Exercise Set 5.3

Practice Exercises

In Exercises 1–8, a point on the terminal side of angle θ is given. Find the exact value of each of the six trigonometric functions of θ.

1. $(-4, 3)$ **2.** $(-12, 5)$ **3.** $(2, 3)$

4. $(3, 7)$ **5.** $(3, -3)$ **6.** $(5, -5)$

7. $(-2, -5)$ **8.** $(-1, -3)$

In Exercises 9–16, evaluate the trigonometric function at the quadrantal angle, or state that the expression is undefined.

9. $\cos \pi$ **10.** $\tan \pi$ **11.** $\sec \pi$

12. $\csc \pi$ **13.** $\tan \dfrac{3\pi}{2}$ **14.** $\cos \dfrac{3\pi}{2}$

15. $\cot \dfrac{\pi}{2}$ **16.** $\tan \dfrac{\pi}{2}$

In Exercises 17–22, let θ be an angle in standard position. Name the quadrant in which θ lies.

17. $\sin \theta > 0, \quad \cos \theta > 0$ **18.** $\sin \theta < 0, \quad \cos \theta > 0$

19. $\sin \theta < 0, \quad \cos \theta < 0$ **20.** $\tan \theta < 0, \quad \sin \theta < 0$

21. $\tan \theta < 0, \quad \cos \theta < 0$ **22.** $\cot \theta > 0, \quad \sec \theta < 0$

In Exercises 23–34, find the exact value of each of the remaining trigonometric functions of θ.

23. $\cos \theta = -\frac{3}{5}, \quad \theta$ in quadrant III

24. $\sin \theta = -\frac{12}{13}, \quad \theta$ in quadrant III

25. $\sin \theta = \frac{5}{13}, \quad \theta$ in quadrant II

26. $\cos \theta = \frac{4}{5}, \quad \theta$ in quadrant IV

27. $\cos \theta = \frac{8}{17}, \quad 270° < \theta < 360°$

28. $\cos \theta = \frac{1}{3}, \quad 270° < \theta < 360°$

29. $\tan \theta = -\frac{2}{3}, \quad \sin \theta > 0$ **30.** $\tan \theta = -\frac{1}{3}, \quad \sin \theta > 0$

31. $\tan \theta = \frac{4}{3}, \quad \cos \theta < 0$ **32.** $\tan \theta = \frac{5}{12}, \quad \cos \theta < 0$

33. $\sec \theta = -3, \quad \tan \theta > 0$ **34.** $\csc \theta = -4, \quad \tan \theta > 0$

In Exercises 35–60, find the reference angle for each angle.

35. $160°$ **36.** $170°$ **37.** $205°$

38. $210°$ **39.** $355°$ **40.** $351°$

41. $\dfrac{7\pi}{4}$ **42.** $\dfrac{5\pi}{4}$ **43.** $\dfrac{5\pi}{6}$

44. $\dfrac{5\pi}{7}$ **45.** $-150°$ **46.** $-250°$

47. $-335°$ **48.** $-359°$ **49.** 4.7

50. 5.5 **51.** $565°$ **52.** $553°$

53. $\dfrac{17\pi}{6}$ **54.** $\dfrac{11\pi}{4}$ **55.** $\dfrac{23\pi}{4}$

56. $\dfrac{17\pi}{3}$ **57.** $-\dfrac{11\pi}{4}$ **58.** $-\dfrac{17\pi}{6}$

59. $-\dfrac{25\pi}{6}$ **60.** $-\dfrac{13\pi}{3}$

In Exercises 61–86, use reference angles to find the exact value of each expression. Do not use a calculator.

61. $\cos 225°$ **62.** $\sin 300°$ **63.** $\tan 210°$

64. $\sec 240°$ **65.** $\tan 420°$ **66.** $\tan 405°$

67. $\sin \dfrac{2\pi}{3}$ **68.** $\cos \dfrac{3\pi}{4}$ **69.** $\csc \dfrac{7\pi}{6}$

70. $\cot \dfrac{7\pi}{4}$ **71.** $\tan \dfrac{9\pi}{4}$ **72.** $\tan \dfrac{9\pi}{2}$

73. $\sin(-240°)$ **74.** $\sin(-225°)$ **75.** $\tan\left(-\dfrac{\pi}{4}\right)$

76. $\tan\left(-\dfrac{\pi}{6}\right)$ **77.** $\sec 495°$ **78.** $\sec 510°$

79. $\cot \dfrac{19\pi}{6}$ **80.** $\cot \dfrac{13\pi}{3}$ **81.** $\cos \dfrac{23\pi}{4}$

82. $\cos \dfrac{35\pi}{6}$ **83.** $\tan\left(-\dfrac{17\pi}{6}\right)$ **84.** $\tan\left(-\dfrac{11\pi}{4}\right)$

85. $\sin\left(-\dfrac{17\pi}{3}\right)$ **86.** $\sin\left(-\dfrac{35\pi}{6}\right)$

Practice Plus

In Exercises 87–92, find the exact value of each expression. Write the answer as a single fraction. Do not use a calculator.

87. $\sin \dfrac{\pi}{3} \cos \pi - \cos \dfrac{\pi}{3} \sin \dfrac{3\pi}{2}$

88. $\sin \dfrac{\pi}{4} \cos 0 - \sin \dfrac{\pi}{6} \cos \pi$

89. $\sin \dfrac{11\pi}{4} \cos \dfrac{5\pi}{6} + \cos \dfrac{11\pi}{4} \sin \dfrac{5\pi}{6}$

90. $\sin \dfrac{17\pi}{3} \cos \dfrac{5\pi}{4} + \cos \dfrac{17\pi}{3} \sin \dfrac{5\pi}{4}$

91. $\sin \dfrac{3\pi}{2} \tan\left(-\dfrac{15\pi}{4}\right) - \cos\left(-\dfrac{5\pi}{3}\right)$

92. $\sin \dfrac{3\pi}{2} \tan\left(-\dfrac{8\pi}{3}\right) + \cos\left(-\dfrac{5\pi}{6}\right)$

In Exercises 93–98, let

$$f(x) = \sin x, \, g(x) = \cos x, \text{ and } h(x) = 2x.$$

Find the exact value of each expression. Do not use a calculator.

93. $f\left(\dfrac{4\pi}{3} + \dfrac{\pi}{6}\right) + f\left(\dfrac{4\pi}{3}\right) + f\left(\dfrac{\pi}{6}\right)$

94. $g\left(\dfrac{5\pi}{6} + \dfrac{\pi}{6}\right) + g\left(\dfrac{5\pi}{6}\right) + g\left(\dfrac{\pi}{6}\right)$

95. $(h \circ g)\left(\dfrac{17\pi}{3}\right)$ **96.** $(h \circ f)\left(\dfrac{11\pi}{4}\right)$

97. the average rate of change of f from $x_1 = \dfrac{5\pi}{4}$ to $x_2 = \dfrac{3\pi}{2}$

98. the average rate of change of g from $x_1 = \dfrac{3\pi}{4}$ to $x_2 = \pi$

In Exercises 99–104, find two values of $\theta, 0 \le \theta < 2\pi$, that satisfy each equation.

99. $\sin \theta = \dfrac{\sqrt{2}}{2}$ **100.** $\cos \theta = \dfrac{1}{2}$

101. $\sin \theta = -\dfrac{\sqrt{2}}{2}$ **102.** $\cos \theta = -\dfrac{1}{2}$

103. $\tan \theta = -\sqrt{3}$ **104.** $\tan \theta = -\dfrac{\sqrt{3}}{3}$

Writing in Mathematics

105. If you are given a point on the terminal side of angle θ, explain how to find $\sin \theta$.

106. Explain why $\tan 90°$ is undefined.

107. If $\cos \theta > 0$ and $\tan \theta < 0$, explain how to find the quadrant in which θ lies.

108. What is a reference angle? Give an example with your description.

109. Explain how reference angles are used to evaluate trigonometric functions. Give an example with your description.

Critical Thinking Exercises

Make Sense? *In Exercises 110–113, determine whether each statement makes sense or does not make sense, and explain your reasoning.*

110. I'm working with a quadrantal angle θ for which $\sin \theta$ is undefined.

111. This angle θ is in a quadrant in which $\sin \theta < 0$ and $\csc \theta > 0$.

112. I am given that $\tan \theta = \frac{3}{5}$, so I can conclude that $y = 3$ and $x = 5$.

113. When I found the exact value of $\cos \frac{14\pi}{3}$, I used a number of concepts, including coterminal angles, reference angles, finding the cosine of a special angle, and knowing the cosine's sign in various quadrants.

Preview Exercises

Exercises 114–116 will help you prepare for the material covered in the next section.

114. Graph: $x^2 + y^2 = 1$. Then locate the point $\left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$ on the graph.

115. Use your graph of $x^2 + y^2 = 1$ from Exercise 114 to determine the relation's domain and range.

116. a. Find the exact value of $\sin\left(\frac{\pi}{4}\right)$, $\sin\left(-\frac{\pi}{4}\right)$, $\sin\left(\frac{\pi}{3}\right)$, and $\sin\left(-\frac{\pi}{3}\right)$. Based on your results, can the sine function be an even function? Explain your answer.

b. Find the exact value of $\cos\left(\frac{\pi}{4}\right)$, $\cos\left(-\frac{\pi}{4}\right)$, $\cos\left(\frac{\pi}{3}\right)$, and $\cos\left(-\frac{\pi}{3}\right)$. Based on your results, can the cosine function be an odd function? Explain your answer.

Section 5.4 Trigonometric Functions of Real Numbers; Periodic Functions

Objectives

1 Use a unit circle to define trigonometric functions of real numbers.

2 Recognize the domain and range of sine and cosine functions.

3 Use even and odd trigonometric functions.

4 Use periodic properties.

1 Use a unit circle to define trigonometric functions of real numbers.
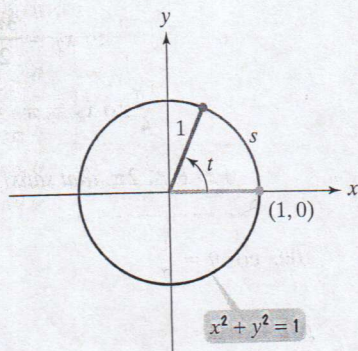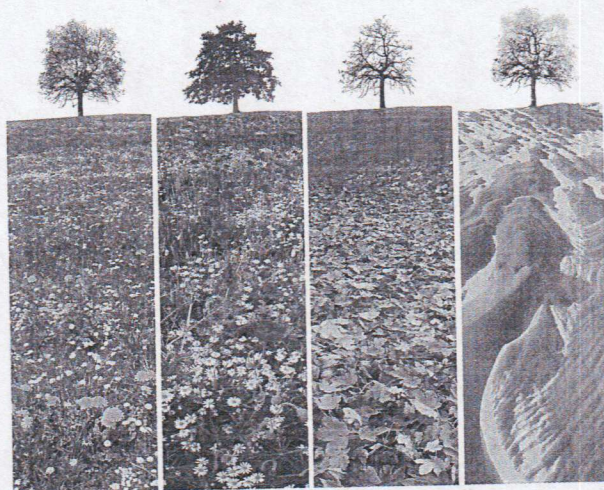

Figure 5.53 Unit circle with a central angle measuring t radians

Cycles govern many aspects of life—heartbeats, sleep patterns, seasons, and tides all follow regular, predictable cycles. In this section, we will see why trigonometric functions are used to model phenomena that occur in cycles. To do this, we need to move beyond angles and consider trigonometric functions of real numbers.

Trigonometric Functions of Real Numbers

Thus far, we have considered trigonometric functions of angles measured in degrees or radians. To define trigonometric functions of real numbers, rather than angles, we use a unit circle. A **unit circle** is a circle of radius 1, with its center at the origin of a rectangular coordinate system. The equation of this unit circle is $x^2 + y^2 = 1$. **Figure 5.53** shows a unit circle in which the central angle measures t radians. We can use the formula for the length of a circular arc, $s = r\theta$, to find the length of the intercepted arc.

$$s = r\theta = 1 \cdot t = t$$

The radius of a unit circle is 1. The radian measure of the central angle is t.