In honor of Anna Sfard
Anna Sfard — one view
Anna — other views
And, because Anna defies representation, yet another:
How do Anna’s ideas about discourse, thinking, mathematics, and communicating inform understanding the work of teaching as communicating to support learning?
1. First, a brief word about discretionary spaces in the work of teaching.
- Teaching is constrained by policies, curriculum, testing regimes, institutionalized procedures.
- Teaching is highly idiosyncratic and individual.

Lipsky (1980), Shulman (1983)
<table>
<thead>
<tr>
<th>Speaker</th>
<th>Talk</th>
</tr>
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<tbody>
<tr>
<td>Teacher</td>
<td>Who would like to try explaining what you think the answer is? And show us your reasoning by coming up to the board...</td>
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<td>Teacher</td>
<td>Okay. Anyone?</td>
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| STUDENT 1 | Playing with hair |
| STUDENT 2 | Laying on arms |

| Teacher | You’re trying to mark what you think this number is and explain how you figured it out. |
| Teacher | Listen closely and see what you think about her reasoning and her answer. (Jumps stands 1/2 by the orange one.) |
| Annap | I put one-seventh because there’s one.
Annap | But she has one-seventh? |
| Teacher | Sure. To Ted: Yeah, (continues to solve). Because there’s seven equal parts, one, two, three, four, five, six, and then seven. (Shows her fingers to count the parts in the number one). |
| Teacher | Before you agree or disagree, I want you to ask questions if there’s something you don’t understand about what she did. No agreeing and disagreeing. Just: All you can do right now is ask any questions. Who has a question for her? |
| Teacher | Okay. Ted: What’s your question for her? |
| Student | You did not. |
| Teacher | Why did (another student) who says something to her from across the room. |
| Teacher | Go ahead, it’s your turn. |
| Teacher | Why did you pick one-seventh? |
| Teacher | Left the table to answer now. That was a very good question. |
Teacher: Who would like to try to explain what you think the answer is? And show us your reasoning by coming up to the board?

Teacher: Okay, anyone?

Tian: Playing with hair

Laying on arms

Other children

Teacher: When someone's presenting at the board, what should you be doing?

Students in class

Looking at them.

Teacher: Looking at that person.

Ample: Uh huh?

Teacher: You want me to write it?

Teacher: You're trying to work what number you think is and explain how you figured it out.

Teacher: Listen closely and see what you think about her reasoning and her answer. (Jomphe writes x/7 on the orange one.)

Ample: I put one seventh because there's one.

Tian: Is this one? Is it because?

Ample: (Blinks to Jomphe) (Continues to write) Because there's seven equal parts, two, one, two, three, four, five, six, and their seven. (Gives Jomphe a piece to count the parts of the number one.)

Teacher: Before you agree or disagree, I want you to ask questions if there's something you don't understand about what she did. No agreeing and disagreeing. Just, if you can do right now is ask ample questions. Who has a question for her?

Teacher: Okay, Tian. What's your question for her?

Tian: Did you?

Dante: (Laughs at another student who says something to her from across the room)

Teacher: Go ahead, it's your turn.

Tian: Why did you pick one-seventh?

Dante: You did not.

Teacher: Let's listen to her answer. That was a very good question.
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<td>Who would like to try to explain what you think the answer is? And show us your reasoning by coming up to the board. If you'd like to come up to the board and try to tell And you know, it might not be right. That's okay because we're learning something new. I'd love to see someone do it and sort of be the teacher and explain how you are thinking about it. Who'd like to try that this evening?</td>
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<td><strong>Teacher</strong></td>
<td>Okay. Anyone?</td>
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<tr>
<td><strong>Student A</strong></td>
<td>Playing with hair</td>
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<tr>
<td><strong>Student B</strong></td>
<td>Laying on arms</td>
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<td><strong>Teacher</strong></td>
<td><em>Where is everyone presenting at the board, what should you be doing?</em></td>
</tr>
<tr>
<td><strong>Students in chair</strong></td>
<td>Looking at them.</td>
</tr>
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<td><strong>Teacher</strong></td>
<td>Looking at that person.</td>
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<td><strong>Teacher</strong></td>
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<td><strong>Annie</strong></td>
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<td><strong>Annie</strong></td>
<td><em>Still the same one seventh</em></td>
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<td><strong>Teacher</strong></td>
<td><em>Scans the board. Continue to scan. Because there's seven equal parts, into one, two, three, four, five, six, and their seven. Uses her fingers to count the parts of the number one.</em></td>
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<td><strong>Teacher</strong></td>
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Teaching is dense with “discretionary spaces.”
What regularly fills the discretionary spaces in teaching?

1. Teachers’ experiences in the broader society.
2. Normalized practices in schools that institutionalize dominant values and habits.

Lortie (1975), Banks, Grant and Koskela, Moll Anyon (1981), Heath, Martin, Tuck Ball (2018)
Discretionary spaces are ubiquitous in the communicating work of teaching.
Teaching is a tangle of moral, intellectual, political, social, and personal communicating work.
Let’s look more closely at the discretionary work involved in communicating in teaching, and its power.
2. The story of Aniyah and Toni—and their classmates—and the communicating work for the teacher in supporting a case of “meta-level learning.”
BEFORE THIS DAY

What fraction of each rectangle below is shaded gray?

What fraction of the rectangle below is shaded orange?

What fraction of the rectangle below is shaded blue?
BEFORE THIS DAY

What fraction of each rectangle below is shaded gray?

What fraction of the rectangle below is shaded orange?

What fraction of the rectangle below is shaded blue?
BEFORE THIS DAY

What fraction of each rectangle below is shaded gray?

What fraction of the rectangle below is shaded orange?

What fraction of the rectangle below is shaded blue?

5. Another student says that \( \frac{8}{8} \) is greater than \( \frac{3}{3} \). The student explains that \( \frac{8}{8} \) is greater than \( \frac{3}{3} \) because 8 is greater than 3.

Do you agree or disagree with this student? Explain why.
Moving from the part-whole world of area and discrete models to the continuous world of the number line
ON THIS DAY . . .

Moving from the part-whole world of area and discrete models to the continuous world of the number line

What is discretionary work of communicating in teaching?
ON THIS DAY . . .

Moving from the part-whole world of area and discrete models to the continuous world of the number line

- Choosing a mathematical task to provoke the need

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What number does the orange arrow point to? Explain how you figured it out.
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What is discretionary work of communicating in teaching?
ON THIS DAY . . .

Moving from the part-whole world of area and discrete models to the continuous world of the number line

- Choosing a mathematical task to provoke the need
- Extending the necessary norms of reasoning

Steps for Naming a Fraction Correctly

1. Figure out what the whole is.
2. Figure out if the whole is divided into equal parts. If not, make equal parts.
3. Count how many equal parts there are.
4. Write \( \frac{1}{d} \) to show one of the equal parts. This is a unit fraction.
5. If more than one of those parts is shaded, count them (n) and write \( \frac{n}{d} \).

\( a \neq d \neq 0 \). \( d \) is a whole number now.
VIDEO: ANIYAH

This video and additional supporting materials are available online here.
What number does the orange arrow point to? \( \frac{1}{3} \)

Explain how you know: "Because it's in

\[ \text{3 parts} \]
What number does the orange arrow point to? \( \frac{1}{3} \)

Explain how you know: Because it's in \( \frac{1}{3} \) parts.

This video and additional supporting materials are available online here.
VIDEO: TONI AND ANIYAH

Toni: Why did you pick one-seventh?
Student: You did not.

This video and additional supporting materials are available online here.
What might a teacher hear and see about the two girls?

ANIYAH

- She has the wrong answer: $\frac{1}{7}$

TONI

- She is playing with her hair and trying to get attention
- She is trying to embarrass Aniyah
In this single discretionary moment, a communicating move could reproduce familiar patterns —

- of marginalization of Black girls.
- of reductionist views of mathematics.
Looking at the micro through the macro: Black girls’ rates of suspension in the U.S.

Epstein, Blake, & González (2017)
FOR ANIYAH —-

NORMALIZED NEXT MOVES

- “Can someone help Aniyah out and show what we call the whole on the number line?”

- “Thumbs up if you agree with Aniyah; thumbs down if you disagree.”

- “What do others think?”

RESULTS

- Aniyah’s answer is signaled to be incorrect and she is positioned as not having contributed to the work.

- Aniyah’s solution is “voted” on by her classmates.

- Aniyah is excluded and her mathematical point is sidelined.
AND FOR TONI —

NORMALIZED NEXT MOVES

- “Toni, when you’re ready to participate appropriately by not playing with your hair and laughing, and have a question to ask, I will come back to you.”
- “You need to be a better listener, Toni. Aniyah already explained why she picked one-seventh. Who else has a real question for Aniyah?”
- “What do others think?”

RESULTS

- Toni is publicly excluded from the discussion.
- Toni is judged to not be listening, her question is judged as not good, and she is excluded from the discussion.
- Toni is excluded and her mathematical point is sidelined.
What would it take to disrupt these patterns of marginalization?

- Seeing Aniyah’s solution as mathematically robust and Toni’s question as key to the class’s work
- Taking as axiomatic the brilliance of Black children, and thus Toni and Aniyah

(Knowing and using mathematics in teaching (MKT)
Interpreting Toni as asking a mathematical question that she means

IN THE WORK OF TEACHING —

Communicating signals who and what is valued.

Communicating either reinforces or disrupts patterns of marginalization and reductive views of mathematics.
IN THE WORK OF TEACHING, TOO —

Communicating supports students to work across differences toward common ground.
LAKEYA

What number does the orange arrow point to? 2

Explain how you know: Because there are equal parts and you're pointing to the second one counts 2.

JAMARI

What number does the orange arrow point to? 5

Explain how you know: First, I thought it was 5 because the zero messed me up.

MARIANA

What number does the orange arrow point to? 1

Explain how you know: How I know it's zero is that there is an interval from one to a zero there was 2 line between 0 and 1.

LARRY

What number does the orange arrow point to? 4

Explain how you know: I count it by 4 and keep going till I got a whole.

Write a complete sentence with one goal for yourself for our math class today. Give an example of what it looks like to do this really well.

Learn more about the number line.
VIDEO: LAKEYA AND ANIYAH

Lakeya: If you start at the zero, how did you get one-seventh?
VIDEO: DANTE AND ANIYAH

What if the orange line wasn't there, and you had to put it where the one is?
Katherine: I think it's one-fourth because this is one whole and there's one, two, three- and there's four equal parts.
MOVING FROM THE PART-WHOLE WORLD TO THE CONTINUOUS WORLD OF THE NUMBER LINE

Aniyah: count intervals on the number line
Toni: determining the denominator on the number line
Lakeya: determining the numerator on the number line
Katherine and Dante: unit interval is 0 to 1
IN TEACHING, COMMUNICATING COORDINATES ACROSS DIFFERENCE TO CREATE COMMON GROUND

- Posing a strategic task
- Eliciting and probing students’ thinking
  - “Tell us one more time how you got one-fourth.”
  - “Did you change your mind?”
- Orchestrating who has the floor and what discourse moves are next
  - “Who has a question for her?”
  - “No agreeing or disagreeing right now. All you can do is ask her a question.”
- Orienting students to one another’s ideas
  - “Talk to her.”
  - “Can everyone be listening to her and make sure you understand her reasoning?”
- Focusing attention on key mathematical ideas or issues
  - Recording different students’ numbers on the board for reference
What number does the orange arrow point to?

3, \( \frac{1}{3} \), \( \frac{2}{3} \), \( \frac{3}{3} \), \( \frac{4}{3} \), \( \frac{5}{3} \), \( \frac{6}{3} \), \( \frac{7}{3} \), \( \frac{8}{3} \), \( \frac{9}{3} \), \( \frac{10}{3} \)

Explain how you know:

First I thought it was \( \frac{5}{3} \) because the zero messes me up.
VIDEO: JAMARI

Jamari: Uh-huh. I think it's one-third because I did not count the zero, and I also counted like one, two, three.
COMMUNICATING TO CREATE COMMON GROUND

ELICITING LEARNERS’ IDEAS

- Can you show us the 1, 2, 3 that you’re looking at?
- And then what? How did you decide it was one-third?
COMMUNICATING TO CREATE COMMON GROUND

ELICITING LEARNERS’ IDEAS

- Can you show us the 1, 2, 3 that you’re looking at?
- And then what? How did you decide it was one-third?

SUPPORTING LEARNERS TO PROBE ONE ANOTHER’S’ IDEAS

- Who has a question for Jamari?
COMMUNICATING TO CREATE COMMON GROUND

ELICITING LEARNERS’ IDEAS

- Can you show us the 1, 2, 3 that you’re looking at?
- And then what? How did you decide it was one-third?

SUPPORTING LEARNERS TO PROBE ONE ANOTHER’S’ IDEAS

- Who has a question for Jamari?
- How did you know to skip the zero?
COMMUNICATING TO CREATE COMMON GROUND

ELICITING LEARNERS’ IDEAS

- Can you show us the 1, 2, 3 that you’re looking at?
- And then what? How did you decide it was one-third?

SUPPORTING LEARNERS TO PROBE ONE ANOTHER’S’ IDEAS

- Who has a question for Jamari?

- How did you know to skip the zero?

“On the steps for naming a fraction, it says that D cannot equal 0.”
COMMUNICATING TO CREATE COMMON GROUND

ELICITING LEARNERS’ IDEAS

- Can you show us the 1, 2, 3 that you’re looking at?
- And then what? How did you decide it was one-third?

HEARING LEARNERS’ PERSPECTIVES

SUPPORTING LEARNERS TO PROBE ONE ANOTHER’S’ IDEAS

- Who has a question for Jamari?
- How did you know to skip the zero?

“On the steps for naming a fraction, it says that D cannot equal 0.”
Aniyah: count intervals on the number line
Toni: determining the denominator on the number line
Lakeya: determining the numerator on the number line
Katherine and Dante: unit interval is 0 to 1
Aniyah: count intervals on the number line
Toni: determining the denominator on the number line
Lakeya: determining the numerator on the number line
Katherine and Dante: unit interval is 0 to 1
Jamari: one-third
COMMUNICATING AS SIGNALING WHO AND WHAT IS VALUED

VALUING

Mathematical competence as—
- Extending definitions
- Revising norms of reasoning
- Asking questions
- Explaining one’s thinking
- Learning from others

- Aniyah, Toni, Lakeya, Dante, Katherine, Jamari
  - as individuals
  - as Black and Brown girls and boys

DISRUPTING COMMON PATTERNS OF VALUE

- Mathematical competence as speed and correct answers
- Deficit views of Black and Brown children
- Patterns of punishment and exclusion of children of color
3. So what have we learned?
How do Anna’s ideas about discourse, thinking, mathematics, and communicating inform understanding the work of teaching as communicating to support learning?
In the discretionary spaces inherent in the work of teaching, communicating moves:

1. Create and build common ground
2. Coordinate and work across difference
3. Signal who and what is valued
COLLEAGUESHIP AS COMMUNICATING

Finding common ground

Working across difference

Valuing and encouraging
Slides will be available on my website
https://deborahloewenbergball.com/
CREDITS

Data on slide 35:

Image on slide 61: