MICHAEL BEND

ABeCeDarian

TEACHER MANUAL

Fractions

Book 1

2018-01 EDITION

a-be-ce-dar-i-an n.

1. One who teaches or studies the alphabet.
2. One who is just learning; a beginner.

a·be·ce·dar·i·an adj.

1. Having to do with the alphabet.
2. Being arranged alphabetically.
3. Elementary or rudimentary.

[Middle English, from Medieval Latin abecedarium, alphabet, from Late Latin abecedarius, alphabetical : from the names of the letters A B C D + -arius, -ary.]

from The American Heritage Dictionary, Third Edition

ABeCeDarian Fractions Book 1

Teacher Manual

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Introduction

The ability to do arithmetic with fractions is a critical foundation for the development of higher math skills. It is also an area that many students, even many capable students, never fully master. The problem lies not so much in the complexity of the topic, but in the rushed and incomplete instruction many receive. The ABeCeDarian Fraction Workbooks provide homeschooling parents, tutors, and teachers working with small groups of students with lessons that are engaging, well-sequenced, and precise and that avoid the limitations of so many other programs. (A modified version of this curriculum for classroom teachers is in development.) As a result, ABeCeDarian students both develop a sound conceptual understanding of fractions and acquire fluent computational skills.

This introduction contains some brief comments about the organizing principles the construction of each lesson, as well as some general comments about how to implement the program. Each chapter also provides detailed directions for presenting each lesson. You can find an in depth presentation of the principles informing the structure and organization of the curriculum, as well as on-going discussion of relevant teaching topics, on the ABeCeDarian website (www.abcdrp.com/math).

General comments about the teaching principles upon which ABeCeDarian lessons are based

A student's skill at mathematical calculation is based on two primary pillars. One pillar is an understanding of fundamental relationships and patterns that is developed through exploration and analysis. The other pillar is the right amount and the right kind of practice doing calculations.

When students understand a mathematical concept well, one might say they understand how a quantity or a shape behaves when manipulated in certain ways. A math student's developing understanding of the behavior of mathematical objects is similar to the way in which an apprentice carpenter learns how different woods respond to different treatments, or the way in which an apprentice chef learns how different foods respond to different techniques in different environments. The best way to aquire such understanding, indeed, perhaps the only real way to do so, is to "play" with the objects. By "play," of course, I don't mean something frivolous. Rather, I'm referring to experimentation, to pushing and pulling something to see how it responds.

In the context of elementary lessons in fraction, this play involves exploring a variety of manipulations done on physical objects such as snap cubes and fraction tiles, as well as on drawn models, such as fraction bars and number lines. This sort of exploration will be at the heart of every ABeCeDarian lesson in which a new concept is introduced.

Understanding doesn't come from "playing with" or manipulating objects by itself, however. In addition, this exploration needs to be accompanied by analysis, that is, the systematic investigation of patterns that allow the student to see not just individual, discrete actions, but underlying, consistent relationships. Thus, almost all ABeCeDarian lessons involve dialogues in which the

teacher helps the student uncover relevant and significant patterns.

Organizing lessons around exploration and analysis allows students to readily incorporate new information within the web of existing patterns and relationships they already know. Furthermore, it requires students to use and develop problem-solving skills and analytical skills in each lesson, so that math lessons are mostly about figuring out how things are related and how they behave, rather than just memorizing often decontextualized and thus easily confused steps and procedures. Indeed, with the right kind of exploration and analysis, students can discover the basic caclulation procedures and start applying them rapidly and with little confusion.

A lesson format organized around exploration and analysis, moreover, allows error to be normalized. As students attempt both to perform the tasks you ask them to do, and try to analyze what they have done, they will make mistakes. But these errors are not faults that must be avoided, but rather, they are necessary components of learning new material. This normalizing of error helps students develop superior problem-solving skills, encourages them to tackle new material, and generally makes math instruction much less stressful.

Another, related feature of ABeCeDarian lessons is that all mathematical symbolization is taught as a way to record certain actions. Given the difficulties some many students have learning math (including so many of us teachers), it is often not very well recognized that the ideas in basic arithmetic, geometry, and algebra are all very accessible and easily understood when presented in regular language, without symbols. Most kindergarten students, in fact, can discuss and understand math generally taught in 6th and 7th grade if it is presented primarily in language and representations that they are already familiar with.

The difficulties with learning mathematical symbolization (and why it is often intimidating) has to do, therefore, not with the fact that the underlying concepts represented by the symbolization are unusually complex, but because the symbols are often presented without helping the students fully recognize what they are representing! In ABeCeDarian, therefore, a new concept is always presented first as a manipulation of physical objects like snap cubes, or of drawn models like fraction bars and number lines, and the manipulation is originally described in ordinary language. Only after this initial presentation is the symbolization presented, and it is presented quite explicitly as a way of recording the action the student just performed.

After a new concept has been explored, analyzed, and understood, the next step is to transform it into a relevant skill that the student can perform easily. To develop skill, of course, the student needs to practice. While just about any form of practice provides some improvement in a student's performance, there are three critical characterstics or features of practice activities that make it optimally efficient.

One important feature of efficient practice is that it needs to be conducted until the student can perform the skill fluently. When one first tries a new task, it requires a great deal of conscious attention as well as a relatively large amount of time. But as one does the task repeatedly, one can do it not only with near 100% accuracy, but also while using very little conscious effort and requiring

very little time. Once skills are fluent, they are readily available to a student as he tackles increasingly complex problems and learns how to combine his basic skills into new, more complex skills.

There is an important caveat about developing a skill to fluency. When students practice only a single, isolated skill, they are practicing it in a limited, restricted way that actually slows the rate at which they can apply the skill appropriately in new contexts. For example, students who have just learned about how to calculate sums with multidigit addends are often given a practice sheet that involves only this skill. In calculating these sums, the students do indeed repeat the necessary procedures, but they have to make no judgments about which procedures among those that they have learned, are appropriate. Initial fluency training, therefore, is not complete until it is incorporated with practice that requires a person to make critical judgments about when to apply which procedures. As you may have noticed, when students have a sheet of a single kind of calculation, they often become automatons and can perform the task successfully with very narrow thinking, usually not even paying attention to whether the problem was written with the addition or subtraction sign!

To most readily develop a skill so that it can be called upon in a variety of contexts, it is necessary to provide students with practice exercises that involve a variety of different skills they have learned, a procedure often referred to as “interleaving.” Therefore, all ABeCeDarian practice exercises (that is, those in the sections marked "Independent Work") are interleaved, that is, the work mixes several different skills the student has been practicing.

The final element of good practice involves how to take something that is in memory and keep it readily available. The fact is that most of our memories are destined to be forgotten. For the overall effectiveness of our minds, this fate is good thing. We are bombarded with an enormous amount of information, such as what clothes we wore, what we had for lunch on a particular day, or the name of a person we meet only once. or what the weather was like on a particular day. Such details, while perhpas useful to remember for a day or so, are not important to remember for a long time. Other things, of course, are important to remember for a much longer tiime. What is the mechanism for rendering some memories more permanent than others?

One of the most important factors is the number of times we try to recall the information. In other words, it is the very act of trying to remember something that helps make the though more durable. The key pedagogical point is that once something has been presented and practiced, it needs to be recalled periodically to keep it readily available. But, it turns out, the recall can occur over increasing intervals of time. That is to say, when a person is first learning some new material, she needs to actively recall it at very short intervals, sometimes as short as seconds and minutes. When the student remembers correctly after these very short intervals, the interval can be increased to hours and then days, and so long as the student is remembering it correctly, the interval can increase. This sort of practice with increasing intervals between recall attempts is called “spaced repetition.” ABeCeDarian regularly and systematically incorporates spaced repetition in all of its practice exercises.

Because students have ample opportunity to explore fundamental patterns and relationships and

then have sufficient practice to make calculations easy for them, they typically can learn new material with relatively little effort. That is because they have a set of well-developed tools to do mathematics based not just on repetition but on understanding as well. Therefore, when given a new task to perform, they often combine their existing skills in a new way with little or no prodding required from the teacher. That is to say, they often discover the new calculation procedures on their own without having to be explicitly taught how to do them.

Here is a summary of these general points about ABeCeDarian:

1. New concepts are initially explored with physical objects such as fraction tiles or drawn models.

2. The teacher describes these concepts in ordinary language.
3. After students have sufficient experience with a new concept presented in this manner, they analyze it. That is to say, they are guided by their teacher to identify fundamental patterns and relationships.

4. Once students have a general understanding of a new concept, they practice the related calculation skill.

5. The practice in ABeCeDarian incorporates three critical features to maximize their efficiency:
   a. Practice is conducted to fluency.
   b. Practice is interleaved.
   c. Practice is presented using spaced repetition.

Preparing to teach the lessons

The most important preparation you can do to teach the ABeCeDarian fraction lessons is to read through the entire manual. Then, some time before you work with your student, you should practice teach the lesson to an older student or an adult. During this review, try to figure out how you will respond to any errors the student makes. (The next section provides guidelines for productive error correction.) At the very least, you should make some notes about what errors you think a student might make and then writing down good responses. However, the best way to figure out how to respond to errors is to have your "practice" student make errors and correct these. This session with errors should be done after you have gone through the lesson initially with the student making no errors. Write down your responses and refine them to make sure you are giving students both precise information and the minimal information necessary for them to make the correction.

I can't stress enough how important it is to practice the lessons in this way. I know, also, that many teachers will be tempted to skip these steps. If you do, however, I can assure you that the quality of your instruction will be much poorer than if you invest in this preparation time. You will be rewarded by smooth-flowing lessons, high engagement, and rapid learning. Indeed, it is

almost a guarentee that the lessons will be both highly effective and quite fun.

Each lesson is divided into two parts. The first part is the lesson proper. All lessons begin by having the student perform some task with manipulatives such as fraction tiles or snap cubes or to perform some task using drawn models, principally number lines or bar models. In many lessons, after students have done some activities, they are asked to look at some equations and discuss with you any patterns they see. It is in the course of these discussions that the student, with your help, figures out how to do calculations without drawing models. After this initial exploration, students are given some tasks to perform that are identified as "Independent Work." This work includes the kinds of tasks and calculations they did in earlier lessons as well as those that they just did together with you. Students should try to do the independent work without any assistance, but if they become confused, you should definitely step in and provide any necessary support.

Students should be able to do the independent work at the end of each lesson easily and with virtually 100% accuracy. If a student makes many mistakes, or takes a great deal of time, or needs assistance, you should provide additional practice on equivalent tasks before moving on. Begin by reviewing the introductory work in the lesson with the student, and then prepare some practice work similar to that presented in the Independent Work section.

If you conduct math sessions 4 or 5 times weekly, it is possible that your student will not need any homework, that is, any work beyond what she does during the lessons themselves. If you wish to provide homework, however, I suggest you make a set of problems similar in number and type to the Independent Work problems presented in workbook. Eventually I will have a separate homework booklet with exactly such problem sets as well as additional practice sheets.

Students should work for 30 to 40 minutes at time on the fraction lessons. Depending upon the lesson and upon the student, you will typically get through one, two, or three lessons in this time. Most students who work for 30 minutes daily five days a week will be able to complete the workbook and master all of the material in 4 to 6 weeks.

How to respond to errors

One of the underlying principles of all ABeCeDarian materials is that errors are necessary for learning. That means, in general, we as teachers, must treat them not as flaws in learning that should be stigmatized, but as an integral, natural part of the students' attempts to apply their knowledge. As the renowned computer scientest and educator, Seymour Papert, urges us, we should think of errors the way a computer programmer views "bugs" in a program, a normal part of working through how to get some procedure to work properly.

If you view errors in this light, then most of the time you will not want to correct an error simply by telling the student he got something wrong. Nor will you usually want to simply tell the student the correct answer. The best first response in most situations, rather, is to give some pertinent information to the student about what he did. For example, if the student was calcu-

lating 2 x 3 and told you that the product was 5, you would say, “You added the numbers.” And if the student was to shade in $2\frac{3}{4}$ bars but shaded in only $1\frac{3}{4}$, you would say, “You shaded in only $1\frac{3}{4}$ bars.”

This sort of response keeps the responsibility for thinking with the student and helps her develop the critical skill of self-reflection and self-correction. It is useful to remind oneself frequently that the goal isn't simply to get the student to say the right answer, but rather to get the student to be able to easily go through all the thinking necessary to arrive at the right answer.

It takes time for teachers to learn how to respond this way. Practice by writing out 2 or 3 errors and corrections in advance and practice until your response comes quickly. (You can put these on flash cards to practice.) Then, as you correct errors during a lesson, jot down a few of the student’s errors and your responses, and go back over these to see if they provided clear feedback to the student about the nature of his error. Revise your responses if you were too wordy or imprecise.

Rest assured, however, that if you can’t think of a simple correction along these lines, it is best to simply state the correct answer explicitly and have the student repeat the problem, first immediately, and then again in a minute or so.

Materials Needed

For the most part, the work the student does is in her workbook. However, there are a few activities that require some additional materials. You need a set of fraction tiles, preferably magnetic fraction tiles.

There is a freely reproducible template available on the ABeCeDarian webiste. You can print out the template, laminate it, and then cut out the pieces. I also recommend that you attach self-adhesive magnets to the fraction tiles. You can also purchase a set from a third party vendor. (If you do order a set of fraction tiles, please make sure that their shape is similar to the tiles in the template on the website. Tiles in the form of rectangles work much better than fraction pieces in other shapes, especially circles.)

Students also need two dice for a recurring activity. One die can be a regular, dotted, six-sided die. The other needs to have the plus sign and the multiplicaton sign on its faces. You can buy blank dice and put these symbols on with a permanent marker. You can also take a regular, dotted die and tape the symbols onto the faces.

Students need snap cubes, also referrred to as linking cubes, for some activities. You should purchase these from a third-party vendor.

Erasable pens are also very useful. It is often useful to do mathematical work in different colors, and erasable pens are generally easier to use than colored pencils. Also, the line remains bright and clear and the point doesn’t break or need sharpening. I have been using the Pilot Frixion line

of erasable pens with great success.

Final thoughts

I think you will find the ABeCeDarian lessons easy to teach and that your students will find the lessons engaging and will readily develop skills and confidence. All lessons, however, should be thought of as provisional. They are provisional because they can and should be refined and improved in response to how students respond to them. Sometimes a direction can be made even clearer and more precise, and sometimes a critical concept has been insufficiently recognized and developed, and so a student struggles unnecessarily.

For this reason, curriculum designers should always be revising their lessons. And the information about what needs to be revised comes from the teachers and students who are using the program. So please let us know about your experiences. We would love to hear about your successess, of course. Your observations about your successes may help other teachers teach the lessons better. But we are especially eager to hear your experiences with your students that will help us improve the lessons. So I hope you will read and comment on the ABeCeDarian blog and write to me at the e-mail address below.

Happy Teaching!

Michael Bend
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Lesson 1

Review of Naming Fractions - Part 1

How to Conduct the Lesson

Review of naming fractions - Part 1

This lesson uses unit “bars” or “rectangles” to review what fractions are and how to write them. Students are given instructions at first to divide some unit bars into a certain number of equal parts and shade in some of the parts and then record the fraction name for the amount of a bar shaded in. After shading in a certain number of parts, students are then directed to shade in some additional parts and record the new number for the total parts of a bar that are shaded.

Students will perform this task a total of 4 times, dividing bars into 4, 5, 3, and 2 equal parts respectively.

In all cases the students will end up with more than one whole bar shaded in. At this point in the program students should write these fractions in improper form. A lesson on mixed numbers is coming shortly. It is not advisable to say anything about mixed numbers at this point.

In the next set of tasks the student is asked to shade in a specific fractional amount of a bar on blank unit bars presented in the workbook.

In the last set of tasks the student is presented a set of unit bars with a part already shaded in. Her task is to identify what fractional part of a bar is shaded in and then to write it.

KEY POINTS: Begin the lesson by reading the first direction out loud to the student.

Have the student make simple vertical cuts with her pencil to divide the bar. You can point out that there are many other ways to divide the bar into 4 equal parts, but usually, you will have her do simple up-and-down cuts because these are easy to draw and the resulting pieces are easy to see. It is also very useful to do each additional shading in a different color. While the models in this manual are fully shaded in, it is advisable to have the student do just a "sloppy" shading, by just putting a scribbled wave in each rectangle she is shading.

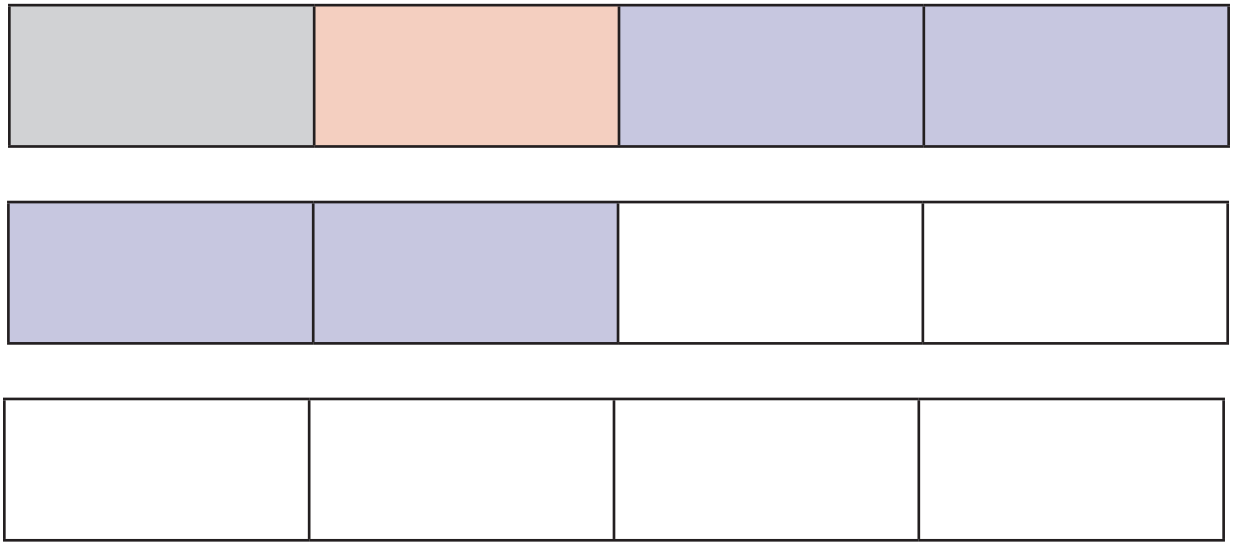
If the student doesn't know how to record what she has done, review the naming of fractions by saying: *When we divide something into 4 equal parts, we call each part a "fourth." How many fourths have you shaded in? Yes, that's right, you shaded in one fourth. Here is how we write one fourth*, etc.

LESSON 1

Review of naming fractions - Part 1

Each bar below represents 1 whole. Divide each bar into 4 equal parts.

(The different colors in the models represent what is shaded in for each of the separte tasks.)



The shaded parts of "bars" are shaded in fully in this manual, but you should have your student shade in bars with a "sloppy" shade, that is just a brief, scribbled wave. This procedure will save considerable time.

Shade in 1 of the parts.

Record how much of a bar is shaded. $\frac{1}{4}$

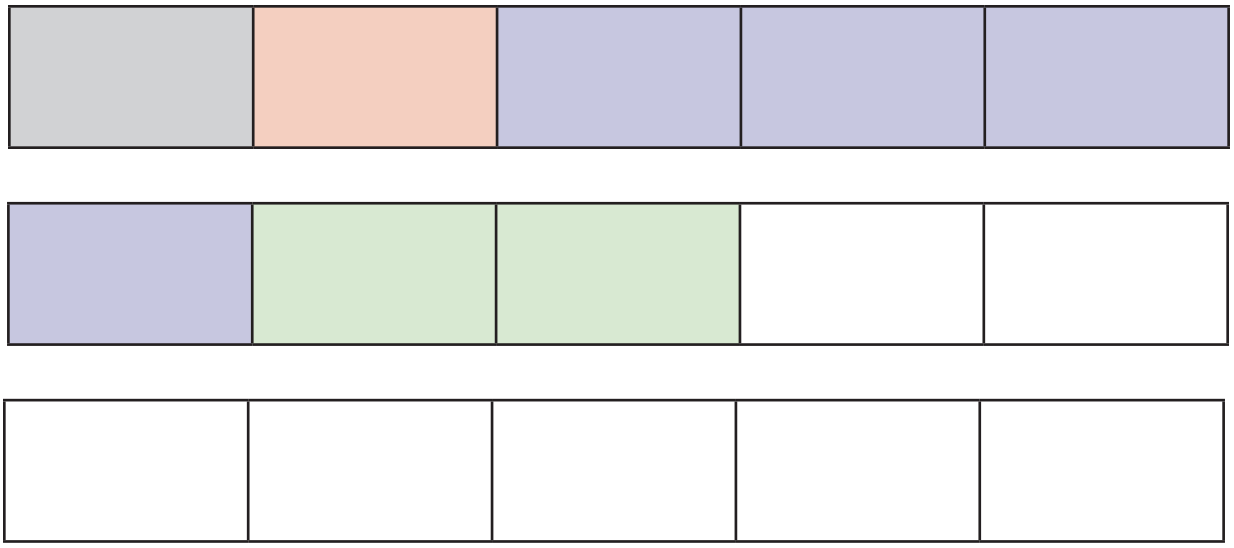
Shade in a second part.

Record how much of a bar is shaded. $\frac{2}{4}$

Shade in four additional parts.

Record how much of a bar is shaded. $\frac{6}{4}$

Each bar below represents 1 whole. Divide each bar into 5 equal parts.



Shade in 1 of the parts.

Record how much of a bar is shaded. $\frac{1}{5}$

Shade in a second part.

Record how much of a bar is shaded. $\frac{2}{5}$

Shade in four additional parts.

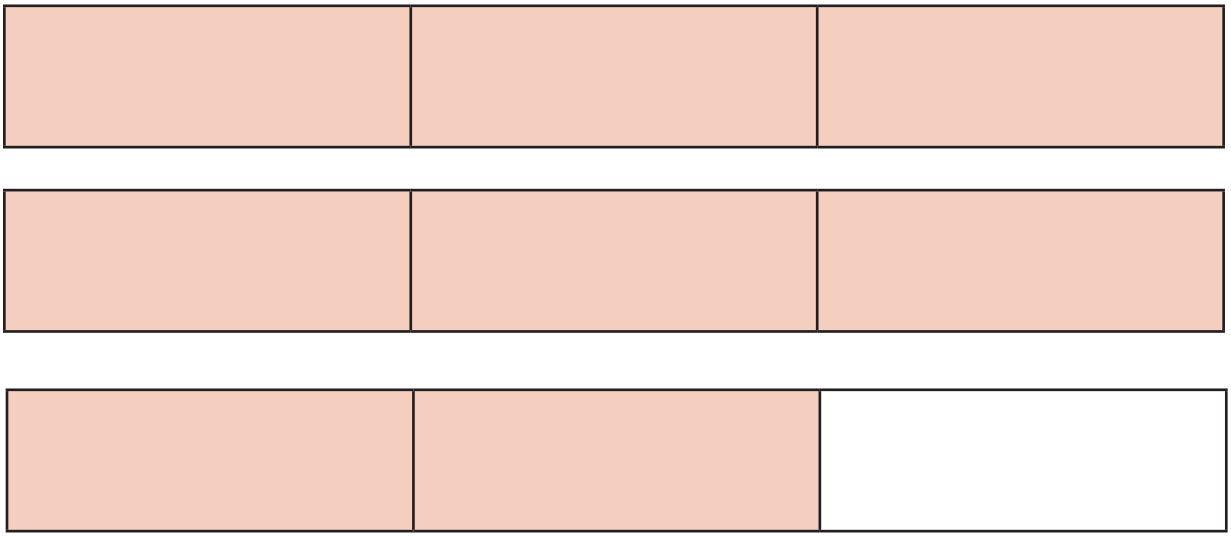
Record how much of a bar is shaded. $\frac{6}{5}$

Shade in two more parts.

Record how much of a bar is shaded. $\frac{8}{5}$

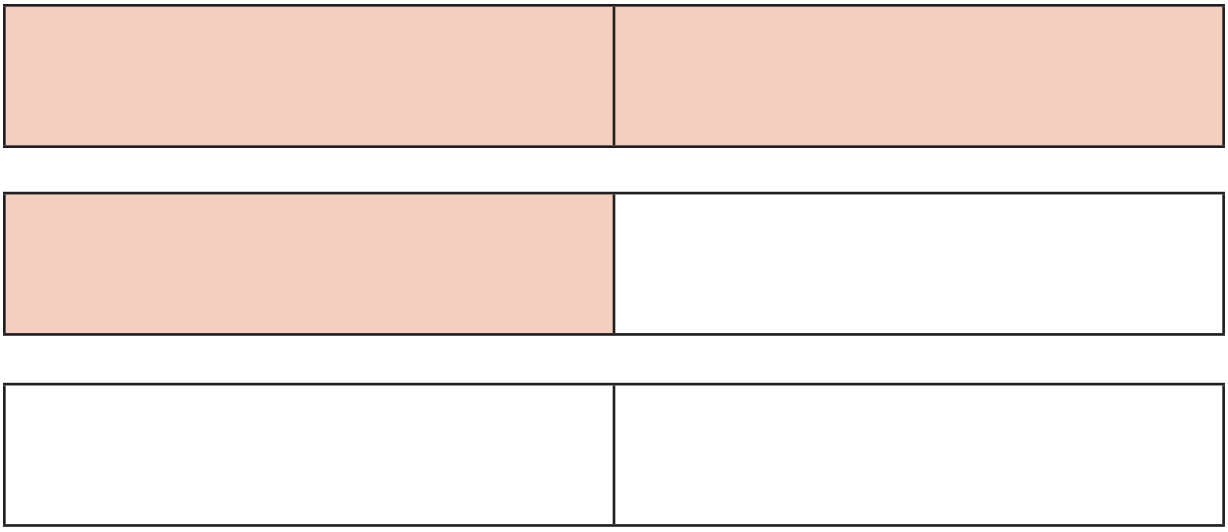
Each bar below represents 1 whole. Divide each bar into 3 equal parts.

Shade in 8 of the parts. Record how much of a bar is shaded. $\frac{8}{3}$

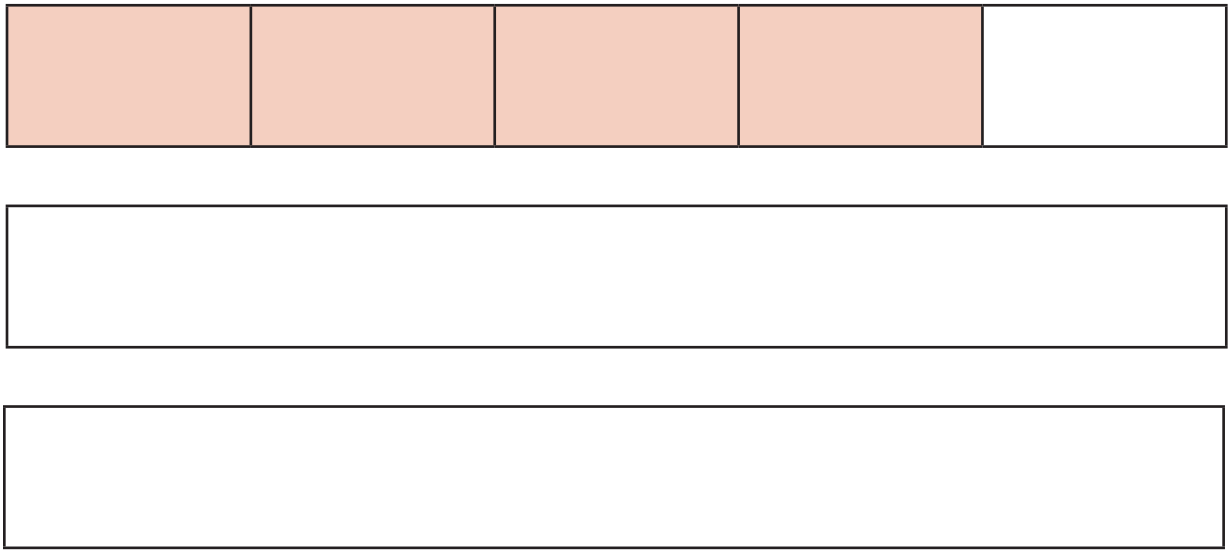


Each bar below represents 1 whole. Divide each square into 2 equal parts.

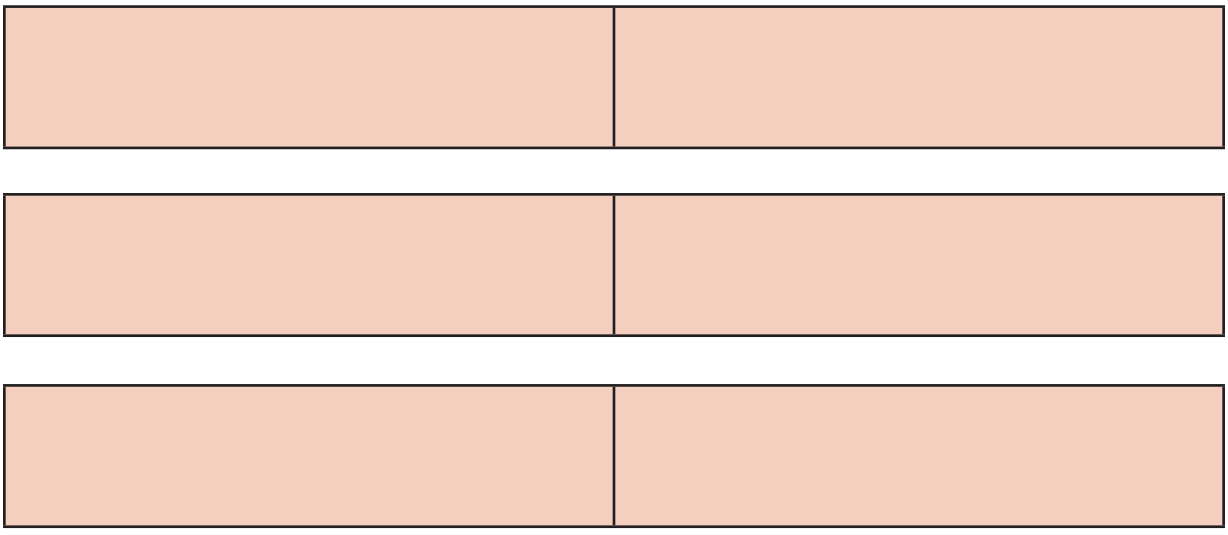
Shade in 3 of the parts. Record how much of a square is shaded. $\frac{3}{2}$



Each bar below represents 1 whole. Shade $\frac{4}{5}$ of a bar.



Each bar below represents 1 whole. Shade $\frac{6}{2}$ of a bar.

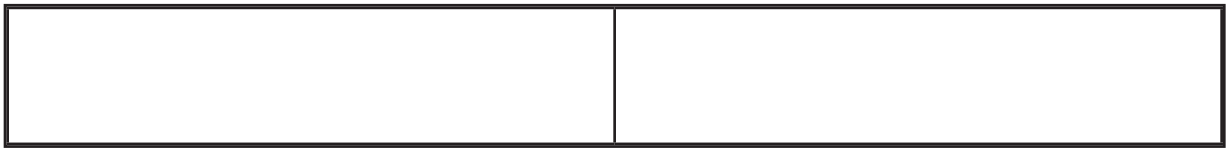


Each bar below represents 1 whole. Record how much of a bar is shaded.

$\frac{5}{3}$

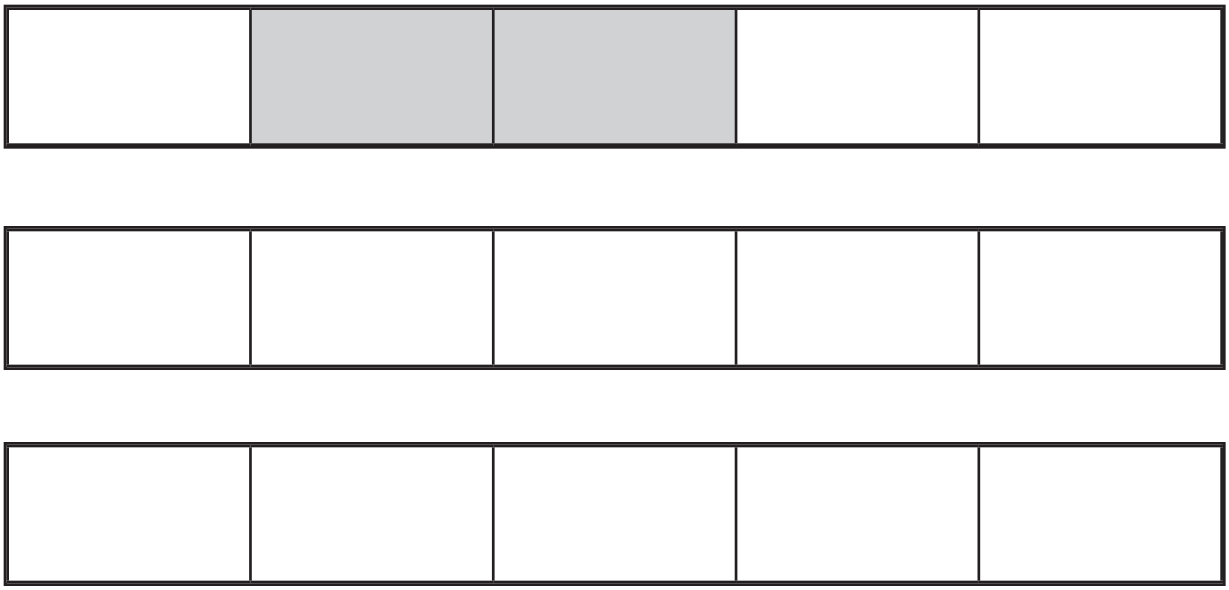
Each bar below represents 1 whole. Record how much of a bar is shaded.

$\frac{1}{2}$



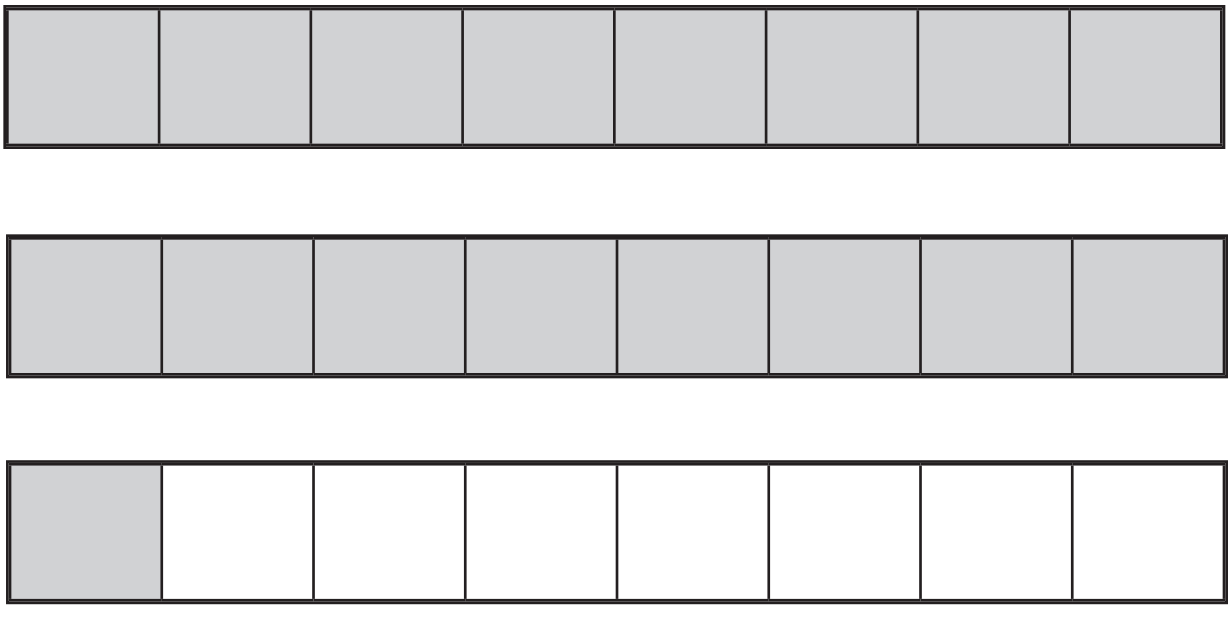
Each bar below represents 1 whole. Record how much of a bar is shaded.

$\frac{2}{5}$



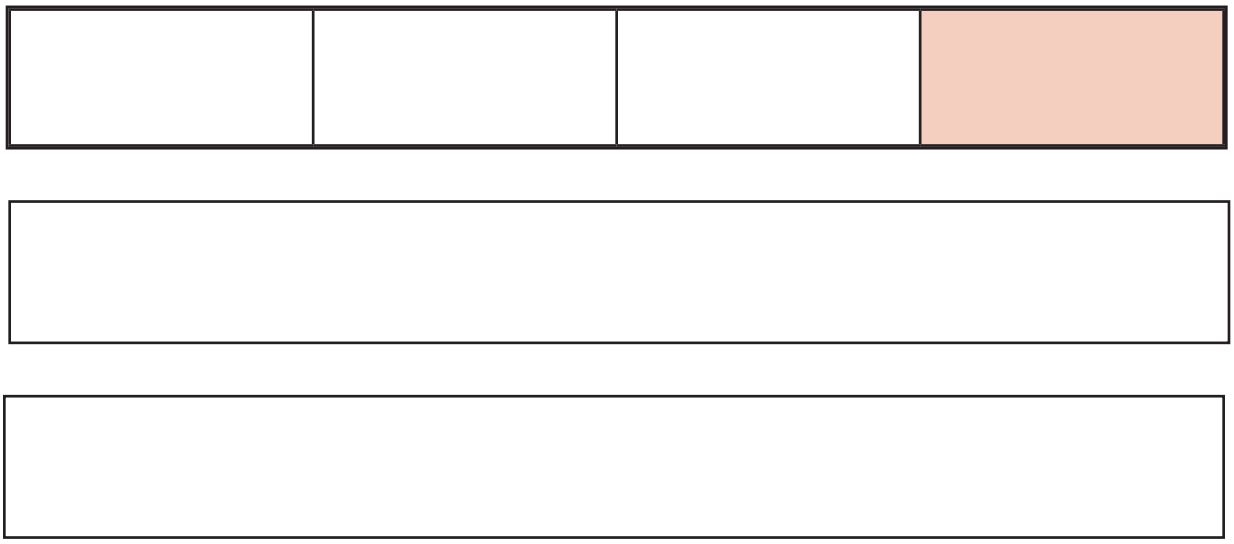
Each bar below represents 1 whole. Record how much of a bar is shaded.

$\frac{17}{8}$

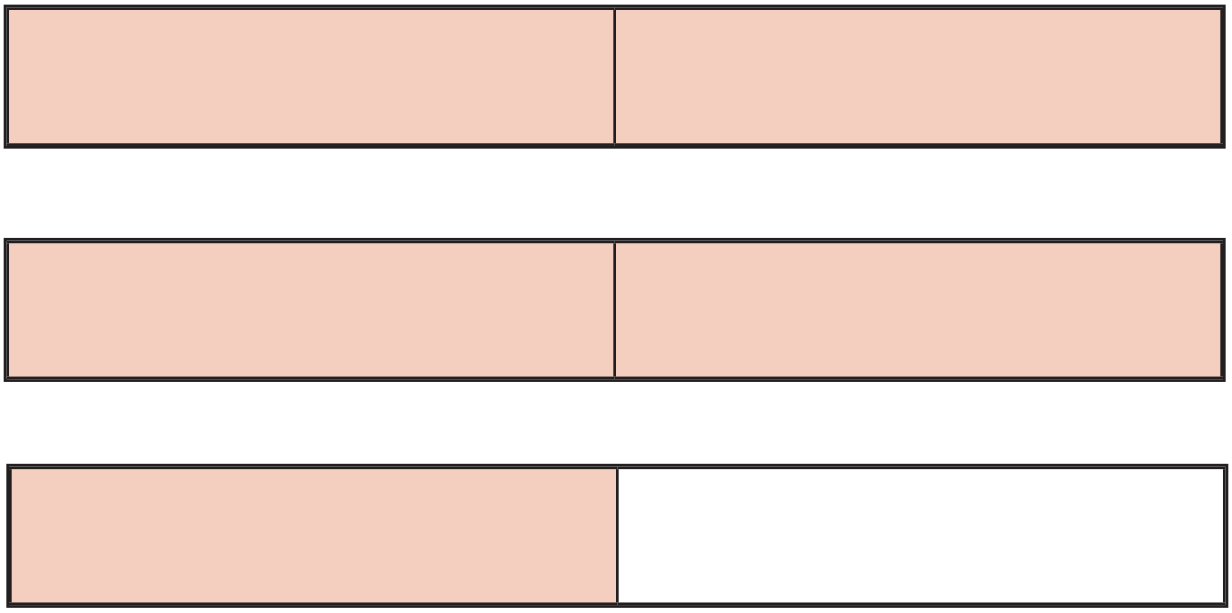


Independent Work

1. Each bar below represents 1 whole. Shade $\frac{1}{4}$ of a bar.

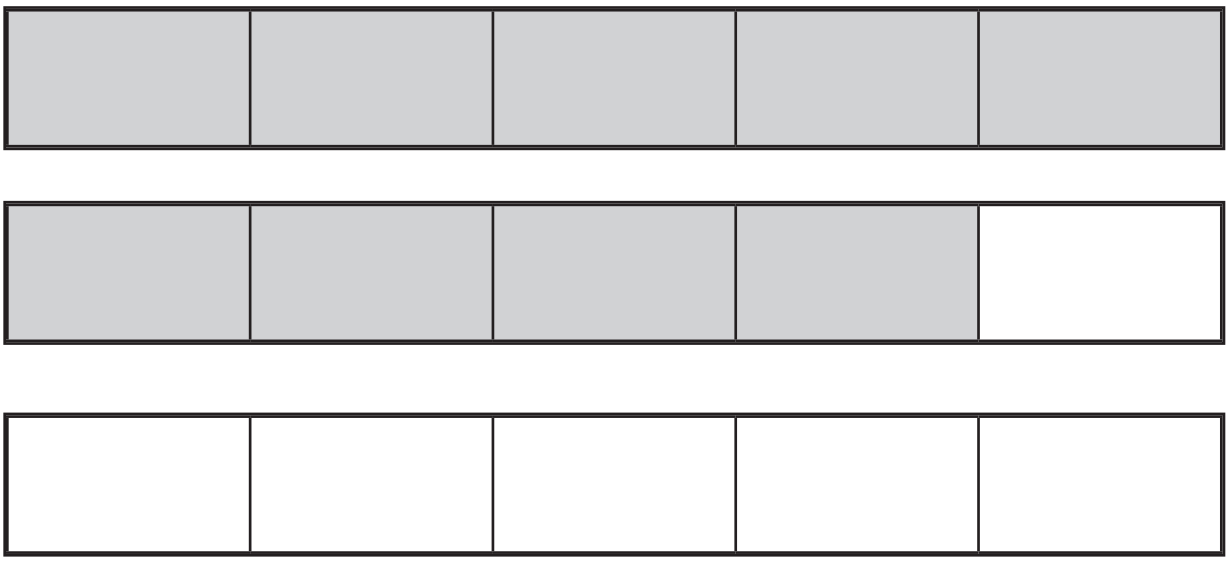


2. Each bar below represents 1 whole. Shade $\frac{5}{2}$ of a bar.



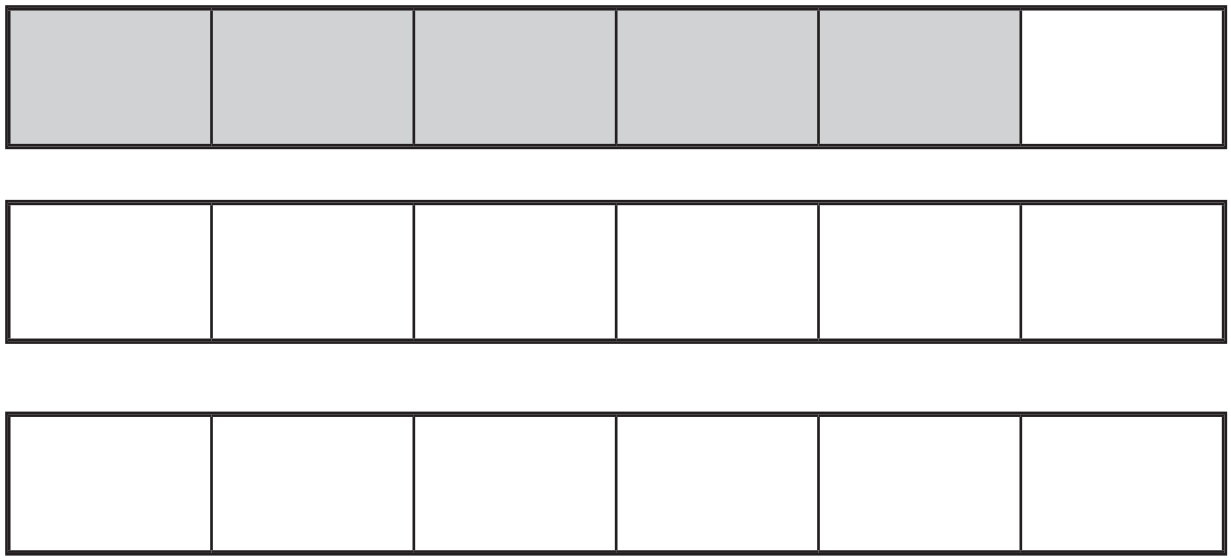
3. Each bar below represents 1 whole. Record how much of a bar is shaded.

$\frac{9}{5}$



4. Each bar below represents 1 whole. Record how much of a bar is shaded.

$\frac{5}{6}$



Lesson 2

Review of Naming Fractions - Part 2

How to Conduct the Lesson

Review of naming fractions - Part 2

In this lesson students learn to represent fractions on a number line instead of using "bars."

Introduce the number line by spreading your thumb and forefinger apart the distance between the 0 and 1 on the number line. Say, "This is 1 whole." Then touch your spread thumb and forefinger to touch "1" and "2" on the number line and say, "This is another whole." Repeat these steps to identify the interval between 2 and 3 and 3 and 4 as wholes. Then have the student do the same, using her spread thumb and forefinger to touch a whole on the number line and saying, "This is a whole."

Read the directions for the first task in the workbook out loud and then tell the student you will start the task. Divide the interval between 0 and 1 into 4 equal parts. Say, "To make 4 parts I need 3 cuts," and make the lines as indicated. Then name and label each cut, including labeling 1 whole as four-fourths.

Then have the student divide each remaining whole on the number line into 4 equal parts and name and label each part.

As the student is recording the fractions for each mark on the number line, tell her that she should write a fraction number not only on the lines she has drawn, but also the lines indicating the units. For instance, on the number line marking fourths, she should write 8/4 above the line indicating 2 wholes, 12/4 above the line indicating 3 wholes, and 16/4 above the line indicating 4 wholes.

If she asks about how to label the mark above 0 wholes, ask her how many fourths are in 0, and then have her write 0/4 above this line.

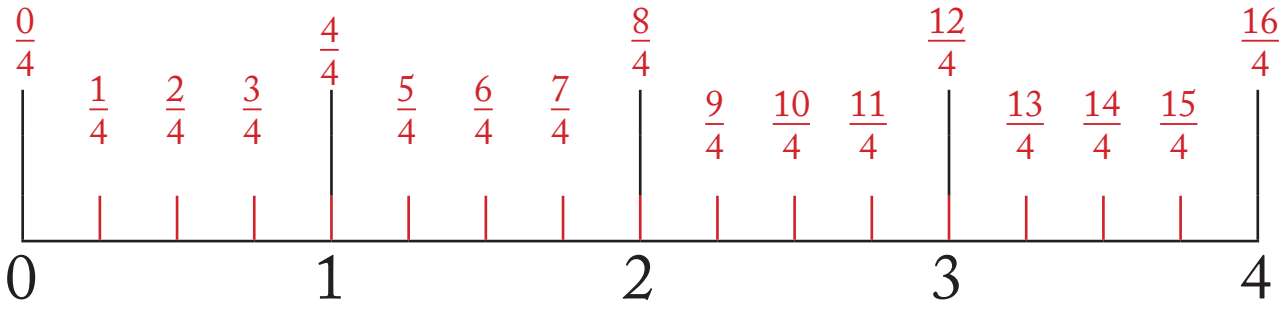
On the first two number lines the student's job is to divide each unit into 4 and 2 parts respectively and then write the appropriate fraction for each mark. On the next four, the student is to identify a particular location on the number line.

NOTE: Number lines are a little bit more abstract than the "bars" used to model fractions in Lesson 1. Bars represent quantity alone. Number lines represent quanity in terms of distance. When the student marks $\frac{3}{4}$ using the bars, she is counting three $\frac{1}{4}$ pieces. When the student marks $\frac{3}{4}$ on the number line, however, she is showing the point $\frac{3}{4}$ of a unit from 0, or the length of three $\frac{1}{4}$ sections.

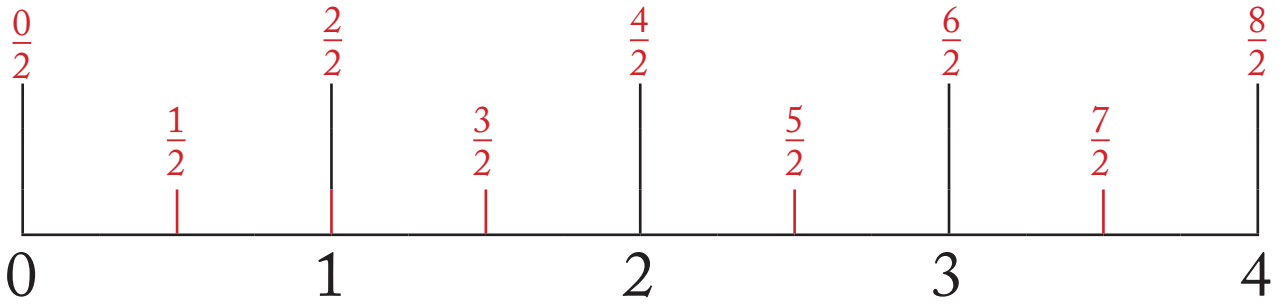
LESSON 2

Review of naming fractions - Part 2

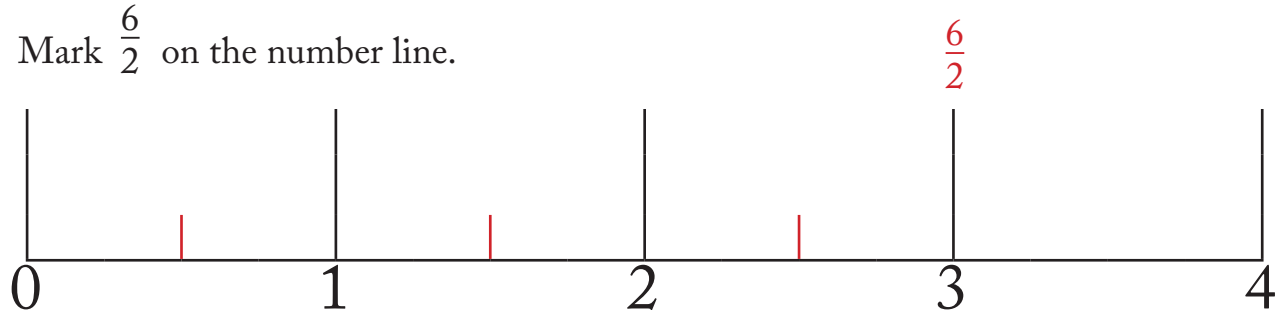
On the number line below draw lines to divide each unit into 4 equal parts. Then record a fraction to identify the value each line represents.



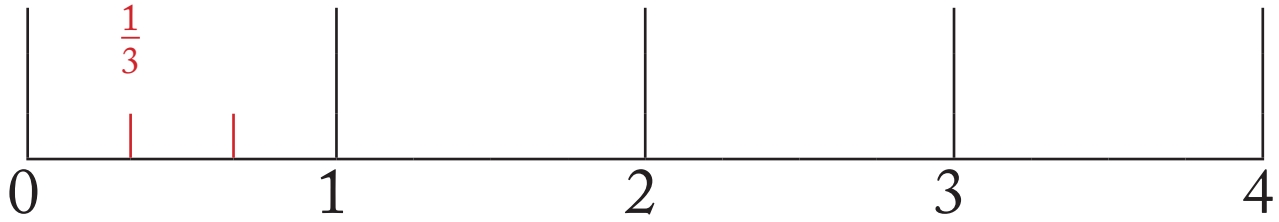
On the number line below draw lines to divide each unit into 2 equal parts. Then record a fraction to identify the value each line represents.



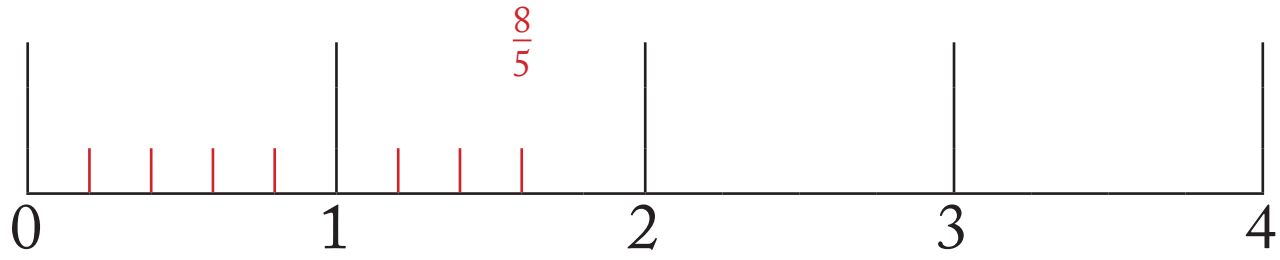
Mark $\frac{6}{2}$ on the number line.



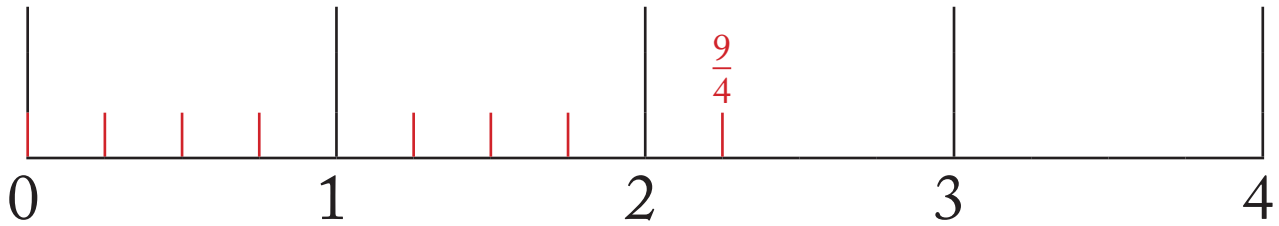
Mark $\frac{1}{3}$ on the number line.



Mark $\frac{8}{5}$ on the number line.

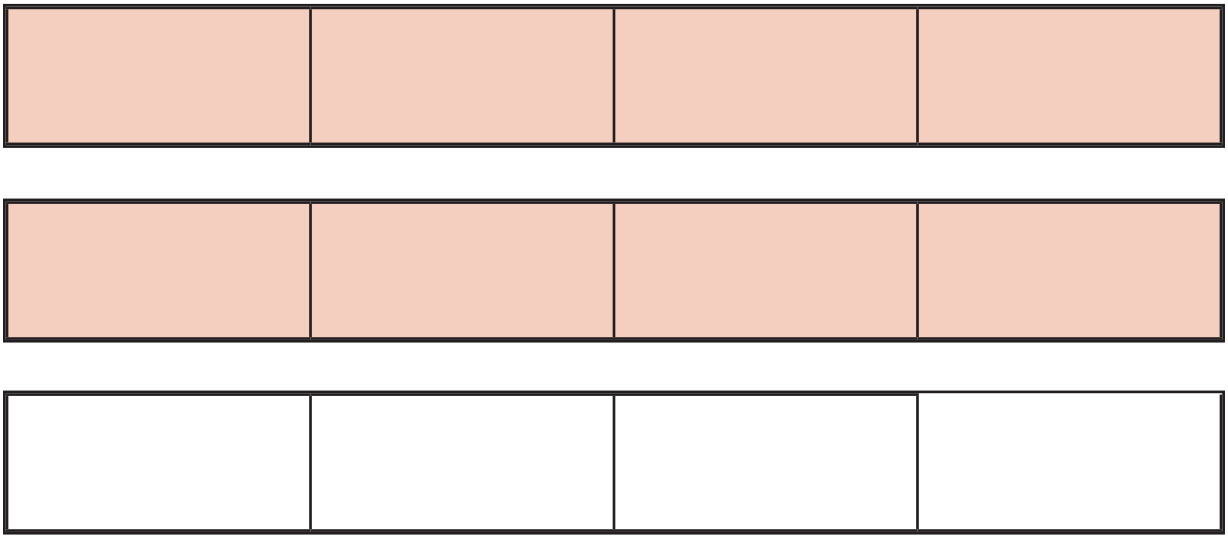


Mark $\frac{9}{4}$ on the number line.



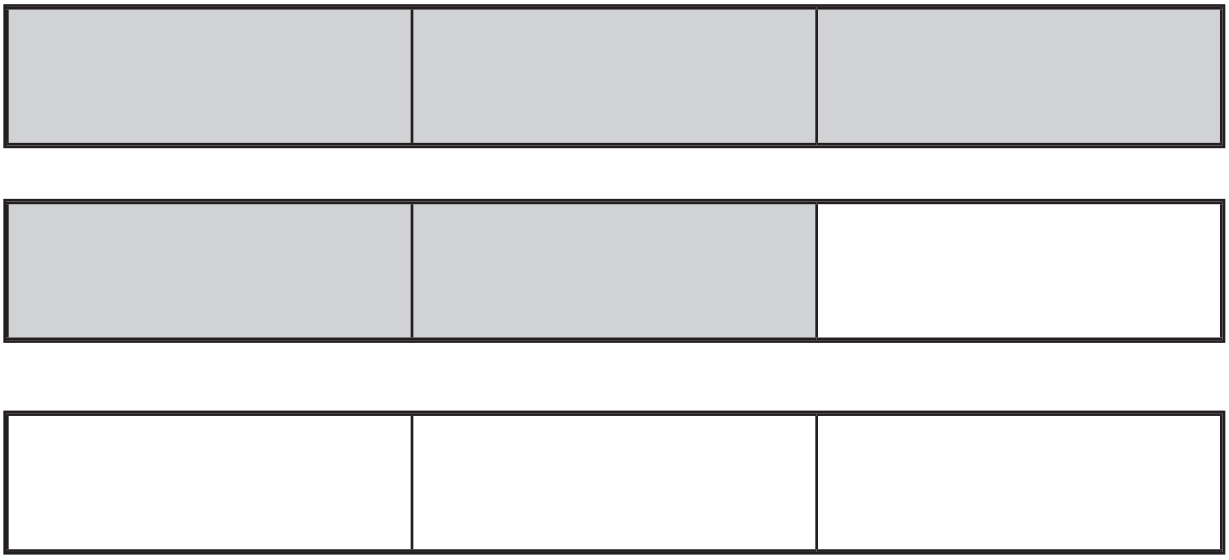
Independent Work

1. Each bar below represents 1 whole. Shade $\frac{8}{4}$ of a bar.



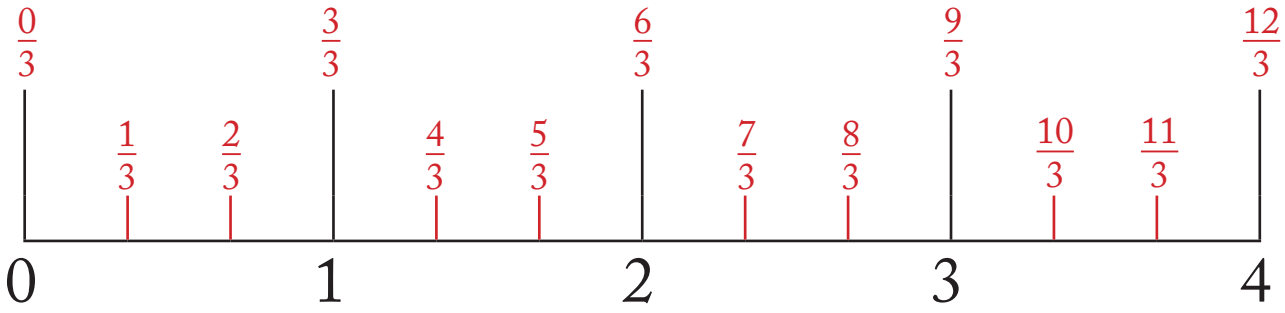
2. Each bar below represents 1 whole. Record how much of a bar is shaded.

$\frac{5}{3}$

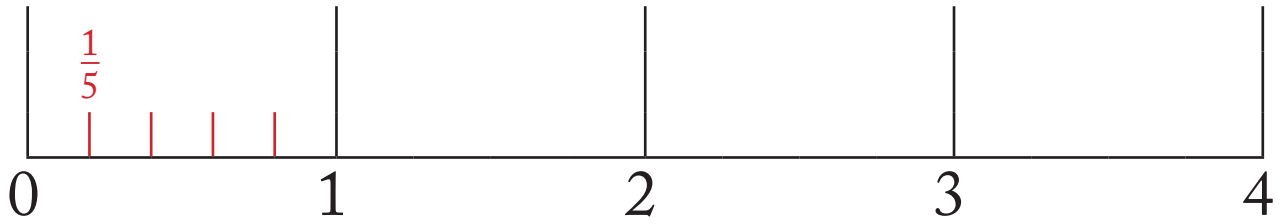


3. On the number line below draw lines to divide each whole into 3 equal parts.

Then record a fraction to identify the value each line represents.



4. Mark $\frac{1}{5}$ on the number line.



5. Mark $\frac{8}{4}$ on the number line.