PROGRESS IN A VITAL QUEST: FROM CONTENT “KNOWLEDGE” TO “CONTENT-WORK OF TEACHING”

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Learning to Teach 12
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FACING THE CHALLENGE OF PREPARING NOVICES WHO KNOW CONTENT WELL ENOUGH TO TEACH IT TO YOUNG PEOPLE

1. Where has the field made progress and what do we know now about “content knowledge for teaching”?

2. What are the most important problems that we have to tackle to ensure that novice teachers are positioned to support young people’s academic growth?
KNOWING MATHEMATICS

\[
\frac{3}{4} \div \frac{1}{2}
\]

or

\[7 \div 0 = \]

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OLD IDEAS ABOUT TEACHER CONTENT KNOWLEDGE, AND A RESIDUAL QUESTION

- Good academic preparation and achievement are necessary for teaching content
- Teachers with higher levels of attainment are more likely to know content for teaching
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- Good academic preparation and achievement is necessary for teaching content
- Teachers with higher levels of attainment are more likely to know content for teaching

But . . .

- Knowledge of the subject area does matter

A better question:
- What kind of content knowledge and insight?
Why “can’t” you divide by 0?

Write a story or situation for $\frac{3}{4} \div \frac{1}{2}$. 
I have two pizzas. My friend eats one quarter of one of the pizzas. I have one and three quarters pizzas left. Then I split it evenly between two of my other friends. Each person gets three and a half pieces of pizza.

1. What is wrong with this?

2. Write a story problem that correctly represents the division.
PEDAGOGICAL CONTENT KNOWLEDGE

- Lee Shulman and colleagues (1980s)
- The most powerful forms of representing the subject matter for learners—metaphors, explanations, examples, stories
CONTENT KNOWLEDGE FOR TEACHING (CKT)

- **Subject Matter Knowledge**
  - Common Content Knowledge (CCK)
  - Horizon Content Knowledge (HCK)
  - Specialized Content Knowledge (SCK)

- **Pedagogical Content Knowledge**
  - Knowledge of Content and Students (KCS)
  - Knowledge of Content and Teaching (KCT)
  - Knowledge of Content and Curriculum (KCC)

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COMMON CONTENT KNOWLEDGE (CCK)

- What is a key theme in Zola Neale Hurston’s *Their Eyes Were Watching God*?
- What is \((-1) - (-3)\)?
- Where do green plants get nutrition?
- What is “voting”?
What is a “theme” of a text? How is it different from the “main idea”?

Explain (-1) – (-3) on a number line in a way that explains what it means, not just show the answer.

Explain how plants get their nutrition.

Choose two contrasting examples of reaching a decision through voting that highlight what it is for.
SPECIALIZED CONTENT KNOWLEDGE: HOW CAN “POSITIVE” AND “NEGATIVE” CUBES REPRESENT SUBTRACTION OF INTEGERS?

positive

negative
\((-1) - (-3)\)
-1 - (-3)
-1 - (-3) = 2
WHAT MATHEMATICAL UNDERSTANDING IS REQUIRED TO USE AND JUSTIFY THE CUBE MODEL?

- Interpretation of subtraction as “taking away” (not as difference)
- Adding opposites (a, -a) equals 0
- Language to explain these ideas
(-1) – (-3)
(-1) – (-3) = 2
WHAT MATHEMATICAL UNDERSTANDING IS REQUIRED TO USE AND JUSTIFY THE NUMBER LINE AS A REPRESENTATION?

- Integers represent distance from 0, and which side of 0
- Interpretation of subtraction as “taking away”
- Understanding what using interpretation of subtraction as “difference” would show
- Language to explain these ideas
KNOWING CONTENT FOR TEACHING

Subject Matter Knowledge
- Common Content Knowledge (CCK)
- Horizon Content Knowledge (HCK)

Pedagogical Content Knowledge
- Specialized Content Knowledge (SCK)
- Knowledge of Content and Students (KCS)
- Knowledge of Content and Teaching (KCT)
- Knowledge of Content and Curriculum (KCC)
PEDAGOGICAL CONTENT KNOWLEDGE

Knowledge of Content and Students (KCS)
1. What are some typical difficulties that students have with understanding photosynthesis?
2. What misconceptions do students often have about voting?
3. What is difficult for students about the idea of “theme”?

Knowledge of Content and Teaching (KCT)
1. Which representation would you use to introduce the meaning of subtraction of negative numbers? Why is “debt” not very effective?
2. What is a simulation that would help students understand voting?
3. What would be a good age-appropriate text to practice considering the notion of “theme”?
WHAT DO WE KNOW ABOUT “CONTENT KNOWLEDGE” AND TEACHING?

- Teachers need to understand the content they are teaching
- But it is a different kind of understanding—and a lot more—than the content knowledge studied in college courses
- This is progress.
- But what big questions remain?
AN INHERENT FACT OF TEACHING

Is that we are always communicating, relating, and making sense across difference, including:

- Age
- Gender
- Race, ethnicity, culture, religion
- Identities
- Language
- Experience
AN INHERENT FACT OF TEACHING

Is that we are always communicating, relating, and making sense across difference, including:

- Age
- Gender
- Race, ethnicity, culture, religion
- Identities
- Language
- Experience

So this means that fundamental to the work of teaching is attuning to other people, and being oriented to others’ ideas and ways of thinking and being.
SOME COMMON TASKS OF TEACHING CONTENT

- Responding to students’ “why” questions
- Unpacking and decomposing content-related ideas
- Explaining and guiding explanation
- Attending to issues of equity (e.g., language, contexts, disciplinary practices)
- Assigning competence
- Using academic language and notation
- Generating examples
- Sequencing ideas
- Choosing and using representations
- Analyzing apparent “errors” or “misconceptions”
- Noticing possible misunderstanding underneath apparently “correct” work
- Interpreting and evaluating alternative solutions and thinking
- Analyzing treatments of content in textbooks
- Making disciplinary practices explicit
BUILDING TOOLS TO “MEASURE”

TEACHERS’ KNOWLEDGE
HOW HAS THE FOCUS ON MEASUREMENT AFFECTED EFFORT TO UNDERSTAND CKT?

**ADVANCES**

1. Established that there are special kinds of knowing for teaching
2. Developed ways to study outcomes of teacher education and professional development

**IMPEDEMENTS**

1. Fell back from practice to knowledge (from sociocultural view to cognitivist view)
2. Not fully dynamic, about what teachers actually have to DO mathematically
3. Compartmentalized teaching — e.g., attention to equity
mathematical knowledge
values, beliefs, PCK, etc.

teaching and learning
mathematical knowledge
values, beliefs, PCK, etc.

mathematical work of teaching

teaching and learning
WHAT IS THE “WORK” OF TEACHING—e.g., MATHEMATICS, HISTORY, SCIENCE, WRITING?
VIDEO: ANIYAH AND TONI

Teacher: Listen closely and see what you think about her reasoning and her answer.

This video and additional supporting materials are available online here.
WHY “MATHEMATICAL WORK OF TEACHING”? 

Mathematical listening, speaking, interacting, acting, fluency, and doing are part of the work of teaching, not just resources for it.
Rising 5th graders.
30 pupils
22 African American, 4 Latinx, 4 White
Low-income community
Most children have been unsuccessful in school mathematics
What number does the orange arrow point to? Explain how you figured it out.
What mathematical work of teaching can you identify?
READING STUDENTS’ WORK

2/4

What number does the orange arrow point to?

Explain how you know:

Because there are equal parts and you are pointing to the second one so it's 2/10.

Give a complete sentence with one goal for yourself for our math class. Give an example of what it looks like to do this really well.

To listen to other people's ideas. I like just because I know the answer that I will still listen to others.
What number does the orange arrow point to? 1

Explain how you know: There's 0 2 spaces
1 2 spaces then 2 2 spaces in those 0 2 spaces are fractions
What number does the orange arrow point to? \[ \frac{1}{4} \]

Explain how you know: because it's divided into 4 units and it's 1 of the parts.
READING STUDENTS’ WORK

1/3 without mathematical explanation
VIDEO: WHAT IS THE MATHEMATICAL WORK OF TEACHING IN THESE THREE MINUTES?

Teacher: Listen closely and see what you think about her reasoning and her answer.

This video and additional supporting materials are available online here.
What is the mathematical work of teaching in these three minutes?

What number does the orange arrow point to? \( \frac{1}{3} \)

Explain how you know: Because it is in \( \frac{3}{3} \) parts
What is the mathematical work of teaching in these three minutes?

What number does the orange arrow point to?

Explain how you know: because there are equal parts and you are pointing to the second one so it's \( \frac{2}{4} \).
What is the mathematical work of teaching in these three minutes?

What number does the orange arrow point to? \( \frac{1}{2} \)

Explain how you know: 

Because if you look at it and count.
The Mathematical Work of Teaching

First example:
- Hearing, seeing, and reading students, in “real time”
  - During a class discussion
  - While circulating in the classroom
  - When reading students’ writing

Second example:
- Assigning competence

What else do you notice that is mathematical work of teaching?
A SECOND EXAMPLE: ASSIGNING MATHEMATICAL COMPETENCE

- Broaden and name what being competent in mathematics means
- Intervene to position who (and what) is seen as competent
- Support individual children to develop their mathematical and academic identities and competence

Sources: E. Cohen and R. Lotan, complex instruction; J. Boaler’s work; Smarter Together: Collaboration and Equity in the Elementary Mathematics Classroom (Featherstone, Crespo, et al., 2011);
What does Aniyah know and what can she do?

What does Toni know and what is she able to do?
VIDEO: WHAT DO ANIYAH AND TONI KNOW AND WHAT CAN THEY DO?

This video and additional supporting materials are available online [here](#).
WHAT DO MANY “HEAR” IN ANIYAH AND TONI?

<table>
<thead>
<tr>
<th>ANIYAH</th>
<th>TONI</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. She has the wrong answer:</td>
<td>1. She is playing with her hair and trying to get attention</td>
</tr>
<tr>
<td>1/7</td>
<td>2. She is trying to embarrass Aniyah</td>
</tr>
</tbody>
</table>
**WHAT DO ANIYAH AND TONI KNOW AND WHAT CAN EACH DO?**

<table>
<thead>
<tr>
<th>ANIYAH</th>
<th>TONI</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Uses the definition for a fraction to explain</td>
<td>1. Listens closely to a classmate’s presentation</td>
</tr>
<tr>
<td>- She identifies the “whole”</td>
<td></td>
</tr>
<tr>
<td>- She makes sure the intervals are equal</td>
<td>2. Uses the definition for a fraction to ask</td>
</tr>
<tr>
<td>- She counts intervals and not tick marks</td>
<td>- How Aniyah decided on 7 parts</td>
</tr>
<tr>
<td>- She knows how to write “one-seventh”</td>
<td>3. Asks a pointed mathematical question</td>
</tr>
<tr>
<td>2. Produces a mathematically well-structured explanation</td>
<td></td>
</tr>
<tr>
<td>3. Presents her ideas clearly</td>
<td></td>
</tr>
</tbody>
</table>
A SECOND EXAMPLE OF THE CONTENT-INTENSIVE WORK OF TEACHING

Examples:

- Hearing students, reading students
- Translating across many differences
- Speaking mathematically fluently and across differences
- Building students’ mathematical identities
- Using mathematical tasks as tools for students’ learning
3.NFA.1: Understand a fraction \( \frac{1}{b} \) as the quantity formed by 1 part when a whole is partitioned into \( b \) equal parts; understand a fraction \( \frac{a}{b} \) as the quantity formed by \( a \) parts of size \( \frac{1}{b} \).

3.NFA.1 and 2: Understand a fraction as a number on the number line; represent fractions on a number line diagram.

MP.1. Make sense of problems and persevere in solving them.

MP.3. Make and critique mathematical arguments.
REASONING ABOUT FRACTIONS

3.NFA.1: Understand a fraction 1/b as the quantity formed by 1 part when a whole is partitioned into b equal parts; understand a fraction a/b as the quantity formed by a parts of size 1/b.

3.NFA.1 and 2: Understand a fraction as a number on the number line; represent fractions on a number line diagram.

MP.1. Make sense of problems and persevere in solving them.

MP.3. Make and critique mathematical arguments.

What are the goals for my students?

But how can I translate this in ways that make sense for my students?
MORE TRANSLATING . . .

- What is a “mathematical argument” at age 10?
- What does it mean to “critique the arguments of others” at this level?

How might my students interpret these terms?

Do they have experiences or language that might help label this practice in our classroom discourse?
TRANSLATING:
DEFINING A FRACTION

Understand a fraction $1/b$ as the quantity formed by 1 part when a whole is partitioned into $b$ equal parts.

1. Figure out what the whole is.
TRANSLATING:
DEFINING A FRACTION

Understand a fraction 1/b as the quantity formed by 1 part when a whole is partitioned into b equal parts.

1. Figure out what the whole is.
2. Figure out if the whole is divided into equal parts.
3. If not, make equal parts.
TRANSLATING: DEFINING A FRACTION

1. Understand a fraction $1/b$ as the quantity formed by 1 part when a whole is partitioned into $b$ equal parts.
2. Figure out what the whole is.
3. Figure out if the whole is divided into equal parts.
4. If not, make equal parts.
5. Count how many equal parts there are. Write $1/d$ to show one of the parts. This is a unit fraction.
TRANSLATING: DEFINING A FRACTION

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5. Count how many equal parts there are. Write $1/d$ to show one of the parts. This is the unit fraction.

Understand a fraction $a/b$ as the quantity formed by a parts of size $1/b$.

1. If more than one of those parts is shaded, count them (n) and write $n/d$. 

Steps for Naming a Fraction Correctly

1. Figure out what the whole is.
2. Figure out if the whole is divided into equal parts. If not, make equal parts.
3. Count how many equal parts there are.
4. Write $1/d$ to show one of the equal parts. This is a unit fraction.
5. If more than one of those parts is shaded, count them (n) and write $n/d$. 

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Understand a fraction $a/b$ as the quantity formed by $a$ parts of size $1/b$.

1. If more than one of those parts is shaded, count them ($n$) and write $n/d$. 

Steps for Naming a Fraction Correctly

1. Figure out what the whole is.
2. Figure out if the whole is divided into equal parts. If not, make equal parts.
3. Count how many equal parts there are.
4. Write $1/d$ to show one of the equal parts. This is a unit fraction.
5. If more than one of those parts is shaded, count them ($n$) and write $n/d$. 

$d \neq 0$ 7. $d$ is a whole number now.
ANOTHER EXAMPLE:
TRANSLATING INTO “LEARNER LANGUAGE”

3.NFA.1: Understand a fraction $1/b$ as the quantity formed by 1 part when a whole is partitioned into $b$ equal parts; understand a fraction $a/b$ as the quantity formed by a parts of size $1/b$.

3.NFA.1 and 2: Understand a fraction as a number on the number line; represent fractions on a number line diagram.

MP.1. Make sense of problems and persevere in solving them.
MP.3. Make and critique mathematical arguments.

But how can I translate this in ways that make sense for my students?
TRANSLATING: DEFINING A FRACTION

Understand a fraction $1/b$ as the quantity formed by 1 part when a whole is partitioned into $b$ equal parts.

1. **Understand a fraction $1/b$** as the quantity formed by 1 part when a whole is partitioned into $b$ equal parts.
2. Figure out what the whole is.
3. Figure out if the whole is divided into equal parts.
4. If not, make equal parts.
5. **Count** how many equal parts there are.
   Write $1/d$ to show one of the equal parts. This is a unit fraction.

Understand a fraction $a/b$ as the quantity formed by $a$ parts of size $1/b$.

1. If more than one of those parts is shaded, count them ($n$) and write $n/d$.

I need to “talk” the idea of $1/d$ in ways that make sense for my students.
CONTENT-INTENSIVE WORK OF TEACHING

Some examples:

- Hearing students, reading students, checking for understanding
- Translating across many differences
- Speaking mathematically fluently and across differences
- Building students’ academic identities in specific subject domains
- Using content tasks as tools for students’ learning
WHAT IS KEY TO THE CONTENT-INTENSIVE WORK OF TEACHING, AND WHAT DO WE NOT YET UNDERSTAND WELL ENOUGH?

- It is fundamentally dynamic, attuned to others, and across differences in social identity
- It is about other people’s thinking (students’)
- It is inside core tasks of teaching
- It requires a special kind of fluency
THE NEW CHALLENGES

- To bring work on equity, teaching practice, and content together (e.g., assigning competence)
- To understand and make explicit the content-work in teaching, not just the teacher’s “knowledge”
- To focus on opportunities to learn and practice the content-work of teaching, including fluency
- To assess candidates’ practice of the content-work of teaching
How might we work together?
Graphic on slides 9 and 20:

Image on slide 26 (cropped and rotated):
“Ruler” by Flickr user Scott Akerman
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