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IDEAL COLOR SPACE

III.

Ideal Color Space Redefined*

Introduction

Conventionally ideal color space has been thought of as a tridimensional array of points, each representing a color, so located that the length of the straight line between any two points is proportional to the perceived size of the difference between the colors represented by the points. By plotting the three variables of the Munsell color system (‹palette› No. 30, p. 22), Munsell hue, Munsell value, and Munsell chroma in cylindrical coordinates, Munsell value vertically, Munsell chroma radially from the black-white axis, and Munsell hue by angle about this axis, it has been supposed that a good approximation to ideal color space is produced.

The Nickerson (1936) evaluation of the relative perceptual importance of steps of hue, value, and chroma, expressed in the Nickerson index of fading, however, indicated for pairs of colors viewed with gray surround that hue differences of less than 10 steps have a practical importance relative to chroma steps of about twice that corresponding to the Euclidian geometry of ideal color space. This experimental indication of the super-importance of hue differences relative to chroma differences was generally disbelieved by many, including myself; and Euclidian formulas for differences between colors expressed in Munsell terms, such as the Balinkin (1939) approximation for differences not involving more than 10 steps, and the Godlove (1951) rigorous formula, came to be used in preference to the Nickerson index of fading.

MacAdam (1942) likewise found experimental evidence for one observer (Nutting) that a valid formula for the perceived size of color differences between pairs of colors viewed with a gray surround could not conform to Euclidian geometry. He wrote the general formula for an ellipse in the (x, y)-chromaticity plane of the 1931 CIE coordinate system for colorimetry:

$$\Delta E = [g_{11}(\Delta x)^2 + 2\,g_{12}(\Delta x)(\Delta y) + g_{22}(\Delta y)^2]^{1/2} \qquad (1)$$

where the metric coefficients, g_{11}, g_{12}, g_{22}, were found by interpolation on the (x, y)-chromaticity diagram among the 25 sets of values derived from Nutting's data. These data corroborated Nickerson's finding of

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the super-importance of hue differences, but only for relatively high chromas. For near-grays, hue differences were found to have the importance relative to chroma steps implied by Euclidian geometry, or even less than this. Because of the care taken by MacAdam to evaluate Nutting's sensibility to chromaticity differences, these data had to be believed valid for Nutting for the experimental conditions used (equi-luminous colors in semicircular shape in juxta-position so as to produce a circle of 2° angular subtense viewed with gray surround of luminance equal to one-half that of the colors being viewed). Whether they apply to the average inspector judging colors of large angular subtense is a separate question. Davidson and Friede (1953) and Ingle and Rudick (1953) found, however, that equation (1) yielded better correlation with appraisals of color difference carried out for the practical conditions of industrial inspection than could be obtained with other then currently proposed color-difference formulas, including the Godlove formula, and (1) has come to be widely used for appraisals carried out under conditions far from those used by MacAdam to evaluate Nutting's sensibility to chromaticity change.

The Nickerson index of fading, which was disbelieved, and the MacAdam-Nutting data, which were believed, alike indicate that ideal color space according to the conventional concept does not exist. Studies of other aspects of color-difference appraisal lead to the same conclusion.

Diminishing Returns in Color-Difference Perception

Another kind of experimental evidence that an ideal color space as originally conceived cannot be constructed might be called the law of diminishing returns in color-difference perception. It has recently been independently pointed out by MacAdam (1963) and Helm (1964) that if three colors, A, B, C, are equally spaced along a geodesic (path of minimum numbers of just noticeable differences), the perceived size of the difference between colors A and C is importantly less than twice the perceived size of that either between colors A and B or between colors B and C. The double difference is perceived as somewhere between 1.4 and 1.8 times the size of either of the two single differences.

Now it is easy to arrange three points in space, one for each of the three colors, A, B, C, so that the distance AC is 1.6 times distance AB or BC. This arrangement corresponds to the apices of an isosceles triangle whose base is 1.6 times as long as either of the two equal sides. We cannot pretend, however, that the space in which this arrangement of points is located corresponds to ideal color space. In ideal color space color geodesics are straight lines; this arrangement of points implies that the color geodesic passing through A, B, and C is a broken line. If we believe the law of diminishing returns, and if we wish to save something of the concept of ideal color space, we might drop the requirement that in ideal color space the distance between two points shall be proportional to the perceived size of the difference between the colors represented by the points, and be satisfied with the less stringent requirement that color differences perceived as equal shall correspond to equal distances in redefined ideal color space. Even the strictly local maps of chromaticity formed by rectifying the MacAdam ellipses correspond to ideal color space only in this less stringent sense, if the law of diminishing returns in color-difference perception be accepted.

MacAdam (1963) has revised his equation (1) to read:

$$\Delta E = [g_{11}(\Delta x)^2 + 2\,g_{12}(\Delta x)(\Delta y) + g_{22}(\Delta y)^2]^{p/2} \quad (2)$$

where p is a number somewhere between 0.3 and 0.8. Any value of p less than unity corresponds to some degree of diminishing returns in color-difference perception.

Influence of Surround Color

The final kind of experimental evidence that ideal color space, as originally conceived, does not exist has to do with the influence on perception of size of color difference exerted by the color surrounding the two whose difference is being appraised. The only way an inspector can avoid having a surround when he compares the sample of delivered goods with the swatch defining the desired color is to have either or both the sample and standard swatch so large that together they fill his entire visual field. Although I have seen standard textile swatches as large as one meter square so as to permit convenient fulfillment of this condition, it is by far more usual that practical considerations force the inspector to view the sample and standard colors against some other color serving as background.

It is well-known that the perception of a target color, though primarily determined by the tristimulus values of the target color, itself, also depends importantly on the tristimulus values of the surround color. Perception of a target color is thus a function of at least six variables. Figure 1 shows a series of five near-gray target colors intended to be spaced equally along the color locus passing through gray and connecting points representing colors of yellowish green and violet hues. This series of five target colors is shown three times; first on a saturated yellowish green surround, then on a gray surround, and finally on a saturated violet surround. You will note that the perceptions of the near-gray colors have been shifted toward violet by the yellowish green surround, and toward yellowish green by the violet surround. Thus it is impossible to predict simply from the tristimulus values of the light reflected from the targets, themselves, what the perceptions will be. A specification of the kind of light coming to the eye of the observer from the surround is also needed. Any color space, like the Munsell space, in which location of the color point is intended to correlate with the perception of the target color, cannot possibly hold for all surround colors. Strictly speaking, it can hold for but one surround color; and practically speaking it can hold approximately only for a limited range of surround colors. Munsell color space correlates with color perception with good approximation for any of the series of surround colors ranging from middle gray to white. For any of these surround colors, Munsell hue corresponds fairly well with the hue of the perception; Munsell value, with its lightness; and Munsell chroma, with its saturation.

How then can we hope to have a color space, like the CIE-$U^*V^*W^*$, the Adams chromatic-value, or the Munsell color space, based only on the tristimulus values of target colors, conform to the characteristic of ideal color space that equally different colors are represented by equally distant points? Do these spaces also refer only to a limited range of surround colors? There is a basis for hoping that the perceived size of the difference between two colors is relatively unaffected by the surround color. It has been noted from Figure 1 that the influence of substituting a chromatic surround for a gray surround is to shift the perception of near-gray target colors toward the complementary color. If the perceptions of both of the nearly identical colors being compared are shifted by nearly identical amounts, the perception of the size of the difference between the two colors might be left nearly unaltered by a change in surround color. If you pay attention to the size of the difference perceived by you between any two contiguous target colors shown on Figure 1, you can see to what extent color-difference perception by you remains unaffected by a change in surround color.

It is, however, recognized by inspectors that there are indeed some very unfavorable choices of surround color. If the standard is a near-white and the tolerance swatch is a slightly different near-white, use of a black surround can make the perceptions of these two colors identical; that is, they both may be perceived to be identical near-white colors. One way to think of this phenomenon is that the black surround induces so much white into both the standard and the tolerance color that the induced white drowns out the small difference between them. Similarly, two slightly different near-blacks can be made to appear identical by using a white surround. It is common practice to select a surround color not much lighter, nor much darker, than the two colors to be compared. The determination of the Munsell notation of any unknown color by comparison with the color chips of the Munsell book can be carried out by means of a surround of middle gray (N 5/); but this determination can be made more precise by using a light-gray mask (say N 9/) for near-whites, and a dark-gray mask (say N 1/) for near-blacks.

In 1933 Schönfelder carried out an experimental study of the influence of the surround color on precision of color matching, and he announced what may be called Schönfelder's law that the most favorable surround color is the average of those being compared. This law was corroborated by Brown (1952), and implies that a large chromatic difference between surround color and the average of the two colors being compared, as well as a large lightness difference, can interfere with color-difference perception. By this law the perceived sizes of the differences between the near-grays shown in Figure 1 for the yellowish green surround should be smaller than those for the gray surround which should be the optimum choice.

If we believe Schönfelder's law, and there seems to be no contrary evidence, we must believe that a color space giving a good approximation to ideal color space for colors of small angular subtense (say 2°), viewed with a white surround, must give a considerably poorer (by at least a factor of two) approximation for colors viewed with a black surround. In spite of the fact that color-

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Five near-grays displayed on three different backgrounds: saturated yellowish green, gray, and saturated violet. Note that the color appearing gray depends on the color of the surround. Note also that the inductive effect of the surround color is about the same for all of the five near-grays. Schönfelder's law implies that the perceived sizes of the differences between successive near grays should be larger for the gray surround than for either of the chromatic surrounds.

difference perception is less dependent on surround than is color perception itself, a marked change in surround color requires a redefinition of the approximation to ideal color space.

Jameson and Hurvich (1961) have established experimentally that to a first approximation the change of color perception of any color induced by a shift of surround color is proportional to the size of that shift. Thus, to a first approximation all of the near-gray target colors shown in Figure 1 should be shifted uniformly by substituting either the yellowish green or the violet surround for the gray, leaving the differences between successive target colors unchanged. This first approximation to the facts does not conform to Schönfelder's law. Recently Takasaki confirmed the validity of the Jameson-Hurvich finding as an approximation, but also found that the change induced in the perceptions of colors of small angular subtense very close either to that of the first surround, or to that of the second, is significantly higher (by at least a factor of two) than the change induced in the perceptions of colors considerably different from either surround color. Takasaki called this phenomenon the crispening effect. The crispening effect is in agreement with Schönfelder's law that the surround color most facilitating the discrimination of two colors is the average of those two colors. Figure 2 shows lightness determined by Takasaki for six observers as a function of Munsell value for a middle gray (N 5/) surround. The crispening effect corresponds to the increase of slope at Munsell value equal to 5/. Kaneko's recent determination (1964) of the lightness scale, also shown on Figure 2, reveals a similar crispening effect.

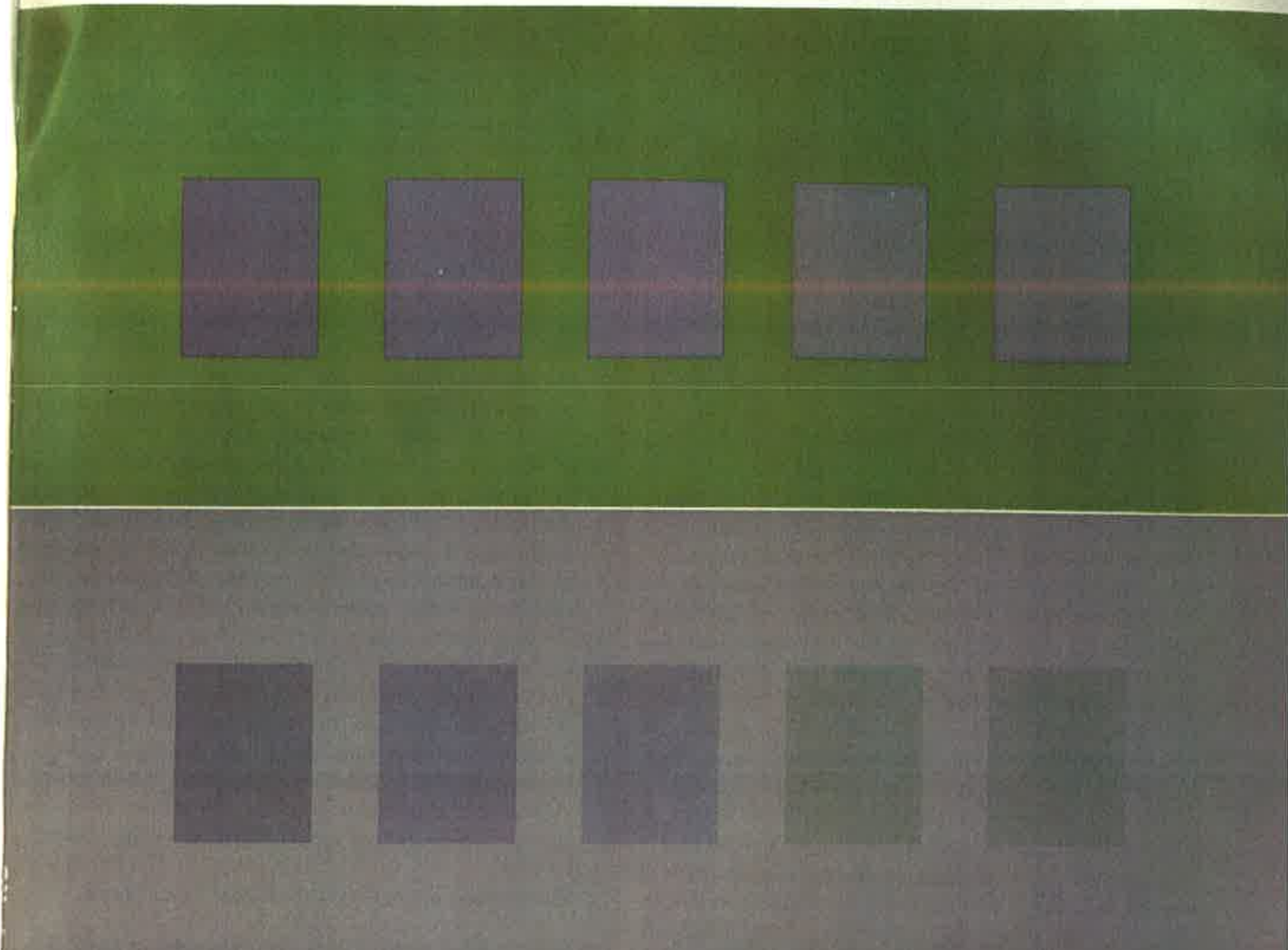
If construction of a color space is planned with the intention of approximating ideal color space for color differences judged against a surround of fixed color, it is obvious that to conform to Takasaki crispening, or to Schönfelder's law, the color of the surround must be a singular point around which the color spacing must be made larger. Furthermore, if construction of another color space is envisaged to conform to color-difference perception for colors viewed with a fixed surround of some different color, a similar singular point must be allowed for, but it will appear at a different location in the space. In a surface corresponding to equi-luminous colors, this singular point would appear at the apex of a dome, thus requiring that surface to have positive curvature in the neighborhood of the singular point. Perhaps the central dome in the MacAdam surface may be ascribed to Takasaki chromaticness crispening. MacAdam used a gray surround whose luminance was one-half that of the target colors. Since Kaneko and Takasaki both found experimental evidence of lightness crispening, the scales in all directions, not just those at constant lightness, from the point representing the surround color would have to be opened up. This must be what is meant by curvature of color space being different from zero (positive) in the neighborhood of a point. Crispening thus also implies that it is impossible to develop ideal color space if the colors to be discriminated are viewed with a surround of fixed color.

Redefinition of the Concept of Ideal Color Space

We have seen that the simple concept of ideal color space, with which we started, cannot be made to conform to the experimental facts. Maybe there is still something useful left in the concept. Perhaps the experimental facts so far established will still permit a less stringent, more complicated, but worthwhile concept to be formulated.

The law of diminishing returns in color-difference perception requires that we give up the requirement that distance in ideal color space be proportional to perceived size of color difference, but we could still hope to develop a space in which equal distances would correspond to equal perceived sizes of color difference.

The influence of surround color on color-difference perception requires that we give up the thought that idea color space can be defined, regardless of the surroun color, simply as a function of the tristimulus values o the light reflected into the inspector's eye from the tw specimens whose color difference is being appraised We may, however, still hope to develop an ideal colo



space for some one specified set of viewing conditions. Schönfelder's law and the Takasaki crispening effect alike indicate that this some one set of conditions cannot include that of a fixed surround color, because such a choice introduces a singular point in the neighborhood of which the color space is positively curved. We could still hope to develop an ideal color space (zero Gaussian curvature) applying to color-difference perceptions made each time either with a surround chosen in accord with Schönfelder's law, or with such large color samples that together they fill enough of the inspector's visual field that it makes little difference what surround color fills the rest of it. Note that according to Brown's (1952) results, little improvement in discrimination is to be expected from fields larger than 12°.

I propose the following redefinition of ideal color space:

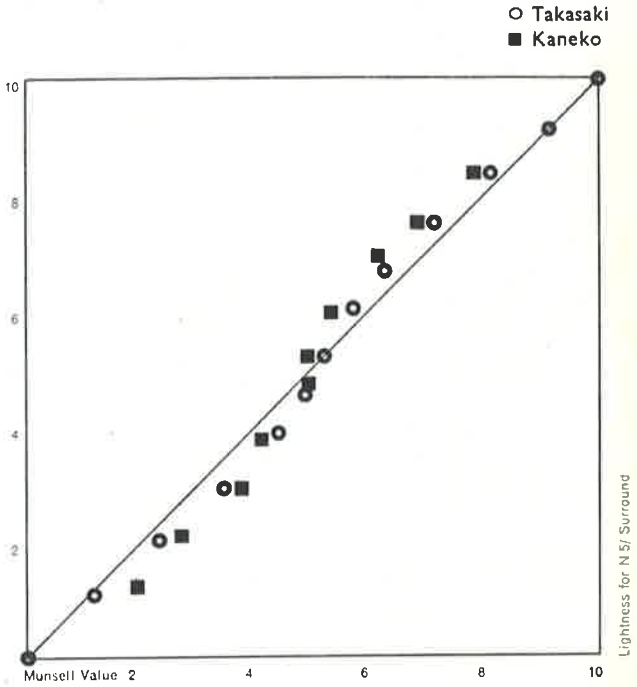
Ideal color space redefined is a tridimensional array of points, each representing a color, such that all pairs of points separated by any fixed distance correspond to pairs of color perceived to differ by the same amount provided that the appraisal of the perceived size be carried out with optimal surround colors chosen in accord with Schönfelder's law that the surround color be the average of the two colors being compared.

To my knowledge, the facts of vision so far established experimentally do not contradict this idea. It is possible that Munsell color space is a good approximation to ideal color space redefined in this way.

Color Spaces Based on Gray Surrounds

The great interest in color spaces based on observations of colors with gray surrounds is that in such spaces a movement of the color point away from the gray axis correlates with color perceptions of increasing saturation, and tangential movements in the space (movements orthogonal to centripetal movements) correspond to perceptions of different hues. The gray surround provides the indispensable reference point. In such spaces, so to speak, the directions indicated by the mile-posts are reliable, but the numbers of miles do not correspond to the distances between the color points, but have to take account both of the law of diminishing returns in color-difference perception and of the superimportance of hue differences. These spaces are therefore somewhat confusing as to distance.

On the other hand redefined ideal color space based on Schönfelder's law permits the numbers of miles to be given reliably in accord with the law of diminishing returns in color-difference perception, but the direction toward gray is uncertain. Helson and Michels (1948)



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Influence of the surround color fixed at middle gray (Munsell N 5/0) on the lightness scale. Kaneko found his lightness scale by choice of grays to produce a scale perceived as having equal lightness steps. Takasaki derived his lightness scale from determinations of the gray viewed with one gray surround that appeared equally light as another gray viewed on a different gray surround. Note that the lightness scale defined by the Munsell value function does not apply to a surround color fixed at middle gray; nor indeed to a surround of any fixed gray.

have shown that for nearly every chromaticity there exists a surround color such as to cause the target color to be perceived as without hue; that is, gray can be nearly anywhere. Each surround color selected in accord with Schönfelder's law yields a different location for gray, which thus may be, not anywhere at all, but anywhere within a large central range. Figure 1 gives a demonstration of the shift in location of gray with surround color. As Carl Foss, to whom I am indebted for the concept of ideal color space redefined, has often remarked, you can get lost in a color space based on surround colors sliding around to conform to the average of the colors being compared; there is no unique place for gray, which is the necessary anchor point for the perception of hue and saturation.

It seems likely that the perceived sizes of color differences appraised with any fixed gray surround could be predicted from ideal Schönfelder color space. It would be necessary only to write the formula for perceived size of color difference from Euclidian geometry with three correction terms. First raise the Euclidian distance to the power, p, as in equation (2); this corrects the perceived size of the color difference in accord with the law of diminishing returns. Second, multiply by a term, f_s, for lightness and chromaticness crispening evaluated by Takasaki. At its June 1967 meeting in Washington, CIE Committee E-1.3.1, Colorimetry, suggested for study an expression for f_s:

$$f_s = \frac{15 + \left[\bar{C}^2 + 16\,(\bar{V} - V_s)^2\right]^{1/2}}{5 + \left[\bar{C}^2 + 16\,(\bar{V} - V_s)^2\right]^{1/2}}, \qquad (3)$$

where $\bar{V}$ and $\bar{C}$ are the average value and chroma for the two samples being compared, and V_s is the Munsell value of the gray surround. Third, multiply the tangential component by a term, f_h, for the super-importance of hue differences relative to chroma (radial) differences. CIE Committee E-1.3.2 has suggested for study an expression for f_h:

$$f_h = \left[\frac{4}{3 - \cos(3.6\,\Delta H)}\right]^2, \qquad (4)$$

where ΔH is the hue difference in Munsell steps between the two colors being compared.

Existing Munsell color space refers to gray surrounds, but not to a fixed gray, or else the Munsell gray scale would show a local expansion near that fixed gray such as shown on Figure 2 near N 5/. Munsell color space thus must be thought of as referring to appraisal of color difference with a gray surround of Munsell value intermediate to those of the two colors being compared. The same remark applies to the CIE-$U^*V^*W^*$ space and to Adams chromatic-value space because both of these spaces are based on the Munsell value function.

For this surround condition (gray of sliding lightness) the formula to predict perceived size of color differences would also be based on Euclidian geometry with three correction terms, but the term, f_s, taking crispening into account would refer only to chromaticness crispening as in the expression obtainable from equation (3) by setting $\bar{V} = V_s$, thus:

$$f_{s,\,\bar{V} = V_s} = \frac{15 + \bar{C}}{5 + \bar{C}}. \qquad (5)$$

Summary

The concept of color space has been redefined to take account of the law of diminishing returns on color-difference perception and Schönfelder's law that the optimum surround color is the average of the two colors being compared. The relation between redefined ideal color space and the formulas by which the perceived size of differences between colors viewed against any fixed gray surround has been indicated.

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