CONTENT KNOWLEDGE FOR TEACHING: HOW IS IT RELATED TO “PEDAGOGICAL CONTENT KNOWLEDGE”? 

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CONTENT KNOWLEDGE FOR TEACHING (CKT)

- Frame: knowledge used in practice
  - “Knowledge entailed by the work of teaching”
- What do we mean by “knowledge”?
  - Mathematical knowledge, skill, habits of mind
- What do we mean by the “work of teaching”?
  - The activities in which teachers engage, and the responsibilities they have, to teach mathematics, both inside and outside of the classroom
KNOWING MATHEMATICS WELL ENOUGH TO TEACH IT: ANALYZING STUDENTS’ THINKING

What mathematical steps produced each of these answers? Why might someone do the problem in this way?

(a)  \[ 49 \times 25 = 1485 \]
(b)  \[ 49 \times 25 = 1275 \]
(c)  \[ 49 \times 25 = 1275 \]

What mathematical steps produced each of these answers? Why might someone do the problem in this way?
MATHEMATICAL KNOWLEDGE FOR TEACHING (MKT)

Subject Matter Knowledge

Common Content Knowledge (CCK)
Horizon Content Knowledge (HCK)

Specialized Content Knowledge (SCK)

Pedagogical Content Knowledge

Knowledge of Content and Students (KCS)
Knowledge of Content and Teaching (KCT)
Knowledge of Content and Curriculum (KCC)
COMMON CONTENT KNOWLEDGE (CCK)

Calculate:

\[
\frac{5}{6} \div \frac{1}{3}
\]
SPECIALIZED CONTENT KNOWLEDGE (SCK)

Is this a fluke?
Does it work in general?
If so, why does it work?

\[
\frac{5}{6} \div \frac{1}{3} = \frac{10}{12} \div \frac{4}{12} = 10 \div 4 = 2 \frac{1}{2}
\]
Which of these can be used to represent \( \frac{5}{6} \div \frac{1}{3} \)?

\[
\begin{array}{cccc}
\frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \\
\frac{1}{3} & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\
\end{array}
\]
Knowledge of Content and Students (KCS)

- What are common errors students make when dividing fractions?
- How do students’ experiences with division of whole numbers support their understanding of division of fractions? How does it confuse them?
- What difficulties do students typically have interpreting the answer to a division of fractions problem?

Knowledge of Content and Teaching (KCT)

- Which representation would you use to introduce the meaning of division of fractions? Or to explain the invert and multiply algorithm?
- What sequence of problems would you use to begin work on division of fractions?
- In a whole-class discussion, what solution methods would you want presented, and in what order?

\[
\frac{5}{6} \div \frac{1}{3} = 2 \frac{1}{2}
\]
A student comments that “if you divide by smaller and smaller fractions, the answers get bigger.” Is the student right? Is this mathematically significant or interesting?

Are there mathematically significant notions that underlie division of fractions?
KNOWLEDGE OF CONTENT AND CURRICULUM (KCC)

- At what grade level are students typically taught to divide fractions?
- How is division of fractions related to division of whole numbers in the school curriculum?
- What are the models for fractions and for division with which students would be familiar?
OTHER TASKS OF TEACHING CONTENT

- Responding to students’ “why” questions
- Unpacking and decomposing content-related ideas
- Explaining and guiding explanation
- Using academic language and notation
- Generating examples
- Sequencing ideas
- Choosing and using representations
- Analyzing errors
- Interpreting and evaluating alternative solutions and thinking
- Analyzing treatments of content in textbooks
- Making disciplinary practices explicit
- Attending to issues of equity (e.g., language, contexts, disciplinary practices)
MAKE ONE EXAMPLE IN ANOTHER CONTENT AREA

Try to show what it means in each domain:

- CCK: Common Content Knowledge
- SCK: Specialized Content Knowledge
- HCK: Horizon Content Knowledge
- KCS: Knowledge of Content and Students
- KCT: Knowledge of Content and Teaching
- KCC: Knowledge of Content and Curriculum