BEYOND “STICK WITH IT”: TEACHING CHILDREN TO PERSEVERE IN MATHEMATICS

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SCHOOL OF EDUCATION
UNIVERSITY OF MICHIGAN

TeachingWorks
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IS THERE A TEACHER WHO HAD A SIGNIFICANT IMPACT ON YOU?
VIDEO OF DA’YANA: MS. REYNOLDS SHOWED US THAT PERSEVERING CAN BENEFIT YOUR LIFE
WHAT IS “PERSEVERANCE”? 

Grit? 

Growth mindset?
TURN AND TALK

What is something you have persevered with in your life, in or out of school?
Place a ring around three of the numbers below so that they sum to 22.
How many different ways can you do this?
How do you know you have found them all?
PERSEVERANCE AS A MATHEMATICAL PRACTICE

MP1: Make sense of mathematics problems and **persevere** in solving them.
Perseverance is contextually situated.

How can their experiences in classrooms support children learning to persevere in mathematics?
CULTIVATING CHILDREN’S PRACTICE OF PERSEVERING IN MATHEMATICS

1. Creating a classroom culture that disrupts broader patterns of exclusion
2. Confronting the “impossible” and the “infinite” in mathematics
3. Using homework to support the development of strong mathematical identity
1. CREATING A CLASSROOM CULTURE THAT DISRUPTS BROADER PATTERNS OF EXCLUSION
CLASSROOM CULTURE AND NORMS

- Cultural notions of what it means to be “good at math” are disrupted.
- Children present their thinking in a variety of ways (e.g., in notebooks, at the board).
- Children are supported to share solutions and reasoning—even those that are incomplete or “wrong.”
- Apparently “wrong” answers are viewed as powerful learning opportunities.
VIDEO OF MEHJABEEN: DISTINGUISHING BETWEEN ENCOURAGING AND “PICKING ON”
What number does the orange arrow point to? Explain how you figured it out.
LAKEYA

What number does the orange arrow point to? __________________________________________________________________________________________

Explain how you know: __________________________________________________________________________________________

JAMARI

What number does the orange arrow point to? __________________________________________________________________________________________

Explain how you know: __________________________________________________________________________________________

MARIANA

What number does the orange arrow point to? __________________________________________________________________________________________

Explain how you know: __________________________________________________________________________________________

LARRY

What number does the orange arrow point to? __________________________________________________________________________________________

Explain how you know: __________________________________________________________________________________________

Write a complete sentence with one goal for yourself for our math class today. Give an example of what it looks like to do this really well.

Learn more about the number line.
What number does the orange arrow point to?

Explain how you know because four seven

equal parts that's why I put seven

down and I put the one down because

that's one equal part out of seven.

So one-seventh
VIDEO: ANIYAH’S EXPLANATION

Aniyah: I put one-seventh because there's-
Toni: Did she say one-seventh?
THE POWER OF THE ENVIRONMENT TO SHAPE CLASSROOM EXPERIENCES

- How do most people see and interpret Aniyah?
- What are likely next moves in classrooms?
WHAT WOULD IT TAKE TO DISRUPT THE PATTERNS THROUGH WHICH A BLACK GIRL IS MARGINALIZED?

- Seeing Aniyah’s solution as mathematically sophisticated and key to the class’s work
- Taking as axiomatic the brilliance of Black girls, and thus Aniyah
- . . . And having something different to do

(Gholson & Martin, 2014, Martin, 2012; Leonard & Martin, 2013)
VIDEO: ANIYAH’S EXPLANATION

This video and additional supporting materials are available online here.
A DIFFERENT MOVE
Ask students to ask questions of the presenter, forestall agreeing/disagreeing.

RESULT
Maintain Aniyah’s authority and agency and position her and her thinking to advance the key mathematical idea.
2. CONFRONTING THE “IMPOSSIBLE” AND THE “INFINITE” IN MATHEMATICS
WHAT IS A SOLUTION SPACE OF A MATHEMATICS PROBLEM?

- The nature of the solutions for a given problem: how many solutions exist for a problem

WHY IS THIS IMPORTANT AND WHAT DOES IT MAKE POSSIBLE?

- Learning to make sense of problems and persevere in solving them
- Developing habits of mind for sizing up a problem and confidence in approaching, working on, and solving problems, and justifying solutions
- Developing sense of control and agency as a thinker
DIFFERENT SOLUTION SPACES

1) No solutions
2) Unique solution (one correct answer)
3) Multiple but finite solutions
4) Infinitely many solutions
<table>
<thead>
<tr>
<th>SOLUTION SPACE TYPES</th>
<th>EXAMPLE</th>
<th>SOLUTIONS</th>
</tr>
</thead>
<tbody>
<tr>
<td>No solutions</td>
<td>Find an even number that when you double it equals 5.</td>
<td>Impossible</td>
</tr>
<tr>
<td>Unique solution</td>
<td>5 + 7 =</td>
<td>Only one possible answer</td>
</tr>
<tr>
<td>Multiple but finite solutions</td>
<td>How many three-digit numbers can you make using the digits 4, 5, and 6 and using each digit exactly once?</td>
<td>Six answers</td>
</tr>
<tr>
<td>Infinitely many solutions</td>
<td>Write equations for 10.</td>
<td>There is no limit to the equations that equal 10</td>
</tr>
</tbody>
</table>
QUESTIONS TO ASK ONESELF ABOUT A MATHEMATICAL PROBLEM

1. Does this problem have a solution?
2. Does this problem have a unique solution?
3. Does this problem have multiple but finite solutions?
4. Does this problem have infinitely many solutions?
Place a ring around three of the numbers below so that they sum to 22.
How many different ways can you do this? How do you know you have found them all?
Write equations for 10.
How many equations are there?
How do you know?
WHAT DO SOLUTION SPACE PROBLEMS AFFORD?

- Disrupt “normal” mathematical experience:
  - One right answer
  - Single entry point of access, exclusionary
  - Narrow views of who is “smart” at math

- Provide alternative and empowering mathematical experience
  - What might count as “solving” a mathematics problem (i.e., some problems are impossible)
  - Multiple entry points, easy access, different degrees and kinds of complexity, but open to all
3. USING HOMEWORK TO SUPPORT THE DEVELOPMENT OF STRONG MATHEMATICAL IDENTITY
HOMEWORK: THREE TYPES OF TASKS TO PRACTICE “DOING MATHEMATICS”

1. Review today’s work
   - You should know how to do this, and if not, see what you need to work on

2. “You Be the Teacher”
   - Teach someone else and assess whether they understand

3. Looking ahead
   - Try things that you were not yet taught
2. YOU BE THE TEACHER

THE TASK

- Pick someone at your house to be your “student.”
- Show this rectangle to the person and ask him/her to tell you what fraction is shaded.
- Explain why some people might answer ¼.
- Explain the correct answer.

TO NOTE:

- Because some students speak Spanish at home, the assignment can be done in English or Spanish.
- This assignment both bridges the gap with the student’s family AND foregrounds the student’s competence.
You Be the Teacher!

Pick someone at your house to be your “student.”

Write the name of your “student” here: ____________________________

Tell your “student” that some people might think that $\frac{1}{3}$ of the figure is correct. Tell your student why $\frac{1}{3}$ is not the right answer. Explain why.

Ask the person you taught this problem to sign his or her name teaching:

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El problema de esta página está escrito en español y en la anterior está en inglés. Puedes escoger resolver el problema en inglés o español, pero sólo tienes que hacerlo una vez.

Tú eres el Profesor!

Escoge a alguien en tu casa para que sea tu “estudiante”.

Escribe su nombre aquí: ____________________________

Muéstralé este rectángulo a la persona escogida y pídele que le diga qué fracción de la figura está sombreada en gris.

Dile a tu “estudiante” que algunas personas podrían pensar que $\frac{1}{3}$ del rectángulo está sombreado de gris pero que esto no es correcto. Dile por qué $\frac{1}{3}$ no es la respuesta correcta. Explicale cuál es la respuesta, y por qué.

Pídele a la persona a la que le estás enseñando el problema que firme con su nombre aquí y que escriba un comentario acerca de lo que le enseñaste.
Tú eres el Profesor:

Escoge a alguien en tu casa para que sea tu "estudiante".

Escribe su nombre aquí: Silvia

Muestra este rectángulo a la persona escogida y pide que te diga que fracción de la figura está sombreada en gris.

\[
\frac{1}{6}
\]

Dile a tu "estudiante" que algunas personas podrían pensar que \( \frac{1}{4} \) del rectángulo está sombreado de gris pero que esto no es correcto. Dile por qué \( \frac{1}{4} \) no es la respuesta correcta. Explicale cuál debería ser la respuesta, y por qué.

¿Cómo se lo explicaste?

May bien. Yo explicale que mi ma

Si sita poler las lineas.

Pídele a la persona a la que le enseñaste el problema que firme con su nombre aquí y escriba un comentario acerca de tu enseñanza:

May bien.
EMELINE

You Be the Teacher!

Pick someone older than you to be your “student.”

Write the first name of your “student” here:  

Today we learned about number sentences. Tell your student what a number sentence is.

Ask your student to write 5 different number sentences for 10. Have your student write the number sentences below:

\[2 \times 5 = 10\]
\[20 \div 2 = 10\]
\[3 + 7 = 10\]
\[5 + 5 = 10\]
\[18 - 8 = 10\]

Tell your student something that you noticed about their number sentences.

Write one thing that your student did well. Or write something that your student needed help with.

She needed help with dividing.

Ask the person you taught this problem to sign below and write a comment about your teaching:

Emeline did a great job explaining to me what a number sentence is... and then she explained the answers clearly.

ANAYJAH

You Be the Teacher!

Pick someone of your choice to be your “student.”

Write the name of your “student” here:  

Show this rectangle to the person and ask him or her to tell you what fraction is shaded gray.

Tell your “student” that some people might think that \(\frac{1}{3}\) of the rectangle is shaded gray but that this is not correct. Tell your student why \(\frac{1}{3}\) is not the right answer. Explain what the answer should be, and explain why.

Ask the person you taught this problem to sign his or her name here and write a comment about your teaching:

She explained why the answer...
3. LOOKING AHEAD

Looking Ahead
We are going to be doing some work with adding positive and negative numbers. Please do not get help from anyone to do these problems. I want to know about your thinking.

Don’t worry if you do not know how to answer some of them. We are going to be working together on adding positive and negative numbers and your work on these problems will help me find out what we should work on.

3. Use the number line to show how to make each equation (number sentence) true.

2 + 3 = ___________

Explain your answer.__________________________________________
VIDEO: “LOOKING AHEAD” PROBLEMS: DEVELOPING PERSEVERANCE AND CONFIDENCE

Finally, do not ask your tutor to help you with the looking ahead problems.
Perseverance is contextually situated.

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VIDEO OF MADDY: LEARNING TO PERSEVERE SHAPED MY LIFE
THANK YOU!

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Slides will be available on my website

https://deborahloewenbergball.com/

(“Google” Deborah Ball)