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Ideal Color Space

I. Curvature of Color Space and Its Implications for Industrial Color Tolerances

Introduction

In the purchase of raw material, of finished articles, but particularly of components to go into a finished article, it is often necessary to specify ahead of time the color that the product or component must have in order to serve its purpose. The simplest way to do this is to supply with the order a sample, swatch, or chip of the desired color so that the vendor may avoid delivering goods that have to be returned as off-color.

It is, of course, impossible to have two articles, or swatches, of precisely the same color. Each raw material, each finished component, has a color range depending on the method of producing it. If the cheapest method of production yields too large a color range for the purchaser's purpose, he must indicate the size of the color range that he can tolerate. The purchaser may indicate this range approximately by giving the supplier two swatches instead of one. The first may be the color standard now thought of as being the central color of the acceptable range, in which case the other swatch represents one of the colors at the boundary of this acceptable range. The supplier will be told that any article that departs in color from the standard swatch by not more than the tolerance swatch will be accepted, and that all others will be rejected. Alternatively, both swatches may represent boundary colors; one on one side of the center of the acceptable range, the other on the opposite side. For this choice of swatches, the supplier will be told that the acceptable range is 'between' the colors of the two swatches.

These two-swatch methods of specifying a color range help the inspector considerably more than the one-swatch method specifying merely the central color of the range. But it is obvious that the guidance given to the inspector by two swatches is far from complete.

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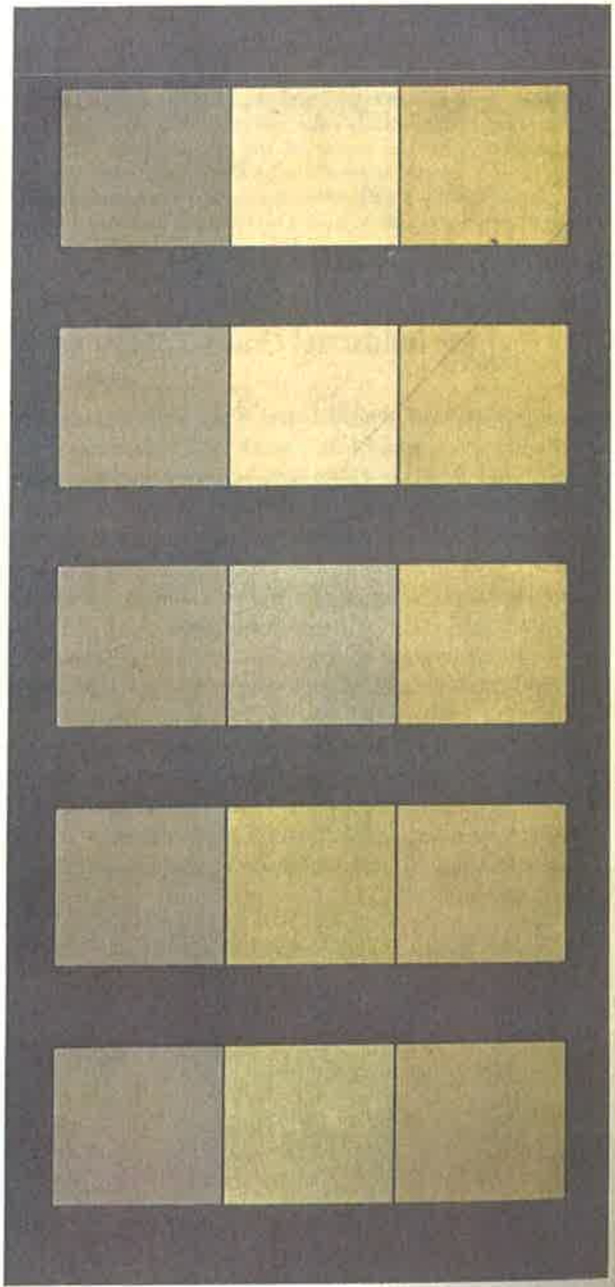
Exactly what is meant by one color being 'between' two other colors? Figure 1 shows five triads of colors, each triad making a horizontal row. The extremes of each triad are intended to be identical; the left extremes are gray; the right ones grayish yellow. These two colors might be limit colors for a rather large acceptable range. The central colors are all different. Which, if any, would you say are between the limit colors? One inspector might consider the central color in the first row to be between the two limit colors; another might say, no, it is lighter than either of them; so it should be rejected.

Similarly, the method of one swatch to define the central color, and another to give a color on the boundary of the acceptable range, also provides the inspector with very incomplete guidance. If the central sample in the bottom row of Figure 1 be taken as the standard for the central color, and if the grayish yellow sample just to the bottom of it is taken to define a color on the boundary of the acceptable range, this boundary shows how far the color of a finished component may depart from the standard in the direction of yellow; but it does not show how much darker, or grayer, or more saturated, or redder, or greener it may be. Complete guidance by color swatch requires an infinite number of swatches, one for each of the infinitely many directions of color difference from the standard color.

Now the terms (range, between, boundary, direction) that we have used to discuss the color-tolerance problem express geometrical ideas; and, indeed, there is no other way to discuss color tolerances with any degree of success. Color is a tridimensional quantity. This we can see from the fact that an inspector with normal color vision can make color distinctions of only three kinds: light–dark, yellow–blue, and red–green. All other color distinctions are combinations of these three. A geometrical model for color can be constructed by finding a method of locating points in space, one point for each color. Such a model can assist the understanding of the color-tolerance problem. The relation of one color to another may be described in terms of the direction and length of the straight line in the model connecting the points corresponding to the two colors.

Ideal color space

Ideal color space is a tridimensional array of points, each representing a color, so located that the length of the straight line between any two points is proportional to the perceived size of the difference between the colors represented by the points. An acceptable color range defining the size of the color tolerance, becomes in ideal color space simply the group of points lying



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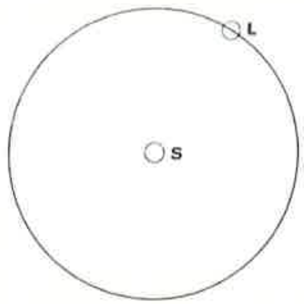


Figure 2
Cross-section of the sphere in ideal color space corresponding to the acceptable range of colors defined by the two-swatch method; one swatch *S* for the center of the range; the other *L* defining the size of the color tolerance. The point for the color standard is at the center of the circular area; that for the tolerance swatch, somewhere on the edge of this area. Colors represented by points within the circular area of any such cross-section are acceptable; colors represented by points outside the circular area are unacceptable.

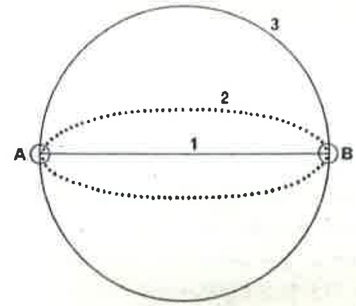


Figure 3
Definitions, in terms of ideal color space, of three possible interpretations of the requirement that to be acceptable a color shall be 'between' two limit colors. Points *A* and *B* represent the two limit colors. The most liberal interpretation *3* of the requirement would accept any color whose point fell within the circular area. The most strict would require the point *1* to fall on the straight line connecting points *A* and *B*. The elliptical area *2* indicates an interpretation of intermediate strictness.

within a sphere whose center is the point corresponding to the color of the first swatch, and whose radius is the distance between the points representing the colors of the two swatches. Figure 2 shows a cross-section of this spherical boundary passed through the point representing the central color.

What might be meant by one color being between two other colors can also easily be made clear in terms of ideal color space. It might mean that the point for the one color must lie somewhere on the straight line connecting the points corresponding to the two other colors, or that the point lies somewhere in the sphere one of whose diameters is this straight line, or something intermediate to these two interpretations. Figure 3 indicates these possibilities by cross-section of the three possible boundaries: the sphere, the straight line, and an intermediate.

Existing approximations of ideal color space

In 1964 the International Commission on Illumination (CIE) recommended the $U^*V^*W^*$ color space formed by plotting on mutually perpendicular axes the variables U^*, V^*, W^*, related to the tristimulus values X, Y, Z, of the color as follows:

$$\begin{aligned} W^* &= 25\,Y^{1/3} - 17, \; 1 < Y < 100 \\ U^* &= 13\,W^*(u - u_0) \\ V^* &= 13\,W^*(v - v_0) \end{aligned} \qquad (1)$$

where u and v are chromaticity coordinates, recommended in 1960 by the CIE, and defined as:

$$u = \frac{4X}{X+15Y+3Z} \qquad v = \frac{6Y}{X+15Y+3Z} \qquad (2)$$

and u_0, v_0 are the chromaticity coordinates of an achromatic (white or gray) color. The approximate perceived size ΔE of the difference between colors X_1, Y_1, Z_1, and X_2, Y_2, Z_2, may be computed as:

$$\Delta E = [(U_1^* - U_2^*)^2 + (V_1^* - V_2^*)^2 + (W_1^* - W_2^*)^2]^{1/2} \qquad (3)$$

which will be recognised as the formula for the distance between the points corresponding to the two colors in $U^*V^*W^*$ color space.

The scales of U^*, V^*, W^* have been adjusted so that the estimates ΔE of perceived size of color difference are given in units about equal to 4 just noticeable differences for optimum viewing conditions. A color difference of the size of one such unit has been found by the National Bureau of Standards to be the largest difference tolerable for many commercial products, and units of about this size have come to be known as NBS units of color difference. Wyszecki has checked the uniformity of color spacing of the CIE-$U^*V^*W^*$ space, and has found that the estimates ΔE computed from Equation (3) correlate well with estimates of the sizes made by actual observers having normal color vision. The degree of correlation that he found was that only 10% of the estimates made by actual observers differed from those computed from $U^*V^*W^*$ space by more than 25%. This degree of correlation is quite sufficient to make $U^*V^*W^*$ space of great practical value in specifying color tolerances and testing delivered goods for compliance to those tolerances. The chief obstacle to universal acceptance of CIE-$U^*V^*W^*$ space for color-tolerance specification is the fact that many industrial firms use other color spaces (Adams chromatic-value space in the cube-root form developed by Glasser et al., or Munsell color space) that they regard as even closer approximations to ideal space, or use another method (MacAdam ellipses) that they have found to correlate with estimates of the sizes of color differences by actual inspectors more closely than any color space yet developed.

MacAdam ellipses

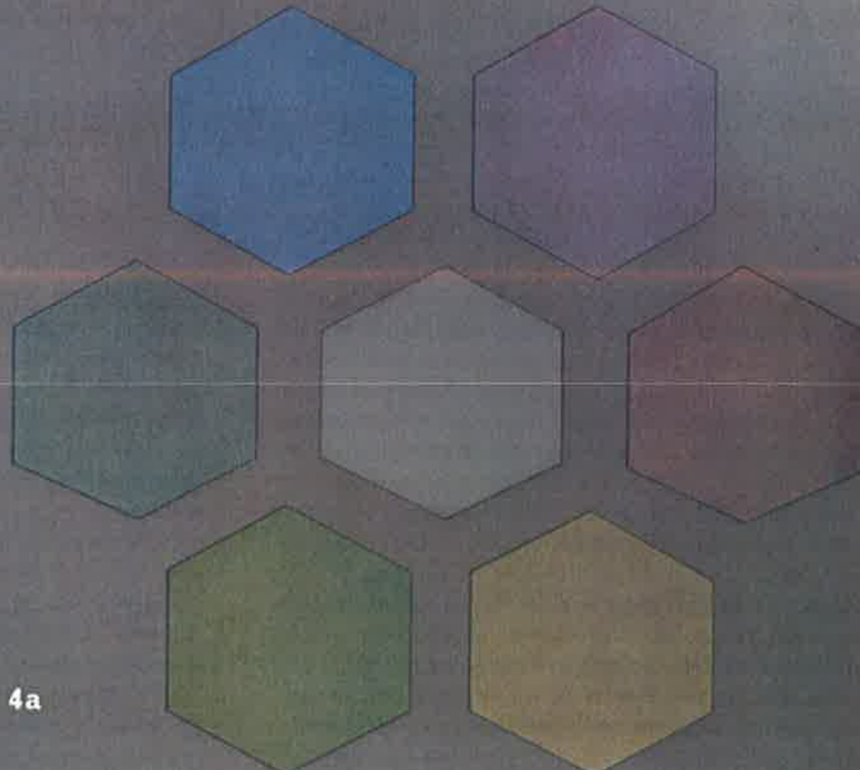
In 1942 MacAdam reported the results of an experimental study of the sensitivity of one observer (P. G. Nutting Jr.) to chromaticity variations among equiluminous colors viewed against a gray surround of one half the luminance of the two colors being compared. The only theoretical assumption that he made was that chromaticities perceived as equally different from a standard chromaticity should correspond to points on an ellipse in the (x, y)-plane centered on the point representing the standard chromaticity. Such ellipses were derived to represent his data for 25 standard chromaticities. Each one of these ellipses could be transformed into the circle demanded by ideal color space by a change in coordinate system. It is necessary only to take one axis of the new coordinate system parallel to the major axis of the ellipse and to take the other axis parallel to the minor axis, and then adjust the scales along these axes to transform the ellipse into a circle. In this way 25 local maps of the two-dimensional diagram representing equiluminous colors were derived such that each local map conformed to a plane in ideal color space. The trouble was that these local maps did not fit together to form a large, all-inclusive plane in ideal color space. Much as local maps of the earth's surface can be shown with sufficient accuracy on plane surfaces, but the map of the whole earth requires a surface of constant positive curvature, so MacAdam found that to map the entire gamut of chromaticities with uniform spacing required a surface far from flat. It had a dome (positive curvature) in the middle, representing near-grays, and ruffles (negative curvature) around the edges.

This finding that equiluminous colors viewed against a gray surround could not be represented with uniform perceptual spacing by points on a plane surface considerably disturbed those who had been thinking about color and color tolerances in terms of ideal color space. If you cannot make a uniform map of equiluminous colors on a plane surface, then how can you make a uniform map of all colors in accord with ideal color space? The answer is: you cannot.

Adherents of ideal color space tended to ascribe to experimental error MacAdam's finding that a curved surface is required for the uniform mapping of chromaticities, and to point out that the data came from one observer only. Maybe data by another observer would indicate a different surface. It was noted by Farnsworth that, by changing Nutting's data only slightly beyond the estimated experimental uncertainties, the transformed ellipses could be fitted onto a plane. MacAdam did not see why his experimental solution for chromaticity tolerances should be forced to fit any preconceived theoretical idea, however attractive this idea might be to others. To get a purely experimental estimate ΔE of the perceived size of the chromaticity difference between two colors of the same luminance, he wrote the general formula for an ellipse in the (x, y)-chromaticity plane:

$$\Delta E = [g_{11}(\Delta x)^2 + 2g_{12}(\Delta x)(\Delta y) + g_{22}(\Delta y)^2]^{1/2} \quad (4)$$

where the metric coefficients, g_{11}, g_{12}, g_{22}, were found by interpolation on the (x, y)-chromaticity diagram among the 25 sets of values derived from Nutting's data. Two color technologists, Davidson and Ingle, particularly concerned with setting industrial color tolerances, independently checked results computed from Equation (4) against their data on acceptable color ranges



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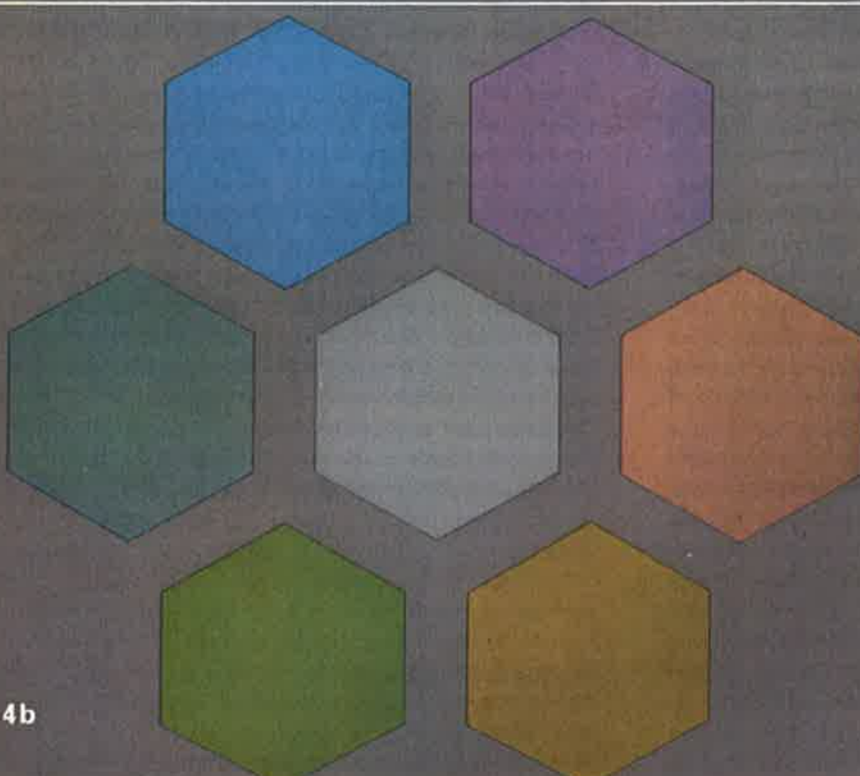

4b

arrived at by consensus of inspectors, and found better correlation than could be obtained by any of the then existing approximations to ideal color space. Since that time a considerable segment of industry in America has used the MacAdam ellipses to set and administer color tolerances. Friele and MacAdam have made remarkable progress toward finding analytical expressions for the metric coefficients as functions of the chromaticity coordinates, x, y, of the average of the two colors being compared against a gray surround. Geometers have a name for the kind of space accommodating Equation (4). Just as the surface required by Equation (4) for uniform mapping of chromaticities is curved, they say that the space required for uniform mapping of colors in accord with Equation (4) is curved, and they call it Riemannian. Ideal color space is the special case having zero curvature, and its geometry is Euclidian.

Practical meaning of Riemannian color space

Figures 4a and 4b show two clusters, each of 7 colors intended to be equiluminous, that is, of the same luminous reflectance, presented on a gray surround of about half that reflectance. This surround color is chosen because the MacAdam ellipses were determined for this condition. The central colors of the two clusters are intended to be identical grays. The six peripheral colors are intended to differ from the central gray by amounts that are perceptually equal, and the hues of the six peripheral colors are intended to be equally different; that is, each peripheral color is intended to differ from each of its two neighbors by one-sixth of the hue circuit. Figure 4a illustrates small departures from gray; Figure 4b shows departures larger by about a factor of 4. If the reproduction of the colors in Figures 4a and 4b accords with the above-stated intentions, and if you perceive saturation differences (differences from the central gray) relative to hue differences (differences among the peripheral colors of the cluster) in accord with ideal color space, you should see the radial color differences in both clusters as precisely the same size as the peripheral differences. If your appraisal accords with the MacAdam ellipses, you should see the radial differences as larger than the peripheral in Figure 4a, but not in 4b. Many people will say that it makes no sense to ask them to compare the perceptual size of a hue difference with that of a saturation difference. They say that the two differences are quite different in nature, which is true, and that the task is impossible. Others will say that the comparison is possible, but can be made only with very low precision, and this is also true. Nevertheless color inspectors faced with the definition of an acceptable range of colors by the two-swatch method, one swatch for the central color, the other for the tolerance, have to make these judgments repeatedly as a part of their everyday task, and the judgments have to be in accord with those made by the ultimate consumer a large fraction of the time or the employer will find somebody else to do the inspecting.

Is Euclidian color space possible?

There are two senses in which it might be said that Euclidian color space is achievable. One view is that the departures from Euclidian color space are not really significant. If you hold this view, you will see the hue spacing not certainly different from the saturation spacing in both clusters of Figure 4. You would grant that the presently available approximations to Euclidian color space (such as CIE-$U^*V^*W^*$, Munsell, or Adams chromatic value) do not have any significant geometrical trouble, but can be adjusted so that any significant nonuniformities in color spacing would disappear.

Another view points to the use by MacAdam of a fixed gray surround, and says that the use of a surround of any fixed color favors the perception of differences between colors near to the surround color. This would account for the region of positive curvature found by MacAdam for near-grays. This view is supported by Schönfelder's 1933 study of the influence of the surround color on precision of color matching, leading to what may be called Schönfelder's law that the most favorable surround color is the average of those being compared. This view is also supported by Takasaki's recent announcement of a phenomenon that he called lightness and chromaticness crispening. He found that the influence of a surround color on the perceived target color was about twice as large for colors nearly matching the surround color as for colors differing greatly from it. By this view, Euclidian color space could be used for the appraisal of color differences between colors either compared with a surround color selected in accord with Schönfelder's law, or between colors subtending so large a fraction of the inspector's visual field that it makes no difference what colors fill the rest of it.

To decide between these views will require the accumulation of a considerable additional body of experimental data. Whether the resulting improved solution to the problem of setting and administering industrial color tolerances will lead to industrial economies commensurate with the cost of the researches is an open question. Such is the scientific interest in the geometry of color space, however, that there is little doubt that many more data will be obtained. These data will determine the nature of the geometry that is to guide the future mechanized industrial practices for setting and administering industrial color tolerances.

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A second article on this subject will appear in the next issue of ‹palette›.