A Model of Credit, Money, Interest, and Prices*

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Abstract

This paper integrates a realistic implementation of monetary policy through the banking system into an incomplete-markets economy with wage rigidity. Monetary policy sets policy rates and alters the supply of reserves. These tools grant independent control over credit spreads and an interest target. Through these tools, monetary policy affects the evolution of real interests rates, credit, output, and the wealth distribution—both in the long and in the short run. We decompose the effects through a combination of the interest and credit channels that depend on the size of the central bank's balance sheet. Monetary policy reaches an expansionary limit when it enters a liquidity trap. The model highlights a trade-off between worse microeconomic insurance (insurance across agents) and greater macroeconomic insurance (insurance across states). The model prescribes that monetary policy should operate with a small balance sheet which tightens credit during booms, and should expand its balance sheet and lower policy rates during busts.

Keywords: Monetary Economics, Monetary Policy, Credit Channel.

JEL: E31-2, E41-4, E52-2

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1 Introduction

In modern economies, monetary policy (MP) operates through the provision of reserves and a corridor of policy rates.[1] A popular view amongst academics is that these tools implement a desired nominal interest rate and, ultimately, this is solely what matters for MP (Woodford, 1998). A view held by practitioners has is that there is more to MP and that the central bank's (CB) balance sheet is important in and of itself, because the supply of reserves impacts real activity through its influence on credit (Bernanke and Blinder, 1988, 1992). Indeed, the heads of the major central banks boldly expanded their balance sheets during the last crises—see Figure 1—acting upon that view, but they did so instinctively, without the backing of a theoretical framework. Today, central bankers debate whether it desirable to permanently operate with large balance sheets (Bindseil, 2016).

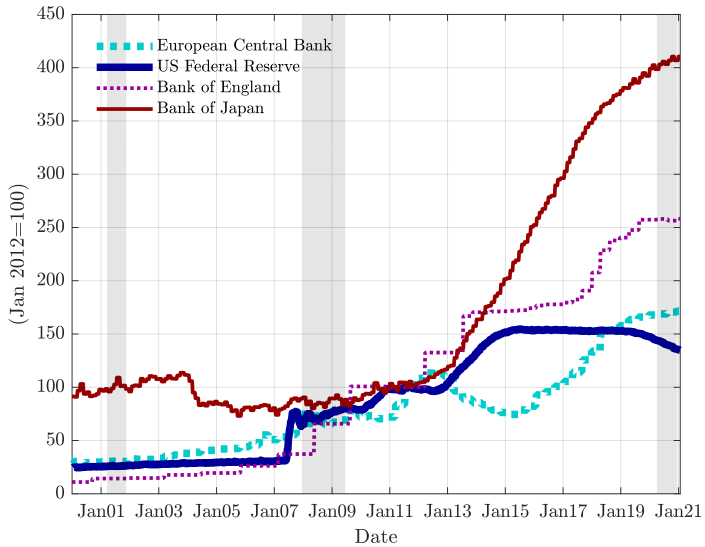


Figure 1: Total Asset Holdings of Major Central Banks

Although the bank-centric view held by practitioners has ample empirical support (e.g., Kashyap and Stein, 2000), its theoretical foundations are still a work in progress. Given the ongoing current policy debates, building those theoretical foundations seems ever more important. This paper studies an economy where credit is intermediated by banks that face settlement frictions. By supplying reserves and by setting policy rates, the central bank can lever on these frictions to influence multiple interest rates. We study the effects of these instruments in the context of an incomplete-markets economy with aggregate-demand externalities.

The paper has two goals. Our first goal is to conduct a positive analysis, namely, to articulate how

[1] A corridor of policy rates is defined by discount rate and an interest rate on reserves. The discount rate is the rate at which a central bank lends reserves. The interest rate on reserves is the rate at which banks are remunerated for holding reserve balances at the central bank. These two rates form a "corridor" that typically contains the interbank market rate in the middle.

the supply of reserves and the choice of policy rates affect credit, interest rates, inflation, and economic activity. In this economy, the rate on reserves sets a floor for all rates, and thus, grants the CB direct control over inflation. Given the interest on reserves, the supply of reserves, which is altered via open-market operations (OMO), grants control over a credit spread between borrowers and savers. The control over inflation relates to well-traveled transmission channels—the interest-rate and inflation-tax channels—whereas the control over spreads is a notion of a credit channel.[2]

MP operates in three possible regimes. The paper clarifies that the effects of the supply of reserves and policy rates depend on the MP regime. In one regime, reserves are scarce for banks and the CB balance sheet is lean. In this regime, the CB operates a *corridor system* with a positive credit spread. Policy rates and OMO carry effects through both a standard interest-rate channel and through the credit channel. In a second regime, reserves are abundant and the CB balance sheet is fat. In this regime, MP operates a *floor system*, in which the spread is zero, OMO are neutral, and only the interest-rate channel is operational. The third regime is activated when the interest on reserves (IOR) is negative and reserves are so ample that the equilibrium deposit rate is zero, hitting a *deposit-zero lower bound* (DZLB). At that point, OMO are irrelevant and reserves are transformed into currency, a notion of a liquidity trap. In that *liquidity trap regime*, reductions in the IOR work backwards by increasing the loans rate, an empirically verified phenomenon (Heider, Saidi and Schepens, 2019; Eggertsson, Juelsrud, Summers and Wold, 2019). Thus, the expansionary power of MP reaches its limit in a liquidity trap regime.

The second goal is to conduct a normative analysis, that is, to prescribe guidelines for an ideal MP. A policy insight that emerges from the paper is that the CB should operate with a lean balance sheet during booms, operating in a corridor system, but expand its balance sheet, operating in a floor system, during busts. This recommendation results from a complex policy tradeoff: In a corridor system, limiting the supply of reserves induces a credit spread. A spread harms microeconomic insurance—the insurance of idiosyncratic income risk. However, a wider spread tightens the amount of credit which increases macro insurance—insurance against the impact of aggregate shocks. Namely, with tighter credit, macroeconomic insurance increases because both the stabilization power of MP is greater and aggregate demand is less responsive to an aggregate shock.

The policy trade-off we highlight is at the core of historical and recurrent debates on what an ideal conduct of MP should look like—as early as Bagehot (1873) and as recently as (Stein, 2018). Throughout history, during booms, it resonates that MP *is sowing the seeds of crises,* whereas during busts, that MP is *pushing on a string.* This paper articulates these views. We contend that a permanent transition to large CB balance sheets surrenders an important policy objective, macroeconomic insurance.

We build this case by exhibiting a model with the following features: Households face undiversified idiosyncratic employment risk, as in Huggett (1993). This is the source of micro insurance. In turn, wage rigidity is the source of an aggregate demand externality. The novelty of the environment is the

[2] A narrative description of different transmission channels of MP is found in Bernanke and Gertler (1995).

introduction of intermediation and money, an aspect that enables MP to control the credit spread. In particular, credit is intermediated by a fringe of competitive banks that issue deposits, make loans, and hold reserves. The power to influence spreads stems from an institutional feature: Whereas loans are permanently held by the issuer bank, deposits circulate. Reserves are used to settle deposit transfers. A potential shortage of reserves by some banks opens the door for interbank credit that operates with frictions (á la Ashcraft and Duffie, 2007; Afonso and Lagos, 2015). As a result, not all reserves deficits can be tapped with interbank credit and some deficits are borrowed at a penalty rate. That cost translates into the credit spread. A greater volume of reserves eases the settlement frictions, which translates into a reduction in spreads. By letting the size of its balance sheet be part of its toolkit, the CB can exploit these frictions to bring the control of spreads into a set of policy targets.

The paper delves into the details of implementation: First, the paper presents closed-form expressions for nominal deposits and loan interest rates. The interest on reserves acts as a floor for both deposits and loan rates, which carry different liquidity premia relative to the IOR. The spread is the difference between these liquidity premia. Thus, the spread is expressed as a function of the CB balance sheet size, relative to the volume of private sector credit.[3]

After detailing the implementation, the paper conducts the positive analysis. Equilibrium in the goods and asset markets is summarized by a single market-clearing condition for real credit that pins down a real interest rate. An important feature is that that clearing condition is affected by the spread, which is ultimately under the control of the CB.

MP has effects both in the short run and in the long run. In the long run, holding a spread as fixed, changes in nominal rates are neutral. However, through the size of its balance sheet, the CB influences spreads and, thus, MP can influence the long-run real interest rate. Along a transition, MP works differently. The dynamics of unemployment and wage rigidity follow a modern incarnation of Barro and Grossman (1971). Due to this nominal rigidity, the real rate is pinned down by the IOR. The variable that adjusts to clear goods market is the unemployment rate. Hence, in the short run, reductions in policy rates operate through the standard interest-rate channel and are expansionary unless MP enters a liquidity trap. Two properties are worth noting. First, the effects of the interest-rate channel are enhanced through the credit channel. Second, the greater the initial spread, i.e., the smaller the CB balance sheet, the greater the power of MP. The intuition behind both properties is that borrowers are less interest rate sensitive than lenders. Hence, when credit is limited by wider spreads, the representative household is more sensitive to changes in interest rates and spreads, rendering both the interest rate and credit channels more powerful.

The final section turns to the normative analysis. We claim that the optimal CB balance sheet size is

[3] A similar implementation appears in Bianchi and Bigio (2020), and in related models (Piazzesi and Schneider, 2018; De Fiore, Hoerova and Uhlig, 2018; Chen, Ren and Zha, 2018; Drechsler, Savov and Schnabl, 2017). Here, bank decisions are simplified, and the pass-through from MP to a target is immediate. Instead, the emphasis is on the responses outside the banking sector.

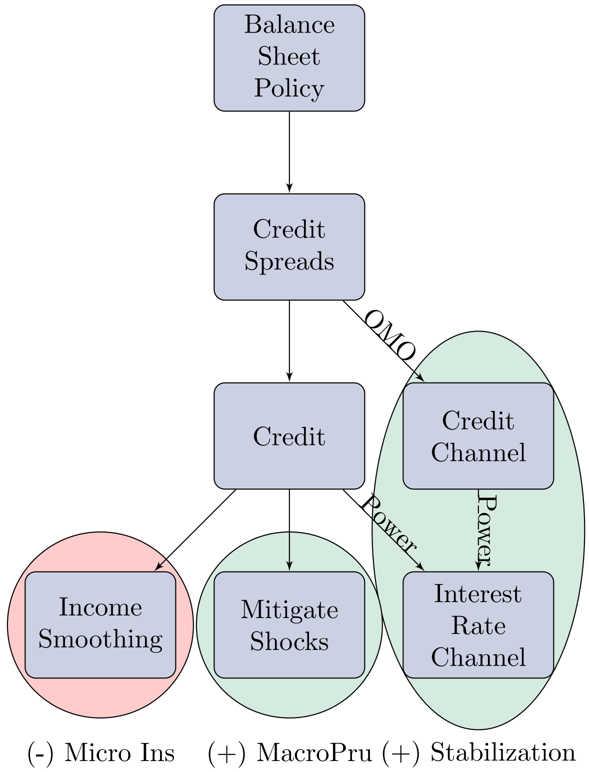


Figure 2: Flow Diagram of Forces in the Model.

governed by a trade-off between greater macro insurance and worse micro insurance, a notion that, although it is costly in terms of financial efficiency, MP should spare "gunpowder" for the future. To showcase this trade-off, the paper studies the problem of an egalitarian CB that expects the economy to suffer an aggregate credit crunch. Upon a credit crunch, MP should lower the IOR and expand the CB balance sheet, up to the point where it triggers a liquidity trap. Prior to the credit crunch, the CB should induce a positive spread by limiting the supply of reserves. Limiting the supply of reserves enhances the stabilization power of MP and mitigates the impact of the credit crunch. However, MP should balance this benefit against the micro-insurance cost. We present a welfare decomposition that clarifies the sources of this trade off.

The message of the paper can be summarized through the flow chart in Figure 1. The CB balance sheet policy has the ability to affect credit spreads—represented in the first flow. Credit spreads, altered through OMO, have direct effects through the credit channel—the flow labeled OMO in the figure. Also, by affecting credit, spreads can enhance the effects of the interest-rate channel, and increase the stabilization power of MP—the flow labeled Power in the figure. As well, by tightening credit, MP can mitigate shocks, a standalone macro-prudential effect. The stabilization power and the macro-prudential benefits contribute to overall macro insurance. The tradeoff is that limiting credit is linked to worse micro insurance.

Connection with the Literature This paper's title emphasizes a connection with the two most common frameworks for MP analysis.[4] One approach emphasizes the relation between *money and prices* and the other between *interest and prices*. In the money and prices approach, money plays a a role as a means of payments (Lucas and Stokey, 1987; Lagos and Wright, 2005) and there is a tight connection between prices and the quantity of (outside) money. The real rate is fixed, so real effects follow because inflation is a transaction tax. The second approach is the new-Keynesian approach where the important connection is between *interest and prices*. Under that framework, MP controls real rates directly because prices are rigid. Neither framework emphasizes the effect of MP on credit, at least not directly. The model here establishes a meaningful connection between *credit, money, interest and prices*. Because the credit channel here can be studied independently of the control of inflation, it only complements the inflation-tax or interest-rate channels germane to these approaches.

Since 2008, there's been an interest in how MP influences credit markets. That gap is being filled, and incomplete market models are a natural starting point. In fact, a first generation of heterogeneous agent models, Lucas (1980) and Bewley (1983) were all about MP, and not about heterogeneity per se. However, neither model established how MP affects credit. Credit, of course, has a tradition in heterogeneous agent models (see the early work of Huggett, 1993; Aiyagari, 1994), but the literature evolved abstracting away from its initial interest in MP.

A recent generation of works has introduced nominal rigidities into Bewley-Lucas models. Guerrieri and Lorenzoni (2017) studies the tightening of debt limits in such an economy.[5] These models are appealing because, as an artifact of incomplete markets, MP responses depend on the distribution of wealth. Auclert (2019) decomposes the responses to policy into different forces in those models. Kaplan, Moll and Violante (2018) introduce illiquid assets which produce high-income elasticities among the wealthy, a feature that modifies the propagation mechanics. Werning (2015) and Bilbiie (2020) provide conditions for aggregate demand amplification. In all these studies, MP operates through the interest rate channel, as in the single agent new-Keynesian model. Here, MP operates in tandem with the credit channel, and this matters for stabilization.[6] Of lesser substance, a different feature here is that we model unemployment and nominal rigidities following Barro and Grossman (1971). The insights on how the distribution of wealth affects the power of MP follow from this body of work. The contribution here is to connect the money supply to credit markets and explain why that matters.

Another stream of recent work in the *money and prices* tradition introduces credit to models where

[4]The title of the paper is reminiscent of a sequence of titles. Don Patinkin added Money to the title "Interest and Prices" to a classic book by Knut Wicksell. Michael Woodford took Money from Patinkin's title, promoting the view that MP can be studied without reference to the money supply. Like much of the work we survey below, we contend that the money supply impacts credit markets and that this matters. Emmanuel Farhi taught us the connection with this sequence of titles.

[5]Following up on that work, McKay, Nakamura and Steinsson (2016) compare the effects of forward-guidance policies.

[6]Partial equilibrium models, Greenwald (2018) and Wong (2019) study interest rate sensitivities to mortgage refinancing options. Our paper sees the interest on mortgages as a separate policy target from the savings rate.

money plays a transactions role. When credit is an imperfect substitute for money, inflation can affect the supply and terms of trade of credit (see for example Berentsen, Camera and Waller, 2007; Williamson, 2012; Gu, Mattesini, Monnet and Wright, 2013; Gu, Mattesini and Wright, 2016; Rocheteau, Wright and Zhang, 2018). Rocheteau, Wong and Weill (2016) bring insights from money-search into an incomplete-market economy like the one here. Allowing for long-term debt, Gomes, Jermann and Schmid (2016) model how MP has effects through debt deflation. Nuno and Thomas (2020) import debt-deflation into an incomplete markets economy with nominal rigidities.

The credit channel here is not new. The MP implementation is inherited from Bianchi and Bigio (2020). The focus of that paper is to introduce the transmission mechanism we employ here with a focus on bank decisions. Here, the banking side is simplified, but the nonfinancial sector is much richer, due to incomplete markets and price rigidity. Piazzesi and Schneider (2018) feature a similar implementation of MP, but their focus is on asset pricing. The model here also shares common elements with Brunnermeier and Sannikov (2012) who study the value of reserves in a Bewley-like economy with aggregate shocks.[7] Benigno and Robatto (2019) study a model where intermediaries issue debt and risky equity in a new Keynesian model. Another close paper is Piazzesi, Rogers and Schneider (2019) which discusses the role of floor and corridor regimes in a New-Keynesian environment. The main distinction is our focus on how incomplete markets which is key to the normative message. On that point, the combination of incomplete markets, nominal rigidity, and a lower bound on policy rates leaves room for normative analysis. This point is established in Farhi and Werning (2016) and Korinek and Simsek (2016) who study demand-driven recessions (as in Eggertsson and Krugman, 2012). Both papers promote the use of debt limits as macro-prudential tools. Here, we argue that MP has enough tools to conduct that countercyclical policy with its balance sheet.

On the normative front, Nuno and Thomas (2020) and Bhandari, Evans, Golosov and Sargent (2020) are among the first papers to study optimal MP under incomplete markets. In Nuno and Thomas (2020), the focus is on inflation as a redistributive channel. In Bhandari et al. (2020), MP balances aggregate demand stabilization against insurance considerations, with a focus on a single instrument. Historically, financial stability has been conceived as a crucial element of MP, as noted in Stein (2012), for example. The normative message here, that MP should actively target spreads, is controversial. Curdia and Woodford (2016) and Arce, Nuno, Thaler and Thomas (2019) suggests that a floor system is ideal. Instead, we take the side of Stein (2012) and Kashyap and Stein (2012) and argue that the control of spreads is crucial for financial stability.

Organization. Section 2 lays out the core model. Section 3 describes the determination of credit, interest and prices and the implementation of MP. Section 4 presents a study on MP regimes. Section 5 studies the benefits of running a corridor system. Section 6 concludes.

[7] Similarly, Lippi, Ragni and Trachter (2015) introduce aggregate shocks into a pure currency economy, and study the optimal helicopter drops. Other related models include Silva (2020) and Buera and Nicolini (2020).

2 Environment

2.1 From Policy Spreads to Credit Spreads

In the model that follows, we embed financial intermediation (by banks) in an environment where money holdings, prices, and rates are determined in general equilibrium. In this introductory section, we present the banking block. We derive a simple formula that maps an MP corridor spread into a real intermediation spread for given monetary aggregates. Later, we show how real spreads determine monetary aggregates, and thus, how the CB has the ability to control spreads. To aid the presentation of this section, Appendix A presents the individual bank balance sheet, and Figure A presents the model timeline with the corresponding T-accounts.

Banks. There is free entry and perfect competition among banks. We consider the static portfolio decision of a bank within an interval of time Δ—below, we take the limit as $\Delta \to 0$ to embed the banking block to the general equilibrium. Banks are owned by households. Because there are no aggregate shocks during the Δ interval, the bank's objective is to maximize static expected profits. Competition guarantees zero expected bank profits.

At the start of the Δ interval, banks choose their supply of nominal deposits, a, their holdings of nominal loans, l, and reserves, m. The aggregate supply of deposits and loans, and holdings of reserves, are denoted by A^b, L^b, and M^b, respectively. Deposits, loans, and reserves earn corresponding rates i^a, i^l, and i^m. Whereas the loan and deposit rates are equilibrium objects, i^m is a policy instrument.

After the portfolio decision is made, banks face random payment shocks, as in Bianchi and Bigio (2020); Piazzesi and Schneider (2018). In particular, within the interval, payment shocks take one of two values, $\omega \in \{-\delta, +\delta\}$ with $\delta \leq 1$. Each possible value occurs with equal probability and is i.i.d across banks. If $\omega = \delta$, a bank receives δa deposits and is credited δa reserves from other banks. If $\omega = -\delta$, the bank transfers δa deposits and δa is debited to other banks. Naturally, if a bank receives a deposit, it absorbs the liability from another bank. If it loses a deposit, another bank absorbs its liability. As a result of the transfer of liabilities, assets need to be transferred to settle the transactions. A key assumption is that within the Δ time interval, loans are illiquid in the sense that they must stay with banks. Therefore, net deposit flows must be settled with reserves, which are cleared at the CB.

Upon the payment shock to a bank, the net reserve balance of a bank at the CB is:

$$b = m + \min\{\omega, 0\} a.$$

That is, if the bank suffers a withdrawal, its balance at the CB is reduced. If the bank experiences an inflow of deposits, its overnight balance is unchanged, although its balance will increase the next day, after the position settles. Notice that deposits never leave the banking system, but a bank that receives a deposit inflow cannot lend the reserves it is owed.

Since ω is random, the reserve balance is not entirely under the control of a bank. For that reason, it is possible that the bank ends up with a negative balance, $b < 0$, provided a bank starts with insufficient reserves. A bank with a negative balance must close this negative position, either by borrowing reserves from banks with a surplus or borrowing from the CB.

Interbank Market. After reserve positions are determined, an interbank market opens and banks borrow and lend reserves to each other. For a balance b, a fraction of those balances are lent (or borrowed, if negative) in the interbank market. In particular, if a bank has a surplus b, it lends the fraction ψ^+ to other banks and, hence, $b - \psi^+ b$ remains idle in a CB account. If the bank has a deficit, $-b$, it borrows only the fraction ψ^- from other banks, and the remainder deficit, $-(b - \psi^- b)$, is borrowed from the CB at a discount window rate i^{dw}. The discount rate is also a policy choice. By convention, borrowed reserves from the CB earn the interest on reserves i^m. Thus, the effective borrowing cost is the policy spread $\iota \equiv i^{dw} - i^m$. The trading probabilities $\{\psi^+, \psi^-\}$ are meant to capture trading frictions in the interbank market. For the rest of the paper, we keep ι fixed.

Integrating b across banks yields expressions for the aggregate deficit and the aggregate surplus balances:

$$B^- \equiv \frac{1}{2} \max\left\{\delta A^b - M^b, 0\right\} \quad \text{and} \quad B^+ \equiv \frac{1}{2}\left(M^b + \max\left\{M^b - \delta A^b, 0\right\}\right).$$

Clearing in the interbank market requires that the total amount of reserve balances lent is equal to the amount borrowed,

$$\psi^- B^- = \psi^+ B^+. \tag{1}$$

Trading frictions, a well-documented empirical feature (see Ashcraft and Duffie, 2007; Afonso and Lagos, 2014), are key in the model to have a pass-through from policy to credit spreads. There are many ways to induce trading frictions. Here, we assume that the interbank market is an over-the-counter (OTC) market in the spirit of Afonso and Lagos (2015), but we adopt the formulation in Bianchi and Bigio (2017) that renders analytic expressions. The interbank market works as follows: The market operates in a sequence of n trading rounds. Given the initial positions $\left\{B_0^-, B_0^+\right\} \equiv \{B^-, B^+\}$, surplus and deficit positions are matched randomly. When a match is formed between two banks, they agree on an interbank market rate for the transaction. The remaining surplus and deficit positions define a new balance, $\left\{B_1^-, B_1^+\right\}$. New matches are formed, and a new interbank market rate emerges. The process is repeated n times, defining a sequence $\left\{B_j^-, B_j^+\right\}_{j \in 1:n}$ until a final round is reached. Whatever deficit remains is then borrowed from the CB at a cost given by ι.

The interbank market rate of a given trading round is determined by a bargaining problem in which banks take into consideration the matching probabilities and trading terms of future rounds. This produces an endogenous average interbank rate, $\bar{i}^f$. Given trading probabilities, the policy rates and the average rate $\bar{i}^f$, the average rates earned on negative and positive positions are respectively:

$$\chi^- = \psi^- \left(\bar{i}^f - i^m\right) + (1 - \psi^-) \cdot \iota, \text{ and } \chi^+ = \psi^+ \left(\bar{i}^f - i^m\right).$$

Banks take into account these costs and benefits when forming their portfolios. To express $\{\chi^-, \chi^+\}$, Bianchi and Bigio (2017) assume that matches are formed on a per-position basis and according to a Leontief matching technology, $\frac{\lambda}{n} \min\left\{B_j^-, B_j^+\right\}$, where λ captures the trading efficiency. Let $\theta = B^-/B^+$ define an initial interbank "market tightness." If $\theta \leq 1$, in the limit $n \to \infty$, trading probabilities across all trading rounds, $\{\psi^+, \psi^-\}$, converge to $\psi^+(\theta) = \theta(1 - \exp(-\lambda))$ and $\psi^-(\theta) = 1 - \exp(-\lambda)$, two expressions consistent with market clearing. With equal bargaining power, the average interbank market rate $\bar{i}^f$ solves

$$\bar{i}^f(\theta, i^m) - i^m = \iota \cdot \frac{\left((\theta + (1-\theta)\exp(\lambda))^{1/2} - 1\right)}{(1-\theta)(\exp(\lambda) - 1)}, \ \theta \leq 1, \tag{2}$$

The corresponding expressions for the average cost functions are found in Appendix B. These coefficients are independent of i^m and only depend on the total gains from trade, $\iota = i^{dw} - i^m$. Of course, i^m affects the direct return of holding reserves. If the CB has the ability to control χ, it will have control over credit spreads.

The interbank market satisfies a symmetry. When $\theta \geq 1$, the limit $n \to \infty$, produces $\{\psi^+, \psi^-\} = \left\{1 - \exp(-\lambda), \theta^{-1}(1 - \exp(-\lambda))\right\}$ and the interbank market

$$\bar{i}^f(\theta, i^m) - i^m = \iota - \bar{i}^f\left(\theta^{-1}, i^m\right), \ \theta \geq 1.$$

The Bank Problem. We turn to the bank's optimal portfolio. The average benefit (cost) of an excess (deficit) reserve balance, b, is:

$$\chi(b; \theta, \iota) = \begin{cases} \chi^-(\theta)\, b & \text{if } b \leq 0 \\ \chi^+(\theta)\, b & \text{if } b > 0 \end{cases}. \tag{3}$$

We label χ the *liquidity yield* function. With this function, we are ready to present the bank's problem:

Problem 1 *[Bank's Problem] A bank maximizes its instantaneous expected profits:*

$$\Pi^b = \max_{\{l,m,a\} \in \mathbb{R}^3_+} i^l \cdot l + i^m \cdot m - i^a \cdot a + \mathbb{E}\left[\chi(b; \theta, \iota)\right]$$

subject to the budget constraint $l + m = a$ where the distribution of reserve balances is:

$$b(a,m) = \begin{cases} m \text{ with probability } 1/2 \\ m - \delta \cdot a \text{ with probability } 1/2 \end{cases}.$$

At the individual level, the bank objective is piece-wise linear and, in particular, linear along a ray in the $\{m,a\}$-space. As in any model with linear firms, banks must earn zero (expected) profits in equilibrium, otherwise they would make infinite profits. Furthermore, at the individual level, banks will be indifferent among different portfolios, within a cone in the $\{m,a\}$-space. However, at the aggregate level, the ratio of reserves to deposits will pin down the $\{i^l, i^m\}$. This feature is similar to what occurs with competitive firms that operate a Cobb-Douglas production technology with two inputs—whereas firms earn zero profits and the individual scale is indeterminate, the ratio of inputs pins down relative prices.

Equilibrium Credit Spreads. Next, we explain how a ratio of monetary aggregates determines the equilibrium loan and deposit rates. To that end, we define the aggregate *liquidity ratio* as $\Lambda \equiv M^b/A^b$, which corresponds to the inverse of the money multiplier.[8] The interbank market tightness can be expressed in terms of this ratio:

$$\theta(\Lambda) \equiv \max\left\{\frac{\delta}{\Lambda} - 1, 0\right\}. \tag{4}$$

The tightness θ is decreasing in the liquidity ratio because with more liquidity, there is less need to borrow. The tightness decreases with Λ, and satisfies $\lim \theta = \infty$ as $\lim \Lambda \to 0$, that and $\theta = 0$ for any $\Lambda \geq \delta$. If we substitute (4) into (3), we can express χ as a function of the policy corridor, ι, and the liquidity ratio, Λ, and do not depend on the level of $\{M^b, A^b\}$. Then, the linearity of the bank's problem, coupled with a free-entry condition, yield corresponding equilibrium nominal rates and a real spread:

Proposition 1 *[Nominal Rates and Real Spread] Consider an aggregate liquidity ratio $\Lambda < \delta$. Then, for given $\{\Lambda, i^m, \iota\}$, any equilibrium with finite loans and deposits must feature the following nominal loan and deposit rates:*

$$i^l \equiv i^m + \underbrace{\frac{1}{2}\left(\chi^+ + \chi^-\right)}_{\textit{reserve liquidity value}} \quad \textit{and} \tag{5}$$

$$i^a \equiv i^m + \underbrace{\frac{1}{2}\left(\chi^+ + \chi^-\right)}_{\textit{reserve liquidity value}} - \underbrace{\frac{\delta}{2}\chi^-}_{\textit{deposit liquidity risk}}. \tag{6}$$

[8] We avoid the term "money multiplier" employed in textbooks because it is misleading. According to the textbook notion, the money multiplier equals the inverse of the ratio of reserve requirements, but there are no reserve requirements here.

If $\Lambda > \delta$, then, $i^l = i^a = i^m$. In the knife edge case where $\Lambda = \delta$, then any rates that satisfy $i^l - i^a \in \left[0, \frac{\delta}{2}\chi^-(0)\right]$ and $i^a = i^m + \left(i^l - i^a\right) \cdot (1-\delta)/\delta\Delta r$ are a possible solution. In all cases, banks earn zero expected profits.

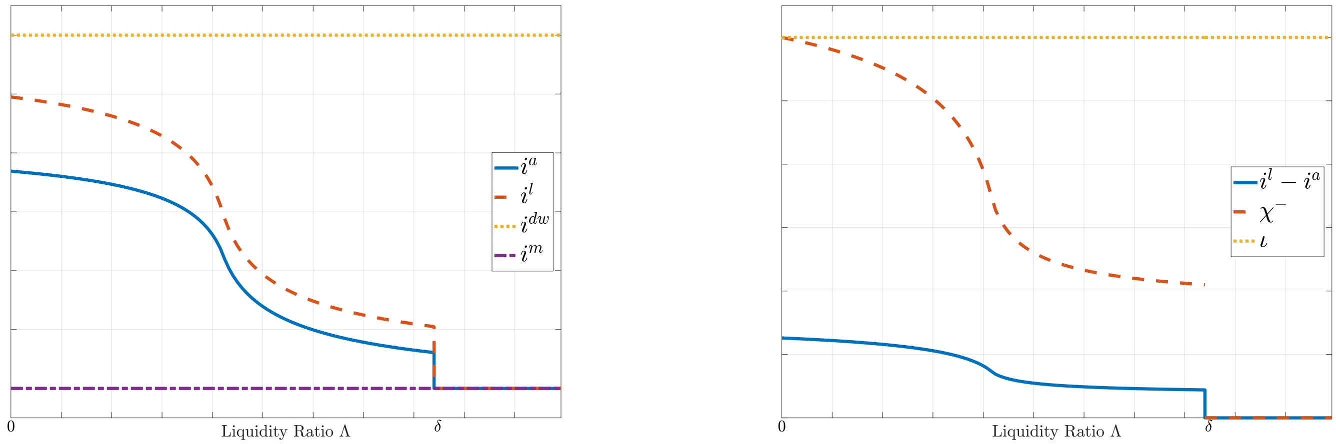
Proposition 1 establishes that the IOR is a basis rate for the nominal borrowing and lending rates, which carry different liquidity premia over the IOR. To understand the formulas, notice that by holding an additional reserve, the bank earns the IOR but also earns the expected liquidity value of reserves. Of the bank ends in surplus, the liquidity value of reserves is the return from lending reserves in the interbank market , χ^+. If the bank ends in deficit, the liquidity value of reserves is that it spares the average cost of borrowing from the interbank market, χ^-. Each scenario occurs with equal probability, so the expected liquidity value of reserves is $\frac{1}{2}\left(\chi^+ + \chi^-\right)$. Loans must earn a premium over reserves because, as an alternative to reserves, their return must compensate the bank for the liquidity value of reserves. In turn, the deposit liquidity premium reflects the liquidity risk of issuing an additional deposit. On the margin, if the bank ends in deficit, an additional deposit increases a reserve deficit by δ, which have a marginal settlement cost of χ^-. Since the probability of ending in deficit is one-half, the risk premium of deposits is $\frac{1}{2}\delta\chi^-$. On the margin, the return on reserves must be equal to the return on deposits. Hence, the deposit liquidity premium equals the liquidity value of reserves minus the liquidity risk of deposits.

The loan deposit spread, a key object for the nonfinancial sector, directly follows from subtracting the deposit rate from the loans rate. The equilibrium credit spread, $i^l - i^a$, is given by,

$$\Delta r = i^l - i^a = \frac{\delta}{2}\chi^-. \tag{7}$$

The spread between two nominal rates is a real object, and thus affects household decisions, regardless of the inflation rate. This credit spread is positive whenever the liquidity ratio is below the amount needed to satisfy the clearing of deficit banks $\Lambda < \delta$, and decreases with the liquidity ratio. Therefore, if the CB can influence that ratio, it will influence real activity. Figure 3 depicts the formulas in Proposition 1 for nominal rates and the spread as functions of Λ, in a region of the space where reserves are scare. The left panel plots $\left\{i^l, \bar{i}^f, i^a\right\}$ as functions of (5) and (6) for fixed policy rates $\{\iota, i^m\}$. Both rates lie in between i^m and i^{dw}. We also see how the credit spread narrows with the liquidity ratio. When reserves are ample, $\Lambda > \delta$, we have that $\{\chi^+, \chi^-\} = 0$. Later, we discuss situations where i^a reaches negative zero, and triggers a DZLB, but for that we need first to present the household's decisions.

The next section embeds bank intermediation into the incomplete markets economy, in the spirit of the early monetary model of Bewley (1983). Before we proceed, we discuss the assumptions encountered so far.



(a) Equilibrium nominal rate given ι and Λ

(b) Equilibrium spread given ι and Λ

Figure 3: Interest Rates and Spread as Functions of Λ

Note: Panel (a) plots the nominal deposit, loan, average interbank rate, and policy rates as functions of Λ; Panel (b) the components of the liquidity yield and the equilibrium spread. The figure is constructed using the calibration presented in Section 4.

Digression: on discount-loan facilities and payment shocks. The discount window rate and the size of payment shocks stand in for missing features. In practice, the cost of reserve shortages can be much larger than the actual discount window rate set by the CB. One reason for this is that discount loans require collateral. If collateral is insufficient, a bank with a negative balance can be intervened (for a related bank model with collateralized discount loans see De Fiore et al., 2018). Another issue is that discount loans can bear a stigma (as in Ennis and Weinberg, 2013). For this reason, the discount window rate in the model should be treated as a much larger cost than the discount rate set by a CB and should not be thought of as being entirely under the control of the CB.

Another feature is that payment shocks are i.i.d. In the data, payment shocks are persistent. To capture the costs of withdrawals, we must increase the size of shocks to compensate for the lack of persistence in the model. Adding persistent shocks would make the model more realistic at the expense of tractability—see Bianchi and Bigio (2020) for a more detailed discussion.

Also, banks here operate without equity. Equity can be introduced into the model with capital requirements or limited capital mobility, features that would produce bank profits. This feature would make equity an aggregate state variable, complicating the message. Likewise, there are no reserve requirements in the model. Since the introduction of sweep accounts, the effective requirement is small, but current bank regulation imposes minimum liquidity requirements and banks may self-impose minimum liquidity requirements to avoid runs. These features are left out of the model.[9]

2.2 General Equilibrium

We now embed intermediation into the general equilibrium model. We take a continuous time limit of the bank's problem. Within a Δ time interval, average profits are $\Delta \cdot \pi^b$—all rates are scaled by Δ

[9]See den Heuvel (2002) for an early model of bank equity capital. Wang (2019) studies the pass-through of MP as a function of the level of bank equity and liquidity requirements.

and the objective is linear. Since bank policy functions are independent of Δ, the equilibrium rates of Proposition 1 also scale with Δ, even as $\Delta \to 0$.[10]

The nonfinancial sector of the economy features a measure-one continuum of heterogeneous households. From their perspective, time is indexed by some $t \in [0, \infty)$. The price of the good in terms of money is P_t. Banks intermediate between borrower households and lender households, but since they make zero profits, they are simply pass-through entities. The CB determines the policy corridor rates, conducts open market operations, and makes/collects (lump sum) transfers/taxes to/from households. Households attempt to smooth idiosyncratic income shocks, via the insurance provided by the intermediation sector.

On a final note, recall that only expected profits, and not realized bank profits, are zero. We assume that households own banks, some of which make profits and some of which make losses, netting out to zero. Thus, the ownership of banks is akin to the ownership of firms with constant returns to scale in other models. Thus, to simplify the model, we abstract from the ownership of banks from now on.

Notation. Individual-level variables are denoted with lowercase letters. Aggregate nominal state variables are denoted with capital letters. Aggregate real variables are written in capital calligraphic font. For example, a_t^h will denote nominal household deposits, A_t^h the aggregate level of deposits, and $\mathcal{A}_t^h$ real household deposits.

Households. Households face a consumption-saving problem. Their preferences are described by:

$$\mathbb{E}\left[\int_0^\infty e^{-\rho t} U\left(c_t\right) dt\right]$$

where $U\left(c_t\right) \equiv \left(c_t^{1-\gamma} - 1\right) / \left(1-\gamma\right)$ is their instantaneous utility. Households receive a flow of real income:

$$dw_t = w_t\left(z\right) dt.$$

This income is the sum of monetary transfers T_t and labor income. Labor income depends on the employment status $z \in \{e, u\}$. If $z = e$, the household is employed and if $z = u$, the household is unemployed. The income of the employed and unemployed are related via

$$w_t\left(e\right) = \left(1 - \tau^l\right) + T_t,\ w_t\left(u\right) = b + T_t$$

[10]The reserve balance b_t, is a random variable. If we were to track b_t as a function of time, this stochastic process would not be well defined. However, treating $b_{t+\Delta}$ as the single realization of the random variable is well defined and so is the limit of the deposit and loans rates.

where b is an exogenous unemployment benefit that measures the degree of exogenous labor market insurance and τ^l is a labor tax that finances the unemployment benefit at steady state. In the expression, we are normalizing the real wage to one. The unemployment benefit b is needed to provide the unemployed with some income.

Households transition from employment to unemployment according to an instantaneous transition probability:

$$\Gamma_t \equiv \begin{bmatrix} \Gamma_t^{eu} \\ \Gamma_t^{ue} \end{bmatrix} = \begin{bmatrix} \nu^{eu} + \phi_t^+ \\ \nu^{ue} - \phi_t^- \end{bmatrix}. \tag{8}$$

Here $\{\nu^{ue}, \nu^{eu}\}$ are fixed parameters that capture natural transition rates and $\phi_t^+ \equiv \max\{\phi_t, 0\}$ and $\phi_t^- \equiv \min\{\phi_t, 0\}$. The variable ϕ_t merits some discussion. This term is an endogenous adjustment that follows from the price rigidity. This feature is a dynamic version of the disequilibrium model of Barro and Grossman (1971). The term adjusts as follows: ϕ_t is positive when there is an excess demand of final goods for a value of $\phi_t = 0$. In turn, ϕ_t is negative when there is an excess supply of final goods for a value of $\phi_t = 0$. Thus, Γ_t captures the endogenous transition rate from state z to state z', where $z \neq z'$. Importantly, the process for ϕ_t is allowed to produce discrete jumps in employment-unemployment upon an unexpected shocks.

Although all financial assets are nominal, the individual state variable, s_t, represents real financial claims. Households store wealth in bank deposits, a_t^h, or as currency, m_t^h, whereas borrowers obtain loans from banks, l_t^h. By convention, $\{a_t^h, m_t^h, l_t^h\} \geq 0$. The real rates of return on deposits and liabilities are $r_t^a \equiv i^a - \dot{P}_t/P_t$ and $r_t^l \equiv i^l - \dot{P}_t/P_t$—currency doesn't earn any nominal interest, so its real return is minus inflation. The law of motion of real wealth follows:

$$ds_t = \left(r_t^a \frac{a_t^h}{P_t} - \frac{\dot{P}_t}{P_t}\frac{m_t^h}{P_t} - r_t^l \frac{l_t^h}{P_t} - c_t + w_t(z) \right) dt, \tag{9}$$

and the balance-sheet identity:

$$\left(a_t^h + m_t^h\right)/P_t = s_t + l_t^h/P_t.$$

From a household's perspective, there is no distinction between holding deposits or currency, beyond their rates of return—there is no transactions demand for money as in the cash-in-advance or money search traditions. Hence, currency is only held when the nominal deposit rate is less than or equal to zero, and both assets yield the same return. Currency is introduced into the model to articulate a DZLB which puts a limit to how expansionary policy can be. Another observation is that households will never hold deposits and loans simultaneously. Combining these observations, we write (9) more succinctly as:

$$ds_t = (r_t(s)\, s - c_t)\, dt + dw_t \text{ where } r_t(s) \equiv \begin{cases} r_t^a \text{ if } s_t > 0 \\ r_t^l \text{ if } s_t \leq 0 \end{cases}. \tag{10}$$

Another important feature is that employment risk cannot be diversified. In particular, credit is limited by a *debt limit* $\bar{s} \leq 0$. This limit determines an absolute lower bound on real debt, $s_t \geq \bar{s}$ where $\bar{s} \leq 0$ is exogenous. Technically, if $s = \bar{s}$, then it must be that $ds_t \geq 0$.

With these features, the household's problem is summarized by the following Hamilton-Jacobi-Bellman (HJB) equation:

Problem 2 *[Household's Problem] The household's value and policy functions are the solutions to:*

$$\rho V(z,s,t) = \max_{\{c\}} U(c) + V_s' \cdot \mu(z,s,t) + \Gamma_t \left[V\left(z',s,t\right) - V(z,s,t)\right] + \dot{V}_t \tag{11}$$

and $\dot{s} \geq 0$ at $s = \bar{s}$ where $\mu(z,s,t) \equiv r_t s - c + w_t(z)$.

Employment and Production. The mass of households adds to one, but among them, a fraction $\mathcal{U}_t$ is unemployed as given by Γ_t. In particular, the mass of unemployed $\mathcal{U}_t$ evolves according to:

$$\dot{\mathcal{U}}_t = \left[\nu^{eu} + \phi_t^+\right] \cdot (1 - \mathcal{U}_t) - \left[\nu^{ue} - \phi_t^-\right] \cdot \mathcal{U}_t. \tag{12}$$

The natural unemployment rate, which coincides with the steady-state unemployment is $\mathcal{U}_{ss} = \nu^{eu} / \left(\nu^{eu} + \nu^{ue}\right)$ which is obtained by setting $\phi_{ss} = \dot{\mathcal{U}}_{ss} = 0$, in this equation. Since labor is the only production input, aggregate output is $Y_t \equiv 1 - \mathcal{U}_t$.

Inflation. The price level evolves according to

$$\dot{\pi}(t) = \rho(\pi(t) - \pi_{ss}) - \kappa(\mathcal{U}_{ss} - \mathcal{U}_t). \tag{13}$$

This is a classic forward-looking Phillips curve (NS), where we use the unemployment rate above/below the natural rate $\mathcal{U}_{ss}$. In the expression, π_{ss} is a long-run expected inflation target implemented by the CB interest-rate policy. As in Werning (2015), solving this equation forward delivers a formula for inflation as a function of the future path of unemployment:

$$\pi(t) = \pi_{ss} + \kappa \int_0^\infty \exp(-\rho s)\left(\mathcal{U}_{ss} - \mathcal{U}_{t+s}\right) ds. \tag{14}$$

Importantly, $\pi(t)$ is not predetermined. Because it depends on the path of future unemployment, it is allowed to jump at time zero. Also, note that inflation is boosted with an intensity κ. When unemployment is below steady state, the economy experiences wage pressure. In that case, wages tend to increase. Similarly, the economy features deflation as the unemployment rate rises above steady state. Like the unemployment flows, this feature of the model is ad hoc.

Wealth Distribution. At each instant t, there's a joint distribution $f(z,s,t)$ of real financial wealth, $s \in [\bar{s}, \infty)$, and employment status z. The cumulative distribution of f is denoted by F. The wealth distribution f satisfies a Kolmogorov-Forward equation (KFE),

$$\begin{aligned}
\frac{\partial}{\partial t} f(e,s,t) &= -\frac{\partial}{\partial s}\left[\mu(e,s,t) f(e,s,t)\right] - \Gamma_t^{eu} \cdot f(e,s,t) + \Gamma_t^{ue} \cdot f(u,s,t), \text{ and} \\
\frac{\partial}{\partial t} f(u,s,t) &= -\frac{\partial}{\partial s}\left[\mu(u,s,t) f(u,s,t)\right] - \Gamma_t^{ue} \cdot f(u,s,t) + \Gamma_t^{eu} \cdot f(e,s,t).
\end{aligned} \tag{15}$$

with the boundary condition $\lim_{s\to\infty} \sum_{z\in\{u,e\}} F(z,s,t) = 1$. As in Achdou, Han, Lasry, Lions and Moll (2020), generically, there may be a positive mass of agents at the debt limit, $F(z,\bar{s},t) \geq 0$. Hence an integral over f refers to the Lebesgue-Stieltjes integral that takes into consideration the mass points. The distribution satisfies the consistency condition:

$$\mathcal{U}_t = \int_{\bar{s}}^{\infty} f(u,s,t)\, ds = 1 - \int_{\bar{s}}^{\infty} f(e,s,t)\, ds.$$

Central Bank. As assets, the CB holds L_t^f, are private loans, and as liabilities, the monetary base, M_t. The CB has matched assets and liabilities, $L_t^f = M_t$. The monetary base is comprised of the sum of reserves, M_t^b, and currency, $M0_t$—without loss of generality, banks do not hold currency. An OMO (or a reverse OMO) is a simultaneous increase (or decrease) in $dM_t = dL_t^f$. Because of interest rate differentials between assets and liabilities and because there is a penalty on discount-window loans, the CB generates operational profits. All profits are distributed to the central government, which in turn, distributes them as transfers. In addition to OMO, the CB also sets the interest on reserves, i_t^m, and the discount window rate, i_t^{dw}, that we introduced earlier. In principle, we could think of $\{i_t^m, i_t^{dw}\}$ as independent instruments, but we leave fixed a corridor spread $\iota = i_t^{dw} - i_t^m \geq 0$.[11]
The CB's operational profits are:

$$\Pi_t^{CB} = i_t^l L_t^f - i_t^m (M_t - M0_t) + \iota_t \left(1 - \psi_t^-\right) B_t^-. \tag{16}$$

The interpretation is that the CB earns i_t^l on L_t^f, and pays i_t^m on the portion of the money supply held as reserves—it earns an interest rate differential—whereas $\iota_t \left(1 - \psi_t^-\right) B_t^-$ is the income earned from discount window lending. The CB's operational income plus the surplus or deficit from the

[11]The CB faces two solvency restrictions: $i_t^{dw} \geq i_t^m$ and $i^{dw} \geq 0$. If either constraint is violated, banks could borrow reserves from the discount window and either hold reserves as currency or reserves and earn an arbitrage.

unemployment benefit, $(1-\mathcal{U}_t)\cdot\tau^l-\mathcal{U}_t\cdot b$, produce a government surplus that is distributed lump sum:

$$P_t T_t = \Pi_t^{CB} + P_t\left(\tau^l\cdot(1-\mathcal{U}_t)-b\cdot\mathcal{U}_t\right). \tag{17}$$

For the rest of the paper, we assume that T_t adjusts to satisfy the balanced budget above (fiscal passive regime), whereas i_t^m follows from a Taylor rule.

A Time-Varying Taylor Rule. To set the interest on reserves, the CB runs a Taylor rule that allows for discretionary short-term deviations, but eventually converges to a standard Taylor rule with long-run commitment:

$$i_t^m = \bar{i}_t^m + \eta_t\cdot(\pi_t-\pi_{ss}). \tag{18}$$

There are several terms in this Taylor rule: η_t is a time-varying response to inflation and $\bar{i}_t^m$ a time-varying interest target that is consistent with a given inflation target. This formulation is flexible enough to allow for isolated changes in policy rates: by letting $\bar{i}_t^m$ change, we can isolate the effects of policy rates while we need to shut down η_t to eliminate the feedback from future inflation to policy rates. At the same time, we force $\{\bar{i}_t^m,\eta_t\}$ to converge to the standard values of a Taylor rule, to abide by the Taylor principle, to circumvent issues related to forward-guidance and lack of commitment. If we do not allow for this discretionary component, we cannot isolate the effects of policies from the endogenous response of the Taylor rule.

Markets. Outside money is held as reserves or currency. Aggregate currency stock is

$$M0_t \equiv \sum_{z\in\{u,e\}}\int_{\bar{s}}^{\infty} m_t^h(z,s)\, f(z,s,t)\, ds,$$

so equilibrium in the money market requires:

$$M0_t + M_t^b = M_t. \tag{19}$$

The credit market has two sides: deposit and loan markets. In the deposit market, households hold deposits supplied by banks. In the loan market, households obtain loans supplied by banks. The deposit market clears when:

$$A_t^b = \sum_{z\in\{u,e\}}\int_0^{\infty} a_t^h(z,s)\, f(z,s,t)\, ds, \tag{20}$$

where $a_t^h(s) \equiv P_t s - m_t^h(s)$, for a positive s. The loans market clears when:

$$L_t^b + L_t^f = \sum_{z \in \{u,e\}} \int_{\bar{s}}^{0} l_t^h(z,s) f(z,s,t)\, ds, \tag{21}$$

where $l_t^h(s) \equiv -P_t s$ for negative s. Finally, the goods market clears whenever:

$$Y_t \equiv 1 - \mathcal{U}_t = C_t \equiv \sum_{z \in \{u,e\}} \int_{\bar{s}}^{\infty} c_t(z,s) f(s,t)\, ds. \tag{22}$$

Equilibrium. A price path is the function $\{P(t), i^l(t), i^a(t)\} : [0,\infty) \to \mathbb{R}_+^3$. A policy path is the function $\{i_t^m, M_t, T_t\} : [0,\infty) \to \mathbb{R}_+^4$. Next, we define an equilibrium.

Definition 1 *[Perfect Foresight Equilibrium.] Given an initial condition for the distribution of wealth f_0, and an initial price level P_0, a policy path, a perfect-foresight equilibrium (PFE) is given by (a) a price path, (b) a path for the real wealth distribution f, (c) a path of aggregate bank holdings $\{L_t^b, M_t^b, A_t^b\}_{t \geq 0}$, (d) unemployment flows, and (e) household's policy $\{c, m^h\}$ and value functions $\{V\}_{t \geq 0}$, such that:*

1. *The path of aggregate bank holdings solves the static bank's problem (1),*
2. *The household's policy rule and value functions solve the household's problem (2),*
3. *The unemployment transitions satisfy (8),*
4. *The law of motion for f is consistent with (15),*
5. *The government's policy path satisfies the budget constraint (16),*
6. *Asset markets and the goods market clear (1,19-22).*

Next, we characterize the equilibrium dynamics. A **steady state** occurs when $\frac{\partial}{\partial t} f(z,s,t) = 0$ and $\{r_t^a, r_t^l\}$ are constant. We use subscripts ss to denote variables at steady state. An important assumption is that we treat P_0 as given. As in any model with nominal assets, the time-zero price determines the real distribution of wealth and thus an equilibrium path. The approach here is to think of P_0 as determined from past MP, and consider that through the nominal rigidity, prices cannot jump at time zero.[12] This approach circumvents the need for refinements that pin down time-zero prices such as the fiscal theory of the price level. A final important feature is that $\mathcal{U}_t$ jumps when there are unexpected shocks.[13]

[12] The idea is to think of the time-zero price as the price level at steady state consistent with a steady state given a nominal monetary base of M_t that was committed a priori.

[13] The interpretation is that ϕ_t jumps to produce the market clearing rate of employment. In the applications, there is only one time-zero jump.

Digression: Model Assumptions. The endogenous labor market dynamics, ϕ_t, and the Phillips curve are admittedly ad-hoc. However, these objects are designed to capture the idea that insufficient aggregate demand translates into unemployment flows—see the related work by Michaillat and Saez (2015). A virtue of this approach is that the business cycle dynamics are driven through the dynamics of firing and hiring rates (Davis, Faberman and Haltiwanger, 2006). Whereas there are several ways to model employment flows and aggregate demand externalities, these typically involve additional state variables that add complications while interfering little with the policy prescriptions. Furthermore, the Phillips curve we adopt has been estimated many times. Thus, we can think of it as an exogenous block of the model that is well understood empirically, although its theoretical foundations are not clear.

Along a transition, the model shares the spirit of the new-Keynesian model: there is a short-run trade-off between inflation and unemployment, but exploiting that trade-off is not desirable along a transition with stable prices. To see this, suppose that the CB lowers the IOR at t. Since inflation is a state variable, the policy lowers the real loans and deposit rates that stimulate consumption. How does the model achieve an equilibrium in the goods and asset markets? For that, ϕ_t falls below zero, producing a decline in $\mathcal{U}_t$. From (14), we have that that increase will provoke an increase in inflation. However, from (13), if inflation remains above a steady-state value, it accelerates the increase. Thus, if the CB wants to stabilize inflation in the future, it needs to compensate for the pressure with an offsetting increase in rates in the future and with an increase in the unemployment rate. Thus, exploiting this tradeoff is undesirable.

The financial architecture in the model captures a fundamental feature of banking. In practice, banks issue deposits in two transactions. The first is a swap of liabilities with the nonfinancial sector. When banks make loans, they effectively credit borrowers with deposits, a bank liability is exchanged for a household liability. This swap is the process of inside money creation. Deposits then circulate as agents exchange deposits for goods. This circulation gives rise to the settlement positions. The second transaction is the exchange of deposits (a bank liability) for currency (a government liability).

A missing element is government bonds. In practice, central banks conduct OMO by purchasing government bonds. Here, negative holdings of L^f are interpreted as government bonds. The implicit assumption is that bonds are as illiquid as private loans. Bianchi and Bigio (2020) introduce bonds that are more liquid than loans, but less so than reserves. The present model can be easily extended to incorporate government bonds.

3 Implementation

A spread between two nominal rates is the spread between the two corresponding real rates. This observation is important because it implies that if a CB can control a spread, it can control real objects such as credit and the distribution of wealth, even in the long-run. We now explain how the

CB implements a desired credit spread by conducting OMOs. Later we explain how the control over credit spreads matters.

Implementation. From (7), we know that Δr_t is a function of the liquidity ratio Λ_t. A natural question is how does the CB control Δr_t? The main result from this section is that the answer depends on different regimes for the choices of the IOR and the size of the CB balance sheet (in real terms), $\left\{i_t^m, \mathcal{L}_t^f\right\}$.

For now, we can focus on a given instant of time t—and suppress the t subindex momentarily. Toward the characterization, it is useful to compute, $\theta^{lb}(i^m)$, the lower bound on the equilibrium interbank market tightness for any given IOR, which is obtained solving:

$$\theta^{lb}(i^m) \equiv \min_{\theta \in [0,\infty)} \theta$$

subject to

$$i^m + \frac{1}{2}\left(\chi^+(\theta) + (1-\delta)\chi^-(\theta)\right) > 0.$$

The constraint in this auxiliary problem just takes into account that the equilibrium deposit rate given by (6) must be positive. The solution to this problem is trivial: when $i^m < 0$, then $\theta^{lb}(i^m) = 0$ since $i^a = i^m$ is positive. When $i^m \geq 0$, then $\theta^{lb}(i^m) > 0$ is the market tightness consistent with a zero deposit rate. This object is useful because it is only a function of a policy variable i^m. With $\theta^{lb}(i^m)$, we can characterize several equilibrium objects as a function of $\{i^m, \mathcal{L}^f\}$. In particular, given a distribution of real wealth f the equilibrium real balances of currency are:

$$\frac{M0}{P} = \mathbb{I}_{[i^m<0]} \cdot \max\left\{\frac{1+\theta^{lb}(i^m)}{1+\theta^{lb}(i^m)-\delta} \cdot \mathcal{L}^f - \frac{\delta}{1+\theta^{lb}(i^m)-\delta} \cdot \int_0^\infty sf(s,t), 0\right\},$$

and, consequently, the equilibrium liquidity ratio is:

$$\Lambda = \left(\frac{\mathcal{L}^f - M0/P}{\sum_{z\in\{u,e\}} \int_0^\infty sf ds - M0/P}\right).$$

The values of i^m and Λ are enough to catalog the three regimes in which MP operates and the effects of policy changes, as shown in the following proposition:

Proposition 2 *[Properties of Equilibrium Rates and Spreads] Consider a distribution of real wealth f, a price level P at a time t. MP operates in either one of the following three regimes:*

Corridor Regime. *If* $\Lambda < \delta / \left(1+\theta^{lb}\left(i^m\right)\right)$, *then* $i^l > i^a > i^m$, *and* $\frac{M0}{P} = 0$. *Furthermore,*

$$\frac{di^a}{di^m} = 1,\ \frac{d\Delta r}{di^m} = 0,\ \frac{di^a}{d\mathcal{L}^f}\mathcal{L}^f = -\frac{1}{2}\frac{\left(\chi_\theta^+ + (1-\delta)\chi_\theta^-\right)}{\Lambda} < 0, \text{ and } \frac{d\Delta r}{d\mathcal{L}^f}\mathcal{L}^f = -\frac{\delta}{2}\frac{\chi_\theta^-}{\Lambda} < 0.$$

Floor Regime. *If* $i^m > 0$ *and* $\Lambda > \delta / \left(1+\theta^{lb}\left(i^m\right)\right)$, *then* $i^l = i^a = i^m$, $\Delta r = 0$ *and* $\frac{M0}{P} = 0$. *Furthermore,*

$$\frac{di^a}{di^m} = 1,\ \frac{d\Delta r}{di^m} = \frac{di^a}{d\mathcal{L}^f}\mathcal{L}^f = \frac{d\Delta r}{d\mathcal{L}^f}\mathcal{L}^f = 0.$$

Liquidity Trap. *If* $i^m < 0$ *and* $\Lambda > \delta / \left(1+\theta^{lb}\left(i^m\right)\right)$, *then* $i^l > i^a = 0$ *and* $\frac{M0}{P} > 0$. *Furthermore,*

$$\frac{d\Delta r}{di^m} = -\delta\frac{\chi_\theta^-\left(\delta\mu-1\right)}{\left(\chi_\theta^+\left(\delta\mu-1\right)+(1-\delta)\chi_\theta^-\left(\delta\mu-1\right)\right)} < 0, \text{ and } \frac{di^a}{di_t^m} = \frac{di^a}{d\mathcal{L}^f}\mathcal{L}^f = \frac{d\Delta r}{d\mathcal{L}^f}\mathcal{L}^f = 0.$$

and $\frac{d}{d\mathcal{L}^f}\left[M0/P\right] = \frac{1+\theta^{lb}(i^m)}{1+\theta^{lb}(i^m)-\delta}$.

Proposition 2 establishes the effects of policy in three regimes that depend on $\{i^m, \Lambda\}$ as illustrated by Figure 4. Liquidity is scarce for banks when $\Lambda_t < \delta / \left(1+\theta^{lb}\left(i^m\right)\right)$ and the interbank market is active. In CB jargon, this regime is a *corridor system* implemented with a *lean* CB balance sheet, with a small enough $\mathcal{L}^f$. In this regime, the credit spread is positive and controlled exclusively through OMO, $d\mathcal{L}^f$. In turn, changes in i^m change the deposit rate one-for-one, without a change in the spread on impact. However, the size of the CB balance sheet does affect the spread. The neutrality of i^m on the spread implies that the CB can control inflation independently from its control over spreads.

The economy enters a *floor system* if $i_t^m > 0$ and $\Lambda_t > \delta / \left(1+\theta^{lb}\left(i^m\right)\right)$. In a floor system, banks are satiated: no bank faces a reserve deficit, so the interbank market is inoperative. The regime is implemented with a *fat* balance sheet, with a large enough $\mathcal{L}^f$. In this regime, OMO are irrelevant (in the sense of Wallace, 1981). Hence, in this regime, the CB loses the ability to affect spreads, but still controls inflation through the IOR.

The economy enters a *liquidity trap* if $i_t^m < 0$ and $\Lambda_t > \delta / \left(1+\theta^{lb}\left(i^m\right)\right)$. In a liquidity trap, the deposit rate is zero. In that region, OMO are also irrelevant, but for a different reason. In a liquidity trap, the CB still purchases bank loans by issuing liabilities, but in equilibrium, the private sector responds by reducing deposits and increasing currency holdings such that the liquidity ratio remains constant. Yet, despite the irrelevance of OMO, in this region, the spread is positive. The reason is that a negative IOR penalizes the issuance of deposits. Since the deposit rate cannot fall below zero, banks require a high lending rate to break even. As a result, a reduction in i_t^m provokes an increase in the loans rate, contrary to the effects in the other regimes. This effect is an interest-rate reversal. Since OMO do not have effects in this region, and reductions in the IOR only increase spreads, we will show that in this region the CB loses the ability to stimulate the economy. Appendix C.1 presents a three dimensional plot that describes the effects of i_t^m and Λ_t on the interest rates.

The interest-rate reversal near a DZLB has been documented by Heider et al. (2019); Eggertsson et al. (2019). Brunnermeier and Koby (2019) and Ulate (2020) also find this effect, but their mechanisms operate through bank capital and monopoly power. To our knowledge, this is the first model to establish a connection between a liquidity trap, negative rates, and the interest-rate reversal. Appendix C.1 also presents a discussion of alternative implementations of spreads through MP and relates these alternative implementation to actual CB practices.

The analysis so far, describes how the pass-through from MP tools to the credit spread depends on the settlement frictions in the interbank market. Because the pass-through depends on predetermined states, $\{f, P\}$, we can think of this pass-through as the unexpected instantaneous effects of MP. Next, we characterize the evolution of the model's state variables, taking the equilibrium spreads as given.

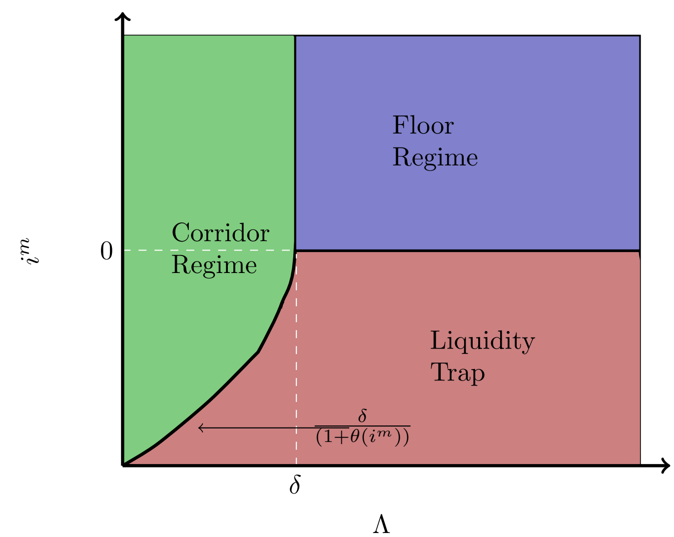


Figure 4: Three Monetary Policy Regimes

Note: The figure presents the MP regimes as a function of the liquidity ratio Λ and the interest on reserves i^m.

Clearing in real terms. It is useful to express the government budget constraint in real terms. If the CB induces a spread, the revenues from the spread must be earned by some agent in the economy. If banks earn zero profits, the only possibility is that the spread is earned by the CB. The next proposition uses this observation to relate the real fiscal effects of the spread:

Proposition 3 *[Real Budget Constraint] Assume that all asset markets clear. Then, transfers are given by:*

$$T_t = \underbrace{\Delta r_t \cdot \sum_{z \in \{u,e\}} \int_0^\infty s f(z,s,t)\, ds}_{\text{CB operational revenue } \Pi_t^{CB}/P_t} + \underbrace{\tau^l (1 - \mathcal{U}_t) - b \cdot \mathcal{U}_t}_{\text{fiscal deficit}}. \quad (23)$$

The proposition gives us the level of transfers for a give spread. Given $\{\Delta r_t, T_t\}$, market clearing in real financial claims is consistent with equilibrium values for r_t^a and ϕ_t. In turn, these variables must be consistent with clearing in a single real asset market, as show by the following proposition:

Proposition 4 *[Real Wealth Clearing and Walras's Law] Let nominal rates be given by (5) and (6) and let transfers be given by (23). Then, market clearing in real wealth, that is,*

$$0 = \sum_{z\in\{u,e\}} \int_{\bar{s}}^{\infty} sf(z,s,t)\, ds \ \text{for } t\in[0,\infty), \tag{24}$$

implies market clearing in all asset markets and, furthermore, clearing in the goods market, (22).

From now on, we refer to the credit market as the market for real wealth, which summarizes the loans, deposit and money markets into a single equation.

Steady State and Long-Run MP Effect. Consider a steady state. Let the CB target a long-run credit spread Δr_{ss}. At steady state, the disturbance in job-separation ϕ_t must be zero because this is the only possibility consistent with a Phillips curve with constant inflation. We also know that inflation has no effect in a steady state—here MP is super-neutral, unlike in a standard new-Keynesian model. Thus, at steady-state, the real interest rate r_{ss}^a solves:

$$0 = \sum_{z\in\{u,e\}} \int_{\bar{s}}^{\infty} sf_{ss}(z,s)\, ds.$$

Once we obtain an equilibrium r_{ss}^a in steady state, which corresponds to the real interest rate, inflation is given by the corresponding Fisher's equation:

$$\pi_{ss} = \pi_\infty = i_\infty^m - r_\infty^a + \frac{1}{2}\left[\chi_\infty^+ + (1-\delta)\,\chi_\infty^-\right].$$

Once inflation is obtained, all nominal variables grow at the rate of inflation. To implement Δr_{ss}, the path of M_t must be consistent with the Λ_{ss} that produces Δr_{ss} according to (7).

An important observation is that the long-run real interest rate, r_{ss}^a, is affected by the CB balance sheet, through the spread. Thus, monetary policy is not long-run neutral, counter to the working assumption of most empirical monetary models. Of course, this is not the only model where MP affects long-run real interest rates. In fact, some classic examples are Lucas (1980); Bewley (1983) or Aiyagari and McGrattan (1998). However, the sources of real long-run effects here are different. In both in Lucas (1980); Bewley (1983) and Aiyagari and McGrattan (1998) the long-run real interest rate depends on the the supply of net outstanding real government liabilities.[14] In this paper, net

[14] In Bewley (1983) real-long run effects follows because households only hold currency and inflation pins down the real rate and the real value of outstanding liabilities. In Aiyagari and McGrattan (1998) private credit co-exists with government liabilities, but the net amount of government liabilities affects real rates.

CB liabilities are zero, $0 = L_t^f - M_t$, but the real long-run effect follows because the gross amount of outstanding government liabilities impact spreads. Like Bewley models, the model here features neutrality, but unlike Bewley models, the model also features super neutrality thanks to the IOR, Ljungqvist and Sargent (2012, Chapter 18.11).

Transitions. Along a transition, things work differently. In particular, π_t is given by (13). Then, given i_t^m and Λ_t, i_t^l and i_t^a are determined by (5-6). The real rates r_t^l and r_t^a follow from the Fisher's equation,

$$r_t^x = i_t^x - \pi_t \text{ for } x \in \{l, a\}. \tag{25}$$

Then, to satisfy clearing in the asset market, ϕ_t adjusts to satisfy (24).

Appendix C.2 discusses the equilibrium restriction imposed on MP along a transition. That appendix also connects the monetary properties of this model with the monetary properties of classic Bewley models, in connection to fiscal and monetary interactions. Appendix F explains how transitions are calculated numerically.

4 Positive Analysis: From Instruments to Channels

This section covers the positive analysis. First, we discuss the effects of MP under a floor system achieved by running a large CB balance sheet that satiates banks with reserves. Then, we study these effects under a corridor system achieved by running a small CB balance sheet that limits the supply of reserves. We also study the effects of MP when it enters a liquidity trap. After we study the effects of policy, as a prelude to the normative analysis, we introduce a credit crunch episode and conclude the section studying the macro insurance effects of different spreads.

Calibration. At this stage, we must present the calibration. The model has many missing elements because the goal is to remain parsimonious. While the spirit is to remain parsimonious, keeping the number of parameters small, we also want to provide a quantitative sense of the operating mechanisms. In that sense, the calibration serves as a guide to inform us of the additional model features that are needed to improve the quantitative fit. The calibration, which is inspired by the US economy of the last decade, is summarized in Table 1.

The reasoning behind the calibration is as follows. Risk aversion, γ, is set to 2, a standard value in the literature. The time discount, ρ, is set to 4%, to yield a steady-state real deposit rate of approximately 3.0%, close to the real return on one-year certificate of deposits. The steady-state IOR is chosen to normalize steady-state inflation at 1.0%. The coefficient of the Taylor rule, η, is set to 1.5, also a standard value. The Phillips-curve coefficient κ is set to 0.1, following the estimates in Hazell et al.

	Value	Description	Target/Reference
γ	2	risk aversion	standard
ρ	0.04	time discount	1.0% real risk-free rate
ν^{eu}	0.4	job separation rate	Shimer (2005)
ν^{ue}	1.2	job finding rate	labor income risk
κ	0.1	Phillips curve coefficient	Hazell, Herreño, Nakamura, Steinsson et al. (2020)
b	0.41	unemployment benefit	UI Replacement
τ^{l}	0.3	labor tax rate	U.S. average labor income tax (OECD)
η	1.5	Taylor-rule coefficient	Taylor (1993)
Δr_{ss}	1%	steady-state credit spread	Bianchi and Bigio (2020)
$\bar{s}$	$-1.5w(u)$	credit limit	precautionary behavior

Table 1: Parameter Values

Note: The table lists the calibrated values of parameters and the corresponding reference/target of calibration.

(2020). We set the interbank-market efficiency, λ, to 2.1, following Bianchi and Bigio (2020) and set the payment shock δ to produce the same steady-state interbank market tightness as in that paper. We target a steady-state credit spread of 1.0% and accordingly set the discount window rate to obtain that target.[15]

We calibrate the income process to strike a balance between fitting the job flows while producing a reasonable income distribution with only four parameters. We set the unemployment benefit, b, to 41% of the real wages. This number matches the average US unemployment insurance replacement rate between 2010 and 2019. The number overstates the actual earnings among the unemployed because unemployment insurance is permanent in the model.[16] The labor tax τ^{l} is set to 0.3, the average labor income tax.[17] A standard approach to calibrate the employment-to-unemployment flows is to use the transition rates of job-hiring and job-firing. Along those lines, we set ν^{eu} to 0.4, following Shimer (2005).[18] For the unemployment-to-employment rate, we set $\nu^{ue} = 1.2$. This number is lower than the corresponding figure in Shimer (2005)—about 5.4. We need longer unemployment spells because otherwise, the distribution of labor income is too concentrated around the median—it is already excessively concentrated under the baseline calibration.

The last parameter is the debt limit, $\bar{s}$ which we set to $\bar{s} = -1.5 \cdot b$, to produce a debt-to-income ratio of 1.5 for the poorest households. This debt limit is tighter than in the typical calibration used in the literature. We chose a tighter debt limit because the precautionary motive is otherwise understated as the model is missing other features that constrain consumption. For example, we are not including

[15]The required spread between the discount window rate and the rate on reserves is much higher than in the data, but as we argued above, this is a stand-in for missing elements such as collateral and stigma (De Fiore et al., 2018).

[16]The UI replacement rate is the ratio of the claimants' weekly benefit amount (WBA) to the claimants' average weekly wage. The average weekly wage is based on the hourly wage of a usual job claiment, normalized to a 40 hour work week. The data is from https://oui.doleta.gov/unemploy/ui_replacement_rates.asp.

[17]The average labor income tax is equal to the U.S. average tax wedge for a single worker from 2000 to 2019. The data is from OECD database https://data.oecd.org/tax/tax-wedge.htm.

[18]This is the annualized value of a monthly average separation rate of 0.034 between 1951 and 2003, used in Shimer (2005).

	Data Income Distribution		Model Income Distribution (year average)	Data Wealth Distribution		Model Wealth Distribution (steady state)
Percentiles	Nominal Value (2019, $)	Relative to Median	Relative to Median	Nominal Value (2019, $)	Deviation from Mean (relative to per capita income)	Deviation from Mean (relative to per capita income)
10%	14,945.45	24.99%	64.08%	-466.58	-901.85%	-51.77%
25%	30,638.18	51.23%	69.68%	12,430.11	-886.29%	-27.59%
50%	59,806.73	100%	100%	121,411.37	-754.77%	-0.38%
75%	105,898.51	177.07%	101.02%	403,283.56	-414.59%	23.80%
90%	166,549.65	278.48%	101.68%	1,219,126.46	569.99%	47.99%
Mean	82,861.41	138.55%	91.47%	746,821	0	0

Table 2: Household Income and Wealth Percentiles (Not-Targeted)

Note: The table lists the percentiles of household income and wealth obtained from U.S. data and model. Column 2 reports the nominal value of U.S. household income percentiles in 2019, column 3 the ratio of percentiles to the median. Column 4 reports the model model analogue. Column 5 reports the nominal value of U.S. net worth percentiles in 2019, and column 6, the deviation of the percentiles from mean, relative to per capita income. Column 7 reports the model analogue. The U.S. income distribution is from the 2019 Current Population Survey (CPS), and the net worth data from the 2019 Survey of Consumer Finances (SCF).

illiquid assets (as in Kaplan et al., 2018) or consumption commitments (as in Chetty and Szeidl, 2007). To benchmark the calibration, Table 2 reports the implied distribution of income in the model vis-á-vis the data. As anticipated, the income distribution in the model is more concentrated than in the data. In particular, the distribution is extremely concentrated above the median household income because the median is on average employed through a typical year—thus, all households above the median earn the same income. Below the median, there is a thicker tail, which as we noted, is still more concentrated than in the data.

Likewise, the wealth distribution is also more concentrated in the model, at both ends of the distribution. Regarding the right tail of the wealth distribution, the model does not feature return shocks, which are necessary to produce a realistic wealth concentration at the top quantiles. However, this feature should not be a concern since the consumption of wealthy households is close to linear in wealth. Hence, the behavior of top quantiles is close to the behavior of a representative agent model—Bilbiie (2020); Debortoli and Gali (2017). Regarding the left tail, it is again more concentrated in the model. In particular, the fraction of households for whom the debt limit is binding is approximately 1.0% at steady state. By comparison, that figure is 10 times larger in Kaplan et al. (2018) due to the presence of illiquid savings in that model. Since the behavior of constrained agents is important for the responses to shocks, and we are understating the population at or near those constraints, we compensate for that missing element with a tighter debt limit.

Finally, the CB operational revenue over output is 0.15%. Thus, the non-Ricardian effects in the model are very small.

Scenario	Shock x	x_{ss}	x_0	T_x	$\bar{\zeta}_x$
I. IOR	η	1.5	0	0	0.2
	$\bar{\iota}^m$	1%	$\{-2\%, -3\%, -4\%\}$	1	50
II. Spread	η	1.5	0	0	0.2
	Δr	0	1%	1	50
III. Credit Crunch	$\tilde{s}$	$\bar{s}$	$0.15\bar{s}$	1	5

Table 3: Logistic Path - Changes in the IOR.

Note: This table lists the baseline calibration of parameters in the logistic paths in each experiments. The value of $\bar{\iota}_0^m$ depends on the experiment.

Logistics. In the experiments that follow, we study events where we shock exogenous variables. In each experiment, we initiate the economy at a steady state and consider $t = 0$ to be the time of an unexpected event. For each variable x_t, we initiate its value at some x_{0^+} where it stays put for an interval after which the variable begins a transition to return to steady state:

$$x_t = \begin{cases} x_0, & \text{if } t \in [0, T_x] \\ x_{ss} + (x_0 - x_{ss}) \cdot \exp\left(-\bar{\zeta}_x \left(t - T_x\right)\right), & \text{if } t > T_x. \end{cases} \tag{26}$$

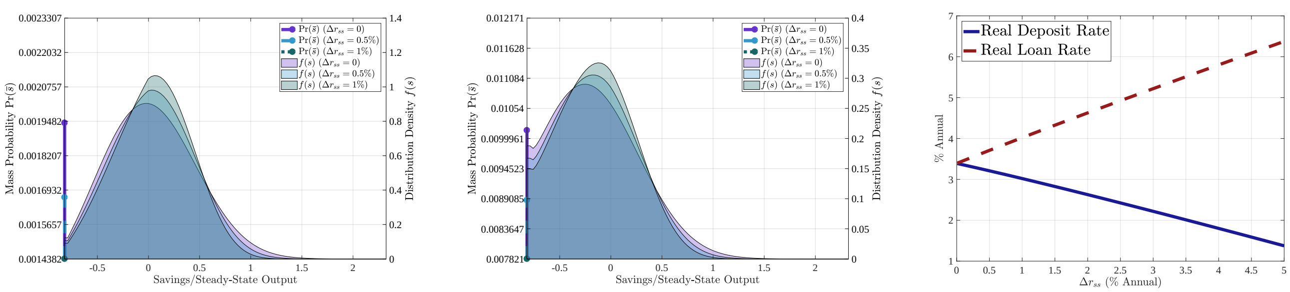
We study the effects of shocks to four variables $x \in \{\eta, \bar{\iota}^m, \Delta r, \tilde{s}\}$—the variable $\tilde{s}$ captures a credit crunch as we explain below. In the formulation, T_x is the time interval where the variable stays put. After T_x, the variable transitions smoothly back to steady-state value, following the logistic (26) with a parameter $\bar{\zeta}_x > 0$ that controls the speed of mean-reversion. Under each experiment, we shock several variables together, as summarized by Table 3, to conveniently disentangle effects.

4.1 Policy Effects

A Floor System and the Fisherian Channels. We now explain the effects of a CB that satiates banks with reserves and operates through a floor system. If the CB satiates banks with reserves, we observed that the only instrument with effects is the IOR. Under flexible prices, changes in the IOR are neutral. In turn, when prices are rigid, changes in the IOR carry real effects akin to those in Guerrieri and Lorenzoni (2017); Kaplan et al. (2018); Auclert (2019). Since, the effects of MP in those papers are well understood, we do not carry our an analysis of their effects in further detail. Next, we abandon a floor system and study the effects of changes in the IOR and OMO.

Corridor System: Steady State CB Balance Sheet and Micro Insurance. In a corridor system, the size of the CB's balance sheet, $\mathcal{L}^f$, affects the spread. We first consider the effects of changes in spreads at steady state. Steady-state output is entirely determined by the natural job flows but the amount of credit and real rates is affected by spreads. Figure 5 reports the real wealth distribution among employed and unemployed households (panels a and b, respectively) and real interests (panel c), for different values of Δr_{ss}. There is a noticeable difference: larger spreads compress the wealth distribution and reduce the mass of agents at the debt limit. The reason for this is that the spread is a tax on credit. Like any tax, a greater spread reduces the real deposit rate while it increases the real loans rates, making both savings and borrowings less attractive. Also like any tax, the spread has an incidence on both sides of the market, on the loan demand or the deposit supply, depending on what side is most elastic. As we can see from panel (b), the incidence on the interest rate on loans is higher. This feature reflects that as agents approach their credit constraints, they become less responsive to changes in interest rates, a result that has been stressed before—see Auclert (2019), for example. This lower interest rate elasticity explains the greater incidence of spreads on borrowers. To clear the credit market, the loans rate has to increase substantially more than the decrease in deposit rates. The greater the relative interest rate elasticity of savers, the greater the effect.[19]

The more concentrated distribution of wealth under a corridor system plays an important role in the model. In an incomplete markets economy, a more concentrated wealth distribution is an indication of worse micro insurance. Take the extreme example of financial repression. Financial repression, leads to a very equal society, where the unemployed suffer excessive consumption risks that lead to ex-ante welfare losses. The increase in spreads produced in a corridor system is a less drastic example, but the same logic applies: greater spreads hurt the insurance of idiosyncratic labor risk, what we call micro insurance.[20]



(a) Employed (b) Unemployed (c) Deposit and Loan Rates

Figure 5: Steady State Effects of Real Spreads.

Note: In panels (a) and (b), the measure of households with assets $\bar{s}$ is a probability mass (left scale), and the measure of households with $s > \bar{s}$ is a probability density (right scale). In panel (c), deposit and loan rates are expressed in annual percentage terms.

We quantify the extent of micro insurance in a given steady state, to get a sense of the cost induced by spreads, through the following metric. We compute the loss in steady state, $\mathcal{L}^{micro}\left(\Delta r_{ss}\right)$, in a complete-markets-no-inequality version of the model that delivers the same utility as the egalitarian welfare criterion. The formula is:

$$\frac{U\left(Y_{ss}\left(1-\mathcal{L}^{micro}\left(\Delta r_{ss}\right)\right)\right)}{\rho} \equiv \sum_{z\in\{e,u\}} \int_{\bar{s}}^{\infty} V\left(s,z,ss;\Delta r_{ss}\right)\cdot f\left(s;\Delta r_{ss}\right) ds. \tag{27}$$

[19] Drechsler et al. (2017) documents that deposit rates are not very sensitive to MP and attributes the effect to market power. We obtain the same result here because savers are more interest rate elastic than borrowers.

[20] The spread has a non-Ricardian effect through transfers, but this effect is quantitatively small.

The value of $\mathcal{L}^{micro}$ measures how much a planner would pay in terms of steady-state output to get rid of the employment fluctuations. We call this loss the micro loss. To get a sense of magnitudes, without a spread, we obtain a loss equivalent to $\mathcal{L}^{micro} = 0.45\%$. If we increase the steady-state spread to 0.25%, we obtain a micro loss of $\mathcal{L}^{micro} = 0.4504\%$, roughly a 0.1 welfare loss. Raising the spread further to 1% produces a $\mathcal{L}^{micro} = 0.4554\%$, increasing the loss by 1.2%. As we should expect, welfare losses are detrimental to ex-ante micro insurance.

Corridor System: Changes in the IOR. We now consider the effects of changes in i_t^m under rigid prices, holding Δr fixed. Figure 6 presents the results of a transition that starts from steady state. The transition is triggered by a 3.0% reduction in i_0^m. The reduction of 3.0% lasts for a year, $T_{\bar{i}^m} = 1$. The value of 3.0% is chosen, for comparison purposes with the shock that eliminates the spread. To isolate the effects of this policy, we neutralize the feedback from the Phillips curve setting η to zero during the experiment, as detailed in Table 3. The reversal of the policy change is immediate.

Three channels operate in tandem: a standard interest-rate channel, a credit channel, and a less important non-Ricardian channel. Because Δr is fixed, the reduction in the IOR leads to reductions in all nominal rates by 3.0%. A first effect is a decline in real rates——panel (b). Given that production is demand determined, the reduction in real rates provokes an increase in output—panel (d)—which occurs mechanically through a jump in employment and a subsequent decrease in unemployment—panel (e). During the transition, unemployment remains above its natural level, and through the Phillips curve, induces a corresponding increase in inflation—panel (c). The response is consistent with the standard real-interest rate channel that operates in the new-Keynesian model. Here, the direct effect of the interest rate is enhanced by the decline in job separations, which in turn stimulates consumption. This additional consumption stimuli occurs through the expectations of higher income by the unemployed. Quantitatively, the passthrough from the policy reduction to accumulated output over the years is approximately 1/3—a one year 1% reduction in the IOR would lead to 0.3% in accumulated output after one year. The quantitative effect is strong.

An important feature is that the volume of credit declines during the experiment. The reason is the greater interest-rate sensitivity of savers. Because the spread is constant, the reduction in real deposit and loan rates is the same. However, the reduction in the deposit rate stimulates the consumption of the saver by more than the consumption of borrowers. If savers desire to save less, but borrowers are less sensitive to interest rates, the economy needs another margin to reestablish an equilibrium. What allows to reestablish an equilibrium is the indirect effect: the expected increase in hiring rates. This increase in income is expected to be temporary, but due to the precautionary motive, it is able to stimulate consumption among the unemployed.

The reduction in credit is important because behind it is an additional amplification effect through the credit channel. Lurking in the background are reverse OMO that allow the CB to maintain a constant spread. As credit falls in response to a reduction in the IOR, if the supply of reserves is unaltered, the liquidity ratio would increase. Thus, to keep the liquidity ratio constant—and isolate the interest-rate channel—we have the CB conduct a reverse OMO. If the CB does not perform the reverse OMO, the interest-rate channel is enhanced through the effect on spreads. Without that correction, in a corridor system, changes in the IOR operate through both the interest rate and credit channels.

We refer readers to Appendix E where we present an analysis of consumption elasticities at different wealth levels, to different variables that are affected by MP. That analysis allows for a decomposition of the strength of the effects of different channels on different agents.

Corridor System: OMO. Now consider an OMO that leads to a reduction in Δr_t, while keeping the IOR constant. In the exercise, an OMO is engineered so Δr_t is brought down from 1% to 0% during a year, $T_{\Delta r}$, following the logistic path in Table 3. Again, we isolate the effects from the endogenous response of the Taylor rule by setting $\eta = 0$ during the year of the response.

The effects are depicted in Figure 7. Panel (a) shows the path of spreads. The reduction in the spread is produced by an OMO that increases the liquidity ratio. Since both the deposit and loan rates carry premia over the IOR that decrease with the liquidity ratio, both nominal rates fall with the OMO. However, note that the reduction in the lending rate is almost twice as large as the decline in the deposit rate. As explained earlier, the policy has a direct effect through the credit channel and another direct effect through the interest-rate channel. The qualitative effects are similar to those of reductions in the IOR. Quantitatively, although the change in real deposit rates is the same as in the previous exercise, the quantitative effect on output is about 15% larger on impact. This additional strength is obtained through the reduction in the spread, which activates the credit channel.

Liquidity trap: Changes in the IOR. Next, we investigate the effects of reductions in the IOR that activate a liquidity trap—as occurs when we move from the green to the brown region in Figure 4. The objective is to show that a reduction in i_t^m that leads to a liquidity trap is contractionary. We study the three IOR reduction scenarios described in Table 3, which differ only in the initial reduction in $\bar{i}_0^m$. In one policy—the solid curve—we reduce $\bar{i}_0^m$ by 2.0%, in another policy, by 3.0%, to the point where nominal deposit rates are just about zero, and in the final exercise, by 4.0% to a point past the value that activates the DZLB in a liquidity trap—the dot-dashed line. In all cases, η is again set to zero.

Figure 8 reports the responses. Panel (a) reports the effects on spreads. Notice that the policy that takes the economy to the liquidity trap induces an increase in spreads by about 25 bps. Panel (b) shows the corresponding deposit rates. We can observe that in a liquidity trap, reductions in the IOR do not affect the real deposit rates because of the DZLB. Panels (d-f) show the effects on output, separations, and the volume of credit. The takeaway is that the policy is expansionary prior to entering the liquidity trap. However, the effect is quantitatively small because, as we have noted, borrowers are less sensitive to changes in the loan rates. This response rationalizes the idea that in a liquidity trap, MP has reached its limits and is akin to pushing on a string. Neither reductions in the IOR nor OMO can stimulate the economy.

4.2 A Credit Crunch and Macro Insurance

Credit Crunch. In the normative section that follows, we will study the advantages of a corridor system when a credit crunch episode is possible. Here, we first study the standalone effects of a credit crunch. In order to introduce a credit crunch, we modify the model. In addition to the *debt limit* $\bar{s}$, we introduce a time-varying *borrowing limit*, $\tilde{s}_t$. The borrowing limit $\tilde{s}_t$ is triggered before the household reaches its debt limit, $\bar{s} \leq \tilde{s}_t \leq 0$. If households exceed their borrowing limit, $s_t \leq \tilde{s}_t$, they are no longer allowed to accumulate more debt principal, but they are still allowed to roll over their debt. Formally, this means that $c_t dt \leq r^l_t s_t dt + dw_t$ in $s \in [\bar{s}, \tilde{s}_t]$. We interpret an increase in $\tilde{s}_t$ as a credit crunch. From now on, we modify the household's HJB equation to include this time-varying constraint.

The distinction between borrowing and debt limits has technical and economic motivation: The technical motivation is that the borrowing limit allows us to study an unexpected credit crunch. Although an unexpected jump in the debt limit is not well-defined mathematically, an unexpected jump in the borrowing limit is.[21] In turn, the economic motivation is that if a bank wants to cut back on credit, it may be convenient to tighten the borrowing limit, but not necessarily to force households to repay debt principal immediately.[22]

Let's discuss the effects of a credit crunch. We introduce a temporal expected increase in $\tilde{s}_t$ starting from $\tilde{s}_{ss} = \bar{s}$ following the parameters of the credit crunch scenario in Table 3. In the scenario, the policy rates are reacting through the Taylor rule—we obtain similar results if we set η to zero. Figure 9 displays the transition. Panels (a) and (b) display the distribution of wealth among the employed and unemployed at steady state and after the crunch is over in $T_{\tilde{s}}$. Notice that the mass of households at the debt limit vanishes by $T_{\tilde{s}}$ and the wealth distribution slightly shifts to the left, particularly among the unemployed. The reduction in credit occurs because the households that violate the borrowing limit cannot increase their debts to smooth consumption. Instead, because they can only roll-over their debts, they accumulate less debt on average. This produces a decrease in the consumption of borrowers whose debt is above the borrowing limit. As a result of the contraction in aggregate consumption, output falls. Due to price rigidities, real rates remain roughly constant—Panel (d). Upon the shock, unemployment jumps and remains high during the credit crunch. The decline in credit is shown in Panel (c).

[21]With an unexpected change in the debt limit, there would be a positive mass of households violating their debt limits. This does not apply to the borrowing limit $\tilde{s}_t$. An alternative approach is to study a gradual shock to debt limits as in Guerrieri and Lorenzoni (2017).

[22]When a bank extends a loan, it increases its liabilities. This is not true about a loan rollover. During crises, banks may want to roll over debt, although they are unwilling to extend loans because the latter consumes regulatory capital. In addition, if loan repayment is suddenly forced, it can trigger default which may lead to costly underwritings.

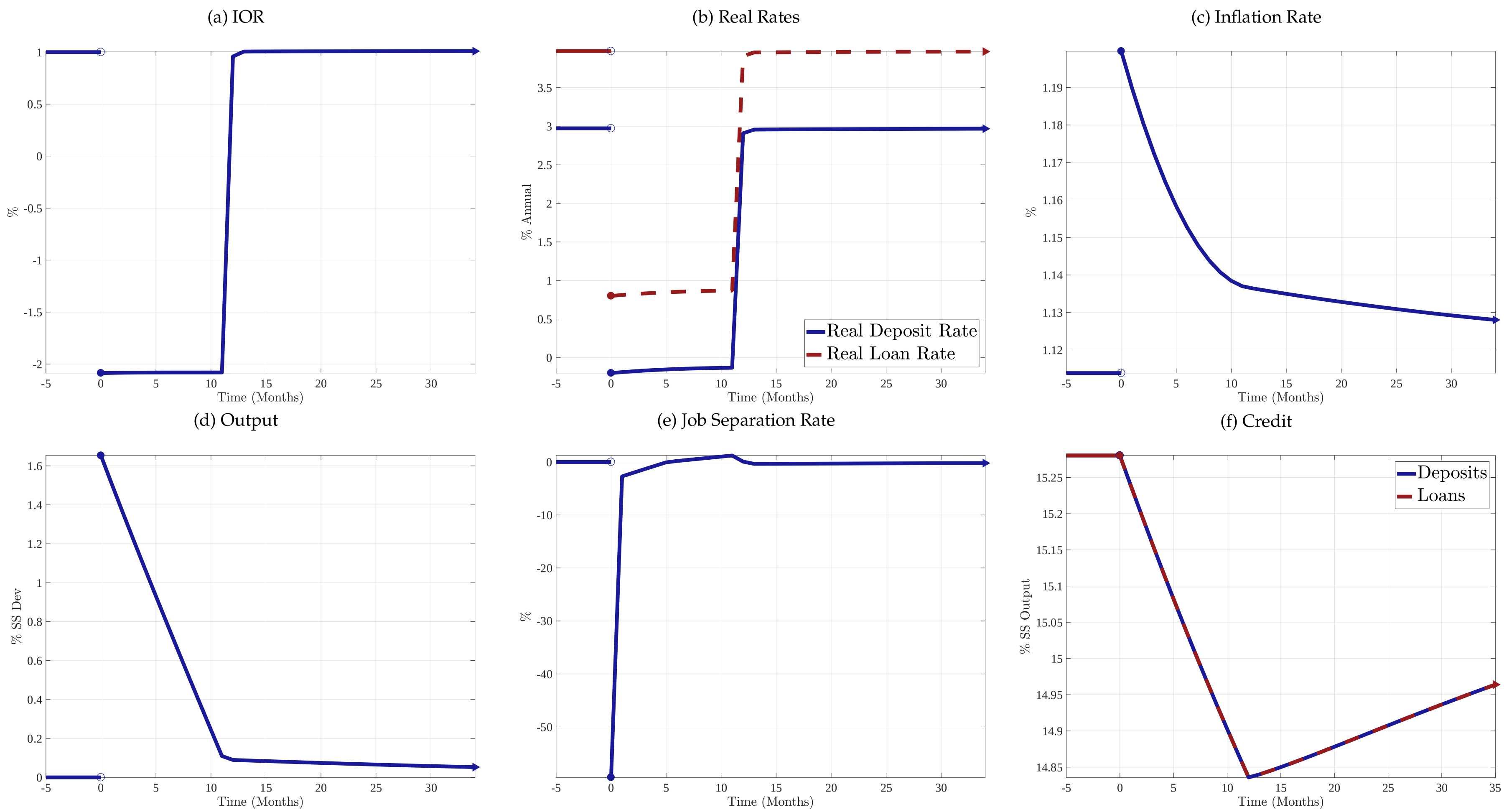


Figure 6: Transition Dynamics after a IOR Reduction (Interest Rate Target).

Note: The figure reports the responses to an unanticipated IOR reduction. The reduction in the IOR is unanticipated at time zero and follows the logistic path equation 26 with calibration in Table 3. All the rates are expressed in annual percentages, the aggregate output is expressed in percentage deviations from the steady state, and the aggregate credit is expressed in the credit-to-steady state output ratio.

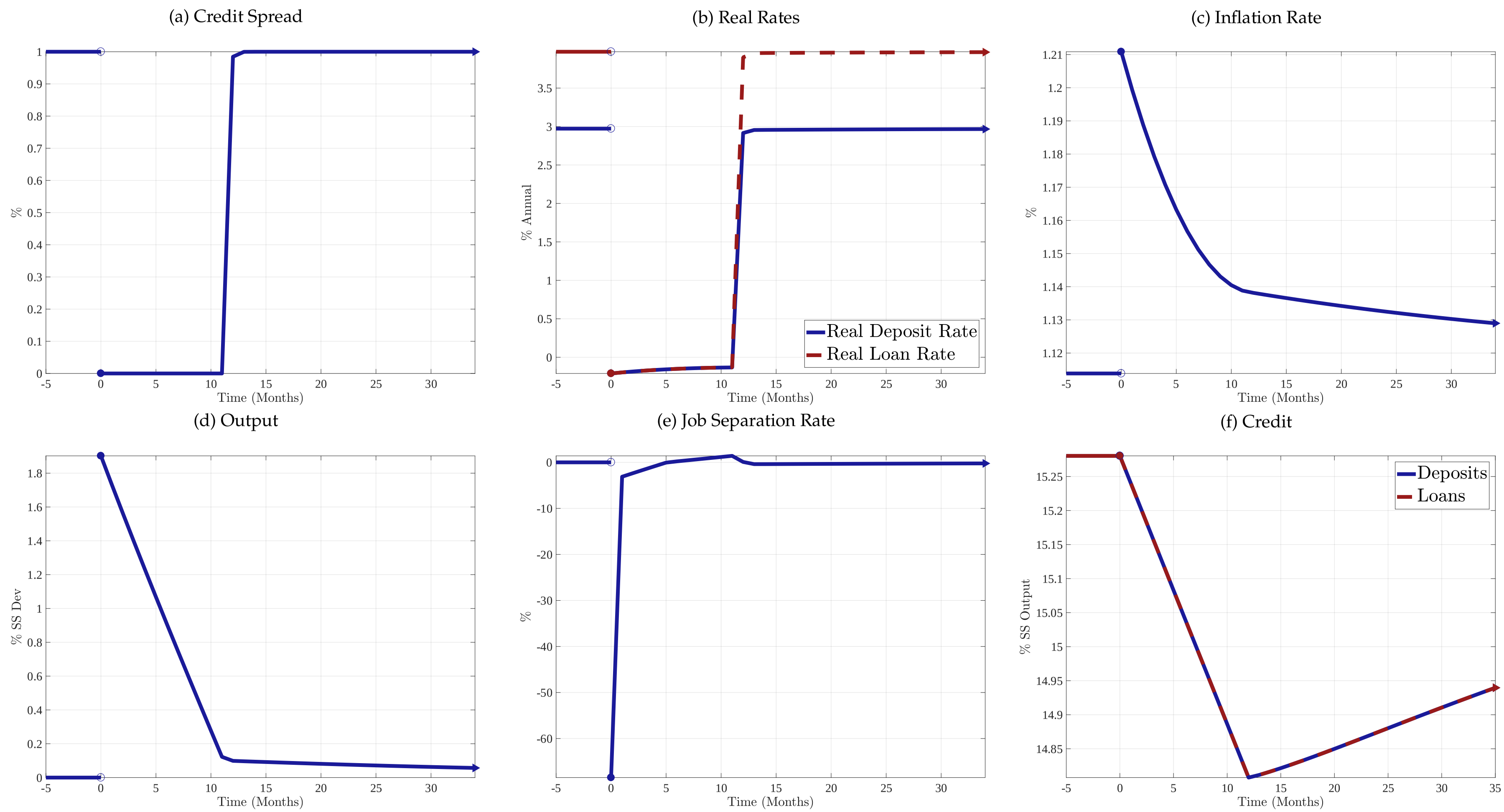


Figure 7: Transition Dynamics after a Credit Spread Reduction (Credit Target).

Note: The figure reports the responses to an unanticipated reduction in the credit spread. The reduction in the credit spread is unanticipated at time zero. Δr_t and η_t follow the calibration in Table 3. The credit spread is fixed at 1% annually before time 0. All the rates are expressed in annual percentages, the aggregate output is expressed in percentage deviations from the steady state, and the aggregate credit is expressed in the credit-to-steady state output ratio.

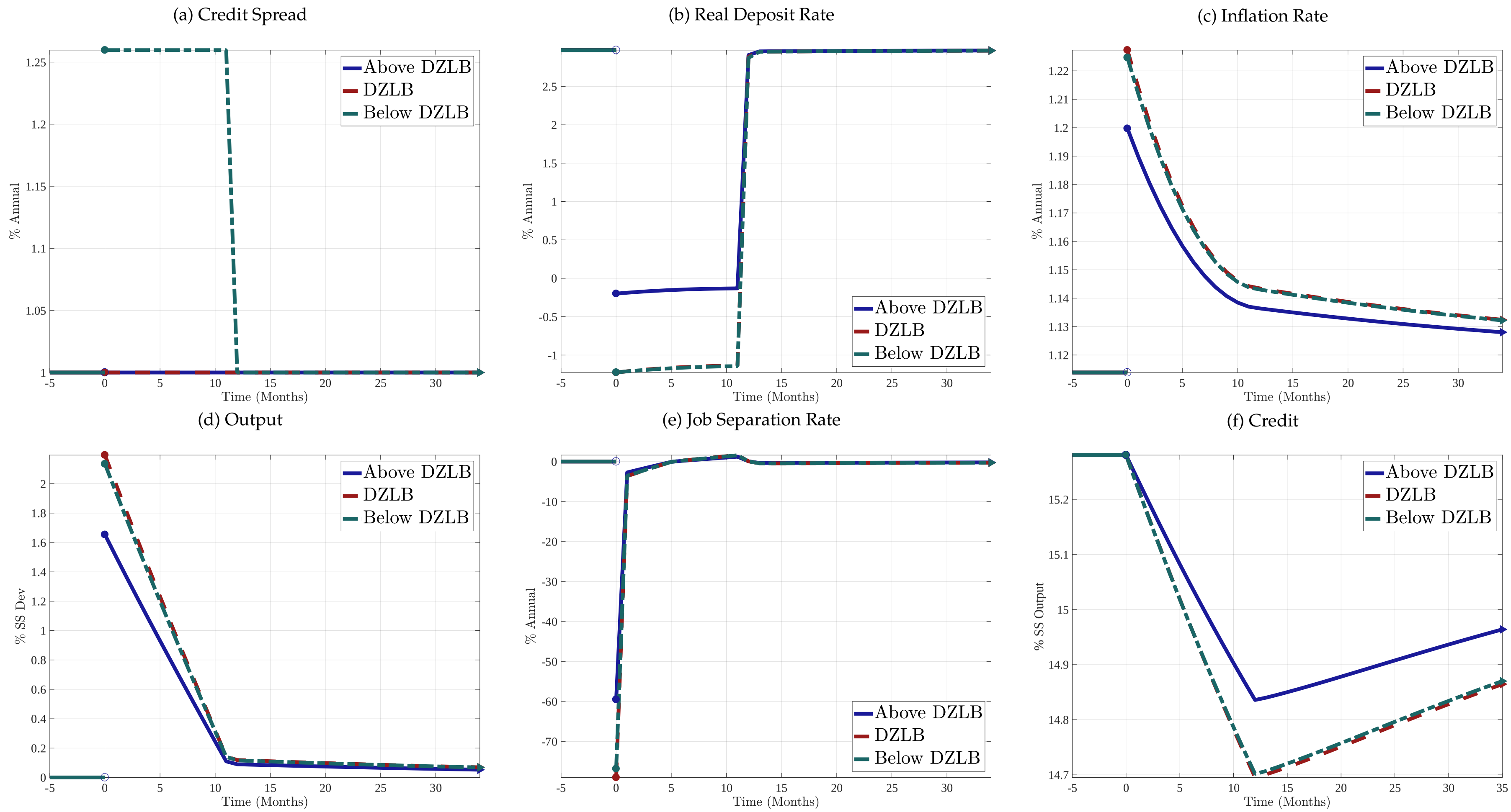


Figure 8: Transition Dynamics after a IOR Reduction (Negative IOR and DZLB).

Note: The figure reports the responses to an unanticipated IOR reductions as in Figure 6 but varying the initial response. In the scenario "DZLB" $\bar{i}_0^m = -2\%$, in "Above DZLB", $\bar{i}_0^m = -1$, and in "Below DZLB" $\bar{i}_0^m = -3$. All rates are expressed in annual percentages, the aggregate output is expressed in percentage deviations from the steady state, and the aggregate credit is expressed in the credit-to-steady state output ratio.

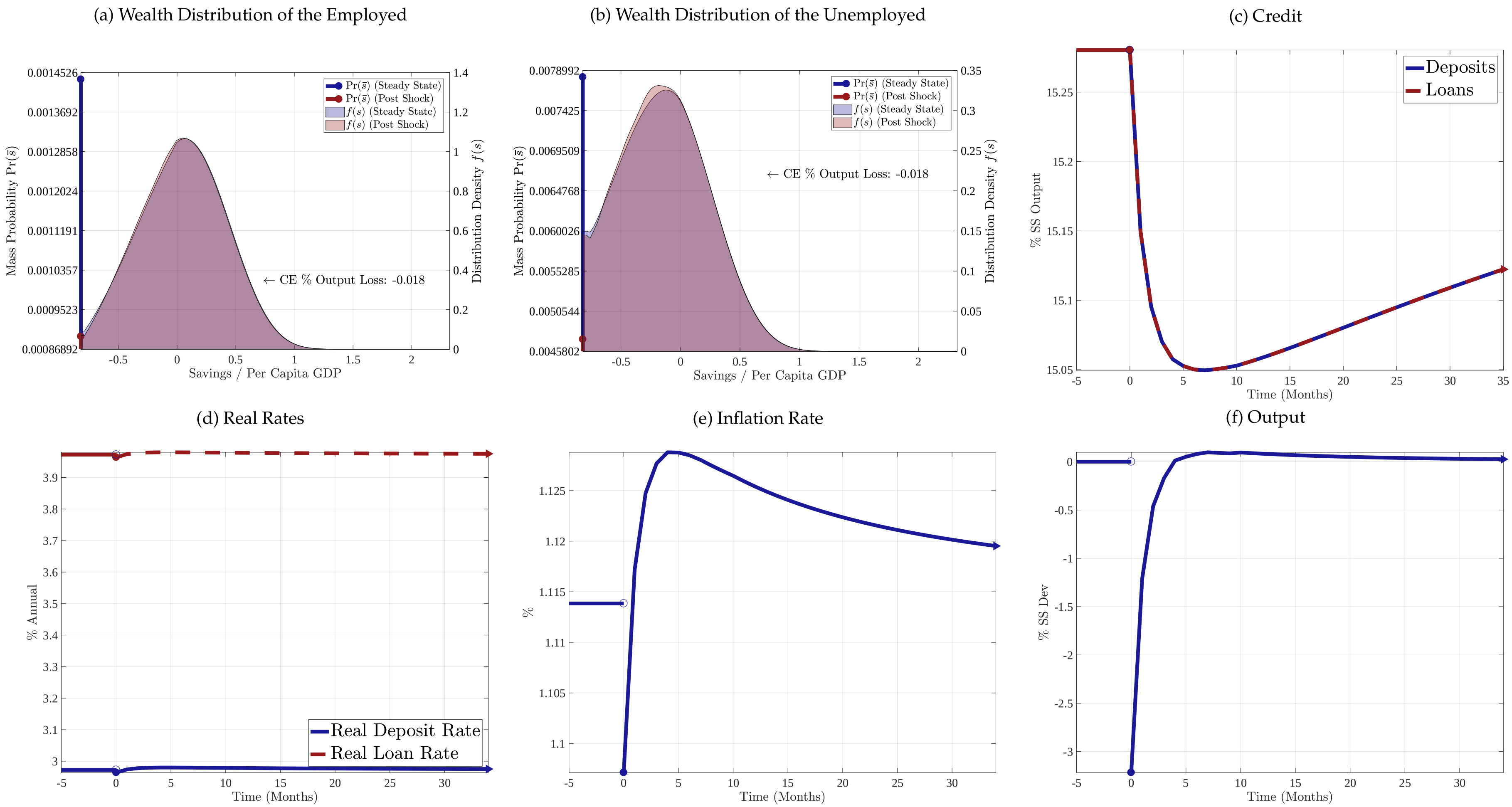


Figure 9: Transition Dynamics after a Credit Crunch.

Note: The figure reports the real wealth distribution, and the responses to credit, rates, inflation and output after an unanticipated credit crunch. In panels (a) and (b), the measure of households with assets $\bar{s}$ is in mass probability (left scale), and the measure of households with $s > \bar{s}$ is in probability density (right scale). The Certainty Equivalent (CE) % loss is expressed in the percentage deviation of the aggregate certainty equivalent after the announcement. The credit crunch is unanticipated at time zero. The net income from credit spread is returned back to households as lump sum transfer. All the rates are expressed in annual percentages, the aggregate output is expressed in percentage deviations from the steady state, and the aggregate credit is expressed in the credit-to-steady state output ratio.

Macro Insurance: stability and macro-prudential power. In this section, we discuss how credit spreads bring welfare benefits through better macro insurance. We refer to macro insurance as the increase in the stabilization power of MP and the ability to mitigate the effects of shocks (macro-prudential power).

For different steady-state values of the spread, Figure 10 displays the transition of output induced by a reduction in the IOR (Panel a), by a reduction in the IOR together with an OMO that takes spreads to zero (Panel b), and by a credit crunch event—as described in Table 3. The takeaway from the figure is that the wider the spread, the greater the macro insurance. In Panel (a) we observe that the wider the initial spread, the greater the output expansion after the reduction in the IOR. The intuition behind the result is that with a wider initial spread, the volume of credit falls, which compresses the distribution of wealth. The stabilization power increases as a result because the mass of agents near their debt limits falls, and these are the agents less sensitive to interest rate cuts. Panel (b) shows that if the reduction in the IOR is coupled with a reduction in spreads, the stabilization is even greater. Finally, panel (c) demonstrates a macro-prudential benefit: With wider initial spreads, because there is less overall borrowing, the interest burden of debt is lower for the average bower. Thus, upon a credit crunch, the consumption by borrowers does not fall as much. Thus, the contraction in aggregate demand in response to the credit crunch event is less severe, even if there is no policy response.

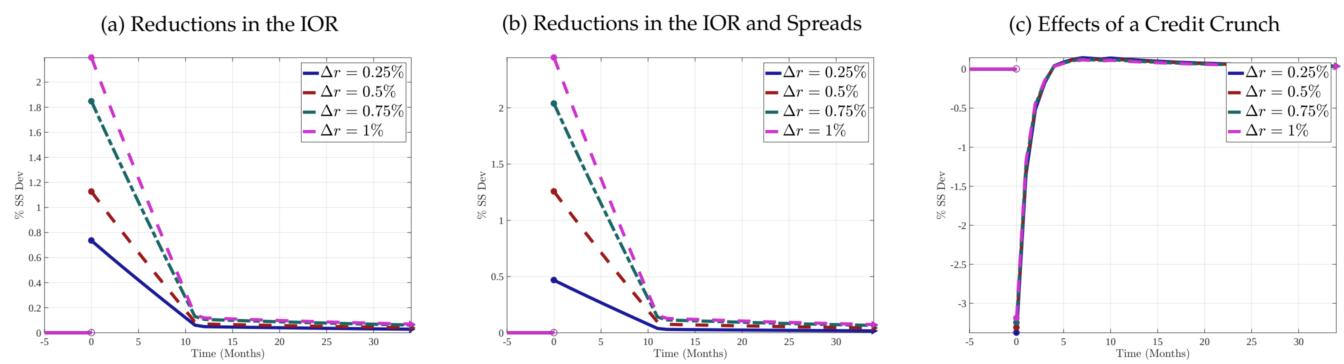


Figure 10: Policy and Credit Crunch for Different Spreads.

Note: The figure reports the responses of aggregate output after an unanticipated reduction in IOR, an unanticipated reduction in IOR and spread, and an unanticipated credit crunch. Aggregate output is expressed in percentage deviations from the steady state. In each panel, we simulate the paths under four different steady-state spreads: $\Delta r_{ss} = \{0.25\%, 0.5\%, 0.75\%, 1\%\}$.

Welfare Loss Decomposition. We construct a metric to measure macro and micro insurance during a transition and use this metric in the normative analysis. First, for a given steady-state spread Δr_{ss}, we compute a metric for the welfare loss associated with the credit crunch, $\mathcal{L}^0$. We define this loss as the output loss that would yield the same welfare in the steady state of a complete-market-and-no-inequality version of the model. In this case, $\mathcal{L}^0$ is obtained by solving:

$$\frac{U\left(Y_{ss}\left(1-\mathcal{L}^0\right)\right)}{\rho} = \sum_{z\in\{e,u\}} \int_{\bar{s}}^{\infty} V\left(s,z,0;\Delta r_{ss}\right)\cdot f^{ss}\left(s;\Delta r_{ss}\right) ds. \tag{28}$$

We are interested in decomposing $\mathcal{L}^0$ into metrics corresponding to macro and micro insurance. To obtain that decomposition we compute $\mathcal{L}^{macro}$ solving:

$$\frac{U\left(Y_{ss}\left(1-\mathcal{L}^{macro}\right)\right)}{\rho} \equiv \int_0^{\infty} \exp\left(-\rho t\right) U\left(Y_t\right) dt. \tag{29}$$

$\mathcal{L}^{macro}$ is the steady-state output loss that would yield the same welfare to a representative agent that consumes the output generated by the model during the transition after the crunch. We add and subtract (29) in (28) to obtain:

$$\begin{aligned}\frac{U\left(Y_{ss}\left(1-\mathcal{L}^{0}\right)\right)}{\rho} &= \underbrace{\frac{U\left(Y_{ss}\left(1-\mathcal{L}^{macro}\right)\right)}{\rho}}_{\text{macro insurance}} \\ &+ \underbrace{\sum_{z\in\{e,u\}}\int_{\bar{s}}^{\infty}\left(\frac{U\left(U^{-1}\left(\rho V\left(s,z,0;\Delta r_{ss}\right)\right)\right)-U\left(Y_{ss}\left(1-\mathcal{L}^{macro}\right)\right)}{\rho}\right)\cdot f^{ss}\left(s;\Delta r_{ss}\right) ds}_{\text{micro-prudential insurance}}. \end{aligned} \tag{30}$$

From this expression, we observe that welfare can be decomposed into a loss that stems from the aggregate output and losses in terms of the deviation of agents from the welfare of the representative agent. We can define the ex-post loss as a residual:

$$\mathcal{L}^{ex-p,micro} = \mathcal{L}^0 - \mathcal{L}^{macro},$$

which is associated with the welfare losses from the second term—these refer to the losses produced by lack of insurance. Clearly, if there was perfect risk sharing, $\mathcal{L}^{ex-p,micro} = 0$.

A final useful metric is a decomposition of $\mathcal{L}^{macro}$ into the loss provoked by the shock and the loss—the gain—attributed to MP stabilization. To obtain that decomposition, we let $\{\tilde{Y}_t\}$ be the output path associated with the credit crunch, without a MP response—as shown in Panel (c) of Figure 10. Let $\mathcal{L}^{macro,pru}$ be obtained by replacing it for $\mathcal{L}^{macro}$ and replacing $\{Y_t\}$ by $\{\tilde{Y}_t\}$ in (29). Thus, $\mathcal{L}^{macro,pru}$ measures how the spread impacts welfare, without a policy response. Thus, it measures the macro-prudential benefits of a given spread. The gain of MP stabilization is the difference between the macro loss and the macro-prudential loss,

$$\mathcal{L}^{stab} \equiv \mathcal{L}^{macro} - \mathcal{L}^{macro-pru}.$$

We present values for these metrics in the next section, as they are critical to understanding the sources of running lean balance sheets.

5 Normative Analysis: Optimal use of the Credit Channel

So far, we analyzed MP from a positive standpoint. A lesson from the steady-state analysis is that spreads hurt micro insurance, but a lesson from the transitional dynamics is that MP is more powerful in an economy where credit was contained by higher spreads. In this section, we investigate if it is optimal to flood banks with reserves to eliminate spreads. That is, should the CB always run a Friedman rule? The generic answer is no. In this model, the CB should operate a corridor system during booms, sacrificing micro insurance for the sake of better future macro insurance.

To build the case for a corridor system, we must allow the crunch to be an anticipated event. The anticipation of shocks is important because the policy objective is to correct an aggregate demand externality. However, if households anticipate shocks, perhaps their own precautionary behavior may be enough to correct the externality. By permitting shocks to be anticipated, we can conduct a proper ex-ante welfare calculation.

At this stage, we face a technical challenge: Computing solutions in incomplete-markets models with aggregate shocks is unfeasible when the state variable is the wealth distribution. This is why the literature employs approximations (like bounded rationality as in for exampe, Krusell and Smith, 1998). Here, we want to say something about optimal policy, which complicates things further. The technical challenge is thus to provide an exact solution to the optimal policy that does not rely on bounded rationality approximations that possibly confound effects. Next, we present an approach that provides insights without the need to compute the entire solution.

The Risky Steady State. Although it is impossible to compute an exact transition with recurrent shocks, we employ an approach to compute solutions when the credit crunch is expected to occur only once. In particular, we employ the risky steady-state (RSS) approach developed in Bigio, Nuno and Passadore (2020a) following Coeurdacier, Rey and Winant (2011): In our context, the RSS is defined as the asymptotic time limit of an economy where households anticipate the realization of a *single credit-crunch event*. That is, we compute the steady state of an economy where the credit crunch is expected, but the shock has not yet been realized. The RSS is the asymptotic limit economy as the waiting time of the shock approaches infinity. After the credit crunch, households do not expect the shock to occur again, so the model solution after the credit crunch is deterministic, but it has the RSS as an initial condition. The assumption of a single event is reasonable, if we consider shocks to be "disaster" events as discussed in Barro and Ursúa (2012). Disaster shocks are large shocks that happen very rarely, and hence it is reasonable to disregard the effect of a second shock on agent behavior. For sufficiently high discounting and events that are far apart enough in time, the deterministic behavior after the single shock should approximate well the behavior under recurrent shocks—akin to the turnpike Theorem.

In the context of this paper, the RSS is characterized as follows: given a Poisson intensity for the realization of an an aggregate credit-crunch shock, θ,and a real spread, Δr, the RSS is characterized by a modified household HJB equation and a wealth-employment distribution:

Definition 2 *[RSS] The risky steady state of the economy is given by:*

a) the solution to the household's value and policy functions at the RSS are the solutions to:

$$\rho\tilde{V}(z,s) = \max_{\{c\}} U(c) + \tilde{V}_s' \cdot \tilde{\mu}(z,s) + \Gamma_{rss}\left[\tilde{V}(z',s) - \tilde{V}(z,s)\right] + \theta\left[V(z,s,0) - \tilde{V}(z,s)\right] \tag{31}$$

and $\dot{s} \geq 0$ at $s = \bar{s}$ where $\tilde{\mu}(z,s) \equiv r_{rss}s - c + w_{rss}(z) + T_{rss}$.

b) the RSS distribution of wealth and employment status is:

$$\begin{aligned} 0 &= -\frac{\partial}{\partial s}\left[\tilde{\mu}(e,s)\tilde{f}_{rss}(e,s)\right] - \Gamma^{eu} \cdot \tilde{f}_{rss}(e,s) + \Gamma^{ue} \cdot \tilde{f}_{rss}(u,s), \text{ and} \\ 0 &= -\frac{\partial}{\partial s}\left[\tilde{\mu}(u,s)\tilde{f}_{rss}(u,s)\right] - \Gamma^{ue} \cdot \tilde{f}_{rss}(u,s) + \Gamma^{eu} \cdot \tilde{f}_{rss}(e,s). \end{aligned} \tag{32}$$

c) In (31), the jump in the value $V(z,s,0)$ is the value function associated with a transition with initial condition $\tilde{f}$.

d) Given a spread target, Δr, a real deposit rate r^a that solves (24) applied to $\tilde{f}$ and T_{rss} is given by (23) applied to $\tilde{f}$ ad Δr.

The technical advantage of the RSS approach is that it can be computed exactly and efficiently, as is clear from the proposition. The only complication is that the distribution of wealth at the start of the crunch, is unknown. However, we no longer need to solve for a fixed point in the space of distributions, but only in the space of functions—the problem is of equal computational complexity as a perfect-foresight transition. To see this, observe that to solve (31), all we need to obtain is a RSS value for the real rate, r_{rss}. With r_{rss}, we obtain consumption rules that solve (31), and from these, we obtain $\tilde{f}_{rss}$ according to (32). Then, $\tilde{f}_{rss}$ is the initial condition for a transition that converges to steady state. Appendix F presents the complete algorithm.

Optimal CB Balance Sheet. To analyze the optimality of a corridor system, we study welfare at the RSS. We compare the ex-ante welfare losses for different policy spreads at the RSS and different stabilization policies during the realization of the crunch episode.

Due to the super-neutrality property of the environment, steady-state output is a constant—although credit varies across different RSS. This feature provides us with a consistent benchmark to compare welfare. We measure welfare using a utilitarian criterion, with equal weights across agents. We express the welfare loss as we did for a steady state, but now computed in the RSS. Thus, $\mathcal{L}^{rss}(\Delta r_{ss})$ solves:

$$\frac{U\left(Y_{ss}\left(1 - \mathcal{L}^{rss}(\Delta r_{ss})\right)\right)}{\rho} = \sum_{z\in\{u,e\}} \int_{\bar{s}}^{\infty} \left(V_{rss}(z,s)\right)\tilde{f}_{rss}(z,s)\,ds.$$

Before analyzing the effects at the RSS, it is instructive to first discuss the welfare loss in a deterministic steady state. In steady state, welfare losses are increasing with the spread. As discussed above, higher spreads compress the steady-state distribution of wealth, and produce less inequality—see Section 4. Since the objective of the planner is concave, an effect other than wealth inequality must produce the greater welfare losses. For that reason, from a steady-state perspective, a lower spread leads to lower welfare losses. In other words, spreads hurt micro insurance. Things turn out to be different if an aggregate shock is anticipated.

We now consider various values for an initial RSS spread prior to a credit crunch shock. In all cases, we start from $i^m_{ss} = 1\%$. For each spread, we compute the RSS and then feed a credit crunch episode. In turn, we consider four policy responses after the shock, corresponding to the policy experiments of the previous sections. Under policy I, we close the spread to 0 and fix the IOR. Under policy II, we reduce the IOR up to the DZLB and leave the spread constant. Under policy III, we reduce the IOR to 1% below the value that triggers the DZLB. Under policy IV, we reduce the IOR only up to the DZLB, and close the spread to 0. In all cases, we set $\eta_0 = 0$. The policy instruments are constant for one year and then follow the logistic paths.

The main takeaways are summarized by Figure 11. The Figure reports the RSS welfare for each spread in the x-axis. Each curve (with the different colors) corresponds to one of the four stabilization policies after the crunch. Comparing across ex-post policies, the greatest loss occurs when only the spread is closed and there is no response in rates. We can also observe if policy only changes the IOR, it induces greater welfare than if it just lowers spreads. This is because the spread is a more powerful tool for a given deposit rate, there is more scope to drop rates because the spread is bounded. The figure also shows that reductions in the IOR beyond the DZLB are counterproductive, as we argued above. Naturally, the best the CB can do is to both eliminate the spread and bring the IOR down to the boundary of the DZLB.

For our discussion, the most relevant comparison is across initial spreads: Regardless of the policy, the lowest losses are for intermediate spreads near 0.75%. Why? In the RSS, the welfare loss is governed by a tradeoff: A higher initial spread induces ex-ante welfare losses due to worse micro insurance. However, during a transition, the effects of the credit crunch are mitigated when the initial spread is higher. This is because higher spreads compress the distribution of wealth which mitigates the direct impact of the credit crunch and increases the power of MP, as we discussed earlier.

To further illustrate the tradeoff between micro and macro insurance, Panel (a) of Figure 12 plots the welfare loss for various policies. The x-axis plots the ex-ante spread and the y-axis plots the IOR immediately after the shock—in all cases we close the spread to zero and keep the IOR fixed during the crunch. The vertical axis plots the RSS welfare loss. We can observe again that the optimal ex-ante spread is positive. In terms of the response of the IOR, the welfare loss is decreasing up to the value that activates the DZLB—at some negative value for the IOR. Panel (b) of Figure 12 reports

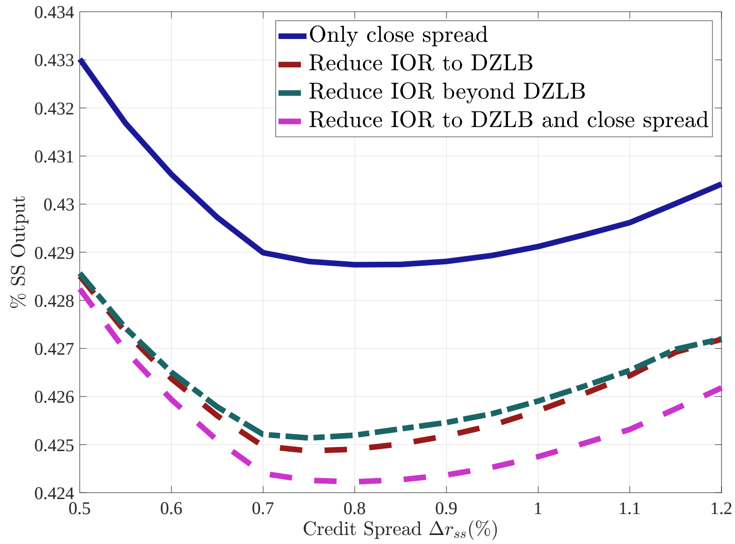
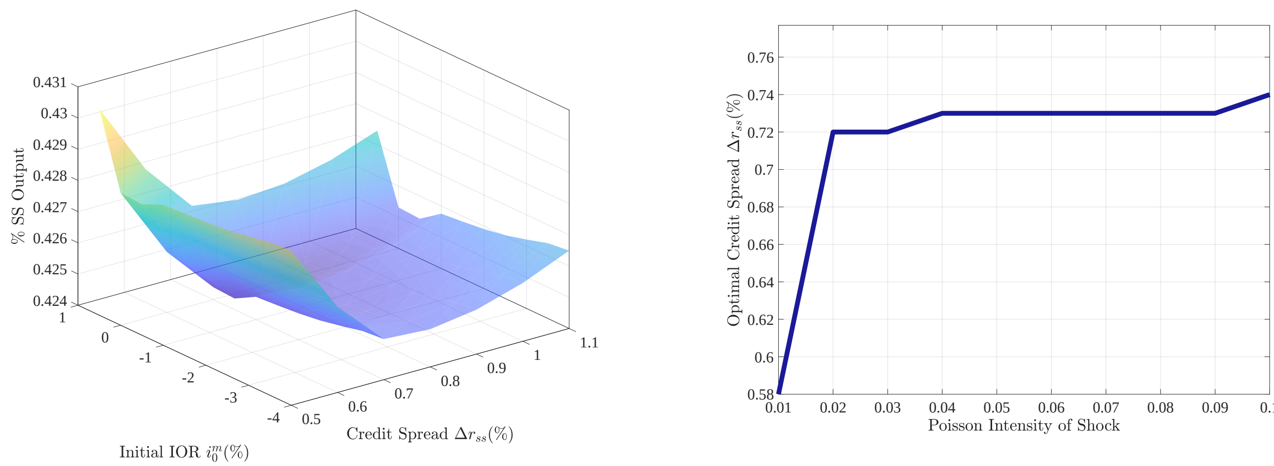


Figure 11: Welfare Loss of Policies During a Credit Crunch (% deviation of Certainty Equivalent from Y_{ss}).

Note: This figure depicts the welfare loss of a credit crunch policy response for different initial spreads. The welfare loss is measured in % deviation of aggregate CE from steady-state output.

the optimal spread as a function of the Poisson intensity of the realization of the credit crunch event. Naturally, the optimal spread is zero if the intensity is zero, as occurs in steady state. As the intensity increases, the optimal spread increases until it reaches a level where there are no further benefits: Above a steady-state spread of 0.72%, the mass of borrowers at their borrowing limits falls to a small value where most of the effect is gone. Beyond that point, there are not further gains of increasing spreads.

Noticeably, the welfare cost of a credit crunch and lack of insurance are small—as in most macroeconomic models. In this model, they only cost around 0.4% of the certainty equivalent in the first best allocation. Operating at the optimal ex-ante spread with a sufficiently small balance sheet can mitigate those welfare losses by about 1%. Naturally, the model needs many more ingredients to increase the welfare losses produced by the inherent frictions.



(a) Risky Steady State Welfare Loss

(b) Optimal Credit Spread

Figure 12: Optimal Credit Spreads.

Note: Panel (a) of the figure plots the RSS welfare loss as a function of the credit spread prior to the shock and the IOR after the shock. Each pair of the initial IOR i_0^m and the credit spread Δr_{ss} in the figure represent the following exercise: in the RSS, the IOR is 1% and the credit spread is equal to Δr_{ss}. When the credit crunch occurs, the government sets the IOR to be i_0^m and reduces the credit spread to 0 for 12 months, and adjusts the paths of the IOR and the credit spread following logistics. Panel (b) plots the optimal credit spread as a function of the Poisson intensity of the credit crunch shock. The optimal credit spread minimizes the RSS welfare loss when the initial IOR i_0^m after the credit crunch is 0.

Welfare Decomposition. Finally, we decompose the welfare loss in a RSS into its micro and macro insurance components. We seek to understand where the benefits are coming from. The egalitarian welfare at the RSS can be decomposed into welfare before and after the shock:

$$\begin{aligned}\frac{U\left(Y_{ss}\left(1-\mathcal{L}^{rss}\right)\right)}{\rho} &= \frac{\rho}{\rho+\phi}\cdot\underbrace{\frac{\sum_{z\in\{e,u\}}\int_{\bar{s}}^{\infty}U\left(c^{rss}\left(s,z;\Delta r_{ss}\right)\right)\cdot f^{rss}\left(s;\Delta r_{ss}\right)ds}{\rho}}_{\text{ex-ante welfare}} \\ &+ \frac{\phi}{\rho+\phi}\underbrace{\frac{\sum_{z\in\{e,u\}}\int_{\bar{s}}^{\infty}U\left(U^{-1}\left(\rho V\left(s,z,0;\Delta r_{ss}\right)\right)\right)\cdot f^{rss}\left(s;\Delta r_{ss}\right)ds}{\rho}}_{\text{ex-post welfare}}.\end{aligned} \tag{33}$$

Thus, welfare is the weighted average of welfare obtained from the expected discounted time at the RSS (ex-ante welfare) and the welfare at the start of the credit crunch. The first term in the summation is akin to steady state welfare for which we can compute an ex-ante micro loss $\mathcal{L}^{ex-a,micro}$, as we did in the steady-state decomposition solving (27). The second term is akin to the ex-post welfare at the beginning of the transition, which we can decompose into the terms, $\{\mathcal{L}^{micro-pru}, \mathcal{L}^{stab}, \mathcal{L}^{macro-pru}\}$ as in the end of the previous section. Thus, using the decomposition in (33) we obtain that the RSS loss can be decomposed into approximately the following term:

$$\mathcal{L}^{rss} \approx \frac{\rho}{\rho+\phi}\mathcal{L}^{ex-a,micro} + \frac{\phi}{\rho+\phi}\left(\mathcal{L}^{ex-p,micro} + \mathcal{L}^{macro-pru} + \mathcal{L}^{stab}\right).$$

Policy	Welfare Loss	Decomposition of Welfare Loss			
Δr_{rss}	$\mathcal{L}^{rss}$	$\mathcal{L}^{ex-a,micro}$	$\mathcal{L}^{ex-p,micro}$	$\mathcal{L}^{macro-pru}$	$\mathcal{L}^{stab}$
0.25%	0.4310	0.4537	0.4260	0.1513	-0.1645
0.5%	0.4286	0.4556	0.4295	0.0773	-0.0999
0.75%	0.4249	0.4601	0.4365	0.0347	-0.0744
1%	0.4257	0.4641	0.4423	0.0259	-0.0731

Table 4: Table: RSS Decomposition of Reducing the IOR to the DZLB during a Credit Crunch (% of Y_{ss})
Note: The table reports the RSS welfare loss decomposition for different initial spreads. The decomposition is performed for four values of steady-state spreads: $\Delta r_{ss} \in \{0.25\%, 0.5\%, 0.75\%, 1\%\}$. The column $\mathcal{L}^{rss}$ reports the egalitarian welfare loss at the risky steady state. The column $\mathcal{L}^{ex-a,micro}$ reports the welfare loss for the ex-ante micro insurance. The column $\mathcal{L}^{ex-p,micro}$ reports the welfare loss for the ex-post micro insurance. The column $\mathcal{L}^{macro-pru}$ reports the post-shock macro-prudential loss under no policy response. The column $\mathcal{L}^{stab}$ reports the post-shock stabilization benefits of a policy response. All losses are expressed in percentage deviations from steady-state aggregate output. In all cases, the error term in the decomposition is negligible.

The numerical results from this decomposition applied to the four values of the spread, $\Delta r_{ss} \in \{0.25\%, 0.5\%, 0.75\%, 1\%\}$, are presented in Table 4. As we noted above, the RSS welfare loss $\mathcal{L}^{rss}$ is lowest for a positive spread of 0.75%. This follows from a trade-off between micro insurance and macro insurance. Prior to the shock, micro insurance always decreases with the spread, as evident from the values of $\mathcal{L}^{ex-a,micro}$, just as it does in a steady state. After the shock, the ex-post $\mathcal{L}^{ex-p,micro}$ insurance is also worse because spreads are closed only temporarily during the crunch. The important message is that the micro losses are compensated by greater macro benefits, which increases in the spread. The macro loss is the sum of the macro prudential loss and the stabilization welfare loss. Observe that the macro-prudential loss falls with the level of spreads. This is because, with tighter spreads, the contraction in aggregate demand is lower after the credit crunch for higher spreads. The stabilization loss is negative because MP improves welfare during a crunch. This benefit decreases as we increase spreads because the stabilization power of MP loses importance because the macro-prudential policy does the job of mitigating the effects of the shock. All in all, an ideal spread is obtained when all these marginal conditions offset each other, which occurs at some intermediate value. Because an intermediate spread is generated by a limited supply of reserves, i.e., a lean CB balance sheet, the model suggests that constantly operating with large CB balance sheets is a bad idea.

Other Shocks: Discount-Factor Shocks. The normative message of the paper is laid out in the context of a credit crunch. One alternative way to view aggregate demand externalities is as an episode where agents face discount factor shocks Eggertsson and Krugman (2012). Discount factor shocks can capture deeper phenomena like sectoral shocks like those of the Covid-19 crisis—–see (Guerrieri, Lorenzoni, Straub and Werning, 2020; Bigio, Zhang and Zilberman, 2020b). Appendix E.3 presents an analogue analysis in the context of discount factor shocks. The message remains the same. Limiting credit is desirable to save MP for the future.

6 Conclusion

In the final paragraph of the introduction to his collected works on monetary economics, Lucas (2013), Robert E. Lucas writes: "Now, toward the end of my career as at the beginning, I see myself as a monetarist. My contributions to monetary theory have been to incorporate the quantity theory into modern modeling. For the empirically well established predictions —long-run links— this job has been accomplished. On the harder questions of monetary economics—the real effects of monetary instability, the roles of inside and outside money, this work contributes examples but little in empirically successful models. It is understandable that in the leading operational macroeconomic models today—the RBC and the new-Keynesian models—money as a measurable magnitude plays no role at all, but I hope we can do better than this in the future."

This paper is one of the many recent attempts to let money play that special role that Lucas alludes to. In fact, the model here is a descendant of one of Lucas's models, Lucas (1980). Here, the supply of reserves—outside money in the language of Lucas—is an input for the supply of credit—inside money. The paper is explicit about the implementation of MP and how different instruments affect different interest rates. If we are open to accepting that idea, we begin to challenge some preconceptions about MP: the idea that MP is neutral is long run neutral or that a single interest rate is all that matters to understand MP.

We furthermore argued that in an incomplete markets economy, with nominal rigidities, MP is more powerful in a regime with scarce reserves. This observation allows us to frame a normative message: managing a countercyclical CB balance sheets is desirable. We hope this study contributes to the debates on the optimal conduct of MP and that Lucas sees a positive slope in this small step.

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Online Appendix

Not for Publication

A List of Acronyms and Accounting Identities

List of Acronyms in the Paper. Along the paper we used the following acronyms:

- MP: Monetary Policy
- CB: Central Bank
- DZLB: Central Bank
- IOR: Interest on Reserves
- OMO: Open-Market Operation

Household Balance Sheet. The household's balance sheet in in nominal terms is structured as:

Assets	Liabilities
m_t^h	l_t^h
a_t^h	$P_t s_t$

Bank Balance Sheet. The balance sheet of an individual bank is structured as:

Assets	Liabilities
m_t^b	a_t^b
l_t^b	

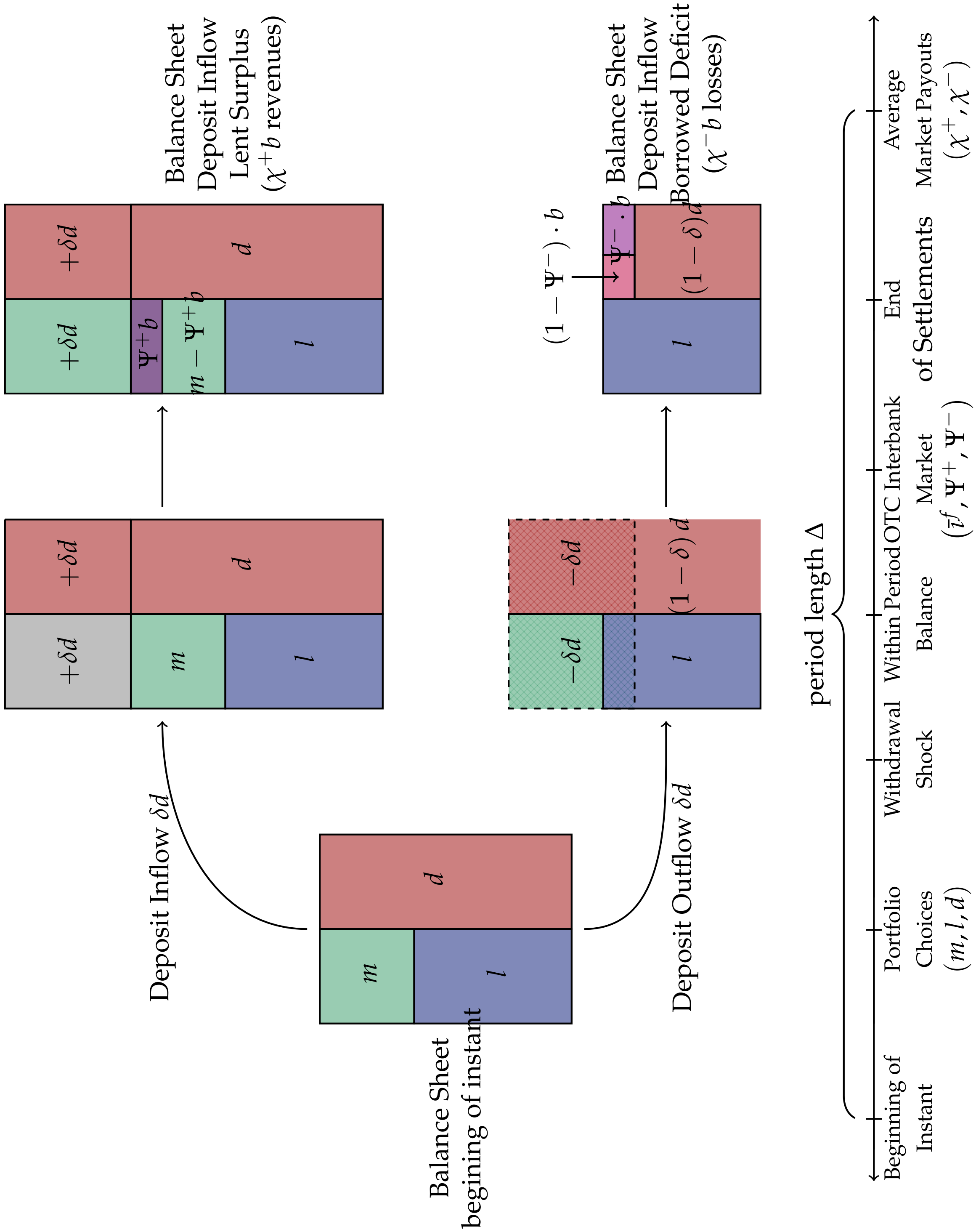
CB Balance Sheet. The balance sheet of the CB is structured as:

Assets	Liabilities
L_t^f	M_t

Monetary Aggregates. The monetary aggregates are given by, M_t, the monetary base, $M0_t$, the currency and $M1_t \equiv A_t^b + M0_t$, the highest monetary aggregate.

Money Multiplier. The money multiplier MM_t is the inverse of the liquidity ratio, $MM_t = \Lambda_t^{-1} = A_t^b / (M_t - M0_t)$.

Timeline of interbank transactions. Figure A presents the accounting for banks, within a Δ time interval. Unlucky banks get hit by negative withdrawal shocks, which can lead them to a negative balance of reserves in the period. That bank mus cover the position by the end of the interval by borrowing funds from other banks, or from the discount window.



Balance Sheet
begining of instant
m
l
d
Deposit Inflow δd
Deposit Outflow δd
$+\delta d$
$-\delta d$
$(1-\delta)\,d$
Balance Sheet
Deposit Inflow
Lent Surplus
($\chi^{+}b$ revenues)
$\Psi^{+}b$
$m-\Psi^{+}b$
Balance Sheet
Deposit Inflow
Borrowed Deficit
($\chi^{-}b$ losses)
$(1-\Psi^{-})\cdot b$
$\Psi^{-}\cdot b$
$(1-\delta)d$
period length Δ
Beginning of Instant
Portfolio Choices (m,l,d)
Withdrawal Shock
Within Period Balance
OTC Interbank Market $(\bar{\iota}^{f},\Psi^{+},\Psi^{-})$
End of Settlements
Average Market Payouts (χ^{+},χ^{-})

B Formulas for the Interbank-Market Payments

According to Bianchi and Bigio (2017), the formulas for the interbank market rate and the trading probabilities depend are broken into two cases, depending on whether $\theta > 1$ or not.

Case I: $\theta \leq 1$. The trading probabilities for surpluses and deficit positions along a trading session are:

$$\psi^{+}(\theta) \equiv \theta\left(1-e^{-\lambda}\right), \qquad \psi^{-}(\theta) \equiv 1-e^{-\lambda}.$$

The expected interbank payments are given by:

$$\chi^{-}(\theta,\eta) = \iota \frac{(\theta+(1-\theta)\exp(\lambda))^{1-\eta}-\theta}{(1-\theta)\exp(\lambda)},$$

and

$$\chi^{+}(\theta,\eta) = \iota \frac{\theta(\theta+(1-\theta)\exp(\lambda))^{1-\eta}-\theta}{(1-\theta)\exp(\lambda)}.$$

The resulting average interbank market rate is:

$$\bar{i}^{f}(\theta,\eta) \equiv i^{m} + \iota \frac{(\theta+(1-\theta)\exp(\lambda))^{1-\eta}-1}{1-\exp(\lambda)} \tag{34}$$

Case II: $\theta \geq 1$. The trading probabilities for surpluses and deficit positions along a trading session are:

$$\psi^{+}(\theta) \equiv \left(1-e^{-\lambda}\right), \qquad \psi^{-}(\theta) \equiv \theta^{-1}\left(1-e^{-\lambda}\right).$$

The expected interbank payments obtained as follows a simple symmetry property:

$$\chi^{-}(\theta,\eta) = \iota - \chi^{+}\left(\theta^{-1}, 1-\eta\right)$$

and

$$\chi^{+}(\theta,\eta) = \iota - \chi^{-}\left(\theta^{-1}, 1-\eta\right)$$

The resulting average interbank market rate is:

$$\bar{i}^{f}(\theta,\eta) \equiv \iota - \bar{i}^{f}\left(\theta^{-1}, 1-\eta\right) \tag{35}$$

Equal Bargaining Power. In the present paper we set $\eta = 1/2$. Thus, we have that for $\theta > 1$, the solution is:

$$\chi^{-}(\theta) = \iota - \chi^{+}\left(\theta^{-1}\right)$$

and

$$\chi^{+}(\theta) = \iota - \chi^{-}\left(\theta^{-1}\right)$$

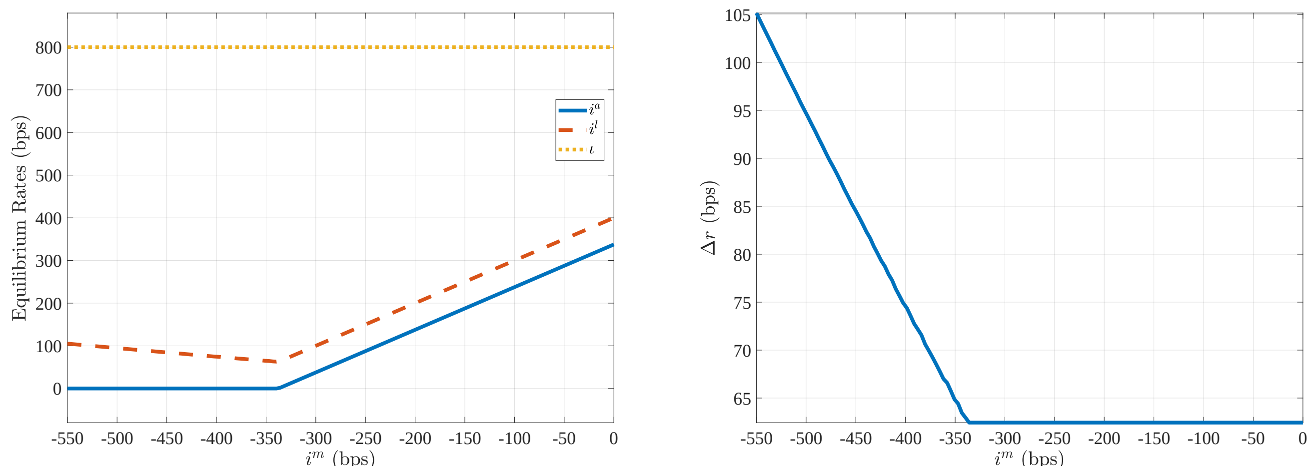
The resulting average interbank market rate is:

$$\bar{i}^f(\theta) \equiv \iota - \bar{i}^f\left(\theta^{-1}\right) \tag{36}$$

C Implementation

C.1 Alternative Implementations

MP implementation in a liquidity trap: Spread and Negative Interest on Reserves. Figure 3 that depicts a map from the liquidity ratio to borrowing and lending rates. That figure is valid when police variables are set within the corridor system regime. In Figure 13 we keep Λ^{MB} constant and show borrowing a lending rates, as we vary i^m. As we can observe, there's an interval of values for i^m such that the spread is constant and both rates move in parallel. Once i^m reaches a sufficiently low value, further reductions in i^m begin to increase spreads while the deposit rate stays fixed. Beyond that point, currency is held by households.



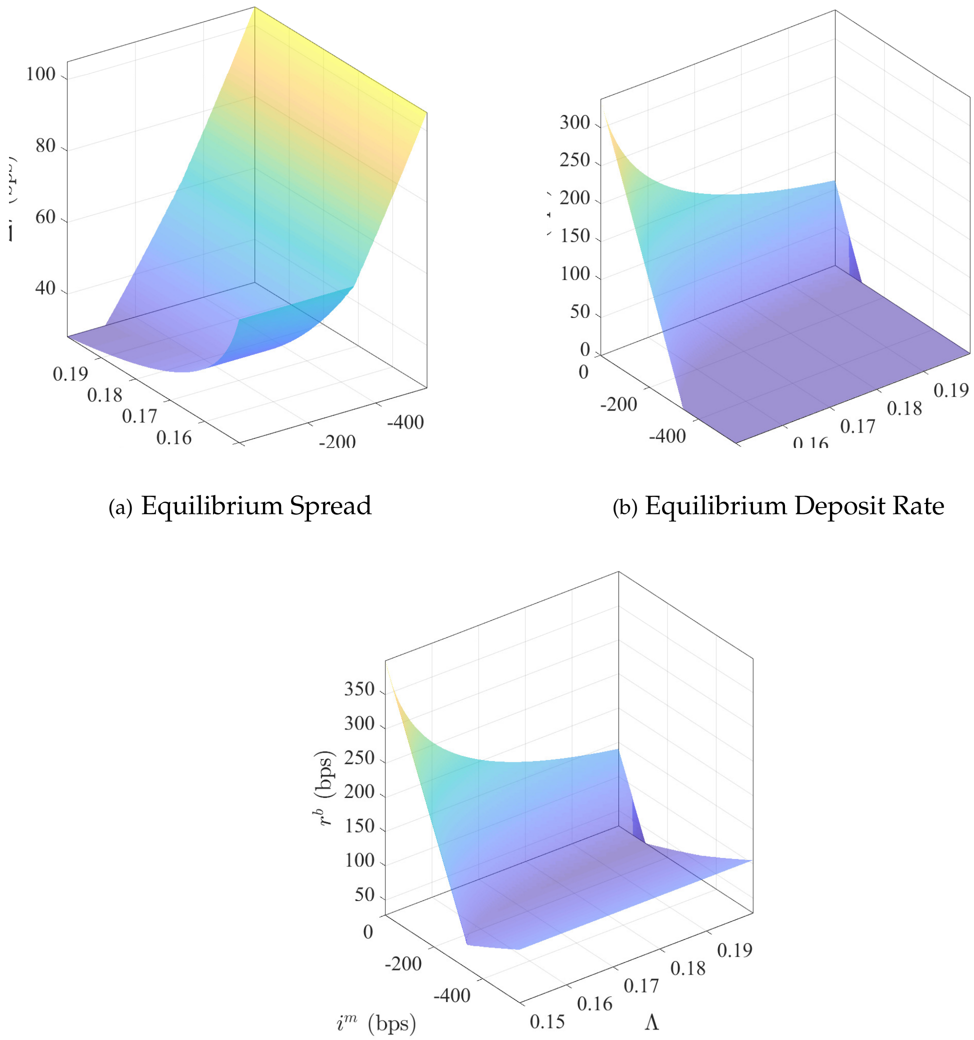
(a) Equilibrium Rates

(b) Equilibrium Spread

Figure 13: Negative Interest on Reserves and the DZLB.

Note: This figure depicts the equilibrium rates and spread as a function of interest on reserves under DZLB. All the rates and spread are expressed in basis points.

In 14 we vary i^m and Λ^{MB} together. Panel (a) shows the spread as a function of both policy variables. There are many combinations that allow to implement the same spread at the DZLB. Panels (b-c) the corresponding deposit and loans rates.



(a) Equilibrium Spread

(b) Equilibrium Deposit Rate

(c) Equilibrium Loan Rate

Figure 14: Negative Interest on Reserves, Liquidity Ratio and the DZLB.
Note: This figure presents the equilibrium spread, deposit rate and loan rate as functions of liquidity ratio and interest on reserves under DZLB. All the rates and spread are expressed in basis points.

Alternative Implementations. In the current formulation, the CB has two tools, $\{i_t^m, M_t\}$. We observed that i^m controls inflation directly and that the size of the balance sheet can achieve a desires spread. We took as given the spread ι. In principle, a desired spread can also be implemented by moving $i^{dw} - i^m$, while keeping M fixed. We could be tempted to argue that these instruments have different fiscal consequences, but they don't:

Corollary 1 *[No Fiscal Consequence of an implementation choice] Consider two policies $\{\iota, \Lambda_t\}$ that implement the same spread, Δr_t. Both are consistent with the same government budget constraint.*

It is worth discussing alternative MP implementations (Bindseil, 2014, reviews cross-country practices.). One way to the control the spread directly is through OMO that targets the interbank rate, $\bar{i}^f$. Because there is a map from $\bar{i}^f$ to

Δr_t, a target for the interbank rate also implements a spread independently of i^m. In practice, most CBs have an explicit interbank rate target, but restrict the way in which they achieve that target: targeting an interbank market at the middle of the corridor, $\bar{i}^f = i^m + \frac{1}{2}\iota$. Other countries keep the rate on reserves at zero, but move ι and maintaining a constant distance between the discount rate and the target. With these additional constraints, CBs simultaneously spreads and inflation when they change policy rates—perhaps inadvertently.

C.2 Implementation Conditions

In the body of the text, we laid out the model. The following proposition describes the set of allocations that can be achieved by a policy with a stationary inflation path.

Proposition 5 *[Implementation Conditions] Consider a desired equilibrium path for $\{r_t^a, \Delta r_t, f_t, \pi_t, \phi_t\}_{t\geq 0}$. To implement the equilibrium path, the CB chooses $\left\{i_t^m, \mathcal{L}_t^f\right\}$ subject to the following restrictions:*

1. $\mathcal{L}_t^f \leq -\int_{\bar{s}}^0 sf(s,t)ds$
2. *The equilibrium liquidity ratio is $\Lambda_t = \min\left\{\Lambda^{zlb}\left(i_t^m, \iota_t\right), \Lambda^{MB}\left(f_t, \mathcal{L}_t^f\right)\right\}$,*
3. *The real transfer, T_t, adjusts to satisfy (23),*
4. *The real spread, Δr_t, satisfies (7) given Λ_t,*
5. *Given i_t^m and Λ_t, the nominal rates $\left\{i_t^l, i_t^a\right\}$ are given by (5-6),*
6. *Given ϕ_t, the unemployment rate $\mathcal{U}_t$ satisfies (8),*
7. *Inflation is consistent with the Phillips curve, (13),*
8. *The real rates $\left\{r_t^l, r_t^a\right\}$ are consistent with Fisher's equation (25),*
9. *The distribution of wealth, f, evolves according to (15); f_0 given,*
10. *Given f, the job separation ϕ_t guarantees the real asset market-clearing condition (24).*

Proposition 5 describes the allocations that can be induced by the CB. These allocations are affected by the CB because it controls the spread and the IOR. The implementation constraint $\mathcal{L}_t^f \leq -\int_{\bar{s}}^0 sf(s,t)ds$ simply tells that there must be enough private liabilities to set $\mathcal{L}_t^f$.

Fiscal-Monetary Interactions in Bewley Models. This is not the only model where MP affects real long-term interest rates. In fact, a first example where that is the case is found in Bewley (1983). However, in that class of models, the effect follows because the CB can affect the long-term value net CB liabilities. Here, net CB liabilities are always zero, but the effect follows from the ability to impact real spread. The model inherits some long-run properties of the Bewley model: the model features neutrality and there is also a continuum of equilibria if we do not fix P_0. Different from Bewley's original model, the economy here is also super neutral because reserves pay interest Ljungqvist and Sargent (2012, see Chapter 18.11 for a discussion of these issues).

D Proofs

D.1 Proof of Proposition 1

Preliminary Steps. We are interested in solutions that satisfy $\{l, m, a\} > 0$. An individual bank takes $\{\chi^+, \chi^-, \theta\}$ and the interest rates $\left\{i^a, i^l\right\}$ as given. Consider the bank's problem:

$$\pi^b = \max_{\{l,m,a\}\in\mathbb{R}^3_+} \left(i^l \cdot l + i^m \cdot m - i^a \cdot a + \mathbb{E}\left[\chi\left(b;\theta,\iota\right)\right]\right)$$

subject to the budget constraint $l + m = a$ and the law of motion for reserve balances at the CB:

$$b\left(a,m\right) = \begin{cases} m \text{ with probability } 1/2 \\ m - \delta \cdot a \text{ with probability } 1/2 \end{cases}.$$

The objective is homogeneous of degree 1. Hence, profits should be zero otherwise the solution is unbounded or zero. Although the solution is unbounded, we can determine the equilibrium portfolio shares consistent with given rates. We also know that the objective is piece-wise linear. Thus, it can be transformed into a linear program. However, here we characterize the solution through the principle of optimality.

To obtain a solution, we substitute out l from the budget constraint to obtain a modified problem:

$$\pi\left(m,a\right) \equiv \max_{\{m,a\}} \left(\left(i^m - i^l\right)\cdot m + \left(i^l - i^a\right)\cdot a + \frac{1}{2}\chi^+ m + \frac{1}{2}\left(\chi^+ \cdot \mathbb{I}_{[\frac{m}{a}>\delta]} + \chi^- \cdot \mathbb{I}_{[\frac{m}{a}\leq\delta]}\right)\left(m - \frac{\delta}{2}a\right)\right),$$

subject to $a \geq 0$ and $m \in [0, a]$. In a solution with $a > 0$, we can factor deposits and write the objective as:

$$\pi\left(m,a\right) \equiv \max_{a\in\mathbb{R}_{++}} a \cdot \left[\left(i^l - i^a\right) + \vartheta\right]$$

where

$$\vartheta \equiv \max_{\mu\in[0,1]} \left[-\frac{\delta}{2}\left(\chi^- \cdot \mathbb{I}_{[\mu\leq\delta]} + \chi^+ \cdot \mathbb{I}_{[\frac{m}{a}>\delta]}\right) + \left(\left(i^m - i^l\right) + \frac{1}{2}\chi^+ + \frac{1}{2}\left(\chi^+ \cdot \mathbb{I}_{[\mu>\delta]} + \chi^- \cdot \mathbb{I}_{[\mu\leq\delta]}\right)\right)\mu\right].$$

and we recover $m = \mu \cdot a$. We further write ϑ as:

$$\vartheta \equiv \max\left\{\vartheta^{scarcity}, \vartheta^{satiation}\right\},$$

where

$$\vartheta^{scarcity} \equiv \sup_{\mu\in[0,\delta)} -\frac{\delta}{2}\chi^- + \left(\left(i^m - i^l\right) + \frac{1}{2}\left(\chi^+ + \chi^-\right)\right)\mu$$

and

$$\vartheta^{satiation} = \max_{\mu\in[\delta,1]} -\frac{\delta}{2}\chi^+ + \left(\left(i^m - i^l\right) + \chi^+\right)\mu.$$

Thus, we break ϑ into two sub-problems, one corresponding to the case where the bank has enough reserves to meet the withdrawal shock and always end with a positive balance (satiation) and another where the bank ends with a reserve deficit if it faces a shock (scarcity).

We have three possible cases depending on the three possible signs of $\left(i^m - i^l\right) + \frac{1}{2}\chi^+$. We describe each of these cases next.

Case 1 (not an equilibrium). If $\left(i^m - i^l\right) + \frac{1}{2}\chi^+ > 0$, we argue that this condition cannot occur in equilibrium. If this case, the solution to $\vartheta^{satiation}$ is to set μ as large as possible. Thus, $\vartheta^{satiation} = \left(i^m - i^l\right) + \left(1 - \frac{\delta}{2}\right)\chi^+$ with $\mu = 1$. Since $\chi^- \geq \chi^+$, the solution to $\vartheta^{scarcity}$ is also to set μ as large as possible, which then yields: $\vartheta^{scarcity} = -\frac{\delta}{2}\chi^- + \left(\left(i^m - i^l\right) + \frac{1}{2}\left(\chi^+ + \chi^-\right)\right)\delta = \left(i^m - i^l\right) + \frac{\delta}{2}\chi^+$. Note that

$$\vartheta^{satiation} - \vartheta^{scarcity} = \chi^+\left(1 - \frac{\delta}{2}\right) - \frac{\delta}{2}\chi^+ = (1-\delta)\,\chi^+ > 0,$$

where the inequality follows form $\delta < 1$.

Thus, under the stated case, it is optimal for the bank to be satiated. Therefore, the solution to the bank's problem is to set, $\vartheta = \vartheta^{satiation}$ with $\mu = 1$. However, since $\mu = 1$, this implies that $a = m$. This cannot occur in an equilibrium with positive loans. Hence, in equilibrium, $\left(i^m - i^l\right) + \chi^+ \leq 0$, two cases we evaluate next.

Case 2 (equilibrium with satiation). Assume that $\left(i^m - i^l\right) + \chi^+ = 0$. Then, $\vartheta^{satiation} = -\frac{\delta}{2}\chi^+ \leq 0$ for any $\mu \in [\delta, 1]$. Also, because $\left(i^m - i^l\right) + \chi^+ = 0$, the value of holding a portfolio with reserves scarcity is:

$$\vartheta^{scarcity} \equiv \sup_{\mu \in [0,\delta)} -\frac{\delta}{2}\chi^- + \frac{1}{2}\chi^-\mu.$$

The objective is increasing in μ, and thus, $\vartheta^{scarcity} = 0$ with a solution $\mu \to \delta$. Hence, $\vartheta = \vartheta^{scarcity} \geq \vartheta^{satiation}$.

We now consider the aggregate conditions, setting $\Lambda = \mu$. Since for any $\mu = \Lambda \geq \delta$, we have that $\theta = 0$, we verify that in any case $\chi^+ = 0$. Thus, $\vartheta^{scarcity} = \vartheta^{satiation} = 0$, and any $[\delta, 1]$ is a solution.Thus, from the stated condition we obtain that:

$$\left(i^m - i^l\right) + \chi^+ = 0 \rightarrow \chi^+ = 0, i^m = i^l.$$

We now turn to the deposit rate. Given that $\vartheta = 0$, and that $\pi = 0$ is an equilibrium condition for any a, it must be that

$$i^m = i^a.$$

Thus, if $\mu \geq \delta$, all banks are satiated and all nominal rates are equal:

$$i^m = i^a = i^l. \tag{37}$$

This case corresponds to the solution under satiation where $\Lambda \geq \delta$.

Case 3 (knife-edge and scarcity equilibria). Finally, assume that $\left(i^m - i^l\right) + \chi^+ < 0$. In this case, the solution to $\vartheta^{satiation}$ is attained when $\mu = \delta$. Thus,

$$\vartheta^{satiation} = \left(i^m - i^l + \frac{1}{2}\chi^+\right)\delta < 0.$$

Now, let's consider the value of $\vartheta^{scarcity}$.

Again, we have to separate the analysis case into three possible cased depending now on the sign of $\left(i^m - i^l\right) + \frac{1}{2}\left(\chi^+ + \chi^-\right)$. We do so in the following steps:

Case 3.a (not an equilibrium). Assume that $\left(i^m - i^l\right) + \chi^+ < 0$ and in addition that $i^m - i^l + \frac{1}{2}\left(\chi^+ + \chi^-\right) < 0$. Then, the solution to $\vartheta^{scarcity} = -\frac{\delta}{2}\chi^-$ obtained when $\mu = 0$. Therefore,

$$\vartheta^{scarcity} - \vartheta^{satiation} = -\left(i^m - i^l + \frac{1}{2}\left(\chi^+ + \chi^-\right)\right)\delta > 0.$$

The inequality follows by hypothesis. Thus, the bank would chose to remain with a reserve scarcity and set $\mu = 0$. However, this solution implies that $m = 0$. Therefore, hence, the case cannot occur with positive reserve holdings.

We furthermore know that:

$$\left(i^l - i^m\right) \in \left[\chi^+, \frac{1}{2}\left(\chi^+ + \chi^-\right)\right],$$

because neither case 3.a nor case 1 can occur in equilibrium.

Case 3.b (knife edge case). Assume that $\left(i^m - i^l\right) + \chi^+ < 0$ and in addition $i^m - i^l + \frac{1}{2}\left(\chi^+ + \chi^-\right) > 0$. Then, the solution to $\vartheta^{scarcity} = \left(i^m - i^l + \frac{1}{2}\chi^+\right)\delta$ obtained $\lim \mu \to \delta$. Thus, $\vartheta^{scarcity} = \vartheta^{satiation}$ and hence, the solution to ϑ requires to set $\mu = \delta$. Since for $\mu = \delta$, we have that $\chi^+ = 0$ then $\vartheta = \left(i^m - i^l\right)\delta$. Thus, combining with the stated hypothesis we obtain:

$$i^m < i^l.$$

From the condition that requires $\pi = 0$, we obtain:

$$i^l - i^a = \left(i^l - i^m\right)\delta.$$

Thus, clearing this condition we obtain:

$$\Delta r \equiv i^l - i^a = \left(i^a - i^m\right)\frac{\delta}{1-\delta}.$$

We re-write the solution to i^a as:

$$i^a = i^m + \Delta r\left(1-\delta\right)/\delta. \tag{38}$$

Then, from $\left(i^m - i^l\right) + \frac{1}{2}\chi^-\left(0\right) > 0$ we obtain that

$$i^m + \frac{1}{2}\chi^-\left(0\right) > i^l = i^a + \dot{\Delta r} \to \frac{1}{2}\chi^-\left(0\right) > \frac{i^a - i^m}{1-\delta} = \frac{\Delta r}{\delta}$$

Thus, in the point where $\Lambda = \delta$, we have that the spread is given by:

$$\Delta r \in \left[0, \frac{\delta}{2}\chi^-\left(0\right)\right]. \tag{39}$$

We arrive at the final case next.

Case 3.c (scarcity solutions). Assume that $\left(i^m - i^l\right) + \chi^+ < 0$ and in addition $i^m - i^l + \frac{1}{2}\left(\chi^+ + \chi^-\right) = 0$. Then, the solution to $\vartheta^{scarcity} = -\frac{1}{2}\chi^+\delta$ obtained by any $\mu \in [0, \delta]$. Thus,

$$\vartheta^{satiation} - \vartheta^{scarcity} = \left(i^m - i^l + \frac{1}{2}\chi^+ + \frac{1}{2}\chi^-\right)\delta = 0,$$

where the equality follows by hypothesis. Thus, the bank is indifferent between level of reserves from $\mu \in [0, \delta]$. Therefore, in this case we have that:

$$i^l = i^m + \frac{1}{2}\left(\chi^+ + \chi^-\right). \tag{40}$$

From the condition that requires $\pi = 0$, we obtain:

$$\left(i^l - i^a\right) + \vartheta = 0 \rightarrow \left(i^l - i^a\right) = \frac{\delta}{2}\chi^-$$

and, thus, we obtain:

$$i^a = i^m + \frac{1}{2}\left(\chi^+ + \chi^-\right) - \frac{\delta}{2}\chi^-. \tag{41}$$

Summary. Thus, taken together, we know that

$$\left\{i^l, i^a\right\} \in \left[i^m, i^m + \frac{1}{2}\left(\chi^+ + \chi^-\right)\right]$$

If an equilibrium features scarcity of reserves, it must fall in case 3.c and satisfy (40) and (41), as stated in the proposition. If the satiation is strict in the sense that $\Lambda > \delta$, then we are in case 2, and the solution is given by (37). Finally, a knife edge case occurs when $\Lambda = \delta$ the satiation is weak in the sense that $\Lambda = \delta$. In this case, there's a range of values as given by Case 3.b. and equations (39) and (38)**. QED.**

D.2 Proof of Proposition 2

We now derive the effects of policy instruments, $\left\{\mathcal{L}_t^f, i_t^m\right\} \in \mathbb{R}_+ \times \mathbb{R}$, on the interest rates and the quantity of currency. Recall that $\{P_t\}$ and $\{f_t\}$, are pre-determined. Thus, we focus on the instantaneous impact of policies, holding fixed prices and wealth. Since effects are static, for the rest of the proof we avoid time subscripts.

Equilibrium conditions. Recall that from Proposition 1 that we have the following subsystem of equilibrium conditions:

$$i^l = i^m + \frac{1}{2}\left(\chi^+(\theta) + \chi^-(\theta)\right) \tag{42}$$

and

$$i^a = i^m + \frac{1}{2}\left(\chi^+(\theta) + (1-\delta)\chi^-(\theta)\right) \geq 0. \tag{43}$$

In turn, the spread is given by:

$$\Delta r = \frac{\delta}{2} \chi^{-}(\theta). \tag{44}$$

Also, note that

$$\theta(\Lambda) \equiv \max\left\{\frac{\delta}{\Lambda} - 1, 0\right\}, \tag{45}$$

with derivative $\theta_{\Lambda}(\Lambda) = -\frac{\delta}{\Lambda^2} < 0$.

The liquidity ratio of banks, considering the monetary base and currency holdings is given by:

$$\Lambda = \frac{M/P}{D/P} = \frac{M/P - M0/P}{\sum_{z \in \{u,e\}} \int_0^\infty sf(z,s,t)\, ds - M0/P} = \frac{\mathcal{L}^f - M0/P}{\sum_{z \in \{u,e\}} \int_0^\infty sf(z,s,t)\, ds - M0/P}. \tag{46}$$

The second equality is obtained by replacing the money-market clearing condition and the deposit-market clearing condition, and then, by replacing the CB balance sheet. Given Λ, the market tightness is given by:

$$\theta(\Lambda) = \delta \left(\frac{\sum_{z \in \{u,e\}} \int_0^\infty sf(z,s,t)\, ds - M0/P}{\mathcal{L}^f - M0/P} \right) - 1. \tag{47}$$

From the household's problem, we also have that

$$M0 \geq 0 \text{ with strict equality if } i^a > i^m. \tag{48}$$

Equations (42-48), represent a subsystem of equilibrium conditions.

Organizing results into Policy Regimes. We study the effects of policy changes in three possible regimes, defined as follows:

- We say that MP is in a *Corridor Regime* if $\left\{\mathcal{L}^f, i^m\right\}$ is such that $i^a > i^m$ and $M0 = 0$.
- We say that MP is in a *Satiation Regime* if $\left\{\mathcal{L}^f, i^m\right\}$ is such that $i^a = i^m > 0$.
- We say that MP is in a *Liquidity Trap* if $\left\{\mathcal{L}^f, i^m\right\}$ is such that $i^a = 0 > i^m$ and $M0 > 0$.

These regions do not overlap and cover the space of policies $\left\{\mathcal{L}^f, i^m\right\} \in \mathbb{R}_+ \times \mathbb{R}$: By definition, the parameters corridor and satiation regimes do not overlap because $i^a > i^m$ and $i^a = i^m$ cannot occur together and $\chi^+(\theta) + (1 - \delta)\chi^-(\theta)$ is a monotone function in θ, thus separating the space in Λ. By definition, a liquidity trap occurs if $\left\{\mathcal{L}^f, i^m\right\}$ induce either (a) $i^a = 0 > i^m$ and $M0 > 0$ or (b) $i^a = i^m = 0$ and $M0 \geq 0$.

We consider the policy effects at the strict interior of these regions. At the boundaries, the system is not differentiable. It is convenient to define the money multiplier as the inverse of the liquidity ratio:

$$\mu \equiv \frac{\sum_{z \in \{u,e\}} \int_0^\infty sf(z,s,t)\, ds}{\mathcal{L}^f}.$$

Case 1 ($i^m > 0$). Assume that $i^m > 0$. Then, because $i^a \geq i^m > 0$, is must be that $M0 = 0$ for any $\mathcal{L}^f$. Note that since $M0 = 0$, combining the CB budget constraint, (16), the money-market clearing condition, (19), and the bank's budget constraint, we obtain that:

$$d\mathcal{L}^f = d\mathcal{M} = d\mathcal{M}^b = -d\mathcal{L}^b, d\mathcal{M}^h = 0.$$

Because $\mathcal{M}^h = 0$, we obtain that (47) is:

$$\Lambda = \frac{\mathcal{L}^f}{\sum_{z\in\{u,e\}} \int_0^\infty sf\left(z,s,t\right)ds}.$$

By Proposition 1, we know that if $\Lambda < \delta$ then, banks must face a reserve scarcity and thus, an equilibrium must feature:

$$i^l > i^a > i^m.$$

We have reserve scarcity only when:

$$\mathcal{L}^f < \delta \int_0^\infty sf\left(s,t\right)ds.$$

We now consider the effects of policy variables when there is scarcity and when there isn't.

Case 1.a ($i^m > 0$ and $\Lambda < \delta$). We now consider the policy effects of changes in $\mathcal{L}_t^f$ and i_t^m when $i^m \geq 0$ and $\mathcal{L}^f < \delta \int_0^\infty sf\left(s,t\right)ds$.

Let's consider first the effects of changes in i_t^m. Taking the differential with respect to i^m in (6) and (5) we obtain:

$$\frac{\partial i^l}{\partial i^m} = 1, \; \frac{\partial i^a}{\partial i^m} = 1, \; \frac{\partial \Delta r}{\partial i^m} = 0.$$

Now, let's consider the differential with respect to $\mathcal{L}^f$. Using (6) we obtain:

$$di^a = \frac{1}{2}\left(\chi_\theta^+\left(\theta\right) + (1-\delta)\chi_\theta^-\left(\theta\right)\right)\frac{\partial\theta}{\partial\Lambda}\frac{\partial\Lambda}{\partial\mathcal{L}^f}d\mathcal{L}^f.$$

Substituting derivatives yields:

$$di^a = -\frac{1}{2}\left(\chi_\theta^+\left(\theta\right) + (1-\delta)\chi_\theta^-\left(\theta\right)\right)\frac{\delta}{\Lambda}\frac{d\mathcal{L}^f}{\mathcal{L}^f} < 0.$$

Similarly, the change in the equilibrium spread is:

$$d\Delta r = -\frac{\delta}{2}\chi_\theta^-\left(\delta\mu - 1\right)\cdot\mu\cdot\frac{d\mathcal{L}^f}{\mathcal{L}^f} < 0.$$

Finally, we obtain:

$$di^l = di^l + d\Delta r = -\frac{\delta}{2}\left(\chi_\theta^+\left(\delta\mu-1\right) + \chi_\theta^-\left(\delta\mu-1\right)\right)\left(\delta\mu-1\right)\frac{d\mathcal{L}^f}{\mathcal{L}^f} < 0.$$

From this expression, we obtain the semi elasticities displayed in the proposition, for the corridor system regime.

Case 1.b ($i^m > 0$ and $\Lambda \geq \delta$). We now consider the policy effects of changes in $\mathcal{L}^f_t$ and i^m_t when $i^m \geq 0$ and $\mathcal{L}^f > \delta \int_0^\infty sf(s,t)\,ds$. Thus, we have that in this case:

$$\frac{\partial i^l}{\partial i^m} = 1,\ \frac{\partial i^a}{\partial i^m} = 1,\ \frac{\partial \Delta r}{\partial i^m} = 0.$$

Therefore, the effects of changes in the IOR are again given by:

$$\frac{\partial i^l}{\partial \mathcal{L}^f}\mathcal{L}^f = 0,\ \frac{\partial i^a}{\partial \mathcal{L}^f}\mathcal{L}^f = 0,\ \frac{\partial \Delta r}{\partial \mathcal{L}^f}\mathcal{L}^f = 0.$$

Case 2 ($i^m < 0$). Assume that $i^m < 0$. In this case, banks cannot be satiated because $0 > i^m = i^a$ implies banks would not hold deposits, a situation that we do not considered in body of the paper. However, it is possible to construct equilibria where deposits are zero. In that case, households only hold currency and the stock of loans is held by the CB. This is ruled out in the paper.

We now define implicitly $\theta^{lb}(i^m)$. Thus function maps and IOR to an interbank market tightness that takes the deposit rate to exactly zero using (6):

$$0 \equiv i^m + \frac{1}{2}\left(\chi^+\left(\theta^{lb}(i^m)\right) + (1-\delta)\chi^-\left(\theta^{lb}(i^m)\right)\right) \geq 0. \tag{49}$$

Also, we define,

$$\Xi = \left(\chi^+(0) + (1-\delta)\chi^-(0)\right).$$

If $i^m \in [-\Xi, 0]$, then since $(\chi^+(\theta) + (1-\delta)\chi^-(\theta))$ is strictly increasing and bounded between $[0, \Xi]$, we obtain that $\theta^{lb}(i^m)$ is well defined in $i^m \in [-\Xi, 0]$. We now substitute (47) with $M0$ into (49) to obtain:

$$\frac{1}{2}\left(\chi^+\left(\delta\left(\frac{\sum_{z\in\{u,e\}}\int_0^\infty sf(z,s,t)\,ds}{\mathcal{L}^f}\right) - 1\right) + (1-\delta)\chi^-\left(\delta\left(\frac{\sum_{z\in\{u,e\}}\int_0^\infty sf(z,s,t)\,ds}{\mathcal{L}^f}\right) - 1\right)\right) = -i^m \leq \Xi.$$

From here, we define $\mathcal{L}^{lb}(i^m, f)$ as the CB balance sheet size such that without currency, the interbank market tightness is exactly $\theta^{lb}(i^m)$. We obtain:

$$\delta\left(\frac{\int_0^\infty sf(s,t)}{\mathcal{L}^{lb}(i^m, f)}\right) - 1 = \theta^{lb}(i^m).$$

Re-arranging yields:

$$\mathcal{L}^{lb}(i^m, f) \geq \frac{\delta}{1 + \theta^{lb}(i^m)} \int_0^\infty sf(s,t).$$

We now analyze the effects of policy in two cases, depending on whether $\Lambda \leq \mathcal{L}^{lb}(i^m, f)$ or $\Lambda > \mathcal{L}^{lb}(i^m, f)$.

Case 2.a ($i^m < 0$ and $\mathcal{L}^f < \mathcal{L}^{lb}(i^m, f)$). Next, we show that if $\mathcal{L}^f < \mathcal{L}^{lb}(i^m, f)$ and $i^m < 0$, the effects of policy are identical to those of Case 1.a. First, observe that,

$$\theta^{lb}(i^m) = \delta\left(\frac{\int_0^\infty sf(s,t)}{\mathcal{L}^{lb}(i^m, f)}\right) - 1 < \delta\left(\frac{\sum_{z\in\{u,e\}}\int_0^\infty sf(z,s,t)\,ds - M0/P}{\mathcal{L}^f - M0/P}\right) - 1 = \delta\mu - 1 = \theta,$$

for any $M0$ where the inequality follows because:

$$\frac{\partial}{\partial M0}\left[\delta\left(\frac{\sum_{z\in\{u,e\}}\int_0^\infty sf\left(z,s,t\right)ds - M0/P}{\mathcal{L}^f - M0/P}\right)\right] = \frac{1}{P}\mu\left(\frac{1}{\mathcal{L}^f - M0/P} - \frac{1}{\sum_{z\in\{u,e\}}\int_0^\infty sf\left(z,s,t\right)ds - M0/P}\right) > 0.$$

and $\mathcal{L}^f < \mathcal{L}^{lb}$. Thus, we have that $\mathcal{L}^f < \mathcal{L}^{lb}$ implies $\theta > \theta^{lb}\left(i^m\right)$. Since this is the case, from Proposition 1, and $\{\chi^-, \chi^+\}$ increasing in θ, we have that $i^a > i^m$, from the bank's problem. Since $i^a > i^m$ implies that $M0/P = 0$, the equilibrium is characterized by the conditions of case 1.a.

Case 2.b ($i^m < 0$ and $\mathcal{L}^f \geq \mathcal{L}^{lb}\left(i^m, f\right)$). Next, we show that if $\mathcal{L}^f \geq \mathcal{L}^{lb}\left(i^m, f\right)$ and $i^m < 0$, the effects of MP are modified. OMO lead to an increase in currency and reductions in rates to an interest rate reversal. Observe that if $\mathcal{L}^f > \mathcal{L}^{lb}\left(i^m, f\right)$ and $M0/P = 0$,then the corresponding market tightness would be:

$$\tilde{\theta} = \delta\left(\frac{\sum_{z\in\{u,e\}}\int_0^\infty sf\left(z,s,t\right)ds}{\mathcal{L}^f}\right) - 1 \leq \delta\left(\frac{\sum_{z\in\{u,e\}}\int_0^\infty sf\left(z,s,t\right)ds}{\mathcal{L}^{lb}\left(i^m, f\right)}\right) - 1 = \theta^{lb}\left(i^m\right).$$

Now, since $\{\chi^+, \chi^-\}$ are increasing in θ, the equilibrium deposit rate obtained from from Proposition 1 would be negative if the market tightness is indeed $\tilde{\theta} < \theta^{lb}\left(i^m\right)$. Thus, it must be that if $\mathcal{L}^f > \mathcal{L}^{lb}\left(i^m, f\right)$, $M0 > 0$ to obtain a tightness such that $\theta = \theta^{lb}\left(i^m\right)$.

In particular, it must be the case that:

$$\theta^{lb}\left(i^m\right) = \delta\left(\frac{\sum_{z\in\{u,e\}}\int_0^\infty sf\left(z,s,t\right)ds - M0/P}{\mathcal{L}^f - M0/P}\right) - 1, \tag{50}$$

and solving $M0/P$, we obtain that:

$$M0/P = \frac{1+\theta^{lb}\left(i^m\right)}{1-\delta+\theta^{lb}\left(i^m\right)}\mathcal{L}^f - \frac{\delta}{1-\delta+\theta^{lb}\left(i^m\right)}\int_0^\infty sf\left(s,t\right) > 0. \tag{51}$$

We now consider the change in currency balances and markets rates to changes in $\mathcal{L}^f$. We have that taking differentials in (51) we obtain:

$$dM0/P = \frac{\mu}{\mu - 1}d\mathcal{L}^f.$$

where we used that:

$$\frac{\mu}{\mu-1} = \frac{\delta\mu}{\delta\mu - \delta} = \frac{1+\theta}{\theta+1-\delta} = \frac{1+\theta^{lb}\left(i^m\right)}{1-\delta+\theta^{lb}\left(i^m\right)}.$$

Next, we produce the effects of policy instruments on the equilibrium rates. We obtain

$$\frac{\partial}{\partial\mathcal{L}^f}\left[\theta^{lb}\left(i^m\right)\right] = 0,$$

and thus, it must be that:

$$\frac{\partial i^l}{\partial\mathcal{L}^f}\mathcal{L}^f = 0,\ \frac{\partial i^a}{\partial\mathcal{L}^f}\mathcal{L}^f = 0,\ \frac{\partial\Delta r}{\partial\mathcal{L}^f}\mathcal{L}^f = 0.$$

Next, consider the effects of changes in the IOR on the market rates and currency holdings. Note that we have from (49) that if the deposit rate is zero:

$$di^m = -\frac{1}{2}\left(\chi_\theta^+ + (1-\delta)\chi_\theta^-\right) d\theta < 0.$$

Then, from the expression for the spread, we have that:

$$d\Delta r = \frac{\delta}{2}\chi_\theta^- d\theta = -\delta \frac{\chi_\theta^-}{\left(\chi_\theta^+ + (1-\delta)\chi_\theta^-\right)} di^m < 0.$$

Thus:

$$\frac{\partial \Delta r}{\partial i^m} = -\delta \frac{\chi_\theta^- \left(\delta\mu - 1\right)}{\left(\chi_\theta^+ \left(\delta\mu - 1\right) + (1-\delta)\chi_\theta^- \left(\delta\mu - 1\right)\right)} < 0.$$

Finally, the effect on currency holdings is given by is:

$$d\theta = \delta\mu \left(\frac{1}{\mathcal{L}^f - M0/P} - \frac{1}{\int_0^\infty sf\left(s,t\right) - M0/P}\right) dM0/P > 0.$$

Thus, we obtain that increases in the IOR produce increases in the reserve balances:

$$\frac{\partial}{\partial i^m}\left[M0/P\right] = -\frac{\delta\mu\left(\frac{1}{\mathcal{L}^f - M0/P} - \frac{1}{\int_0^\infty sf(s,t) - M0/P}\right)}{\frac{1}{2}\left(\chi_\theta^+ + (1-\delta)\chi_\theta^-\left(\theta\right)\right)} < 0.$$

General solution to currency balances. We obtain a general solution to the currency holdings. The solution is given by:

$$M0/P = \mathbb{I}_{[i^m<0]} \cdot \max\left\{\frac{1+\theta^{lb}\left(i^m\right)}{1-\delta+\theta^{lb}\left(i^m\right)}\mathcal{L}^f - \frac{\delta}{1-\delta+\theta^{lb}\left(i^m\right)}\int_0^\infty sf\left(s,t\right), 0\right\}.$$

Note that when $i^m < 0$, then $\theta^{lb}\left(i^m\right) > 0$. Thus, the term on first entry is positive when:

$$\mathcal{L}^f > \frac{\delta}{1+\theta^{lb}\left(i^m\right)},$$

which coincides with (51) when i^m.

Regimes that are not considered in the paper. There could be equilibria where $\mathcal{L}^f = \mathcal{L}^b = \frac{M0}{P} = \sum_{z\in\{u,e\}}\int_0^\infty sf\left(z,s,t\right) ds$. That is, equilibria in which the CB does all the intermediation. In this case deposits are zero and bank balance sheets are empty. This will occur if the interest in reserves is so low that banks cannot hold deposits, $-i^m > \frac{\iota}{2}$. At that point, the deposit rate is zero and the loans rate is also zero. We do not consider this case.

QED.

D.3 Flow of Funds Identity

In the proofs that follow, we make use of the following Lemma.

Lemma 1 *If the deposit, loans and money markets clear, then:*

$$P_t \sum_{z\in\{u,e\}} \int_{\bar{s}}^{\infty} sf\left(z,s,t\right) ds = 0. \tag{52}$$

Proof. The deposits and loans markets clearing conditions require:

$$A_t^b = \sum_{z\in\{u,e\}} \int_0^{\infty} a_t^h(s,z) f(s,z,t) ds \tag{53}$$

$$L_t^b + L_t^f = \sum_{z\in\{u,e\}} \int_{\bar{s}}^{0} l_t^h(s,z) f(s,z,t) ds, \tag{54}$$

and clearing in the money market requires:

$$M_t^b + M0_t = M_t. \tag{55}$$

If we aggregate the budget constraint—the balance sheet identity—of banks, we obtain:

$$A_t^b = L_t^b + M_t^b. \tag{56}$$

Once we combine (53), (54), and (55) into (56), we obtain:

$$\sum_{z\in\{u,e\}} \int_0^{\infty} a_t^h(s) f(s,z,t) ds = \sum_{z\in\{u,e\}} \int_{\bar{s}}^{0} l_t^h(s) f(s,z,t) ds + M_t - M0_t - L_t^f. \tag{57}$$

Nominal deposits and currency are related to real wealth via:

$$P_t \sum_{z\in\{u,e\}} \int_0^{\infty} sf(s,z,t) ds = \sum_{z\in\{u,e\}} \int_0^{\infty} a_t^h(s,z) f(s,z,t) ds + M0_t. \tag{58}$$

and, similarly for loans:

$$-P_t \sum_{z\in\{u,e\}} \int_{\bar{s}}^{0} sf(s,z,t) ds = \sum_{z\in\{u,e\}} \int_{\bar{s}}^{0} l_t^h(s) f(s,z,t) ds. \tag{59}$$

This condition can be expressed in terms of real household wealth, with the use of (58) and (59):

$$P_t \sum_{z\in\{u,e\}} \int_0^{\infty} sf(s,z,t) ds - M0_t = P_t \sum_{z\in\{u,e\}} \int_{\bar{s}}^{0} sf(s,z,t) ds + M_t - M0_t - L_t^f.$$

Thus, using that $M_t = L_t^f$, we obtain:

$$\sum_{z\in\{u,e\}} \int_0^{\infty} sf(s,z,t) ds = \sum_{z\in\{u,e\}} \int_{\bar{s}}^{0} sf(s,z,t) ds.$$

Thus, clearing in all nominal asset markets implies clearing in a single real asset market, (52). **QED.**

D.4 Proof of Proposition 3

The nominal profits of the CB are given by:

$$\Pi_t^{CB} = i_t^l L_t^f - i_t^m \left(M_t - M0_t\right) + \iota_t \left(1 - \psi_t^-\right) B_t^- .$$

Note that the earnings from discount-window loans equal the average payment in the interbank market, and thus:

$$\iota_t \left(1 - \psi_t^-\right) B_t^- = -\mathbb{E}\left[\chi_t \left(b\left(A_t, A_t - L_t\right)\right)\right]. \tag{60}$$

By Proposition 1, banks earn zero profits in expectation. Thus,

$$-\mathbb{E}\left[\chi_t \left(b\left(A_t, A_t - L_t\right)\right)\right] = i_t^l L_t^b + i_t^m M_t^b - i_t^a A_t^b. \tag{61}$$

Thus, substituting (60) and (61) into the expression for π_t^f above yields:

$$\begin{aligned}\Pi_t^{CB} &= i_t^l L_t^f - i_t^m \left(M_t - M0_t\right) + i_t^l L_t^b + i_t^m M_t^b - i_t^a A_t^b.\\ &= i_t^l L_t^h - i_t^a A_t^h,\end{aligned}$$

where we used the clearing condition in the money market, $M_t^b + M_t^0 = M_t$, the deposit market, $A_t^b = A_t^h$, and the loans market, $L_t^h = L_t^b + L_t^f$. Now, observe that:

$$\Pi_t^{CB} = -i_t^l P_t \sum_{z\in\{u,e\}} \int_{\bar{s}}^{0} sf(z,s,t)ds - i_t^a \left(P_t \sum_{z\in\{u,e\}} \int_0^{\infty} sf(z,s,t)ds - M0_t.\right),$$

but we know from the household's problem that $i_t^a M0_t = 0$. Hence, profits are given by:

$$\Pi_t^{CB} = -i_t^l P_t \sum_{z\in\{u,e\}} \int_{\bar{s}}^{0} sf(z,s,t)ds - i_t^a P_t \sum_{z\in\{u,e\}} \int_0^{\infty} sf(z,s,t)ds.$$

Then, since $i^a = i^l - \Delta r$, we have that:

$$\Pi_t^{CB} = -i_t^l P_t \left(\sum_{z\in\{u,e\}} \int_{\bar{s}}^{\infty} sf(z,s,t)ds\right) + \Delta r_t P_t \sum_{z\in\{u,e\}} \int_0^{\infty} sf(z,s,t)ds.$$

Thus, from Lemma 1, we obtain:

$$\Pi_t^{CB} = P_t \cdot \Delta r_t \sum_{z\in\{u,e\}} \int_0^{\infty} sf(z,s,t)ds.$$

Now, we turn to the government's budget constraint, (17), we have that:

$$P_t T_t = P_t \cdot \left(\Delta r_t \sum_{z\in\{u,e\}} \int_0^{\infty} sf(z,s,t)ds\right) + P_t \left(\tau^l \cdot (1 - \mathcal{U}_t) - b \cdot \mathcal{U}_t\right),$$

and dividing by the price level we obtain

$$T_t = \Delta r_t \sum_{z\in\{u,e\}} \int_0^{\infty} s f(z,s,t) ds + \tau^l \cdot (1-\mathcal{U}_t) - b \cdot \mathcal{U}_t$$

as stated by the proposition. **QED.**

D.5 Proof of Proposition 4

Proof of Clearing in all markets. Lemma 1 shows that if all asset markets clear, then there is clearing in real wealth (24). We now proof the converse. That is, if (24) holds, then, the deposit loans, and money markets must clear.

The proof is by contradiction. We start by taking (24) as given. Next, we multiply by P_t in both sides and, by definition, obtain:

$$\sum_{z\in\{u,e\}} \int_{\bar{s}}^{0} l_t^h(s) f(z,s,t) ds = \sum_{z\in\{u,e\}} \int_0^{\infty} a_t^h(s) f(z,s,t) ds + M0_t.$$

From the central bank's balance sheet, we obtain that:

$$\sum_{z\in\{u,e\}} \int_{\bar{s}}^{0} l_t^h(s) f(z,s,t) ds = \sum_{z\in\{u,e\}} \int_0^{\infty} a_t^h(s) f(z,s,t) ds + M_t - M_t^b, \text{ for } t \in [0,\infty). \tag{62}$$

We now substitute the balance sheet of the CB, $M_t = L_t^f$, and the consolidated balance sheet of banks, $M_t^b = A_t^d - L_t^b$, to obtain. In this case, we obtain:

$$\sum_{z\in\{u,e\}} \int_{\bar{s}}^{0} l_t^h(s) f(z,s,t) ds - L_t^f - L_t^b = \sum_{z\in\{u,e\}} \int_0^{\infty} a_t^h(s) f(z,s,t) ds - A_t^d.$$

This equation guarantees that if there is no clearing in the loans market, there is no clearing in the deposit market by that same amount. Assume there is a deviation from market clearing in the amount ε. Then, an income $\Delta r \cdot \varepsilon$ would not be accounted for by. However, since all the spread is earned by the CB, following Proposition (23), it must be that $\varepsilon = 0$. **QED.**

Proof of Walras's Law. Next, we prove that if (24) holds, then the goods market clears, which is a derivation of Walras's law in the continuous-time setting.

Recall that f satisfies the following KFE equations:

$$\begin{aligned}
\frac{\partial}{\partial t} f(e,s,t) &= -\frac{\partial}{\partial s}\left[\mu(e,s,t) f(e,s,t)\right] - \Gamma_t^{eu} \cdot f(e,s,t) + \Gamma_t^{ue} \cdot f(u,s,t), \text{ and} \\
\frac{\partial}{\partial t} f(u,s,t) &= -\frac{\partial}{\partial s}\left[\mu(u,s,t) f(u,s,t)\right] - \Gamma_t^{ue} \cdot f(u,s,t) + \Gamma_t^{eu} \cdot f(e,s,t).
\end{aligned}$$

A similar KFE holds for the cumulative distributions:

$$\begin{aligned}
\frac{\partial}{\partial t} F(e,s,t) &= -\mu(e,s,t) f(e,s,t) - \Gamma_t^{eu} \cdot F(e,s,t) + \Gamma_t^{ue} \cdot F(u,s,t), \text{ and} \\
\frac{\partial}{\partial t} F(u,s,t) &= -\mu(u,s,t) F(u,s,t) - \Gamma_t^{ue} \cdot F(u,s,t) + \Gamma_t^{eu} \cdot F(e,s,t).
\end{aligned}$$

Recall that the integrals in the clearing conditions, are Lebesgue integrals. It is convenient to be explicit about the mass points at the debt limit in (24):

$$0 = \sum_{z\in\{u,e\}} \left[\bar{s}F(z,\bar{s},t) + \lim_{\sigma\to\bar{s}^+} \int_{\sigma}^{\infty} sf(z,s,t)ds \right],$$

so that the first integral is in the Riemann sense. Then, taking time derivatives:

$$0 = \sum_{z\in\{u,e\}} \left[\sum_{z\in\{u,e\}} \bar{s}\frac{\partial}{\partial t}F(z,\bar{s},t) + \frac{\partial}{\partial t}\left[\lim_{\sigma^+\to\bar{s}} \int_{\sigma}^{\infty} sf(z,s,t)ds \right] \right]. \tag{63}$$

Substituting the KFE equations into the first term, we obtain:

$$\begin{aligned} \sum_{z\in\{u,e\}} \bar{s}\frac{\partial}{\partial t}F(z,\bar{s},t) &= -\bar{s}\cdot \sum_{z\in\{u,e\}} \Big(\mu(z,s,t) f(z,\bar{s},t) + \Gamma_t^{zz'} \cdot F(z,s,t) - \Gamma_t^{z'z} \cdot F(z',s,t) \Big). \\ &= -\sum_{z\in\{u,e\}} \bar{s}\mu(z,s,t) f(z,\bar{s},t). \end{aligned} \tag{64}$$

The second line follow from: $\sum_{z\in\{u,e\}} \Gamma_t^{zz'} \cdot F(z,s,t) - \Gamma_t^{z'z} \cdot F(z',s,t) = 0.$[23]
Substituting the KFE equations into the second term of (63), we obtain:

$$\sum_{z\in\{u,e\}} \frac{\partial}{\partial t}\left[\lim_{\sigma\to\bar{s}} \int_{\sigma}^{\infty} sf(z,s,t)ds \right] = \sum_{z\in\{u,e\}} \int_{\bar{s}}^{\infty} \left[\underbrace{-s\frac{\partial}{\partial s}\left[\mu(s,t) f(z,s,t)\right]}_{\mathcal{A}\equiv} - s \underbrace{\Big(\Gamma_t^{zz'} \cdot f(z,s,t) + \Gamma_t^{z'z} \cdot f(z',s,t) \Big)}_{\mathcal{B}\equiv} \right] ds.$$

We analyze each term in the integral. First, notice that $\mathcal{B} = 0$, because again:

$$\sum_{z\in\{u,e\}} \Big[\Gamma_t^{zz'} \cdot f(z,s,t) - \Gamma_t^{z'z} \cdot f(z',s,t) \Big] = 0,$$

Second, we use integration by parts, to obtain that $\mathcal{A}$ is given by:

$$-\sum_{z\in\{u,e\}} \lim_{\sigma\to\bar{s}} \int_{\sigma}^{\infty} s\frac{\partial}{\partial s}\left[\mu(s,t) f(z,s,t)\right] ds = \sum_{z\in\{u,e\}} \underbrace{-s\cdot\mu(s,t) f(z,s,t)|_{\bar{s}}^{\infty}}_{\mathcal{A}.1\equiv} + \sum_{z\in\{u,e\}} \underbrace{\int_{\bar{s}}^{\infty} \mu(s,t) f(z,s,t)\, ds}_{\mathcal{A}.2\equiv}.$$

Importantly, to use integration by parts, in evaluating the definite integral, we use the Lebesgue integral. Thus, $\mathcal{A}.2$ is in the Lebesgue sense again.
Evaluating the terms$\mathcal{A}.1$ yields,

$$\lim_{s\to\infty} f(z,s,t) = 0 \text{ and } \lim_{\sigma\to\bar{s}} \sigma\cdot\mu(\sigma,t) f(z,\sigma,t) = \bar{s}\frac{\partial}{\partial t}F(z,s,t).$$

[23] The employment status is independent of z and population is preserved. Thus, the condition says that within a wealth level, the mixing from employment to unemployment does not change wealth.

Thus, summing the terms (64) and $\mathcal{A}.1$, we obtain that (63), is equivalent to:

$$0 = \sum_{z\in\{u,e\}} \int_{\bar{s}}^{\infty} \mu\left(s,t\right) f\left(z,s,t\right) ds.$$

Next, we compute the integral $\mathcal{A}.2$. Recall that:

$$\mu\left(s,t\right) = \left[r_t\left(s\right)\left(s - m^h\left(z,s,t\right)/P_t\right) - \dot{P}_t/P_t \cdot m^h\left(z,s,t\right)/P_t - c\left(z,s,t\right) + w_t\left(z\right)\right].$$

From the household's problem, $i_t^a \cdot m^h\left(z,s,t\right) = 0$ for $s > 0$ and $m^h\left(z,s,t\right) = 0$ for any $s \leq 0$. Hence, we have that:

$$\left(r_t\left(s\right) + \dot{P}_t/P_t\right) m^h\left(z,s,t\right)/P_t = 0.$$

Thus, we can freely add the term above into the drift, since this term is always zero, hence:

$$\mu\left(s,t\right) = \left[r_t\left(s\right) \cdot s - c\left(z,s,t\right) + w_t\left(z\right)\right].$$

Thus, $\mathcal{A}.2$ reduces to:

$$\sum_{z\in\{u,e\}} \int_{\bar{s}}^{\infty} \mu\left(s,t\right) f\left(z,s,t\right) ds = \sum_{z\in\{u,e\}} \int_{\bar{s}}^{\infty} \left[r_t\left(s\right) s - c\left(z,s,t\right) + w_t\left(z\right)\right] f\left(z,s,t\right) ds. \tag{65}$$

We have that:

$$\sum_{z\in\{u,e\}} \int_{\bar{s}}^{\infty} \left(w_t\left(z\right) - c\left(s,t\right)\right) f\left(z,s,t\right) ds = \left(1-\tau^l\right)\left(1-\mathcal{U}_t\right) + b\cdot\mathcal{U}_t + T_t - C_t,$$

and using $Y_t = \left(1-\mathcal{U}_t\right)$ we have:

$$\sum_{z\in\{u,e\}} \int_{\bar{s}}^{\infty} \left(w_t\left(z\right) - c\left(s,t\right)\right) f\left(z,s,t\right) ds = Y_t - C_t - \tau^l\cdot\left(1-\mathcal{U}_t\right) + b\cdot\mathcal{U}_t + T_t$$

In turn, we have that:

$$\begin{aligned}
\sum_{z\in\{u,e\}} \int_{\bar{s}}^{\infty} r_t\left(s\right) s\cdot f\left(z,s,t\right) ds &= r_t^l \sum_{z\in\{u,e\}} \int_{\bar{s}}^{\infty} s\cdot f\left(z,s,t\right) ds - \Delta r_t \sum_{z\in\{u,e\}} \int_{0}^{\infty} s f\left(z,s,t\right) ds \\
&= -\Delta r_t \sum_{z\in\{u,e\}} \int_{0}^{\infty} s f\left(z,s,t\right) ds.
\end{aligned}$$

where the second line follows from the market clearing condition, Lemma 1.

Thus, summing the last two equations above, we obtain that (65) is:

$$\begin{aligned}
0 &= \sum_{z\in\{u,e\}} \int_{\bar{s}}^{\infty} \left[r_t\left(s\right) s - c\left(z,s,t\right) + w_t\left(z\right)\right] f\left(z,s,t\right) ds \\
&= Y_t - C_t + T_t - \tau^l\cdot\left(1-\mathcal{U}_t\right) + b\cdot\mathcal{U}_t - \Delta r_t \sum_{z\in\{u,e\}} \int_{0}^{\infty} s f\left(z,s,t\right) ds,
\end{aligned}$$

but recall that (23) implies that $T_t = \tau^l \cdot (1 - \mathcal{U}_t) - b \cdot \mathcal{U}_t + \Delta r_t \sum_{z \in \{u,e\}} \int_0^\infty s f(z,s,t)\, ds$. Thus, we obtain that (65) implies:

$$0 = Y_t - C_t.$$

This expression verifies Walras's Law.

D.6 Proof of Corollary 1

The discount window profits are equal to $\Delta r_t \int_0^\infty s f(s,t)\, ds$ since banks are competitive and earn zero profits. Given the same real credit spread Δr_t, the equilibrium real wealth distribution $f(s,t)$ is also same. Thus Corollary 1 is established. **QED.**

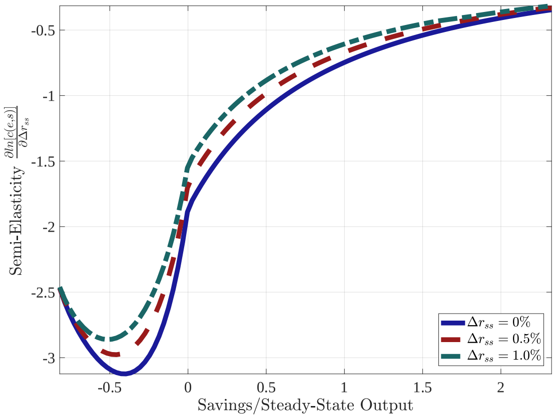
E Marginal Propensities to Consume

E.1 Marginal Propensities to Consume

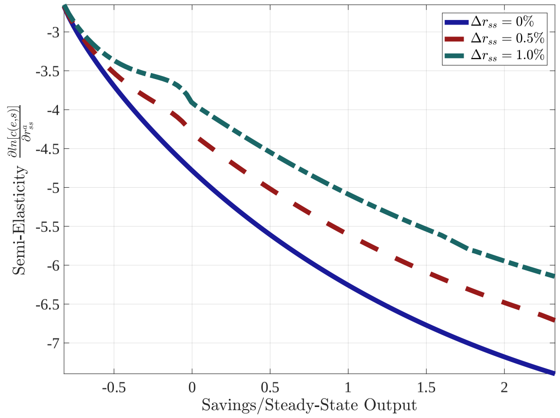
Decomposition of the channels. To gauge the sensitivity to we decompose the semi elastisity of consumption to changes in spreads as follows:

$$\frac{d\ln c\,(z,s)}{d\Delta r_{ss}} = \underbrace{\frac{\partial \ln c\,(z,s)}{\partial \Delta r_{ss}}}_{\text{credit channel}} + \underbrace{\frac{\partial \ln c\,(z,s)}{\partial r^a_{ss}} \cdot \frac{\partial r^a_{ss}}{\partial \Delta r_{ss}}}_{\text{interest-rate channel}} + \underbrace{\frac{\partial \ln c\,(z,s)}{\partial T_{ss}} \cdot \frac{\partial T_{ss}}{\partial \Delta r_{ss}}}_{\text{Ricardian effect}}$$

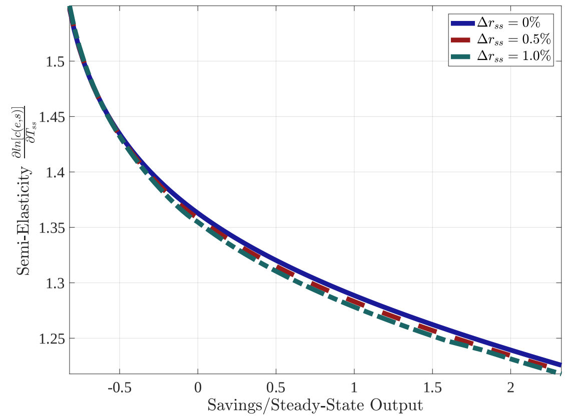
where $\frac{\partial r^a_{ss}}{\partial \Delta r_{ss}} < 0$ and $\frac{\partial T_{ss}}{\partial \Delta r_{ss}} > 0$. The semi elasticities $\frac{\partial \ln c(z,s)}{\partial \Delta r_{ss}}$, $\frac{\partial \ln c(z,s)}{\partial r^a_{ss}}$ and $\frac{\partial \ln c(z,s)}{\partial T_{ss}}$ are plotted in Figure 15. The key takeaways are that each term is associated with a transmission channel. From the figure, we can observe that different households respond differently to the different channels. First, let's consider the non-Ricardian channel. Consistent with work on fiscal transfers, the poor are most sensitive to transfers and among them, the unemployed. Consider now the interest-rate channel: increases in rates impact much more the wealthy than the poor. Finally, in terms of the credit channel, it impacts every household, even savers, through their expectations. Naturally, the spread impacts the consumption of the poor the most, except those that are very close to their debt limits, which do not adjust their consumption anyways.



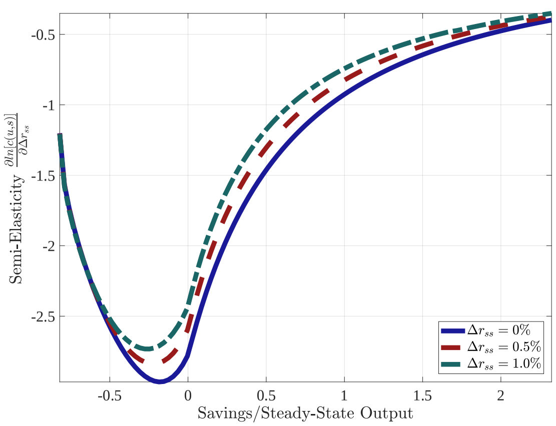
(a) Semi-Elasticity $\frac{\partial \ln c(e,s)}{\partial \Delta r_{ss}}$ of the Employed Households under Different Spreads



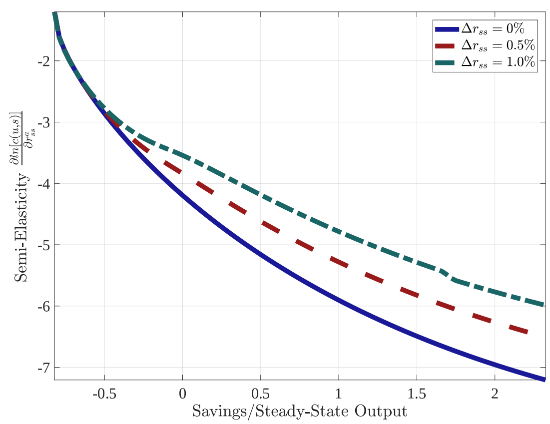
(b) Semi-Elasticity $\frac{\partial \ln c(e,s)}{\partial r^d_{ss}}$ of the Employed Households under Different Spreads



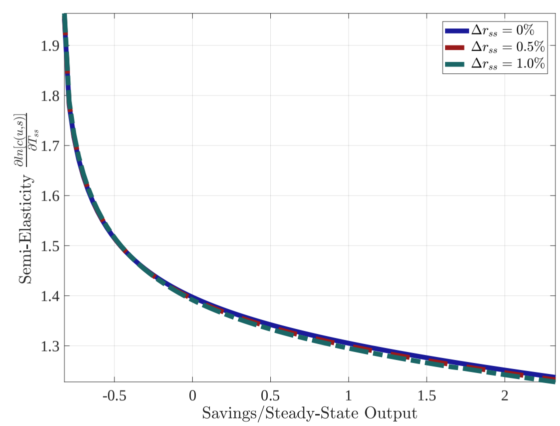
(c) Semi-Elasticity $\frac{\partial \ln c(e,s)}{\partial T_{ss}}$ of the Employed Households under Different Spreads



(d) Semi-Elasticity $\frac{\partial \ln c(u,s)}{\partial \Delta r_{ss}}$ of the Unemployed Households under Different Spreads



(e) Semi-Elasticity $\frac{\partial \ln c(u,s)}{\partial r^d_{ss}}$ of the Unemployed Households under Different Spreads

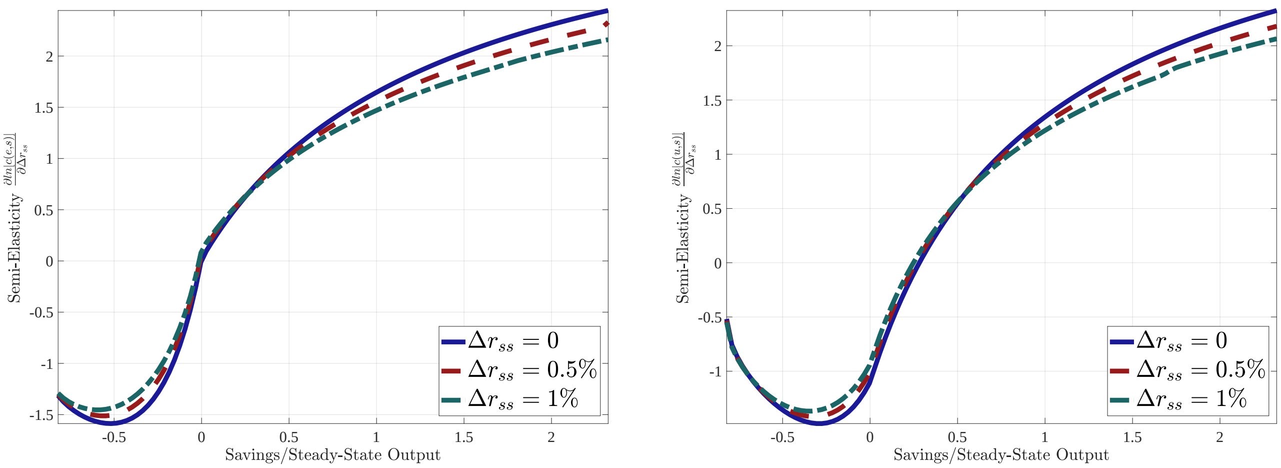


(f) Semi-Elasticity $\frac{\partial \ln c(u,s)}{\partial T_{ss}}$ of the Unemployed Households under Different Spreads

Figure 15: Steady State Effects of Real Spreads on Marginal Propensity to Consume.

Note: This figure depicts the semi-elasticities of household consumption to real spreads and real deposit rates for different values of real spreads. The semi-elasticity of consumption to real spreads is defined as $\frac{\partial \ln c(z,s)}{\partial \Delta r_{ss}}$ for $z = e, u$. The semi-elasticity of consumption to real deposit rates is defined as $\frac{\partial \ln c(z,s)}{\partial r^d_{ss}}$. For all panels, the real spread is expressed in annual percentage terms.

Figure 16 sums up the effects through all channels. We can observe that increases in the spread have very different effects among borrowers and savers. For savers, the rate channel dominates, and increases their consumption. For the poor, the credit channel dominates and an increase in spreads impacts their consumption negatively. Among these, the employed are the most sensitive.



(c) Semi-Elasticity $\frac{\partial \ln c(e,s)}{\partial \Delta r_{ss}}$ of the Employed Households under Different Spreads

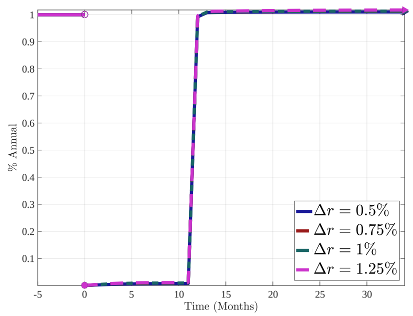
(d) Semi-Elasticity $\frac{\partial \ln c(u,s)}{\partial \Delta r_{ss}}$ of the Unemployed Households under Different Spreads

Figure 16: (General Equilibrium) Steady State Effects of Real Spreads on Marginal Propensity to Consume.

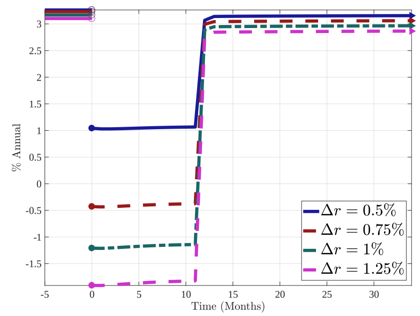
Note: This figure depicts the instantaneous marginal propensities to consume (MPC) function and the semi elasticities of household consumption to real spreads for different values of spreads. The instantaneous MPC is defined as $\frac{\partial c(z,s)}{\partial s}$ for $z = e, u$. The semi-elasticity of consumption to real spreads is defined as $\frac{\partial \ln c(z,s)}{\partial \Delta r_{ss}}$ for $z = e, u$. For all panels, the real spread is expressed in annual percentage terms.

E.2 Transition Paths in Normative Analysis

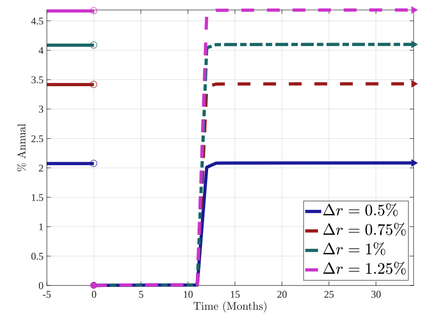
In this appendix, we report the transition paths for the risky-steady-state equilibrium, where we reduce IOR to DZLB and close spread during an anticipated credit crunch. These transition paths correspond to the paths that lead to the welfare measures in 11.



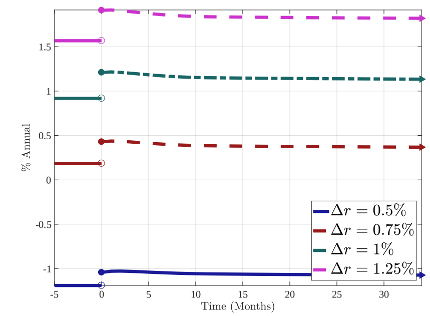
(1) IOR i_t^m



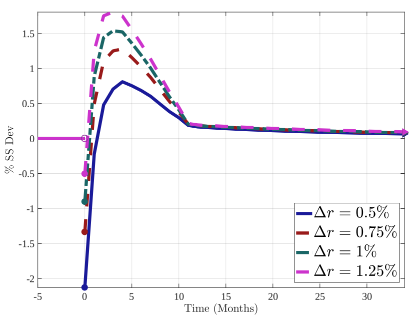
(2) Real Deposit Rate r_t^a



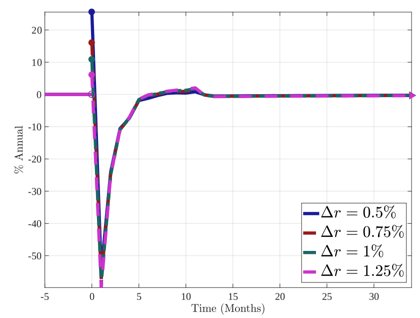
(3) Nominal Deposit Rate i_t^a



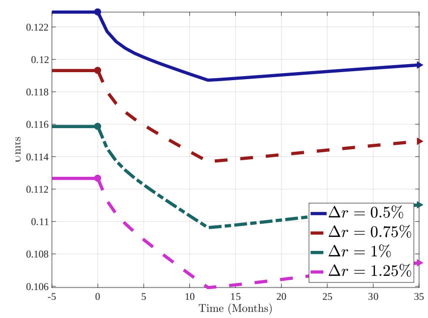
(4) Inflation Rate π_t



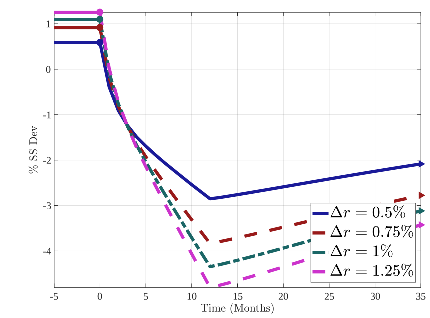
(5) Output (% Deviation from Steady State)



(6) Job Separation Rate ϕ_t



(7) Credit B_t (Level)



(8) Credit B_t (% Deviation from Steady State)

Figure 17: Transition Paths of Reducing IOR to DZLB and Closing Spread During Credit Crunch

Note: The figure reports the paths of IOR, real and nominal deposit rate, inflation, output, job separation rate and credit after an anticipated credit crunch shock with a simultaneous reduction in credit spread and IOR. The paths of borrowing limit, credit spread and IOR follow the logistic paths (26) calibrated in Table 3. All panels report the paths under four levels of initial spreads: $\Delta r_{ss} = 0.5\%, 0.75\%, 1\%, 1.25\%$. In panels (5) and (8), the aggregate output and credit are expressed in percentage deviations from the steady state. In panel (7) the credit is expressed in absolute values. In all other panels the variables are expressed in annual percentages.

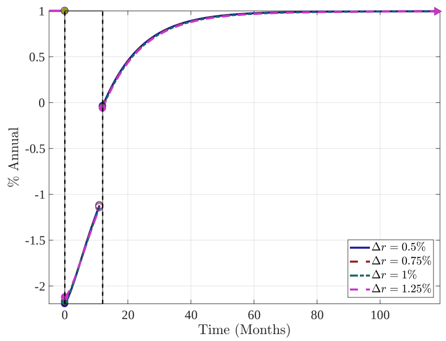
E.3 Discount Factor Shocks

In this section, we study the optimality of an ex-ante spread, when the discount factor shock takes the form: $e^{-\rho t}\delta(t)$, where $\delta(t)$ is a U-shape curve over time, and assume $\delta(t) = 1$ if there is no discount factor shock. We multiply the households utility by $\delta(t)$ in the HJB equation to produce this patience shock. Figure 6 plots the path of $\delta(t)$.

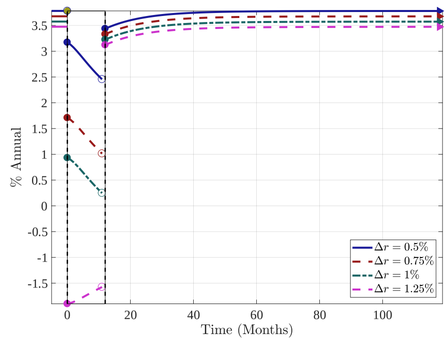
Reduction of IOR to DZLB and No Spread. As in Section 5, we simulate starting from four values of the spread, $\Delta r_{ss} = \{0.5\%, 0.75\%, 1\%, 1.25\%\}$. For the long-run monetary policy we set $i^m_{ss} = 1\%$ for all scenarios. For the monetary policy during shock, we set $\bar{i}^m_0 = 0$ and $\Delta r_t = 0$, so the nominal deposit rate $i^a_t \equiv 0$ during this period. Out of the shock, $\Delta r_t \equiv \Delta r_{ss}$. The following table reports the welfare loss (in terms of certainty equivalence) at time 0 and the following figure plots the transition paths of all scenarios. We we can observe, the same tradeoff emerges when we consider a patience shock.

Table 8: Welfare Loss of Reducing IOR to DZLB and Closing Spread During Discount Factor Shock

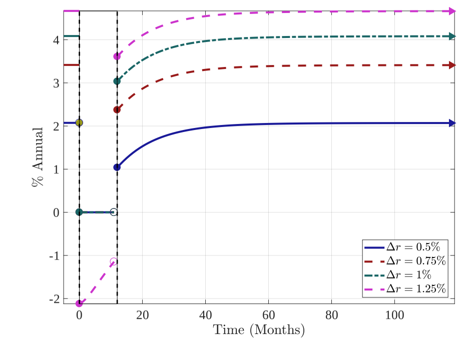
Scenario of Δr	0.5%	0.75%	1%	1.25%
$\mathcal{L}^0(\Delta r_{ss})$	0.5361	0.5349	0.5378	0.5308
$\mathcal{L}^{ss}(\Delta r_{ss})$	0.0238	0.0284	0.0326	0.0365



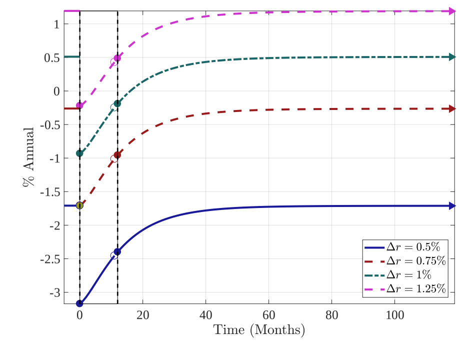
(1) IOR i_t^m



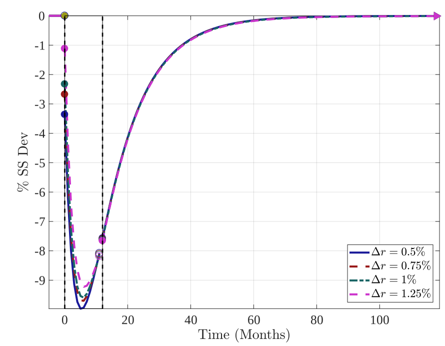
(2) Real Deposit Rate r_t^a



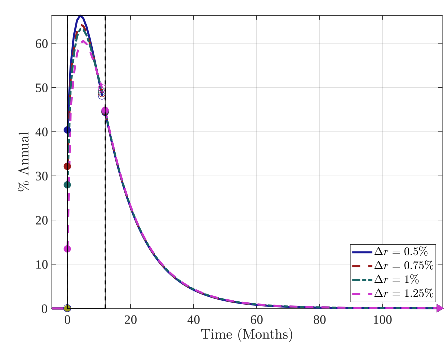
(3) Nominal Deposit Rate i_t^a



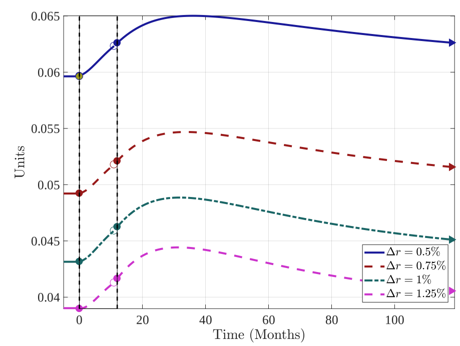
(4) Inflation Rate π_t



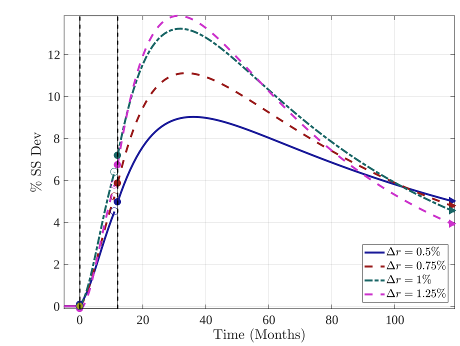
(5) Output (% Deviation from Steady State)



(6) Job Separation Rate ϕ_t



(7) Credit B_t (Level)



(8) Credit B_t (% Deviation from Steady State)

Figure 18: Transition Paths of Reducing IOR to DZLB and Closing Spread During Discount Factor Shock

Note: The figure reports the paths of IOR, real and nominal deposit rate, inflation, output, job separation rate and credit after an unanticipated discount factor shock with a simultaneous reduction in credit spread and IOR. During the credit crunch we set $\Delta r_t = \bar{i}_0^m = 0$. Out of the credit crunch we set $\Delta r_t \equiv \Delta r_{ss}$ for all scenarios and increases back to pre-shock level after that. All panels report the paths under four levels of initial spreads: $\Delta r_{ss} = 0.5\%, 0.75\%, 1\%, 1.25\%$. In all the scenarios, the size of credit crunch is 99%, i.e. $\tilde{s} = 0.01 \cdot \bar{s}$. For the long-run monetary policy we set. In panels (5) and (8), the aggregate output and credit are expressed in percentage deviations from the steady state. In panel (7) the credit is expressed in absolute values. In all other panels the variables are expressed in annual percentages.

F Solution Algorithm

The computational method follows (Achdou et al., 2020) closely. The main differences are the presence of the net asset position and the spread. Propositions 1, 3 and 4 are the objects we need to solve the model. They allow us to solve the model entirely by solving for the equilibrium path of a single price. For example, we can solve the model by solving the path for a real deposit rate r_t^a. The spread Δr_t follows immediately from Proposition 1 if we know the path for ι_t and Λ_t set by the CB. The real spread gives us r_t^l. To solve the household's problem, we need the path for $\{r_t^a, r_t^l, T_t\}$. The path for T_t must be consistent with (23). Then, the evolution of $f(s,t)$ obtained from the household's problem yields the right-hand side of equation (24). The equilibrium rate r_t^a must be the one that solves (24) implicitly.

Note that in the steady state of the model, given the real credit spread Δr, the HJB equation (11), KFE equation (15) and the real market clearing condition (24) imply that the equilibrium solution to the real markets is independent of implementation and nominal variables. Thus, we divide the solution algorithm into two parts: the part of real market and the part of implementation. For the part of real market, the path of credit spreads is taken as given. For the part of implementation and nominal variables, we take the IOR i^m as given and use the equations (6), (7) and the Fisher equation to pin down the steady-state interbank market tightness θ, nominal deposit rate i^a and inflation rate π. However, in solving the transition dynamics, the real market variables are connected to implementation and nominal variables via the Phillips curve and the Taylor Rule. Therefore, given the initial IOR i_0^m and the path of the real credit spreads Δr_t, we solve the real deposit rate r_t^a and the endogenous adjustment rate ϕ_t jointly using the real market clearing condition and the Fisher equation. Our algorithm closely follows the finite difference in Achdou et al. (2020).

F.1 Solution Algorithm: Stationary Equilibrium

We need to compute the value of the deposit rate that satisfies the real market clearing condition (24) in steady state. We focus on the stationary equilibrium where the stead-state job finding rate and job separation rate are the natural rates calibrated in Table 1. Therefore, the endogenous adjustment rate ϕ_{ss} is 0 in steady state. This implies that the steady-state values of total output, the employment and unemployment population are $Y_{ss} = e_{ss} = \frac{\nu^{ue}}{\nu^{eu}+\nu^{ue}}$ and $u_{ss} = \frac{\nu^{eu}}{\nu^{eu}+\nu^{ue}}$.

To solve the steady-state real deposit rate r_{ss}^a, we use an iteration algorithm that proceeds as follows. Let us denote $z \in \{e, u\}$ as the household's employment status, $z' = \{e, u\} \setminus z$, and $s \in [\bar{s}, \infty)$ as the household's asset holdings. First, we take the real credit spread Δr as given, consider an initial guess of deposit rate $r^{a,0}$ and fiscal transfer $T^{0,0}$, and set the iteration index $j, k := 0$. Then:

1. **Individual household's problem.** Given $r^{a,k}$ and $T^{j,k}$, for each $z \in \{e, u\}$, solve the household's value function $V^{j,k}(z,s)$ from HJB equation (11) using a finite difference method. Calculate the consumption function $c^{j,k}(z,s)$ and the asset accumulation rate $\mu^{j,k}(z,s) = r^k(s) \cdot s - c^{j,k}(z,s) + w^{j,k}(z)$,

where

$$r^k(s) = \begin{cases} r^{a,k}, & \text{if } s > 0, \\ r^{a,k} + \Delta r_{ss}, & \text{if } s \le 0, \end{cases}$$

$w^{j,k}(e) = 1 - \tau^l + T^{j,k}$, and $w^{j,k}(u) = b + T^{j,k}$.

2. **Aggregate distribution.** Given $\mu^{j,k}(z,s)$ and $c^{j,k}(z,s)$, solve the KF equation (15) for $f^{j,k}(z,s)$ using a finite difference method.

3. **Fiscal transfer and total output.** Given $c^{j,k}(z,s)$, $f^{j,k}(z,s)$, calculate fiscal transfer

$$T^{j+1,k} = \Delta r_{ss} \cdot \int_0^\infty s \left[f^{j,k}(e,s) + f^{j,k}(u,s) \right] ds + \tau^l \cdot e_{ss} - b \cdot u_{ss}.$$

If $T^{j+1,k}$ is close enough to $T^{j,k}$, proceed to 4. Otherwise, set $j := j+1$ and proceed to 1.

4. **Equilibrium real deposit rate.** Given $f^{j,k}(z,s)$, compute the net supply of real financial claims

$$S\left(r^{a,k}\right) = \int_{\bar{s}}^\infty s \left[f^{j,k}(e,s) + f^{j,k}(u,s) \right] ds$$

and update the interest rate: if $S\left(r^{a,k}\right) > 0$, decrease it to $r^{a,k+1} < r^{a,k}$ and vice versa. If $S\left(r^{a,k}\right)$ is close enough to 0, stop. Otherwise, set $k := k+1$ and $j = 0$, and proceed to 1.

5. **Equilibrium implementation and nominal variables.** Given the exogenous credit spread Δr_{ss} and IOR rate i^m_{ss}, the steady-state interbank market tightness θ_{ss}, the nominal deposit rate i^a_{ss} and inflation rate π_{ss} are given by

$$\begin{cases} \Delta r_{ss} = \frac{\delta}{2} \chi^-(\theta_{ss}), \\ i^a_{ss} = i^m_{ss} + \frac{1}{2}\left[\chi^+(\theta_{ss}) + (1-\delta)\chi^-(\theta_{ss})\right], \\ \pi_{ss} = i^a_{ss} - r^a_{ss}. \end{cases}$$

F.1.1 Solution to the HJB equation

The household's HJB equation is solved using an upwind finite difference scheme similar to Achdou et al. (2020). It approximates the value function $V(z,s)$ on a finite grid with step $\Delta s : s \in \{s_1, ..., s_I\}$, where $s_i = s_{i-1} + \Delta s = s_1 + (i-1)\Delta s$ for $2 \le i \le I$. The bounds are $s_1 = \bar{s}$ and $s_I = s^{max}$, such that $\Delta s = (s^{max} - \bar{s}) / (I-1)$. The upper bound s^{max} is an arbitrarily large number such that $f(z,s,t) = 0$ for all $s > s^{max}$. We use the short-hand notation $V_{z,i} \equiv V(z,s_i)$, and similarly for the policy function $c_{z,i}$ and $\mu_{z,i}$.

Note that the HJB involves the first and second derivatives of the value function, $V'_{z,i} = V'_s(z,s_i)$ and $V''_{z,i} = V''_s(z,s_i)$. The first derivative is approximated with either a forward (F) or a backward (B)

approximation,

$$V'_{z,i} \approx \partial_F V_{z,i} \equiv \frac{V_{z,i+1} - V_{z,i}}{\Delta s}, \tag{66}$$

$$V'_{z,i} \approx \partial_B V_{z,i} \equiv \frac{V_{z,i} - V_{z,i-1}}{\Delta s}. \tag{67}$$

The second-order derivative is approximated by a central difference:

$$V''_{z,i} \approx \partial_{ss} V_{z,i} \equiv \frac{V_{z,i+1} - 2V_{z,i} + V_{z,i-1}}{(\Delta s)^2}. \tag{68}$$

Let the superscript n be the iteration counter. The HJB equation is approximated by the following upwind scheme,

$$\frac{V_{z,i}^{n+1} - V_{z,i}^{n}}{\Delta} + \rho V_{z,i}^{n+1} = U\left(c_{z,i}^{n}\right) + \partial_F V_{z,i}^{n+1} \cdot \left(\mu_{z,i,F}^{n}\right)^{+} + \partial_B V_{z,i}^{n+1} \cdot \left(\mu_{z,i,B}^{n}\right)^{-} + \Gamma^{zz'}\left[V_{z',i}^{n+1} - V_{z,i}^{n+1}\right], \tag{69}$$

where

$$\mu_{z,i,F}^{n} = r(s_i) \cdot s_i - \left(\partial_F V_{z,i}^{n}\right)^{-1/\gamma} + w(z), \tag{70}$$

$$\mu_{z,i,B}^{n} = r(s_i) \cdot s_i - \left(\partial_B V_{z,i}^{n}\right)^{-1/\gamma} + w(z). \tag{71}$$

The optimal consumption is set to

$$c_{z,i}^{n} = \left(\partial V_{z,i}^{n}\right)^{-1/\gamma}, \tag{72}$$

where

$$\partial V_{z,i}^{n} = \partial_F V_{z,i}^{n} \mathbf{1}_{\mu_{z,i,F}^{n} > 0} + \partial_B V_{z,i}^{n} \mathbf{1}_{\mu_{z,i,B}^{n} < 0} + \partial \bar{V}_{z,i}^{n} \mathbf{1}_{\mu_{z,i,F}^{n} \leq 0} \mathbf{1}_{\mu_{z,i,B}^{n} \geq 0}.$$

In the above expression, $\partial \bar{V}_{z,i}^{n} = \left(\bar{c}_{z,i}^{n}\right)^{-\gamma}$ where $\bar{c}_{z,i}^{n}$ is the consumption level such that $\mu_{z,i}^{n} = 0$, i.e.,

$$\bar{c}_{z,i}^{n} = r(s_i) \cdot s_i + w(z).$$

Substituting the definition of the derivatives (66), (67) and (68), equation (69) is

$$\frac{V_{z,i}^{n+1} - V_{z,i}^{n}}{\Delta} + \rho V_{z,i}^{n+1} = U\left(c_{z,i}^{n}\right) + \frac{V_{z,i+1}^{n+1} - V_{z,i}^{n+1}}{\Delta s} \cdot \left(\mu_{z,i,F}^{n}\right)^{+} + \frac{V_{z,i}^{n+1} - V_{z,i-1}^{n+1}}{\Delta s} \cdot \left(\mu_{z,i,B}^{n}\right)^{-} + \nu^{zz'}\left[V_{z',i}^{n+1} - V_{z,i}^{n+1}\right].$$

Collecting terms with the same subscripts on the right-hand side

$$\begin{cases} \frac{V_{z,i}^{n+1}-V_{z,i}^{n}}{\Delta}+\rho V_{z,i}^{n+1}=U\left(c_{z,i}^{n}\right)+\alpha_{z,i}^{n}V_{z,i-1}^{n+1}+\beta_{z,i}^{n}V_{z,i}^{n+1}+\zeta_{z,i}^{n}V_{z,i+1}^{n+1}+\nu^{zz'}V_{z',i}^{n+1} \\ \alpha_{z,i}^{n}=-\frac{\left(\mu_{z,i,B}^{n}\right)^{-}}{\Delta s} \\ \beta_{z,i}^{n}=-\frac{\left(\mu_{z,i,F}^{n}\right)^{+}}{\Delta s}+\frac{\left(\mu_{z,i,B}^{n}\right)^{-}}{\Delta s}-\nu^{zz'} \\ \zeta_{z,i}^{n}=\frac{\left(\mu_{z,i,F}^{n}\right)^{+}}{\Delta s} \end{cases} \tag{73}$$

Note that $\alpha_1 = 0$, and we set $\zeta_I = 0$ for the stability of the algorithm. Equation (73) is a system of $2I$ linear equations which can be written in the following matrix form:

$$\frac{1}{\Delta}\left(\mathbf{V}^{n+1}-\mathbf{V}^{n}\right)+\rho\mathbf{V}^{n+1}=\mathbf{U}^{n}+\mathbf{A}^{n}\mathbf{V}^{n+1}$$

where

$$\mathbf{A}^{n}=\begin{bmatrix} \beta_{e,1}^{n} & \zeta_{e,1}^{n} & 0 & \cdots & 0 & \nu^{eu} & 0 & 0 & \cdots & 0 \\ \alpha_{e,2}^{n} & \beta_{e,2}^{n} & \zeta_{e,2}^{n} & 0 & \cdots & 0 & \nu^{eu} & 0 & 0 & \cdots \\ 0 & \alpha_{e,3}^{n} & \beta_{e,3}^{n} & \zeta_{e,3}^{n} & 0 & \cdots & 0 & \nu^{eu} & 0 & 0 \\ \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \vdots \\ 0 & \ddots & \ddots & \alpha_{e,I}^{n} & \beta_{e,I}^{n} & 0 & 0 & 0 & 0 & \nu^{eu} \\ \nu^{ue} & 0 & 0 & 0 & 0 & \beta_{u,1}^{n} & \zeta_{u,1}^{n} & 0 & 0 & 0 \\ 0 & \nu^{ue} & 0 & 0 & 0 & \alpha_{u,2}^{n} & \beta_{u,2}^{n} & \zeta_{u,2}^{n} & 0 & 0 \\ 0 & 0 & \nu^{ue} & 0 & 0 & 0 & \alpha_{u,3}^{n} & \beta_{u,3}^{n} & \zeta_{u,3}^{n} & 0 \\ 0 & 0 & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots \\ 0 & \cdots & \cdots & 0 & \nu^{ue} & 0 & \cdots & 0 & \alpha_{u,I}^{n} & \beta_{u,I}^{n} \end{bmatrix}, \tag{74}$$

and

$$\mathbf{V}^{n+1}=\begin{bmatrix} V_{e,1}^{n+1} \\ \vdots \\ V_{e,I}^{n+1} \\ V_{u,1}^{n+1} \\ \vdots \\ V_{u,I}^{n+1} \end{bmatrix},\ \mathbf{U}^{n}=\begin{bmatrix} U\left(c_{e,1}^{n}\right) \\ \vdots \\ U\left(c_{e,I}^{n}\right) \\ U\left(c_{u,1}^{n}\right) \\ \vdots \\ U\left(c_{u,I}^{n}\right) \end{bmatrix}.$$

The system in turn can be written as

$$\mathbf{B}^n \mathbf{V}^{n+1} = \mathbf{d}^n \tag{75}$$

where $\mathbf{B}^n = \left(\frac{1}{\Delta} + \rho\right) \mathbf{I} - \mathbf{A}^n$ and $\mathbf{d}^n = \mathbf{U}^n + \frac{1}{\Delta}\mathbf{V}^n$.

The algorithm to solve the HJB is as follows. We take the interest rate $\{r(s_i)\}_{i=1}^{I}$ and fiscal transfer T as given and begin with an initial guess $\left\{V_{e,i}^0, V_{u,i}^0\right\}_{i=1}^{I}$. Set $n = 0$. Then:

1. Compute $\left\{\partial_F V_{z,i}^n, \partial_B V_{z,i}^n\right\}_{i=1}^{I}$ using (66) and (67).
2. Compute $\left\{c_{z,i}^n\right\}_{i=1}^{I}$ using (72) and $\left\{\mu_{z,i,F}^n, \mu_{z,i,B}^n\right\}_{i=1}^{I}$ using (70) and (71).
3. Find $\left\{V_{z,i}^n\right\}_{i=1}^{I}$ solving the linear system of equations (75).
4. If $\left\{V_{z,i}^{n+1}\right\}$ is close enough to $\left\{V_{z,i}^n\right\}$, stop. Otherwise set $n := n+1$ and proceed to step 1.

F.1.2 Solve KFE in Stationary Equilibrium

The stationary distribution of real wealth satisfies the Kolmogorov Forward equation:

$$0 = -\frac{\partial}{\partial s}\left[\mu(z,s) f(z,s)\right] - \nu^{zz'} \cdot f(z,s) + \nu^{z'z} \cdot f\left(z',s\right), \tag{76}$$

$$1 = \int_{\bar{s}}^{\infty} \left[f(e,s) + f(u,s)\right] ds. \tag{77}$$

We also solve the equation using a finite difference scheme. We use the notation $f_{z,i} \equiv f(z,s_i)$ .The system can be expressed as

$$\begin{aligned} 0 = &- \frac{f_{z,i}\left(\mu_{z,i,F}^n\right)^+ - f_{z,i-1}\left(\mu_{z,i-1,F}^n\right)^+}{\Delta s} - \frac{f_{z,i+1}\left(\mu_{z,i+1,B}^n\right)^- - f_{z,i}\left(\mu_{z,i,B}^n\right)^-}{\Delta s} \\ &- \nu^{zz'} f_{z,i} + \nu^{z'z} f_{z',i}, \end{aligned}$$

or equivalently

$$f_{z,i-1}\zeta_{z,i-1} + f_{z,i}\beta_{z,i} + f_{z,i+1}\alpha_{z,i+1} + f_{z',i}\nu^{z'z} = 0.$$

The linear equations system can be written as

$$\mathbf{A}^{\mathbf{T}}\mathbf{f} = \mathbf{0}, \tag{78}$$

where $\mathbf{A^T}$ is the transpose of $\mathbf{A} = \lim_{n\to\infty} \mathbf{A}^n$. Notice that $\mathbf{A}^n$ is the approximation of the operator $\mathcal{A}$ and $\mathbf{A^T}$ is the approximation of the adjoint operator $\mathcal{A}^*$. In order to impose the normalization constraint (77), we replace one of the entries of the zero vector in equation (78) by a positive constant. We solve the system (78) and obtain a solution $\hat{\mathbf{f}}$. Then we renormalize as

$$f_{z,i} = \frac{\hat{f}_{z,i}}{\sum_{i=1}^{I} \sum_{z\in\{e,u\}} \hat{f}_{z,i} \Delta s}.$$

The algorithm to solve the stationary distribution is as follows.

1. Given the interest rate $\{r(s_i)\}_{i=1}^{I}$ and fiscal transfer T, solve the HJB equation to obtain an estimate of the matrix $\mathbf{A}$.

2. Given $\mathbf{A}$, find the aggregate distribution $\mathbf{f}$.

F.2 Solution Algorithm: Transition Dynamics

The equilibrium transition path is solved in finite horizon $[0, \bar{T}]$, assuming that the terminal state of the economy is steady state. The finite horizon is discretized evenly into $N_{\bar{T}}$ points in time dimension. We use an iterative algorithm as follows. Given the initial distribution of real wealth $f_0(z,s)$ and the path of exogenous shocks (e.g., equation (26), guess a path of real deposit rate $r_t^{a,0}$, endogenous adjustment rate ϕ_t^0 total output Y_t, and fiscal transfer T_t, and set the iteration index $j,k := 0$. Then

0. **The asymptotic steady state.** The asymptotic steady-state value function and real wealth distribution are calculated from Section F.1.

1. **The aggregate output, employment and unemployment.** Given the path of ϕ_t^k and the terminal condition $\mathcal{U}_{\bar{T}}^k = u_{ss}$, solve the law of motion of unemployed mass (12) backwards in time to compute the path of unemployed mass $\mathcal{U}_t^k$. Calculate the path of aggregate output $Y_t^k = 1 - \mathcal{U}_t^k$.

2. **Individual household's problem.** Given $r_t^{a,k}, \phi_t^k, \mathcal{U}_t^k$ and $T_t^{j,k}$, and the terminal condition $V^{j,k}(z,s,\bar{T}) = V_{ss}(z,s)$, solve the HJB equation (11) backwards in time to compute the path of $V^{j,k}(z,s,t)$. Calculate the consumption policy function $c^{j,k}(z,s,t)$ and the rate of asset accumulation $\mu^{j,k}(z,s,t)$.

3. **Aggregate distribution.** Given $c^{j,k}(z,s,t)$ and $\mu^{j,k}(z,s,t)$, solve the Kolmogorov Forward equation (15) with initial condition $f^{j,k}(z,s,0) = f_0(z,s)$ forward in time to compute the path for $f^{j,k}(z,s,t)$.

4. **Fiscal transfer and total output.** Given $c^{j,k}(z,s,t)$, $f^{j,k}(z,s,t)$ and $\mathcal{U}_t^k$ calculate the path of fiscal transfer

$$T_t^{j+1,k} = \Delta r_t \cdot \int_0^\infty s\left[f^{j,k}(e,s,t) + f^{j,k}(u,s,t)\right] ds + \tau^l \cdot \left(1 - \mathcal{U}_t^k\right) - b \cdot \mathcal{U}_t^k.$$

If $\left\{T_t^{j+1,k}\right\}_{t=0}^{\bar{T}}$ is close enough to $\left\{T_t^{j,l}\right\}_{t=0}^{\bar{T}}$, proceed to 5. Otherwise, set $j := j+1$ and proceed to 2.

5. **Equilibrium inflation rate and nominal deposit rate.** Given the path of aggregate unemployed mass $\mathcal{U}_t^k$ and the terminal condition of inflation $\pi_{\bar{T}}^k = \pi_{ss}$, solve the Phillips curve (13) backwards in time to compute the path of the inflation rate π_t^k. Next, given the paths of discretionary rate $\bar{i}_t^m$, Taylor parameter η_t and the inflation rate π_t^k, use the Taylor rule (18) to calculate the path of IOR $i_t^{m,k}$. Then given the path of credit spread Δr_t, back out the path of interbank market tightness θ_t using

$$\Delta r_t = \frac{\delta}{2}\chi^-(\theta_t).$$

Finally, compute the nominal deposit rate using the implementation equation (6), i.e.,

$$i_t^{a,k} = i_t^{m,k} + \frac{1}{2}\left[\chi^+(\theta_t) + (1-\delta)\chi^-(\theta_t)\right].$$

6. **Equilibrium real deposit rate and endogenous adjustment rate.** Given $f^{j,k}(z,s,t)$, $i_t^{a,k}$ and π_t^k, calculate

$$S^r\left(r_t^{a,k}, \phi_t^k, t\right) = \int_{\bar{s}}^{\infty} s\left[f^{j,k}(e,s,t) + f^{j,k}(u,s,t)\right] ds$$

and

$$S^\phi\left(r_t^{a,k}, \phi_t^k, t\right) = i_t^{a,k} - r_t^{a,k} - \pi_t^k.$$

We update $\left\{r_t^{a,k}, \phi_t^k\right\}_{t=0}^{\bar{T}}$ to $\left\{r_t^{a,k+1}, \phi_t^{k+1}\right\}_{t=0}^{\bar{T}}$ using the Broyden's method. However, one can use alternative numerical methods for finding roots in $2N_{\bar{T}}$ variables. If

$$\max_t\left\{\max\left\{\left|S^r\left(r_t^{a,k}, \phi_t^k, t\right)\right|, \left|S^\phi\left(r_t^{a,k}, \phi_t^k, t\right)\right|\right\}\right\}$$

is close enough to 0, stop. Otherwise, set $k := k+1$ and $j = 0$, and proceed to 1.

F.2.1 Solution to the HJB Equation

The dynamic HJB equation (11) can be approximated using an upwind scheme as

$$\rho\mathbf{V}^n = \mathbf{U}^{n+1} + \mathbf{A}^{n+1}\mathbf{V}^n + \frac{1}{\Delta t}\left(\mathbf{V}^{n+1} - \mathbf{V}^n\right),$$

where $\mathbf{A}^{n+1}$ is defined in an analogous fashion to (74), and $\Delta t = T/N$ denotes the time length of each discrete period. We start with the terminal condition $\mathbf{V}^N = \mathbf{V}_{ss}$ and solve the path of value function backward, where $\mathbf{V}_{ss}$ denotes the solution to stationary equilibrium obtained from Section F.1. For each $n = 0, 1, ..., N-1$, define $\mathbf{B}^n = \left(\frac{1}{\Delta t} + \rho\right)\mathbf{I} - \mathbf{A}^{n+1}$ and $\mathbf{d}^{n+1} = \mathbf{U}^{n+1} + \frac{1}{\Delta}\mathbf{V}^{n+1}$, and we can solve

$$\mathbf{V}^n = \left(\mathbf{B}^n\right)^{-1} \mathbf{d}^{n+1}.$$

F.2.2 Solution to the KF Equation

Let $\{\mathbf{A}^n\}_{n=1}^{N-1}$ be the solution obtained from Section F.2.1. It is the approximation to the operator $\mathcal{A}$. Using a finite difference scheme similar to the one we employed in Section F.1.2, we obtain:

$$\frac{\mathbf{f}^{n+1} - \mathbf{f}^n}{\Delta t} = \left(\mathbf{A}^n\right)^{\mathbf{T}} \mathbf{f}^{n+1},$$

which implies

$$\mathbf{f}^{n+1} = \left(\mathbf{I} - \Delta t \left(\mathbf{A}^n\right)^{\mathbf{T}}\right)^{-1} \mathbf{f}^n,\ n = 0, 1, ..., N-1. \tag{79}$$

We start from the initial period condition $\mathbf{f}^0 = \mathbf{f}_0$ and solve the KFE forward using (79).

F.3 Solution Algorithm: Risky Steady State Equilibrium

The risky steady state equilibrium consists of the post-shock transition path and the pre-shock steady state. We solve the two parts simultaneously based on the algorithm in Section F.1 and F.2 as follows. Set the iteration index $k := 0$. Then

1. Use the algorithm in Section F.1 to solve the post-shock steady state. Use the post-shock steady-state distribution $f_{ss}(z,s)$ as the guess of initial wealth distribution $f^k(z,s,0)$ at time 0, and use the algorithm in Section F.2 to solve the post-shock transition path and time-0 value function $V^k(z,s,0)$.

2. Use $V^k(z,s,0)$ in step 1 as the input into the following risky steady state HJB:

$$\rho V_{rss}(z,s) = \max_{\{c\}} U(c) + \frac{\partial V_{rss}(z,s)}{\partial s} \cdot \mu(z,s) + \nu^{zz'} \left[V_{rss}(z',s) - V_{rss}(z,s)\right] + \chi_{rss}\left[V(z,s,0) - V_{rss}(z,s)\right].$$

Solve the risky steady state solution $V^k_{rss}(z,s)$ and $f^k_{rss}(z,s)$ using the above HJB together with the KF equation (15) and the real market clearing condition (24) according to the algorithm in Section F.1.

3. Set $f^{k+1}(z,s,0) = f^k_{rss}(z,s)$ as the initial wealth distribution at time 0, and then use the algorithm in Section F.2 to solve the post-shock transition path and time-0 value function $V^{k+1}(z,s,0)$.

4. Iterate step 2 and 3 until $\left\{f^k_{rss}(z,s), V^k_{rss}(z,s)\right\}$ converges.