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UNCOVERING THE SPECIAL MATHEMATICAL WORK OF TEACHING
What does David Broussard claim is the “worst way to teach”? Why?
A FEW HARSH REALITIES

- University and college mathematics departments fail about 20–30% of their students.
- These students are disproportionately students of color, women, and first generation students.
These students don’t put in the effort; they don’t work or come to office hours.

They are underprepared and lack basic knowledge and skills.
WHAT DOES THE EVIDENCE SUGGEST?

- Stereotype threat
- Instruction that does not intervene on stereotype threat
- Instruction, in whatever format, unattuned to what students know and can do
WHAT IS TEACHING?

Cohen, Raudenbush, and Ball (2003)
WHAT IS THE WORK OF TEACHING?

Taking responsibility for deliberately maximizing the quality of these interactions . . .

. . . in ways that maximize the probability that students learn

. . . worthwhile content and skills

high-leverage teaching practices
A CORE DILEMMA OF TEACHING

- Teaching that is most likely to help students learn is attuned to what students know, can do, understand, and that engages them in authentic mathematical thinking and work.
- Teaching that reaches students is **attuned** to them as learners.
- But your students are not **you**.
- Therefore, generalizing from what you think is clear, helpful, interesting, will work only sometimes.
## GOOD TEACHING IS UNNATURAL

### EVERYDAY LIFE

<table>
<thead>
<tr>
<th>Action</th>
<th>Teaching</th>
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<tr>
<td>Asking questions to which you do not know the answers</td>
<td>Asking questions to which you often do know (at least part of) the answer</td>
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<td>Telling and showing others, doing things for people</td>
<td>Listening to and watching others, helping others do</td>
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<td>Assuming you know what others mean</td>
<td>Probing others’ ideas</td>
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<tr>
<td>Correcting and smoothing over mistakes</td>
<td>Provoking disequilibrium and error</td>
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<tr>
<td>Assuming others experience things as you do</td>
<td>Not presuming shared identity; seeking to learn others’ experiences and perspectives</td>
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USING MATHEMATICS AS A SOURCE OF HIGH-LEVERAGE TEACHING PRACTICES IN LARGE OR SMALL FORMAT CLASSES

- To strengthen students’ mathematical flexibility and confidence
- To intervene on stereotype threat and reproduction of inequity and marginalization
- To expand ALL students’ sense of what “being good (or smart) at mathematics” is
USING MATHEMATICS AS A SOURCE OF HIGH-LEVERAGE TEACHING PRACTICES

① Making mathematics transparent: Identifying and using “conditions” of a problem
② Foregrounding and using “error”
③ Assigning mathematical competence
① MAKING MATHEMATICS TRANSPARENT: IDENTIFYING AND USING CONDITIONS OF A PROBLEM
A KEY MATHEMATICAL PRACTICE

Making sense of and interpreting problems.

- What does this involve?
  - Careful reading
  - Identifying conditions

Making this explicit and transparent is a high-leverage mathematical teaching practice.
THE MINICOMPUTER

(Papy Minicomputer)
Using exactly one positive and one negative checker, find all the numbers that can be represented on this minicomputer board.

Prove your answer and explain why.
IDENTIFYING AND USING PROBLEM “CONDITIONS”

CONDITIONS OF THE PROBLEM
1. Use exactly one positive and one negative checker.
2. Must be a number that can be made on the Minicomputer.

PROPOSED SOLUTIONS

3. **YES, because it can be made with a positive on the 4 and a negative on the 1. This (1) uses exactly one positive and one negative checker and (2) can be made on the Minicomputer.**

9. **NO, because it cannot be made with exactly one positive and one negative checker on the Minicomputer.**
THE ROLE OF CONDITIONS OF A PROBLEM

- Identifying conditions can help in making sense of and interpreting a problem
- Using the conditions can help in persevering in solving a difficult problem
- Conditions are useful in constructing a mathematical argument
- Referring to the conditions is useful in critiquing an argument
“WARM-UP” PROBLEM

- How many different three-digit numbers can you make using the digits 1, 2, and 3, and using each digit only once?
- Show all the three-digit numbers that you found.
- How do you know that you found them all?
VIDEO:
MAKING ANALYSIS OF CONDITIONS EXPLICIT
Pose Questions About Conditions

3. A student is working on this problem:

Use exactly one positive checker and one negative checker to make the number 6 on the minicomputer.

This is one solution that the student makes:

Is this answer correct? YES NO

Explain why or why not. ____________________________________________

__________________________
② FOREGROUNDING AND USING “ERROR”
THE TASK: BEGINNING TO UNDERSTAND FRACTIONS AS NUMBERS

What number does the orange arrow point to? Explain how you figured it out.

← 0 1 2 →
VIDEO: ANIYAH AND TONI

Teacher: Listen closely and see what you think about her reasoning and her answer.
FOREGROUNDING AND USING “ERRORS”

- Pausing on “apparently incorrect” answers
  - Actually not incorrect
  - Answer to a different (and reasonable) question
  - More correct than incorrect
WHAT EXAMPLE WOULD YOU USE NEXT?

- What mathematical goal do you have with that example?
- How is your goal focused on what you think the key mathematical issue is?
- What role are you playing with your example, and what role do you want students to take?
FOREGROUNDING AND USING “ERRORS”

- Pausing on “apparently incorrect” answers
  - Actually not incorrect
  - Answer to a different (and reasonable) question
  - More correct than incorrect
- Deliberately inviting or featuring “stuck” or “wrong” solutions
  - Sharing and analyzing “wrong” interpretations, answers, method
  - Designing problems that entail appraisal of a solution
FOREGROUNDING AND USING “ERROR”
RESPONDING TO OTHERS’ CLAIMS

**BETA**: The bike is going up a very steep hill between 30 and 45 minutes into the ride.

**DELTA**: The bike is going up a very steep hill in the last 15 minutes of the ride.
INSTRUCTIONAL ACTIVITIES THAT FOREGROUND “ERROR”

1. **Analyze and respond**: Discuss the two answers with a person sitting next to you:
   - What is each person (Beta, Delta) thinking?
   - With whom do you agree? Why?
   - What would you do to explain this to ___?

2. **Make up a next example**: Make a specific graph to present to Beta and Delta next.
   - Explain why you would choose that one.
   - What point would you be trying to make explicit?
INSTRUCTIONAL ACTIVITY TYPE #3: WHAT QUESTIONS WOULD YOU ASK A CLASS?
WHAT IS INSIDE OF “ERROR”? 

\[
\frac{3}{4} \div \frac{1}{2}
\]
WRITE A STORY OR DRAW A REPRESENTATION

\[
\frac{3}{4} \div \frac{1}{2}
\]
I have two pizzas. My friend eats one quarter of one of the pizzas. I have one and three quarters pizzas left. Then I split it evenly between two of my other friends. Each person gets three and a half pieces of pizza.

1. What is wrong with this?
2. Write a story problem that correctly represents the division.
HOW WE TALK ABOUT “ERROR” MATTERS

- Making the environment “safe” is not all there is
- “Errors” are a necessary part of mathematical work
- Being “meta” about mathematical work is an important mathematical competence
  - Dwelling on things that go wrong or make you stuck
  - Analyzing solutions or methods that do not work, are not right
ASSIGNING MATHEMATICAL COMPETENCE
All this talk about not focusing on students’ deficits—why does this matter so much? Isn’t our job to figure out what students don’t know and help them grow?

1. Learning occurs through a process of building on prior knowledge and experience.
2. Strong academic and mathematical identities are a means to developing competence. They are also instructional goals.
3. For people from historically marginalized groups, stereotype threat and other biases interfere with and impede their performance.

So—focusing on students’ strengths is crucial for effective and equitable instruction and confronting implicit bias.
VIDEO: WHAT DO YOU SEE AND HEAR ABOUT ANIYAH AND TONI IN THIS SHORT CLIP?

Teacher: Listen closely and see what you think about her reasoning and her answer.
WHAT DO MANY “HEAR” IN ANIYAH AND TONI?

ANIYAH
- She has the wrong answer: 1/7

TONI
- She is playing with her hair and trying to get attention
- She is trying to embarrass Aniyah
WHAT DO ANIYAH AND TONI KNOW AND WHAT CAN EACH DO?

ANIYAH
- Uses the definition for a fraction to explain
  - She identifies the “whole”
  - She makes sure the intervals are equal
  - She counts intervals and not tick marks
  - She knows how to write “one-seventh”
- Produces a mathematically well-structured explanation
- Presents her ideas clearly

TONI
- Listens closely to a classmate’s presentation
- Uses the definition for a fraction to ask
  - How Aniyah decided on 7 parts
- Asks a pointed mathematical question
DILEMMAS OF LEARNING TO SEE AND HEAR STUDENTS’ RESOURCES

1. Feeling committed to students as sensemakers who bring many strengths and feeling pressure to make sure students get it “right”

2. Using yourself yet also suspending assumptions based on what you would mean or feel

3. Knowing mathematics well enough to see “mathematics” in students’ talk, representations, etc. while also not letting your own mathematical knowledge overtake your capacity to see and hear what they are saying or showing

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SEEING STUDENTS’ RESOURCES AND ASSIGNING MATHEMATICAL COMPETENCE

A set of practices that deliberately deploy the power of teaching to:

- Broaden and label what being competent in mathematics means
- Intervene to position who (and what) is seen as competent in math class
- Support individual students to develop their mathematical and academic identities and competence

Sources: E. Cohen and R. Lotan, complex instruction; J. Boaler’s work; Smarter Together: Collaboration and Equity in the Elementary Mathematics Classroom (Featherstone, Crespo, et al., 2011);
WHAT DOES “ASSIGNING COMPETENCE” REQUIRE IN TEACHING?

1. Broaden and label what being competent in mathematics means
2. Intervene to position who (and what) is seen as competent in math class
3. Support individual students to develop their mathematical and academic identities and competence

1. Be able to see what is “mathematical” and what is “competent”
2. Have techniques for making these moves to intervene
3. Strategically using these techniques with particular students in authentic and well-timed ways

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WRAPPING UP
USING MATHEMATICS AS A SOURCE OF HIGH-LEVERAGE TEACHING PRACTICES

① Making mathematics transparent: Identifying and using “conditions” of a problem
② Foregrounding and using “error”
③ Assigning mathematical competence

and there are others!
SEEING TEACHING: WHY IS IT DIFFICULT—AND DOES IT MATTER?

- Much of teaching is invisible work
- Student perspective and experience often unprobed
- Language for describing teaching is thin and imprecise
- Rush to judgment and evaluation.

But: Teaching has powerful impact on learning; seeing teaching is necessary for helping to develop and improve it. It matters.
THANK YOU!

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Slides will be available on my website
(deborahloewenberbgball.com)

NEW! AVAILABLE 8/8/16!
Graphic on slides 6 and 7:

Graphic on slides 14–16 and 20: