 1 & 1 & 1 \\ 3 & 5 & 4 \\ 3 & 6 & 5 \end{bmatrix}$

69. $\begin{bmatrix} 0 & -1 & 0 & 1 \\ -1 & -5 & 0 & 3 \\ -2 & -4 & 1 & -2 \\ -1 & -4 & 0 & 1 \end{bmatrix}$

70. $\begin{bmatrix} \frac{3}{5} & 0 & -\frac{2}{5} & \frac{1}{5} \\ \frac{1}{5} & 0 & \frac{1}{5} & -\frac{1}{10} \\ 0 & 1 & 0 & 0 \\ -\frac{6}{5} & 0 & \frac{4}{5} & \frac{1}{10} \end{bmatrix}$

71. $\{(2, 3, -5)\}$ 72. $\{(1, 2, -1)\}$ 73. $\{(1, 2, -1)\}$ 74. $\{(5, 4, -1)\}$ 75. $\{(2, 1, 3, -2, 4)\}$ 76. $\left\{ -\frac{22}{5}, \frac{17}{5}, \frac{11}{5}, \frac{14}{5} \right\}$

79. does not make sense

80. makes sense

81. makes sense

82. does not make sense

83. false

84. true

85. false

86. false

87. false

88. false

90. A

91. $a = 3$ or $a = -2$

93. 2

94. 6

95. -31

Section 9.5

Check Point Exercises

1. a. -4 b. -17 2. $\{(4, -2)\}$ 3. 80 4. -24 5. $\{(2, -3, 4)\}$ 6. -250

Exercise Set 9.5

1. 1 2. -16 3. -29 4. -17 5. 0 6. -22 7. 33 8. 2 9. $-\frac{7}{16}$ 10. $\frac{2}{3}$ 11. $\{(5, 2)\}$ 12. $\{(2, -1)\}$

13. $\{(2, -3)\}$ 14. $\{(-1, -3)\}$ 15. $\{(3, -1)\}$ 16. $\left\{ -1, \frac{5}{2} \right\}$ 17. The system is dependent. 18. $\left\{ 4, \frac{1}{3} \right\}$ 19. $\{(4, 2)\}$

20. $\{(-2, -1)\}$ 21. $\{(7, 4)\}$ 22. $\{(1, -2)\}$ 23. The system is inconsistent. 24. The system is dependent.

25. The system is dependent. 26. The system is inconsistent. 27. 72 28. 28 29. -75 30. -76 31. 0 32. -20

33. $\{(-5, -2, 7)\}$ 34. $\{(-2, 3, 4)\}$ 35. $\{(2, -3, 4)\}$ 36. $\{(2, 3, 5)\}$ 37. $\{(3, -1, 2)\}$ 38. $\{(-1, 3, 2)\}$ 39. $\{(2, 3, 1)\}$

40. $\{(0, 4, 2)\}$ 41. -200 42. 78 43. 195 44. 48 45. -42 46. -407 47. $2x - 4y = 8; 3x + 5y = -10$

48. $2x - 3y = 8; 5x + 6y = 11$ 49. -11 50. 26 51. 4 52. 3 53. 28 sq units 54. 26 sq units 55. yes 56. yes

57. The equation of the line is $y = -\frac{11}{5}x + \frac{8}{5}$. 58. The equation of the line is $y = \frac{1}{3}x + \frac{10}{3}$.

68. -2100 69. 13,200 70. Gauss-Jordan elimination 71. does not make sense 72. does not make sense 73. does not make sense

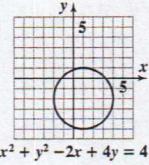
74. does not make sense 75. a. a^2 b. a^3 c. a^4 d. Each determinant has zeros below the main diagonal and a 's everywhere else.

- e. Each determinant equals a raised to the power equal to the order of the determinant. 76. 48 77. The sign of the value is changed when

- 2 columns are interchanged in a 2nd order determinant. 78. For both systems, $x = \frac{c_1b_2 - c_2b_1}{a_1b_2 - a_2b_1}$ and $y = \frac{a_1c_2 - a_2c_1}{a_1b_2 - a_2b_1}$.

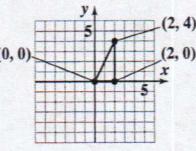
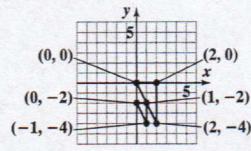
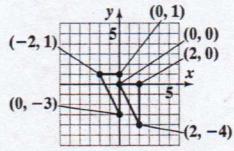
79. $\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} = x(y_1 - y_2) - y(x_1 - x_2) + (x_1y_2 - x_2y_1) = 0$; solving for y , $y = \frac{y_1 - y_2}{x_1 - x_2}x + \frac{x_1y_2 - x_2y_1}{x_1 - x_2}$, and $m = \frac{y_1 - y_2}{x_1 - x_2}$ and $b = \frac{x_1y_2 - x_2y_1}{x_1 - x_2}$.

81. a. -3 and 3 b. -2 and 2 82. $\frac{x^2}{16} + \frac{y^2}{25} = 1$ 83. $(x - 1)^2 + (y + 2)^2 = 9$; center: $(1, -2)$; radius: 3

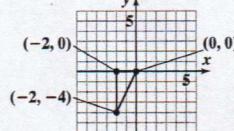


Chapter 9 Review Exercises

1. $\begin{bmatrix} 1 & 2 & 2 \\ 0 & 1 & -1 \\ 0 & 0 & 9 \end{bmatrix}$ 2. $\begin{bmatrix} 1 & -1 & \frac{1}{2} \\ 1 & 2 & -1 \\ 6 & 4 & 3 \end{bmatrix}$ 3. $\{(1, 3, -4)\}$ 4. $\{(-2, -1, 0)\}$ 5. $\{(2, -2, 3, 4)\}$
6. a. $a = -2$; $b = 32$; $c = 42$ b. 2:00 p.m.; 170 parts per million
8. \emptyset 9. $\{(2t + 4, t + 1, t)\}$ 10. $\{(-37t + 2, 16t, -7t + 1, t)\}$
12. a. $x + z = 750$ b. $\{(-t + 750, t - 250, t)\}$ c. $x = 350$; $y = 150$ 13. $x = -5$; $y = 6$; $z = 6$
16. $\begin{bmatrix} -4 & 4 & -1 \\ -2 & -5 & 5 \end{bmatrix}$ 17. Not possible since B is 3×2 and C is 3×3 .
21. $\begin{bmatrix} -1 & -16 \\ 8 & 1 \end{bmatrix}$ 22. $\begin{bmatrix} -10 & -6 & 2 \\ 16 & 3 & 4 \\ -23 & -16 & 7 \end{bmatrix}$ 23. $\begin{bmatrix} -6 & 4 & -8 \\ 0 & 5 & 11 \\ -17 & 13 & -19 \end{bmatrix}$ 24. $\begin{bmatrix} 10 & 5 \\ -2 & -30 \end{bmatrix}$
26. $\begin{bmatrix} 7 & 6 & 5 \\ 2 & -1 & 11 \end{bmatrix}$ 27. $\begin{bmatrix} -6 & -22 & -40 \\ 9 & 43 & 58 \\ -14 & -48 & -94 \end{bmatrix}$ 28. $\begin{bmatrix} -2 & -6 \\ 3 & \frac{1}{3} \end{bmatrix}$ 29. $\begin{bmatrix} 2 & 2 & 2 \\ 1 & 2 & 1 \\ 1 & 2 & 1 \end{bmatrix}$
31. $\begin{bmatrix} -2 & 0 & 0 \\ 1 & 1 & -3 \end{bmatrix}$ 32. $\begin{bmatrix} 0 & 1 & 1 \\ -2 & -2 & -4 \end{bmatrix}$

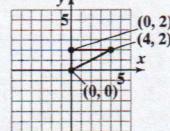


34. $\begin{bmatrix} 0 & -2 & -2 \\ 0 & 0 & -4 \end{bmatrix}$



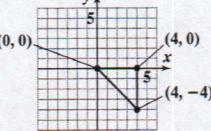
The effect is a reflection over the y -axis

35. $\begin{bmatrix} 0 & 0 & 4 \\ 0 & 2 & 2 \end{bmatrix}$



The effect is a 90° counterclockwise rotation about the origin

36. $\begin{bmatrix} 0 & 4 & 4 \\ 0 & 0 & -4 \end{bmatrix}$



The effect is a horizontal stretch by a factor of 2.

37. $AB = \begin{bmatrix} 1 & 7 \\ 0 & 5 \end{bmatrix}$; $BA = \begin{bmatrix} 1 & 0 \\ 1 & 5 \end{bmatrix}$; $B \neq A^{-1}$

38. $AB = I_3$; $BA = I_3$; $B = A^{-1}$

39. $\begin{bmatrix} 3 & 1 \\ 2 & 1 \end{bmatrix}$

40. $\begin{bmatrix} -\frac{3}{5} & \frac{1}{5} \\ 1 & 0 \end{bmatrix}$

41. $\begin{bmatrix} 3 & 0 & -2 \\ -6 & 1 & 4 \\ 1 & 0 & -1 \end{bmatrix}$

42. $\begin{bmatrix} 8 & -8 & 5 \\ -3 & 2 & -1 \\ -1 & -1 & 1 \end{bmatrix}$

43. a. $\begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & 3 \\ 3 & 0 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 7 \\ -2 \\ 0 \end{bmatrix}$

b. $\{(-18, 79, -27)\}$

44. a. $\begin{bmatrix} 1 & -1 & 2 \\ 0 & 1 & -1 \\ 1 & 0 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 12 \\ -5 \\ 10 \end{bmatrix}$

- b. $\{(4, -2, 3)\}$ 45. The encoded message is 96, 135, 46, 63; The decoded message is 18, 21, 12, 5 or RULE.

48. -86 49. -236 50. 4 51. 16 52. $\left\{ \left(\frac{7}{4}, -\frac{25}{8} \right) \right\}$ 53. $\{(2, -7)\}$ 54. $\{(23, -12, 3)\}$ 55. $\{(-3, 2, 1)\}$

56. $a = \frac{5}{8}$; $b = -50$; $c = 1150$; 30- and 50-year-olds are involved in an average of 212.5 automobile accidents per day.

Chapter 9 Test

1. $\left\{ \left(-3, \frac{1}{2}, 1 \right) \right\}$ 2. $\{(t, t - 1, t)\}$ 3. $\begin{bmatrix} 5 & 4 \\ 1 & 11 \end{bmatrix}$ 4. $\begin{bmatrix} 5 & -2 \\ 1 & -1 \\ 4 & -1 \end{bmatrix}$ 5. $\begin{bmatrix} \frac{3}{5} & -\frac{2}{5} \\ \frac{1}{5} & \frac{1}{5} \end{bmatrix}$ 6. $\begin{bmatrix} -1 & 2 \\ -5 & 4 \end{bmatrix}$ 7. $AB = I_3$; $BA = I_3$