49th Annual NCSM Conference
Engaging in the Mission of Mathematics Education Leadership:
High Quality, Meaningful, and Relevant Mathematics for All

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UNCOVERING THE SPECIAL
MATHEMATICAL WORK OF TEACHING
OUR NATION, AND OUR FUTURE

- There are 78,000,000 people under the age of 18 in our country.
- Almost 25% of the nation’s population.
- 50,000,000 are in schools (1.3 million homeless).

How can we teach mathematics so that people stop hating and killing one another?

- Adapted from Maisha Winn, inspired by Ihab Hassan
THE POWER OF TEACHING AS A FORCE FOR A MORE JUST WORLD
THE POWER OF TEACHING AS A FORCE FOR A MORE JUST WORLD

What does achieving this require of us?
THE POWER OF TEACHING AS A FORCE FOR A MORE JUST WORLD

1. We need to understand in much more close professional detail the work of teaching.

2. Teaching mathematics is deeply mathematical work.

3. Persistent and determined disruption of inequities and attention to mathematics are fundamentally intertwined.
WHAT IS THE “WORK” OF MATHEMATICS TEACHING?
WHAT IS INSTRUCTION?

Instruction is co-constructed

- . . . in broad socio-political, historical, economic, cultural, community, family environments
- . . . through the interpretations and interactions of teachers, students, and “content”

Cohen, Raudenbush, and Ball (2003)
WHAT IS THE WORK OF TEACHING?

Taking responsibility for deliberately maximizing the quality of these interactions . . .

- in ways that maximize the probability that students learn
- worthwhile content and skills
- and that advance a just society

Cohen, Raudenbush, and Ball (2003)
WHY “WORK” OF TEACHING”?

1. To focus our attention on what teachers DO and to distinguish this from other features of classrooms, such as instructional formats, classroom culture and norms, what students are doing, how the curriculum is designed.

But what about small group work, open-ended problems, “grit,” etc.? Aren’t those what teachers DO?
WHY “WORK” OF TEACHING”?

1. To focus our attention on what teachers DO and to distinguish this from other features of classrooms, such as instructional formats, classroom culture and norms, what students are doing, how the curriculum is designed

2. To honor the effortful and deliberate nature of teaching and not to leave it invisible, implicit, and taken for granted
WHY “MATHEMATICAL” WORK OF TEACHING”?

To look at how mathematical listening, speaking, interacting, acting, fluency, and doing are part of the work of teaching, not just resources for it.

This is what I mean by “the special mathematical work of teaching.”

Let’s look!
EXAMPLE #1:
TRUSTING STUDENTS TO LEARN AND
HOLDING HIGH EXPECTATIONS OF THEM IN
REAL TIME
THE MINICOMPUTER

- Abstract mathematical context for work on number relationships, mathematical structure, arithmetic properties (e.g., distributive property), even and odd numbers
- Also a setting for developing skills of mathematical argument and analysis, as well as proof
- Novel and complex mathematical environment for children

(Papy Minicomputer)
THE MINICOMPUTER

-4

12

17
MEET VIRSHAWN

10 years old, Black male
In fourth grade:

- He was often sent out of the room, to the hall or the principal’s office
- He was in trouble often
- He wasn’t doing well in math
- He was articulate and liked to write
SEEING VIRSHAWN

Virshawn between 11:19 – 11:25

Paper airplane

What do you see of Virshawn?
At 11:26, the teacher says, “You know what, Virshawn, I am going to need you to come up here closer where you can see and hear and won’t be distracted.”
Over the next minute, Virshawn is raising his hand to answer questions.
At 11:27, the teacher says, “Virshawn, you get to come up and make a number because you are the closest person to the board.” “You can have two checkers and you can put them wherever you want, and make a number, and then you can call on somebody.”
AT 11:28, VIRSHAWN TAKES THE ROLE OF THE TEACHER
VIRSHAWN, ONE WEEK LATER

THE TWO-CHECKER PROBLEM

What numbers are possible to make on the Minicomputer with exactly two positive checkers?
ADDING A NEGATIVE CHECKER!

The children found that 7, 11, 13, 14, and 15 were impossible with exactly two checkers.

EXTENSION:
Can you make 7, 11, 13, 14, and 15 if you have a negative checker?
You must use both positive checkers and you can use one negative checker if it is useful.
VIDEO: HOW DOES THIS DAY RELATE TO WHAT HAPPENED ONE WEEK EARLIER?

Then the purple is a four, the red is a two, and the white is a one, so you couldn’t do it without a- without if- without another negative number.

I think the negative checker is awesome.
WHAT IS THE (MATHEMATICAL) WORK OF TRUSTING STUDENTS TO LEARN AND HOLDING HIGH EXPECTATIONS OF THEM IN REAL TIME IN THIS EXAMPLE?

- Trusting Virshawn to be mathematically engaged
- Making explicit positive mathematical roles that actively include and support children (e.g., “being the teacher”)
- Creating opportunities to “practice mathematics”
- Making available an opportunity to develop agency through “proving the impossible”
- Focusing on mathematics as a context for positive roles and identity
EXAMPLE #2: READING CHILDREN’S WORK "ON THE FLY"
What number does the orange arrow point to? Explain how you know.
What are the children thinking about this? How shall I stage a discussion of this task?
**TONI**

What number does the orange arrow point to? \( \frac{1}{3} \)

Explain how you know: Because it's \( \frac{1}{3} \) points.

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**MARIANA**

What number does the orange arrow point to? \( \frac{1}{2} \)

Explain how you know: How I know it's zero is that it's a number from one to one, there was a line between 0 and 1.5.

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**ASHTON**

What number does the orange arrow point to? \( \frac{1}{2} \)

Explain how you know: 

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**MAKAYLA**

What number does the orange arrow point to? \( \frac{1}{3} \)

Explain how you know: Count from zero, one then make it equal and then it's equal then count from the 1 I saw the one then \( \frac{1}{3} = \frac{1}{3} \). Maybe not.

Write a complete sentence with one goal for yourself for our math class today. Give an example of what it looks like to do this really well.
ANiyah:
What number does the orange arrow point to? 

Explain how you know: because this segment is drawn into 7 equal parts. That's why I put 7/7 and I put the one down because that's one equal part out of seven. So one-seventh is 7/7.

Parker:
What number does the orange arrow point to? 

Explain how you know: Because if you count by ones you get zero over five.

Mariana:
What number does the orange arrow point to? 

Explain how you know: How I know it's zero to one is that there is an interval from zero to one, there was 2 time between 0 and 1.

Dante:
What number does the orange arrow point to? 

Explain how you know: Because if you count it and count.
WHAT IS THE (MATHEMATICAL) WORK OF READING CHILDREN’S WRITING “ON THE FLY” IN THIS EXAMPLE?

1. Considering in advance the mathematical issues embedded in the task
   1. Interpreting the number line
      - The whole as 0 to 1 or some other whole
      - Counting intervals vs. tick marks
      - $\frac{1}{2}$ as the “universal fraction”

2. Anticipating what children might think, do, or write
   2. Interpreting “number” as “whole number”; 1 as after 0

3. Being fluent with reading children’s written expression
   3. Reading spelling, filling in missing words, looking non-linearly
WHAT ABOUT THE (MATHEMATICAL) WORK OF STAGING A DISCUSSION IN THIS EXAMPLE?

- What is my goal in discussing this task in whole group?
  - To make the fraction definition more usable
  - To make features of the number line more explicit
  - To make clear that a fraction is a number

- What might be the most productive order to take up different proposed answers?
  - What are the mathematical issues that each answer makes visible? e.g., 1/7, 2/4
  - Who explained or represented in ways that make visible productive mathematical practices?
  - Whom do I want to position as contributing key mathematical ideas or practices?
EXAMPLE #3:
TRANSLATING FORMAL MATHEMATICAL IDEAS INTO LANGUAGE ACCESSIBLE TO LEARNERS
3.NFA.1: Understand a fraction $1/b$ as the quantity formed by 1 part when a whole is partitioned into $b$ equal parts; understand a fraction $a/b$ as the quantity formed by $a$ parts of size $1/b$.

3.NFA.1 and 2: Understand a fraction as a number on the number line; represent fractions on a number line diagram.

How can I translate this in ways that make sense for my students?
TRANSFORMING: DEFINING A FRACTION

Understand a fraction $1/b$ as the quantity formed by 1 part when a whole is partitioned into $b$ equal parts.

- Figure out what the whole is.
- Figure out if the whole is divided into equal parts.
- If not, make equal parts.
- Count how many equal parts there are.
- Write $1/d$ to show one of the equal parts. This is a unit fraction.
TRANSLATING: DEFINING A FRACTION

Understand a fraction $\frac{1}{b}$ as the quantity formed by 1 part when a whole is partitioned into $b$ equal parts.

- Figure out what the whole is.
- Figure out if the whole is divided into equal parts.
- If not, make equal parts.
- Count how many equal parts there are.
- Write $\frac{1}{d}$ to show one of the equal parts. This is a unit fraction.

Understand a fraction $\frac{a}{b}$ as the quantity formed by a parts of size $\frac{1}{b}$.

- If more than one of those parts is shaded, count them (n) and write $\frac{n}{d}$.

Steps for Naming a Fraction Correctly

1. Figure out what the whole is.
2. Figure out if the whole is divided into equal parts. If not, make equal parts.
3. Count how many equal parts there are.
4. Write $\frac{1}{d}$ to show one of the equal parts. This is a unit fraction.
5. If more than one of those parts is shaded, count them (n) and write $\frac{n}{d}$. 
WHAT IS THE (MATHEMATICAL) WORK OF TRANSLATING MATHEMATICS INTO LANGUAGE ACCESSIBLE TO LEARNERS IN THIS EXAMPLE?

1. Decomposing the mathematical ideas into key components
2. Considering the formal mathematical language carefully
3. Identify mathematical terms to be taught and others to be translated
4. Attending to mathematical issues that might arise in careless translation

1. The whole, equal parts, number, number line, 1/b
2. Quantity, 1/b, fraction, number
3. Whole, use of a variable (should it be $b$? maybe use $d$ and $n$?)
4. 0 in the denominator! $n$ or $d$ as only whole numbers?
TRANSLATING:
DEFINING A FRACTION

Understand a fraction 1/b as the quantity formed by 1 part when a whole is partitioned into b equal parts.

- Figure out what the whole is.
- Figure out if the whole is divided into equal parts.
- If not, make equal parts.
- Count how many equal parts there are.
- Write 1/d to show one of the equal parts. This is a unit fraction.

Understand a fraction a/b as the quantity formed by a parts of size 1/b.

- If more than one of those parts is shaded, count them (n) and write n/d.
TRANSLATING: DEFINING A FRACTION

Understand a fraction $1/b$ as the quantity formed by 1 part when a whole is partitioned into $b$ equal parts.

- Figure out what the whole is.
- Figure out if the whole is divided into equal parts.
- If not, make equal parts.
- Count how many equal parts there are.
- Write $1/d$ to show one of the equal parts. This is a unit fraction.

Understand a fraction $a/b$ as the quantity formed by $a$ parts of size $1/b$.

- If more than one of those parts is shaded, count them ($n$) and write $n/d$.

I need to “talk” the idea of $1/d$ in ways that make sense for my students.
THE IMPERATIVE FOR US:
MAKING VISIBLE—AND LEARNING TO DO—
THE MATHEMATICAL WORK OF TEACHING

Some examples:

- Hearing students, reading students
- Disrupting inequities that marginalize minoritized groups
- Translating across many differences
- Speaking mathematically fluently and across differences
- Building students’ mathematical identities
- Using mathematical tasks as tools for students’ learning
HOW DO WE PROVIDE MORE OPPORTUNITIES TO LEARN TO DO THE MATHEMATICAL WORK OF TEACHING?
2017 ELEMENTARY MATHEMATICS SUMMER PROGRAM

TeachingWorks website:  http://teachingworks.org/

July 31 – August 11, 2017

Professional development programs: July 31 – August 4, 2017

Specific URL:  
http://www.teachingworks.org/training/LaboratoryClasses/2017EML

April 5, 2017
THANK YOU!
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Slides will be posted on my website
deborahloewenbergball.com
(Google Deborah Ball)
Graphic on slides 7 and 8:

Graphic on slides 13, 14, 21, 22, and 23: