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A Taxonomy for Teaching Music Theory: J. S. Bach and Lessons in Invertible Counterpoint

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J. S. Bach's pedagogical legacy has been widely discussed and examined.[1] What is worthy of further examination is how his instructional methods are relevant to current thinking in higher education, particularly in the undergraduate music theory classroom. At least two aspects of Bach's teaching are still applicable today. First, Bach taught new concepts through the study of complete compositions, aligning with current trends toward teaching directly from repertoire over extensive practice with acontextual exercises.[2] Second, Bach presented new material in a systematic fashion, gradually progressing from simple to more complex concepts.[3] Efforts to reconstruct Bach's practices as a teacher will necessarily require some modification, since there is not a direct correlation between what Bach probably taught in his private

I would like to thank Christina Fuhrmann, Philip Stoecker, and the anonymous reviewers, for their thoughtful comments and suggestions.

[1] See, for example, Stephen Roe, "Sons, Family and Pupils," in *The Routledge Companion to Johann Sebastian Bach*, ed. Robin A. Leaver (New York: Routledge, 2017), 458–63; and Robin A. Leaver and Derek Remeš, "J. S. Bach's Chorale-Based Pedagogy: Origins and Continuity," *BACH: Journal of the Riemenschneider Bach Institute* 48, no. 2 and 49, no. 1 (2018): 116–50. For more general accounts of Bach as teacher, see *The New Bach Reader*, ed. Hans T. David and Arthur Mendel, revised and expanded by Christoph Wolff (New York: W. W. Norton & Company, 1998); and Christoph Wolff, *Johann Sebastian Bach: The Learned Musician* (New York: W. W. Norton & Company, 2013).

[2] "Bach did not believe in teaching isolated elements or fragments of music. He avoided using 'species' in teaching counterpoint. He never wrote an exercise in composition or velocity that was not a fully rounded composition. Even where he set out to demonstrate a specific technical feature, he was never only a teacher." *New Bach Reader*, 22. On current trends in music theory pedagogy, see Elizabeth West Marvin, "The Core Curricula in Music Theory: Developments and Pedagogical Trends," *Journal of Music Theory Pedagogy* 26 (2012): 255–64.

[3] C. P. E. Bach's well-known account of his father's teaching methods, summarized in a letter to Johann Nikolaus Forkel, suggests a thoughtful progression of materials that gradually increases in difficulty: "His pupils had to begin their studies by learning pure four-part thorough bass. From this he went to chorales; first he added the basses to them himself, and they had to invent the alto and tenor. Then he taught them to devise the basses themselves. … In teaching fugues, he began with two-part ones, and so on." *New Bach Reader*, 399.

lessons and what modern-day instructors can achieve with large groups of students. Nevertheless, attempts to emulate Bach's pedagogical practices can certainly yield fruitful results in the twenty-first-century classroom.

Recent studies that take a historically informed approach toward music theory pedagogy, those that reconstruct Bach's practices for teaching specific topics, are convincing and appealing in their own right.[4] In contrast to these studies, however, I propose organizing lessons and activities around a modified version of Benjamin S. Bloom's *Taxonomy of Educational Objectives*.[5] This approach offers greater flexibility for music instructors since it can be applied to any topic. Part 1 of this essay draws inspiration from Bach's systematic method of instruction, and introduces a revised taxonomy for teaching written theory. Part 2 is similarly influenced by Bach's preference for teaching from real musical compositions. To demonstrate a concrete application of the taxonomy, I provide a series of activities designed to teach the topic of invertible counterpoint. Each stage of the taxonomy is accompanied by one or more examples from either Bach's two-part inventions or the *Well-Tempered Clavier.*

A Revised Taxonomy for Teaching Written Theory

Bloom's *Taxonomy of Educational Objectives* was first published in 1956 and has been widely influential in education and assessment standards ever since. A revised taxonomy was published in 2001 and continues to play a central role in setting educational standards and objectives.[6] Although neither version specifically addresses music

[4] See Austin Gross, "The Improvisation of Figuration Preludes and the Enduring Value of Bach Family Pedagogy," *Journal of Music Theory Pedagogy* 27 (2013): 19–46; Vasili Byros, "Prelude on a Partimento: Invention in the Compositional Pedagogy of the German States in the Time of J. S. Bach," *Music Theory Online* 21, no. 3 (2015); Derek Remeš, "J. S. Bach's Chorales: Reconstructing Eighteenth-Century German Figured-Bass Pedagogy in Light of a New Source," *Theory and Practice* 42 (2017): 29–54; Remeš, "Chorales in J. S. Bach's Pedagogy: A Method for Teaching Undergraduate Music Theory Inspired by a New Source," *Journal of Music Theory Pedagogy* 31 (2018): 65–92; and Remeš and Leaver, "J. S. Bach's Chorale-Based Pedagogy."

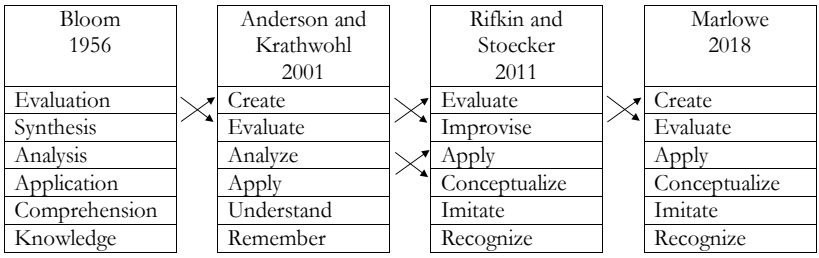
[5] Benjamin S. Bloom, ed., *Taxonomy of Educational Objectives: The Classification of Educational Goals*, 1st ed. (New York: Longmans, Green, 1956).

[6] Lorin Anderson and David R. Krathwohl, eds., *A Taxonomy for Learning, Teaching, and Assessing: A Revision of Bloom's Taxonomy of Educational Objectives* (New York: Longman, 2001). A useful summary of Bloom's Taxonomy and subsequent developments can be found in Patricia Armstrong, "Bloom's Taxonomy," Vanderbilt University Center for Teaching, https://cft.vanderbilt.edu/guides-sub-pages/blooms-taxonomy.

education, the categories are flexible and recent scholarship has adapted the model to various musical contexts.[7] Of these, the study by Deborah Rifkin and Philip Stoecker is most relevant to the present discussion: the authors thoroughly examine the taxonomy categories as they relate to teaching undergraduate aural skills, then present a series of lesson plans to demonstrate how they apply their revised taxonomy in their courses.

The taxonomy categories and their subsequent revisions are summarized in figure 1.[8] The processes are arranged in ascending order, where the lowest row reflects the simplest cognitive process and progresses to the highest row, which reflects the most complex process in the taxonomy. Arrows indicate where each subsequent study has reordered various stages of the process.

Figure 1. Comparison of learning taxonomies



The revised taxonomy put forth by Lorin Anderson and David R. Krathwohl suggests that the most meaningful learning results when progressing from knowledge retention (*Remember*), a past-based process, to knowledge transfer (*Create*), a future-based process, in which students are able to apply learned material to new situations.[9] Rifkin and Stoecker,

[7] James Caldwell, "Using Bloom's Taxonomy to Develop an Approach to Analysis," *Journal of Music Theory Pedagogy* 3 (1989): 223–32; Wendell Hanna, "The New Bloom's Taxonomy: Implications for Music Education," *Arts Education Policy Review* 108 (2007): 7–16; Deborah Rifkin and Philip Stoecker, "A Revised Taxonomy for Music Learning," *Journal of Music Theory Pedagogy* 25 (2011), 155–90; and Kevin R. Burke, "Hacking the Listening Guides: Bloom's Taxonomy and Aural Learning," *Engaging Students: Essays in Music Pedagogy* 1 (2013), http://flipcamp.org/engagingstudents.

[8] Figure 1 builds on the comparison chart provided by Rifkin and Stoecker, "A Revised Taxonomy for Music Learning," 158.

[9] Anderson and Krathwohl, *Taxonomy*, 62–65. Figure 1 presents processes associated with the "Cognitive Process Dimension," but the taxonomy also includes a four-part "Knowledge Dimension" that interacts with each of the six categories listed. Essentially, the "Knowledge Dimension" ranges from the simplest tasks, being able to explain *what* something is, to more complex tasks, being able to demonstrate *how* to do something or *how* to apply learned skills to new situations. Many, though not all, of the activities I suggest in this essay operate within the higher levels of the "Knowledge Dimension."

with substantive support from findings in music cognition studies, recommend renaming and reordering certain processes to reflect more accurately the learning experience in an aural skills setting. For instance, they consider *Apply* to be a higher-order objective than *Conceptualize* (formerly named *Analyze*), since "music students often need some sort of conceptual map, usually a visual or kinesthetic representation before they can apply and synthesize a new musical concept."[10] They also reverse the order of the final two objectives, suggesting that "[p]lacing *Evaluate* at the top of the taxonomy encourages a circular process in which a student uses the insights from their evaluation stage as the starting point for their next learning objective," and thus "reinvigorate[s] the learning process for the next learning task."[11]

My taxonomy largely follows the model set forth by Rifkin and Stoecker; however, I suggest reversing the final two processes (yet again) for written theory, and reposition *Create* at the top of the learning taxonomy. In an aural skills setting, the student must create something (*Improvise*) to initiate the evaluation stage. The act of trying an activity and assessing how well it worked is the best method for developing critical listening skills. In written theory, however, evaluation can, and often does, occur without creating something first (students can study works by other composers), whereas the task of creating (typically a model composition or analysis essay) is not complete without evaluation (editing and revising). Ultimately, it seems that the reason scholars continue to rearrange these two processes in their respective taxonomies is that they are highly interactive and thus do not necessarily fit into a linear model.[12]

An intriguing aspect of this taxonomy is that the categories reflect many details of what we know about Bach's teaching methods. We know that Bach encouraged his students to study works by master composers, including many of his own compositions (*Recognize*).[13] We also know that his students learned by copying his manuscripts (*Imitate*).[14] There

[10] Rifkin and Stoecker, "A Revised Taxonomy for Music Learning," 161.

[11] Rifkin and Stoecker, "A Revised Taxonomy for Music Learning," 162.

[12] Anderson and Krathwohl, *Taxonomy*, 85; Rifkin and Stoecker, "A Revised Taxonomy for Music Learning," 166.

[13] "[Bach's pupils'] sense of purity, order, and connection in the parts must first have been sharpened on the inventions of others, and have become in a manner habitual to them before [Bach] thought them capable of giving these qualities to their own inventions." *New Bach Reader*, 454.

[14] "Copying music was then considered the main way of learning to compose, and

is evidence that Bach thought about contrapuntal possibilities before committing anything to writing. For instance, he could examine a fugal subject and understand all of its contrapuntal potential before composing, and one could certainly imagine this skill being passed on to his students (*Conceptualize*).[15] Finally, although we know that Bach often had a clear view of a work before writing it down, there is evidence that he later revised some of his compositions, suggesting interaction between the final two stages of the taxonomy, *Evaluate* and *Create*.[16] The taxonomy will not reconstruct precisely how Bach taught, of course, but the parallels with Bach's methods certainly bolster this approach.

The Taxonomy in Practice: Teaching Invertible Counterpoint with Examples from J. S. Bach

To demonstrate how the taxonomy functions in the written theory classroom, the remainder of this essay explains how I teach invertible counterpoint in my eighteenth-century counterpoint course. My course primarily consists of performance and composition majors in their third or fourth year of undergraduate study, and knowledge of theory fundamentals and tonal harmony and voice leading is assumed. The class typically has around twenty-five students, and we meet once per week for one hour and forty minutes. Invertible counterpoint is challenging for students to master, but I find that students learn the concept best through engagement with these lessons and activities. Because I teach a dedicated class in eighteenth-century counterpoint, I have the luxury of spending multiple class meetings on this topic. To that end, some of the materials I provide will be too advanced or time-consuming for a core theory course, but several activities can easily be adapted for other music theory topics. I hope that this demonstration will inspire theory teachers to consider applying the taxonomy in their own teaching.

The decision to draw examples primarily from Bach's two-part inventions and the *Well-Tempered Clavier* is informed by several pedagogical considerations. First, we know that Bach composed

for this the Inventions and *The Well-tempered Clavier* were the core works, judging by the number of student copies." David Ledbetter, *Bach's* Well-tempered Clavier: *The 48 Preludes and Fugues* (New Haven, CT: Yale University Press, 2000), 138.

[15] "It was Carl Philipp Emanuel, too, who reported to Forkel that when Father Bach had heard the beginning of a fugue he would at once state 'what contrapuntal devices it would be possible to apply, and which of them the composer by rights ought to apply.'" *New Bach Reader*, 20.

[16] On Bach's compositional process, see Wolff, *Johann Sebastian Bach*, 382–89.

these pieces for instructional purposes.[17] Second, although invertible counterpoint appears in many textures, keyboard texture encourages students to focus on learning the technique without the added challenge of reading scores for larger ensembles. (Scores for other instrumental and vocal works are incorporated into later class discussions once students have gained familiarity with the concept.) Finally, the culminating project is a model composition of a two-part invention for keyboard in the baroque style.

The examples discussed below are thus part of a larger corpus study of Bach's fifteen two-part inventions. After analyzing an invention together during class, I assign each of the remaining inventions to small groups of students for homework. Each group is asked to report on their assigned piece and, as a result, the students are able to catalogue a large amount of information. These observations, in combination with in-class discussion of invertible counterpoint and other contrapuntal techniques, ultimately prepare students for their final composition project. Each lesson is designed gradually to develop specific skills and techniques; even the composition is introduced in smaller stages, with clearly articulated goals and components. This is an example of "scaffolding," which Elizabeth West Marvin describes as "both a 'design feature' of an activity or course and an 'interactional process' between teacher and student, where a series of graduated pedagogical steps in a supportive environment leads to an eventual hand-off, in which the student—now adequately prepared—can take over his or her learning. … As the student nears the end of the project, the scaffold is removed."[18]

[17] Roe, "Sons, Family and Pupils," 444. We also know that Bach arranged his two-part inventions in a systematic order for instructional purposes. See, for example, Courtney S. Adams, "Organization in the Two-Part Inventions of Johann Sebastian Bach, Part I," *BACH: Journal of the Riemenschneider Bach Institute* 13, no. 2 (1982): 6–16; Adams, "Organization in the Two-Part Inventions of Johann Sebastian Bach, Part II," *BACH: Journal of the Riemenschneider Bach Institute* 13, no. 3 (1982): 12–19; and Ellwood Derr, "The Two-Part Inventions: Bach's Composers' Vademecum," *Music Theory Spectrum* 3 (1981): 26–48.

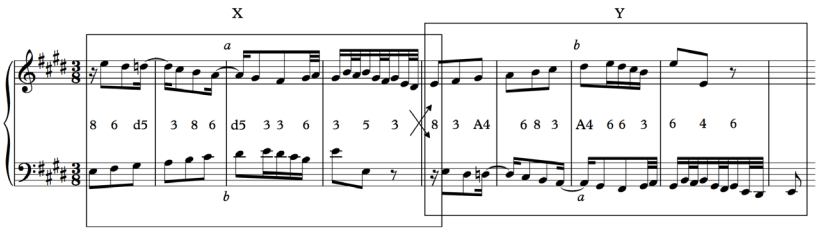
[18] "What I Know Now: Reflections on Music Theory Pedagogy," in *The Norton Guide to Teaching Music Theory*, ed. Rachel Lumsden and Jeffrey Swinkin (New York: W. W. Norton and Company, 2018), 372.

Recognize (Remember)[19]

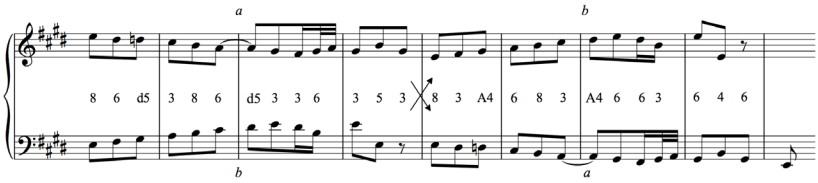
The initial stage of the taxonomy requires that students be able first to define and then to recognize invertible counterpoint in a real musical texture. To accomplish this task, they must remember specific contrapuntal combinations and identify where they return later in a composition. The most logical starting point, then, is a composition where the repetition occurs as early as possible and the pitch content remains invariant. Bach's Invention No. 6 in E Major BWV 777 (exs. 1a–d) contains two clearly identifiable instances of invertible counterpoint at the octave (IC8).[20] I ask the students to analyze mm. 1–9 first and provide a rhythmic normalization below the score so they can analyze the intervallic content more easily (exs. 1a-b).[21] Because invertible counterpoint occurs immediately before the piece modulates, the pitches remain invariant and students are able to identify the inverted lines easily.

Example 1. J. S. Bach, Invention No. 6 in E Major BWV 777

a. mm. 1-9, score



b. mm. 1-9, rhythmic normalization



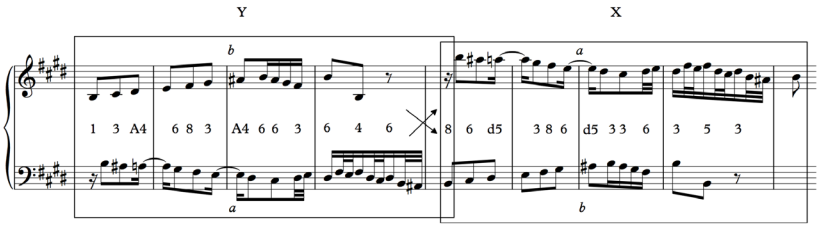
[19] In the following discussion, I include the taxonomy terms outlined by Anderson and Krathwohl in parentheses. Using terms universally applied across disciplines will prove advantageous for instructors needing to explain how music theory instruction aligns with assessment criteria endorsed by their respective institutions.

[20] I use the abbreviation IC for invertible counterpoint followed by 8, 10, or 12 to represent invertible counterpoint at the octave (IC8), tenth (IC10), or twelfth (IC12).

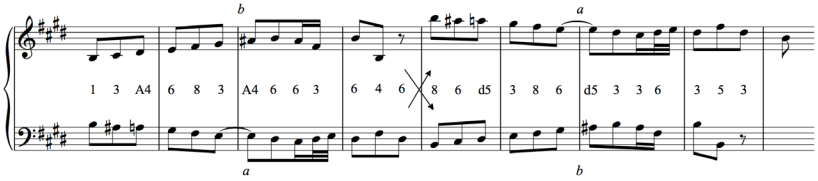
[21] I do not require that my students distinguish between compound and simple intervals in this type of activity.

Example 1 (con.)

c. mm. 21-29, score



d. mm. 21-29, rhythmic normalization



Imitate (Understand)

Rifkin and Stoecker explain that aural skills students must "recall and repeat previous music events," or *imitate* what was previously heard.[22] For written theory, students complete written exercises to recreate something observed in a prior example. Invertible counterpoint (IC8) will occur in the same key (exs. 1a–b), or it can appear in a different key later in a composition (exs. 1c–d). The next two written activities reinforce this observation.

I present the excerpt in examples 1c-d as a worksheet. I provide the score for mm. 21–24 of the E-major invention, and ask students to generate IC8 for mm. 25–29 (they may not choose the same registers as Bach, but should be able to transpose each line up and down by an octave). Next, I provide the full score and, asking them to compare the excerpts from examples 1a and 1c, we begin discussing the concept in more general terms. I challenge them to think about the process more deeply and ask them to consider the usefulness for learning invertible counterpoint. Peter Schubert writes, "[k]nowing how composers thought about their music is essential to understanding it, and counterpoint provided many of the basic structures for all their music."[23] For performers, the ability to

[22] Rifkin and Stoecker, "A Revised Taxonomy for Music Learning," 160.

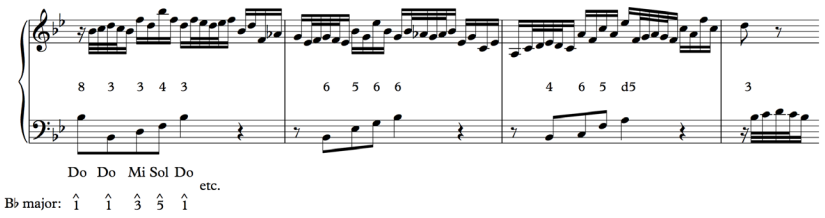
[23] Peter Schubert, "Teaching Historical Counterpoint," in *The Norton Guide to Teaching Music Theory*, ed. Rachel Lumsden and Jeffrey Swinkin (New York: W. W. Norton and Company, 2018), 23.

recognize what Schubert calls "blocks of polyphony" is useful for learning and memorizing new music quickly. In example 1, where the initial focus was on individual melodic lines (*a* and *b*), we now observe the combined lines as vertical "blocks," labeled X and Y. For composers, it is useful to observe how Bach varies the sound and generates "more music" simply by inverting the melodic lines.

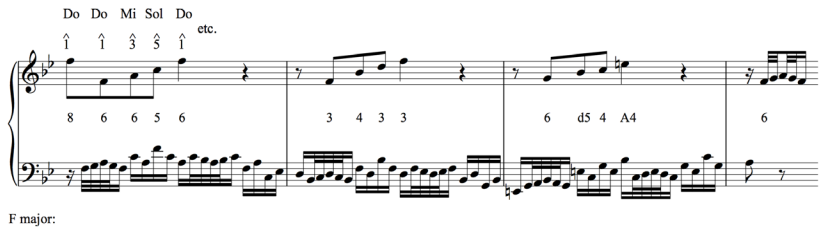
Example 1 also illustrates how invertible counterpoint may or may not occur in the original key later in a composition. To complete the worksheet for examples 1c-d, students simply transposed each line up and down an octave, but example 1c also shows the material transposed to the key of the dominant. They should now consider other strategies for generating IC8 in a new key. One way to achieve this is through analysis of tonal function. Singing examples 1a and 1c on moveable-do solfège (or scale-degree numbers) will highlight that the same scale degrees occur in each excerpt; reversing the process, we find that scale-degree function is an efficient way to generate IC8 in a new key. To practice this, I provide an excerpt from the Invention No. 14 in B-flat Major BWV 785 (ex. 2a) and ask students to compose IC8 in the key of the dominant (ex. 2b). One could produce the same result through first inverting, then transposing the lines into the key of F major, but scale-degree function makes the task much less cumbersome.

Example 2. J. S. Bach, Invention No. 14 in B-flat Major BWV 785

a. mm. 1-4



b. mm. 6-9



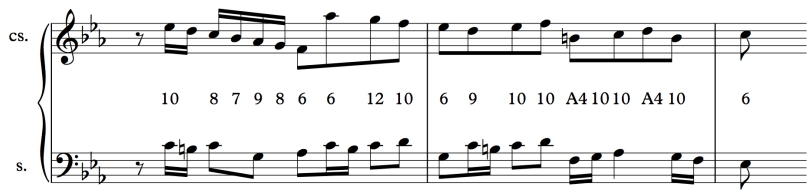
Conceptualize (Analyze)

In *Teaching Approaches in Music Theory: An Overview of Pedagogical Philosophies*, Michael Rogers writes, "music reading and meaningful learning are totally dependent on the ability to conceptualize what is heard."[24] Rifkin and Stoecker explain that, in aural skills, students must first develop a "conceptual map" before applying a new concept to new situations. The same is true in written theory, and an added benefit of renaming this category *Conceptualize* is that it cannot be confused with the task of music analysis. Rather, the focus of this stage of the taxonomy is to explore a particular concept or technique and later *Apply* it to a real musical context.

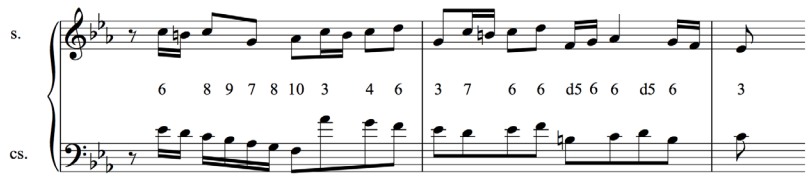
Invertible counterpoint typically occurs at three transposition levels in tonal music: invertible counterpoint at the octave (IC8), the tenth (IC10), and the twelfth (IC12). The process is the same for each of these, but different contrapuntal combinations result. Example 3 is a worksheet I use for in-class examination of IC8, IC10, and IC12.[25] The activity is designed to reinforce the process for generating invertible counterpoint, this time at three different transposition levels, and encourages students to explore how each of these transpositions impacts the counterpoint.

Example 3. Composition exercise using themes from J. S. Bach, Fugue in C Minor from the *Well-Tempered Clavier* Book I

a. score, mm. 7–9 (outer voices)



b. student solution, IC8

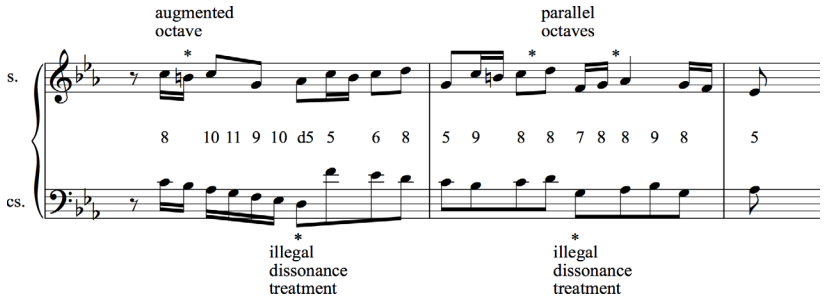


[24] 2nd ed. (Carbondale: Southern Illinois University Press, 1984), 33.

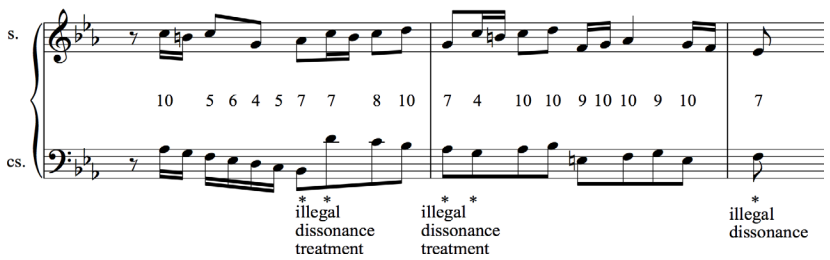
[25] Although I encourage my students to think about the properties of IC10 and IC12 during class, I do not require them to compose with IC10 or IC12. Their culminating project requires only IC8.

Example 3 (con.)

c. student solution, IC10



d. student solution, IC12



Example 3a provides the subject and countersubject from Bach's Fugue in C Minor from the *Well-Tempered Clavier* Book I, mm. 7–9.[26] I divide the class into three groups. Each group transposes the countersubject to generate IC8, IC10, or IC12, and analyzes the harmonic intervals. I always stress that the goal is to "sound stylistic" and not to assess whether or not something "sounds good." The students are therefore evaluating these solutions against a list of rules and procedures for writing tonal counterpoint, *not* assessing which solution they think sounds the best. After playing their results for the class, it is immediately obvious that IC8 is the only option that produces correct tonal counterpoint. They continue by discussing the contrapuntal problems that occur with IC10 and IC12.

The *Conceptualization* stage thus provides students with the opportunity to engage more deeply with a particular concept, but also

[26] The inner voice is omitted for clarity during this part of the lesson. It should also be noted that the excerpt in example 3a is actually the first occurrence of IC8 in the C-minor fugue, not the first time the subject and countersubject appear together (this occurs in m. 3). Choosing the contrapuntal combination in m. 7 eliminates the added complication of dealing with a tonal answer; the topic is revisited during a later discussion of the entire fugue.

provides an important link between the lower- and higher-order processes in the taxonomy.[27] Example 3, for instance, engages with both the *Imitate* and *Evaluate* processes. It highlights the transitional function of this stage within the taxonomy, and furthermore demonstrates how the process is not always linear. According to Rifkin and Stoecker, "[t]o conceptualize a musical event is to analyze it and create a way to concretize it in one's mind."[28] Although the simplicity of the earlier examples may be initially deceiving, the worksheet in example 3 demonstrates that careful planning is absolutely necessary when composing with invertible counterpoint, setting the stage for a discussion about compositional strategies for IC8, IC10, and IC12.

After experiencing the problems that arise in different transposition levels, I show students the interval charts that theorists typically use to examine invertible counterpoint (fig. 2). These charts offer a quick overview of what occurs under IC8, IC10, and IC12, and are useful for composition and analysis. In composition, the charts are used as a reference tool for isolating interval combinations that should and should not be emphasized. The charts also introduce students to the concept of the "magic number." The "magic number" always equals one number greater than the interval of inversion (because the unison is labeled as "1"). In analysis, we add related intervals to derive the "magic number," which then tells us the interval of inversion.[29]

Figure 2. Invertible counterpoint charts

Invertible counterpoint at the octave (IC8)

1	2	3	4	5	6	7	8
8	7	6	5	4	3	2	1
9	9	9	9	9	9	9	9

Invertible counterpoint at the tenth (IC10)

1	2	3	4	5	6	7	8	9	10
10	9	8	7	6	5	4	3	2	1
11	11	11	11	11	11	11	11	11	11

Invertible counterpoint at the twelfth (IC12)

1	2	3	4	5	6	7	8	9	10	11	12
12	11	10	9	8	7	6	5	4	3	2	1
13	13	13	13	13	13	13	13	13	13	13	13

[27] Anderson and Krathwohl, *Taxonomy*, 79.

[28] Rifkin and Stoecker, "A Revised Taxonomy for Music Learning," 161.

[29] Robert Gauldin, *A Practical Approach to Eighteenth-Century Counterpoint*, rev. ed. (Long Grove, IL: Waveland Press, 2013), 114–15. See also Peter Schubert and Christoph Neidhöfer, *Baroque Counterpoint* (Upper Saddle River, NJ: Pearson Prentice Hall, 2006), 131; and Douglass Green and Evan Jones, *The Principles and Practice of Tonal Counterpoint* (New York: Routledge, 2016), 70.

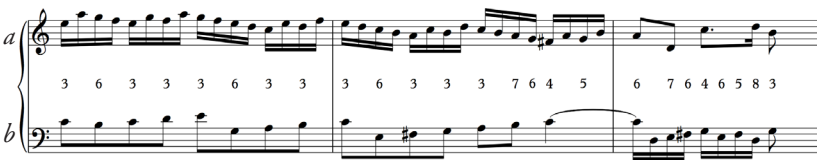
Apply

Examples 4 and 5 encourage students to apply the interval charts (fig. 2) in analysis. Anderson and Krathwohl break the *Apply* process down into two subcategories: "*executing*—when the task is an exercise (familiar)—and *implementing*—when the task is a problem (unfamiliar)."[30] Examples 4a-b provide the more familiar task of analyzing IC8 and help students solidify their understanding of how to apply the interval charts in analysis. Examples 5a-e contain invertible counterpoint at multiple transposition levels, and will require more careful application of the technique.

To begin, I distribute the full score for Bach's C-major Invention, and ask the students to identify where invertible counterpoint occurs. Since they have already been exposed to the topic for three prior lessons, the new challenge is locating the technique within a complete composition, which can be quite difficult. Students are usually quick to identify IC8 in mm. 1 and 7, but the technique also appears in the excerpts shown in examples 4a-b. The latter relationship is often overlooked because, unlike the previous examples, it occurs after the start of a new section and does not involve direct statements of the opening motives. Once they have identified this instance of invertible counterpoint, the students label the keys for each passage (C major modulating to G major; D minor modulating to A minor) and sing each line on solfège. They confirm that both passages utilize the same melodic material in spite of the key change, and therefore feature IC8.[31]

Example 4. J. S. Bach, Invention No. 1 in C Major BWV 772

a. mm. 3–5

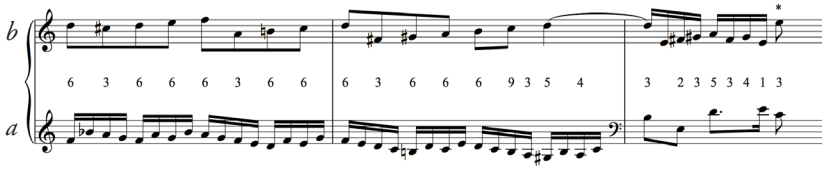


[30] Anderson and Krathwohl, *Taxonomy*, 77.

[31] Singing with moveable-do solfège will necessitate changes to some syllables (*mi* to *me*, for instance) since one passage is in major and the other is in minor. Nevertheless, both excerpts present the same scale-degree functions.

Example 4 (con.)

b. mm. 11–13



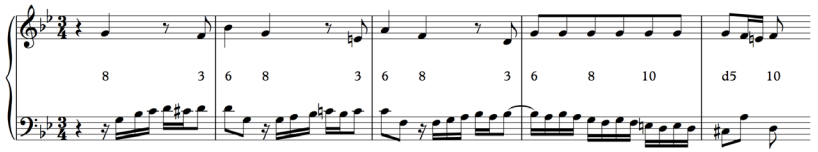
After labeling the harmonic intervals, I demonstrate two ways to apply the interval charts in analysis. The first approach simply compares interval transformations between passages (thirds later appear as sixths, perfect fifths as perfect fourths, etc.), which will align with one of the three inversion charts. The second approach involves adding intervals in parallel passages to determine the "magic number." For example, the first four harmonic intervals in m. 3 are 3–6–3–3, and the first four intervals in m. 11 are 6–3–6–6. Each interval pair equals nine, and therefore must be an instance of IC8. This method is faster, although less comfortable for many students.

Examples 5a-e present several excerpts from Bach's Fugue in G Minor from the *Well-Tempered Clavier* Book II. This fugue features IC8, IC10, and IC12, and I challenge students to use the interval charts to identify each type.[32] Since this activity is more challenging and more time-consuming, I prefer to isolate the relevant excerpts, rather than provide the full score, so students can focus on applying the new analytical technique. Example 5a provides the answer and countersubject as they initially appear in mm. 5–9. I begin by having the class sing each contrapuntal combination on solfège, and students quickly realize that the syllables no longer remain the same in each excerpt as they did in previous exercises. The first takeaway from this activity is that scale-degree function will only remain invariant under IC8. Singing each line on a moveable system thus serves as an initial test for determining the transposition level of a passage. With the exception of a slight modification at the start, example 5b features the same solfège syllables and pitch content as example 5a. This is quickly identified as IC8, and we move on to the more challenging excerpts.

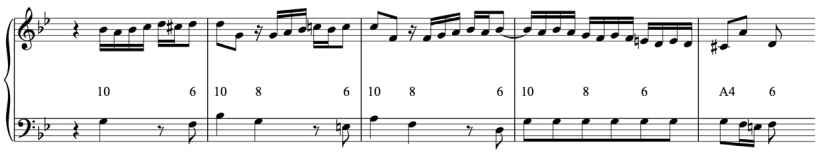
[32] For a more detailed discussion of invertible counterpoint in this fugue, see Sarah Marlowe, "Fugue in Context: A Schenkerian Approach to Select Works by J. S. Bach and Dmitri Shostakovich" (PhD diss., Eastman School of Music, University of Rochester, 2013), 81–100.

Example 5. J. S. Bach, Fugue in G Minor from the *Well-Tempered Clavier* Book II

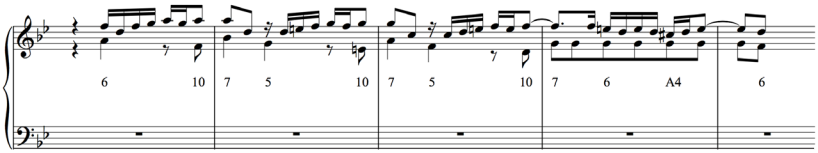
a. mm. 5–9



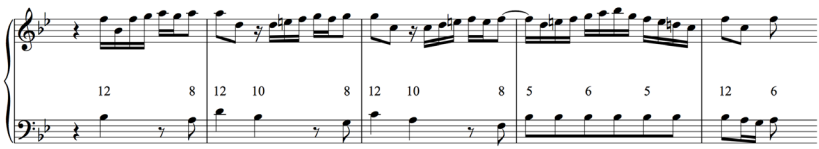
b. mm. 13–17



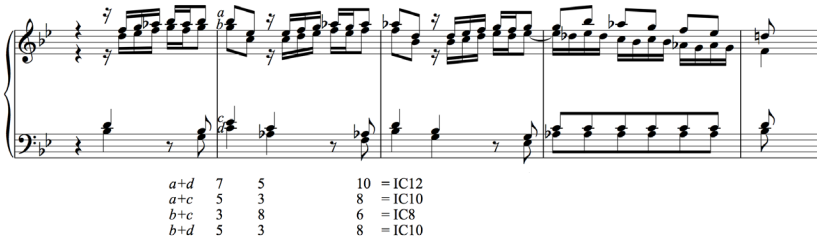
c. mm. 28–32



d. mm. 36–40



e. mm. 59–63



There is another factor to consider before applying the interval charts here. To determine the "magic number" successfully, we need to compare identical combinations. But the G-minor fugue contains a tonal answer,

and the subject's beginning may be altered in some of the excerpts. Comparing interval combinations in the second measure will ensure correct results because the lines normalize (i.e., are consistently the same) here. Students then analyze examples 5c (IC12) and 5d (IC10) in small groups using the interval charts. Finally, I direct the class to observe example 5e, where both the subject and countersubject are embellished with parallel thirds. For the purposes of discussion, each line is labeled *a, b, c*, and *d* in the score (m. 60). The harmonic intervals labeled below the score reveal that all three types of invertible counterpoint can be derived from this passage, and students are typically very excited to discover this.

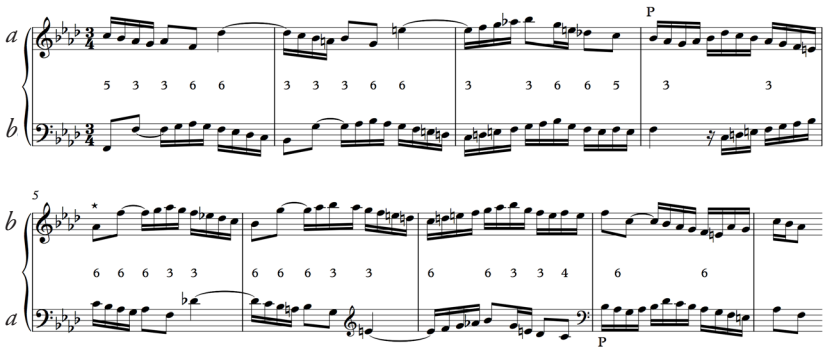
Evaluate

In part 1 of this essay, I explained my reasons for reversing the final two stages of Rifkin and Stoecker's taxonomy. In written theory, it is important for students to learn how to evaluate a composition against the rules and procedures they have learned. They must first think critically about existing works, so they can effectively critique their own work in the culminating stage of the process. With a firm understanding of how to generate and identify invertible counterpoint, students are ready for the more sophisticated task of defending why a composer might diverge from an expected outcome. Examples 6 and 7 present cases where Bach does not simply transpose lines to create invertible counterpoint; he also makes modifications to the lines, and the students must offer plausible reasons for his decisions.

In example 6, lines *a* and *b* (mm. 1–4) are imitated almost exactly when they are inverted in mm. 5–8.[33] Line *b* is slightly modified on the downbeat of m. 5, however, and I ask students to suggest why this modification is necessary. Line *a* begins on $\hat{5}$, so m. 5 would begin with a dissonant perfect fourth if everything was transposed exactly. In two-voice counterpoint, a perfect fourth implies a $\frac{6}{4}$ chord, which should be avoided in this style. Then I ask students to consider other ways this could be avoided and to supply reasons why Bach decided to alter line *b* instead of line *a*. Since the opening of this invention is sequential, changing line *a* would necessitate a change in m. 6 as well; the contour of line *b* is otherwise unaffected by the modification in m. 5, so perhaps Bach chose to preserve as much of the original melodic content as possible.

[33] A detailed analysis of the F-minor invention is provided in Green and Jones, *The Principles and Practice of Tonal Counterpoint*, 87–91; see also Schubert and Neidhöfer, *Baroque Counterpoint*, 209–10.

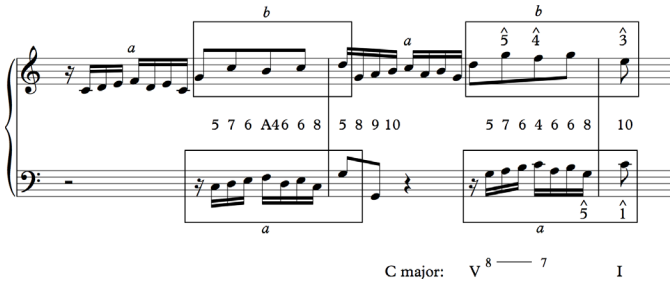
Example 6. J. S. Bach, Invention No. 9 in F Minor BWV 780, mm. 1–8



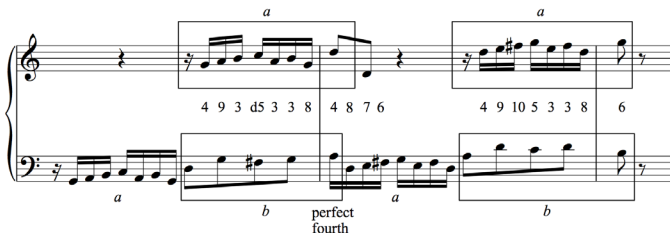
Examples 7a-c revisit the C-major invention to promote a slightly more complex discussion. The students are asked to recompose the material in mm. 1–3 (ex. 7a) to generate IC8 in the key of G major; the (correct) student solution is shown in example 7b. I then ask them to critique the results and compare their solution with what Bach actually does in mm. 7–9 (ex. 7c). Three pitches marked with a star indicate where Bach modified the original contrapuntal combination.

Example 7. J. S. Bach, Invention No. 1 in C Major BWV 772

a. score, mm. 1–3

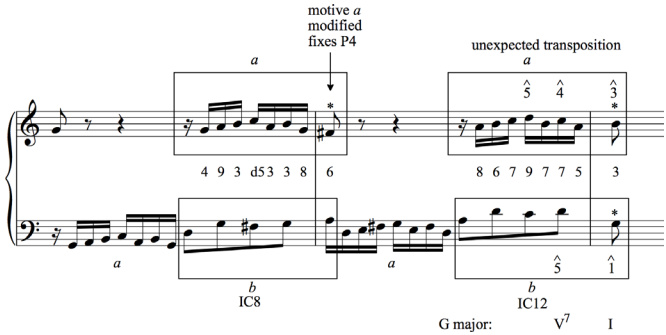


b. student solution, IC8 transposed to G major



Example 7 (con.)

c. score, mm. 7–9



Bach avoids the dissonant perfect fourth on the downbeat of m. 8 by altering motive *a*. Avoidance of the perfect fourth might also support Bach's decision to transpose motive *a* in the second half of m. 8, but since he chose to preserve this relationship in the second half of m. 7, this is probably not the best justification. Instead, I encourage students to consider the implied harmony. Both excerpts in examples 7a and 7c close with $\hat{5}$–$\hat{1}$ motion in the lower voice and $\hat{4}$–$\hat{3}$ motion in the upper voice, requiring a modification to motive *b* and additional transposition of motive *a* in mm. 8–9. Here, it seems that Bach prioritizes the authentic cadence over exact motivic repetition. He could have achieved this by altering motive *b* on its own, but the lines would have converged on a perfect octave rather than a tenth in m. 9 (observe motive *a* at the end of example 7b). Transposing motive *a* by a twelfth rather than an octave preserves the voice leading in each cadence, and is more likely the reason why his solution is not what we anticipated in example 7b. The contrapuntal combination at the end of m. 8 is still an instance of invertible counterpoint, but now features IC12 instead of IC8.

Examples 6 and 7 do not present the most complex situations that one will encounter, but are helpful introductions to how composers apply techniques in their compositions. Recognizing not only where the technique of invertible counterpoint dominates a musical passage (ex. 5e), but also where a composer may decide to make slight modifications, either for melodic (ex. 6) or harmonic reasons (ex. 7c), helps students make informed decisions about the works they are studying and in their own writing. Anderson and Krathwohl suggest that "*critiquing* involves judging a product or operation based on externally imposed criteria and standards.

In *critiquing*, a student notes the positive and negative features of a product and makes a judgment based at least partly on those features. *Critiquing* lies at the core of what has been called critical thinking."[34] The *Evaluate* stage encourages students to identify contrapuntal errors, but more importantly exposes them to the notion that these techniques allow for compositional choice as well. They will later apply this thinking to their own work.

Create

I reposition *Create* as the final stage of the taxonomy, since the culminating task for written theory is often a model composition or analysis paper. Both activities demand that students combine several concepts and techniques to form an original product. Anderson and Krathwohl break this cognitive process down into three tasks: *Generate*, a divergent phase where students are encouraged to brainstorm and develop multiple ideas; *Plan*, a convergent phase where students select and organize the best options; and *Produce*, where students combine their ideas and construct a complete and original work.[35] The culminating experience for my counterpoint course is a model composition of a two-part invention for keyboard, which requires the successful use of IC8 as well as several other contrapuntal techniques discussed in class. Example 8 presents how I teach my students to *Generate* ideas, *Plan* ways to incorporate the required components, and then *Produce* a complete composition. This process reactivates the *Evaluate* stage, but students are now asked to think critically about their own work.

The most challenging aspect of composing a two-part invention (or any imitative work) is writing a suitable motive or theme. Bach's themes feature slow, simple harmonic rhythm and are typically no longer than two measures. If a student composes a theme that is too complex, it will make the remainder of the assignment unnecessarily challenging. I offer two solutions to this problem. First, I work through the process of composing a theme with my students during class, which will be discussed below. Second, for the purposes of the project, I provide a list of precomposed themes that students are allowed to borrow if they have difficulty getting started.[36]

Example 8 is the product of an in-class discussion I had with my counterpoint students. Using Bach's C-major invention as a model, we

[34] Anderson and Krathwohl, *Taxonomy*, 84. Italics in original.

[35] Anderson and Krathwohl, *Taxonomy*, 85–86.

[36] For a useful list of sample themes, see Schubert and Neidhöfer, *Baroque Counterpoint*, 89.

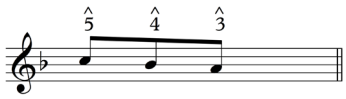
set out to compose the first section of a two-part invention, incorporating imitation at the octave and IC8. When demonstrating how to compose material like this, I find it beneficial to rely on student suggestions rather than to supply them with a preplanned solution. Since students are not given the opportunity to prepare in advance, it is likely that things will not work perfectly on the first try. Imperfect suggestions are perfect for class demonstrations since they provide the impetus for discussing problem-solving together as a class. According to Forkel, Bach was known to have encouraged experimentation as well:

> With all his strictness in this point, he allowed his pupils, in other respects, great liberties. … he let them dare whatever they would and could, only taking care to admit nothing which would be detrimental to the musical euphony and the perfectly accurate and unequivocal expression of the intrinsic sense, for the sake of which all purity of harmony is sought. As he himself attempted everything possible in this respect, he liked to see his scholars do the same. Other teachers of composition before him, for instance, {Angelo} Berardi, {Antonio Maria} Bononcini, and Fux, did not allow so many liberties. They were afraid that their pupils might thereby get entangled in dangers, but thus evidently prevented them from learning to overcome dangers.[37]

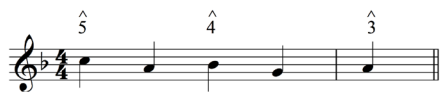
Experiential learning like this allows students to internalize the process for working and problem-solving, which aligns with the *Generate* and *Plan* components of the final stage of the taxonomy. When students learn how to problem-solve, they will be ready to *Produce* a final product on their own.

Example 8. Composing the first section of a two-part invention

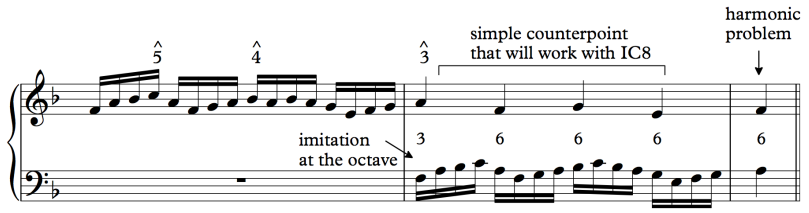
a. linear framework



b. simple embellishment with quarter notes



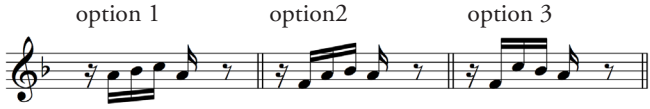
c. initial attempts at embellishment



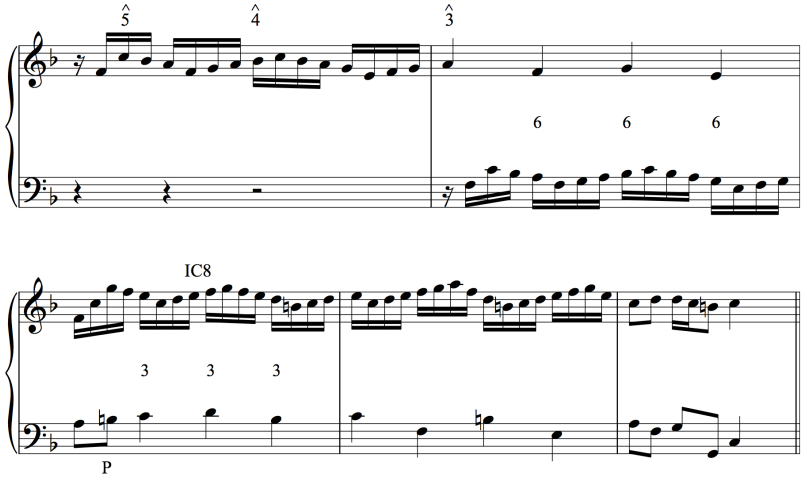
[37] *New Bach Reader*, 455.

Example 8 (con.)

d. revising the theme so it begins with a rest: three options



e. complete outline



The first step in creating a new motive or theme is to think of a melodic framework.[38] Since many baroque melodies emerge from stepwise frameworks, and since many of Bach's themes (especially his fugue subjects) feature $\hat{5}$–$\hat{4}$–$\hat{3}$, I suggest this as a starting point for class discussion (ex. 8a).[39] Example 8b adds quarter notes, and the pattern is organized so that $\hat{3}$ arrives on the downbeat of m. 2 (metric positioning will have been pointed out in previous discussions of Bach's inventions). Example 8c provides a first attempt at embellishing the line with sixteenth notes. Students recall the potential problems for themes beginning on $\hat{5}$ (ex. 6), so the theme is now modified to begin on $\hat{1}$. This is followed by

[38] Linear frameworks for baroque melodies are also discussed in Gauldin, *18th-Century Counterpoint*, 27ff. My approach is modeled after Gauldin, but my example provides a demonstration more accessible for students with little or no compositional background.

[39] For a detailed discussion of specific linear frameworks that occur in Bach's fugue subjects, see William Renwick, *Analyzing Fugue: A Schenkerian Approach* (Stuyvesant, NY: Pendragon Press, 1995).

imitation at the octave and simple counterpoint featuring thirds and sixths (ideal intervals to emphasize when composing with IC8). Unfortunately, the downbeat of m. 3 implies a tonic harmony and prevents immediate repetition of the contrapuntal combinations in the key of the dominant (like Bach does in his C-major invention). Of course, one could delay the next thematic statement, but I challenge the class to find a solution where an immediate restatement of the theme is possible. I point out that Bach's C-major invention begins with a sixteenth rest, and this slight alteration offers greater harmonic flexibility in an imitative texture. Simply removing the first sixteenth note, however, will not always be the best solution. Returning to the *Generate* phase, example 8d proposes three options for modifying the beginning of the theme. Option 1 simply removes the first note. There is nothing inherently wrong with this, but I recommend experimenting with many ideas. Additionally, some students feel that F major is no longer emphasized strongly enough. Option 2 removes the fourth sixteenth note, but the upper-neighbor figure causes the melody to lose its forward momentum. For these reasons, we decide that option 3 would work best.[40] There is a key-confirming leap from F to C, and the passing motion, C–B♭–A, creates a nicer line.

Example 8e shows how the revised theme allows for a quick transition from tonic to dominant in m. 3. The transition to C major feels quite abrupt, so a passing tone (B♮) is added in the left-hand part in m. 3 as well. Having properly outlined imitation and IC8, example 8e continues toward a cadential goal. Bach's first section of the C-major invention modulates to G major (V), so this is the tonal goal in example 8 as well. We include a descending-fifths sequence to make a smoother transition between the onset of C major (m. 3) and the key-confirming cadence (m. 5). Having devised a clear *Plan*, I then encourage small groups to experiment with the *Produce* phase by expanding the harmonies, adding more embellishments to the texture, and so on. The exercise outlined in examples 8a-e thus presents a concrete procedure that students can follow when drafting their own compositions.

Conclusion

Bach's renown as a music teacher is well documented, and much of what we know about his pedagogical methods remains relevant today. In this essay, I draw inspiration from Bach's method of instruction and,

[40] There are certainly more than three possible options. The class generated these options with the goal of minimizing the number of modifications to the original ideas.

building on Rifkin and Stoecker's work with aural training, I propose a revised taxonomy suitable for the written theory classroom. In contrast to recent studies that take a historically informed approach, my revised taxonomy can be applied to any music theory topic, providing greater flexibility for the modern-day theory instructor. The lessons provided in part 2 of the essay have been tested in multiple semesters, and I find that this systematic progression of materials is the most successful way to promote meaningful learning in my counterpoint course. My students react positively to these activities, and they feel confident applying their counterpoint skills independently, as evidenced by the creation of many successful model compositions over the years. My hope is that instructors will use the taxonomy in their own teaching, and perhaps try a few of my lessons as well.

Abstract

Many aspects of J. S. Bach's pedagogical tradition remain relevant to current thinking in undergraduate music theory instruction. This essay draws inspiration from Bach as a teacher, particularly his systematic presentation of ideas and his preference for teaching from compositions rather than abstract exercises. Part 1 explores the potential for applying Bloom's Taxonomy to music theory instruction. Building on recent applications of the taxonomy to music education, particularly work by Deborah Rifkin and Philip Stoecker, I propose a revised taxonomy for teaching undergraduate written theory. This approach provides a great deal of flexibility in the classroom since it can be applied to any theory topic. To demonstrate its effectiveness, Part 2 presents a series of detailed activities for teaching invertible counterpoint following the six categories of the taxonomy. Each activity includes excerpts either from Bach's *Well-Tempered Clavier* or two-part inventions.