BEFORE THE WASHINGTON

UTILITIES & TRANSPORTATION COMMISSION

WASHINGTON UTILITIES AND TRANSPORTATION COMMISSION,

Complainant,

v.

AVISTA CORPORATION d/b/a AVISTA UTILITIES,

Respondent.

DOCKETS UE-170485 & UG-170486 (*Consolidated*)

RESPONSE TESTIMONY OF DAVID J. GARRETT (DJG-1T)

ON BEHALF OF

PUBLIC COUNSEL

OCTOBER 27, 2017

RESPONSE TESTIMONY OF DAVID J. GARRETT (DJG-1T)

DOCKETS UE-170485 and UG-170486 (*Consolidated*)

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Exhibit DJG-2	Curriculum Vitae
Exhibit DJG-3	Roger A. Morin, *New Regulatory Finance* 449-450 (Public Utilities Reports, Inc. 2006) (1994).
Exhibit DJG-4	Cost of Capital and Capital Structure Analysis
Exhibit DJG-5	A. Lawrence Kolbe, James A. Read, Jr. & George R. Hall, *The Cost of Capital: Estimating the Rate of Return for Public Utilities* 21 (The MIT Press 1984)
Exhibit DJG-6	Aswath Damodaran, *Investment Valuation: Tools and Techniques for Determining the Value of Any Asset* 62-63 (3rd ed., John Wiley & Sons, Inc. 2012)
Exhibit DJG-7	Zvi Bodie, Alex Kane & Alan J. Marcus, *Essentials of Investments* 149 (9th ed., McGraw-Hill/Irwin 2013)
Exhibit DJG-8	John R. Graham, Scott B. Smart & William L. Megginson, *Corporate Finance: Linking Theory to What Companies Do* 179-80 (3rd ed., South Western Cengage Learning 2010)
Exhibit DJG-9	Beta by Sector (January 5, 2017) http://www.stern.nyu.edu/~adamodar/New_Home_Page/data.html
Exhibit DJG-10	Eugene F. Fama, *Efficient Capital Markets: A Review of Theory and Empirical Work*, Vol. 25, No. 2 The Journal of Finance 383 (1970)
Exhibit DJG-11	Congressional Budget Office Long-Term Budget Outlook, https://www.cbo.gov/publication/51580
Exhibit DJG-12	William F. Sharpe, *A Simplified Model for Portfolio Analysis* 277-93 (Management Science IX 1963)
Exhibit DJG-13	Elroy Dimson, Paul Marsh & Mike Staunton, *Triumph of the Optimists: 101 Years of Global Investment Returns* 4 (Princeton University Press 2002)
Exhibit DJG-14	2015 Ibbotson Stocks, Bonds, Bills, and Inflation Classic Yearbook 91 (Morningstar 2015)
Exhibit DJG-15	Aswath Damodaran, *Equity Risk Premiums: Determinants, Estimation and*

EXHIBITS LIST (CONTINUED)

	Implications – The 2015 Edition 17 (New York University 2015)
Exhibit DJG-16	John R. Graham and Campbell R. Harvey, *The Equity Risk Premium in 2016*, at 3 (Fuqua School of Business, Duke University 2014)
Exhibit DJG-17	Pablo Fernandez, Pablo Linares & Isabel F. Acin, *Market Risk Premium used in 171 Countries in 2016: A Survey with 6,932 Answers*, at 3 (IESE Business School 2015)
Exhibit DJG-18	Myron J. Gordon and Eli Shapiro, *Capital Equipment Analysis: The Required Rate of Profit* 102-10 (Management Science Vol. 3, No. 1 Oct. 1956)
Exhibit DJG-19	Rolf W. Banz, *The Relationship Between Return and Market Value of Common Stocks* 3-18 (Journal of Financial Economics 9 (1981))
Exhibit DJG-20	Vitali Kalesnik and Noah Beck, *Busting the Myth About Size* (Research Affiliates 2014), available at https://www.researchaffiliates.com/Our%20Ideas/Insights/Fundamentals/Pages/284_Busting_the_Myth_About_Size.aspx
Exhibit DJG-21	Adrien M. Mckenzie Workpaper Excerpt - 2017 Market DCF Analysis

I. INTRODUCTION

Q: State your name and occupation.

A: My name is David J. Garrett. I am a consultant specializing in public utility regulation. I am the managing member of Resolve Utility Consulting, PLLC. I focus my practice on the primary capital recovery mechanisms for public utility companies: cost of capital and depreciation.

Q: Summarize your educational background and professional experience.

A: I received a B.B.A., with a major in Finance, an M.B.A., and a Juris Doctor from the University of Oklahoma. I worked in private legal practice for several years before accepting a position as assistant general counsel at the Oklahoma Corporation Commission in 2011. At the Oklahoma Commission, I worked in the Office of General Counsel in regulatory proceedings. In 2012, I began working for the Public Utility Division as a regulatory analyst providing testimony in regulatory proceedings. I am a Certified Depreciation Professional with the Society of Depreciation Professionals. I am also a Certified Rate of Return Analyst with the Society of Utility and Regulatory Financial Analysts. I have testified in many regulatory proceedings on cost of capital, depreciation, and other issues. A more complete description of my qualifications and regulatory experience is included in my curriculum vitae.[1]

Q: On whose behalf are you testifying in this proceeding?

A: I am testifying on behalf of the Public Counsel Unit of the Washington Office of Attorney General (Public Counsel).

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[1] Exh. No. DJG-2.

Q: Describe the scope and organization of your testimony.

A: The purpose of my testimony is to present an independent analysis and opinion of the cost of equity capital and a prudent capital structure for Avista Corp. (Avista or the Company). Based on my estimates of the Company's weighted average cost of capital, I present a recommendation for the allowed rate of return for the Company. My testimony responds to the Direct Testimony of Adrien M. McKenzie.

II. OVERVIEW OF COST OF CAPITAL RECOMMENDATIONS

Q: Explain the weighted average cost of capital and how the Company's cost of equity and capital structure affect this equation.

A: The term "cost of capital" refers to the weighted average cost of all types of securities within a company's capital structure, including debt and equity. Determining the cost of debt is relatively straight-forward. Interest payments on bonds are contractual, "embedded costs" that are generally calculated by dividing total interest payments by the book value of outstanding debt. Determining the cost of equity, on the other hand, is more complex. Unlike the known, contractual cost of debt, there is no explicit "cost" of common equity. To determine the appropriate cost of equity capital, companies must estimate the return their equity investors will demand in exchange for giving up their opportunity to invest in other securities or postponing their own consumption, in light of the level of risk associated with the investment. Thus, the overall weighted average cost of capital (WACC), includes the cost of debt and the estimated cost of equity. It is a "weighted average," because it is based upon the Company's relative levels of debt and equity. Companies in the competitive market often use their WACC as the discount rate to determine the value of various capital projects. The basic WACC equation used in regulatory proceedings is presented below:[2]

**Equation 1:
Weighted Average Cost of Capital**

$$WACC = \left(\frac{D}{D+E}\right)C_D + \left(\frac{E}{D+E}\right)C_E$$

where: $WACC$ = *weighted average cost of capital*
D = *book value of debt*
C_D = *embedded cost of debt capital*
E = *book value of equity*
C_E = *market-based cost of equity capital*

Thus, the term "cost of capital" is synonymous with the "weighted average cost of capital," which includes both debt and equity components. As discussed further below, the Commission's determination of a fair awarded rate of return should be based on a reasonable estimate of the Company's weighted average cost of capital.

In this Application, the Company has proposed a cost of equity of 9.9 percent, as discussed in the direct testimony of Mr. McKenzie. The Company has also proposed a cost of debt of 5.62 percent and a debt ratio of 50 percent. These three factors equate to a proposed weighted average cost of capital of 7.7 percent for the Company. In the sections below, I discuss several significant errors upon which the Company's requested weighted average cost of capital is based.

Q: Summarize your analyses and conclusions regarding Avista's Cost of Equity.

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[2] *See* Exh. DJG-3 (Roger A. Morin, *New Regulatory Finance* 449-450 (Public Utilities Reports, Inc. 2006)). The traditional practice uses current market returns and market values of the company's outstanding securities to compute the WACC, but in the ratemaking context, analysts usually employ a hybrid computation consisting of embedded costs of debt from the utilities books, and a market-based cost of equity. Additionally, the traditional WACC equation usually accounts for the tax shield provided by debt, but taxes are accounted for separately in the ratemaking revenue requirement.

A: In formulating my recommendation, I performed thorough independent analyses to calculate Avista's cost of equity. To do this, I selected a proxy group of companies that represents a relevant sample with asset and risk profiles similar to those of Avista. Based on this proxy group, I evaluated the results of two widely-accepted financial models for calculating cost of equity: (1) the Discounted Cash Flow (DCF) model; and (2) the Capital Asset Pricing Model (CAPM). I evaluated these models to ensure a balanced approach that meets the legal standards, objective market considerations, and regulatory goals for establishing an appropriate awarded return for Avista. Based on my quantitative and qualitative analyses, as discussed throughout my testimony below, I recommend an awarded return on equity of 9.0 percent, which represents the midpoint within a reasonable a range of 8.75 percent and 9.25 percent. While Avista's actual cost of capital is much lower, my recommendation represents a gradual, rather than abrupt move toward true cost of capital.

Q: Summarize your analyses and conclusions regarding Avista's capital structure.

A: The Company's capital structure includes 51.5 percent long-term debt and 48.5 percent common equity. While it is often reasonable for utilities to have higher amounts of debt in their capital structures, my analysis in this case indicates that Avista's proposed capital structure is reasonable.

Q: What is the impact of your cost of capital recommendations?

A: My cost of capital recommendations are illustrated in the following figure:

Figure 1:
Awarded Rate of Return Recommendation[3]

Source	Capital Structure	Cost Rates	Weighted Cost
Long-term Debt	51.50%	5.62%	2.89%
Common Equity	48.50%	9.00% 8.75% 9.00%	4.37% 4.24% 4.37%

	7.14%	**7.26%**	**7.26%**
Recommended Range for Awarded Rate of Return	**LOW**	**RECOMMENDED**	**HIGH**

As shown in this figure, an awarded return on equity of nine percent with an equity ratio of 48.5 percent results in overall awarded return of 7.26 percent.

Q: Provide an overview of the problems you have identified with the Company's cost of capital estimate.

A: In this case, Mr. McKenzie recommends a very high awarded rate of return of 9.9 percent. Mr. McKenzie recommendations are based on several models, including the CAPM and DCF Model, however, several of his key assumptions and inputs to these models violate fundamental, widely-accepted tenants in finance and valuation. In the sections below, I will discuss my concerns regarding the Company's requested cost of capital in further detail. However, the key areas of concern are summarized as follows:

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[3] Exh. No. DJG-4 at 1.

1. In his DCF Model, Mr. McKenzie's long-term growth rate for the proxy group is more than twice the long-term growth rate for the entire U.S. economy. It is a fundamental concept in finance that, in the long run, a company cannot grow at a faster rate than the aggregate economy in which it operates; this is especially true for a regulated utility with a defined service territory.

3. Mr. McKenzie's estimate for the Equity Risk Premium ("ERP"), the single most important factor in estimating the cost of equity, is nearly twice as high as the estimate reported by thousands of other experts evaluating companies in a myriad of industries across the country. Mr. McKenzie inappropriately bases his equity risk premium estimate in part on awarded returns in other jurisdictions – a non-market factor that bears no meaningful relationship to the market-based ERP.

4. Mr. McKenzie suggests that Company-specific risk factors have an increasing effect on its cost of equity. However, this overlooks the fundamental concept that the market does not reward diversifiable, firm-specific risk; therefore, investors do not expect a return for such risk. Mr. McKenzie also erroneously suggests that the Company's relative size should have an increasing effect on its cost of equity despite the overwhelming evidence confirming that the "size premium" phenomenon was short-lived and has not been seen for over a quarter-century.

In short, the assumptions employed by Mr. McKenzie skew the results of his financial models such that they do not reflect the economic realities of the market upon which cost of equity recommendation should be based. In the testimony below, I demonstrate how correcting the various erroneous assumptions in the DCF and CAPM financial models results in appropriate ROE recommendations which better align with today's market and Avista's risk profile.

III. LEGAL STANDARDS FOR ESTABLISHING COST OF CAPITAL

Q: Discuss the legal standards governing the allowed rate of return on capital investments for regulated utilities.

A: In *Wilcox v. Consolidated Gas Co. of New York*, the U.S. Supreme Court first addressed the meaning of a fair rate of return for public utilities.[4] The Court found that "the amount of risk in the business is a most important factor" in determining the appropriate allowed rate of return.[5] Later, in two landmark cases, the Court set forth the standards by which public utilities are allowed to earn a return on capital investments. In *Bluefield Water Works & Improvement Co. v. Public Service Commission of West Virginia*, the Court held:

> A public utility is entitled to such rates as will permit it to earn a return on the value of the property which it employs for the convenience of the public . . . but it has no constitutional right to profits such as are realized or anticipated in highly profitable enterprises or speculative ventures. The return should be reasonably sufficient to assure confidence in the financial soundness of the utility and should be adequate, under efficient and economical management, to maintain and support its credit and enable it to raise the money necessary for the proper discharge of its public duties.[6]

In *Federal Power Commission v. Hope Natural Gas Company*, the Court expanded on the guidelines set forth in *Bluefield* and stated:

> From the investor or company point of view it is important that there be enough revenue not only for operating expenses ***but also for the capital costs of the business***. These include service on the debt and dividends on the stock. By that standard the return to the equity owner should be commensurate with returns on investments in other enterprises having corresponding risks. That return, moreover, should be sufficient to assure confidence in the financial integrity of the enterprise, so as to maintain its credit and to attract capital.[7] [emphasis added]

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[4] *Wilcox v. Consolidated Gas Co. of New York*, 212 U.S. 19 (1909).

[5] *Id*. at 48.

[6] *Bluefield Water Works & Improvement Co. v. Public Service Comm'n of West Virginia*, 262 U.S. 679, 692-93 (1923).

[7] *Fed. Power Comm'n v. Hope Nat. Gas Co*., 320 U.S. 591, 603 (1944) (emphasis added).

The cost of capital models I have employed in this case are in accord with all of the foregoing legal standards.

Q: Is it important that the "allowed" rate of return be based on the Company's actual cost of capital?

A: Yes. The Supreme Court in *Hope* makes it clear that the allowed return should be based on the cost of capital. Under the rate base rate of return model, a utility should be allowed to recover all of its reasonable expenses, its capital investments through depreciation, and a return on its capital investments sufficient to satisfy the required return of its investors. The "required return" from the investors' perspective is synonymous with the "cost of capital" from the utility's perspective. Scholars agree that the allowed rate of return should be based on the cost of capital:

> Since by definition the cost of capital of a regulated firm represents precisely the expected return that investors could anticipate from other investments while bearing no more or less risk, and since investors will not provide capital unless the investment is expected to yield its opportunity cost of capital, the correspondence of the definition of the cost of capital with the court's definition of legally required earnings appears clear.[8]

The models I have employed in this case closely estimate the Company's true cost of equity. If the Commission sets the awarded return based on my lower, and more reasonable rate of return, it will comply with the Supreme Court's standards, allow the Company to maintain its financial integrity, and satisfy the claims of its investors. On the other hand, if the Commission sets the allowed rate of return much *higher* than the true

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[8] Exh. No. DJG-5 (A. Lawrence Kolbe, James A. Read, Jr. & George R. Hall, *The Cost of Capital: Estimating the Rate of Return for Public Utilities* 21 (The MIT Press 1984)).

cost of capital, it arguably results in an inappropriate transfer of wealth from ratepayers to shareholders. This point is underscored as follows:

> [I]f the allowed rate of return is greater than the cost of capital, capital investments are undertaken and investors' opportunity costs are more than achieved. Any excess earnings over and above those required to service debt capital accrue to the equity holders, and the stock price increases. In this case, the wealth transfer occurs from ratepayers to shareholders.[9]

Thus, it is important to understand that *awarded* returns and *actual* cost of capital are two separate concepts. Awarded returns are set through the regulatory process and may be influenced by a number of factors other than objective market drivers. Cost of capital, on the other hand, should be evaluated objectively and closely tie to the economic market realities. In other words, cost of capital it is driven by stock prices, dividends, growth rates, and most importantly – it is driven by risk. Cost of capital can be estimated through the use of financial models used by firms, investors, and academics around the world for decades. The problem is, with respect to regulated utilities, there has been a trend in which awarded returns fail to closely track with actual market-based cost of capital. To the extent this occurs, the results are detrimental to ratepayers and the state's economy.

Q: If the Commission sets the allowed return at a level far greater than the market-based cost of capital, will this permit an excess transfer of wealth from Washington ratepayers to Company shareholders and the federal government?

A: Yes. As discussed further in the sections below, Mr. McKenzie's recommendation of a 9.9 percent awarded ROE is far higher than Avista's true cost of capital based on

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[9] Exh. No. DJG-3 at 23-24.

objective market data and risk profiles of comparable firms. If the Commission were to adopt the Company's position in this case, it would be permitting an excess transfer of wealth from Washington customers to Company shareholders. In addition, it would be permitting an excess transfer of wealth from Washington citizens to the Internal Revenue Service. The detrimental impact to ratepayers and the state's economy is clear. Establishing an awarded return based on flawed assumptions which overstate the cost of capital effectively prevents the awarded returns from changing along with economic conditions. As shown in the figure below, awarded returns for public utilities have been well above the average required market return for at least ten years. Due to the fact that utility stocks are consistently far less risky than the average stock in the marketplace, the cost of equity for utility companies are *less* than the required return on the market.

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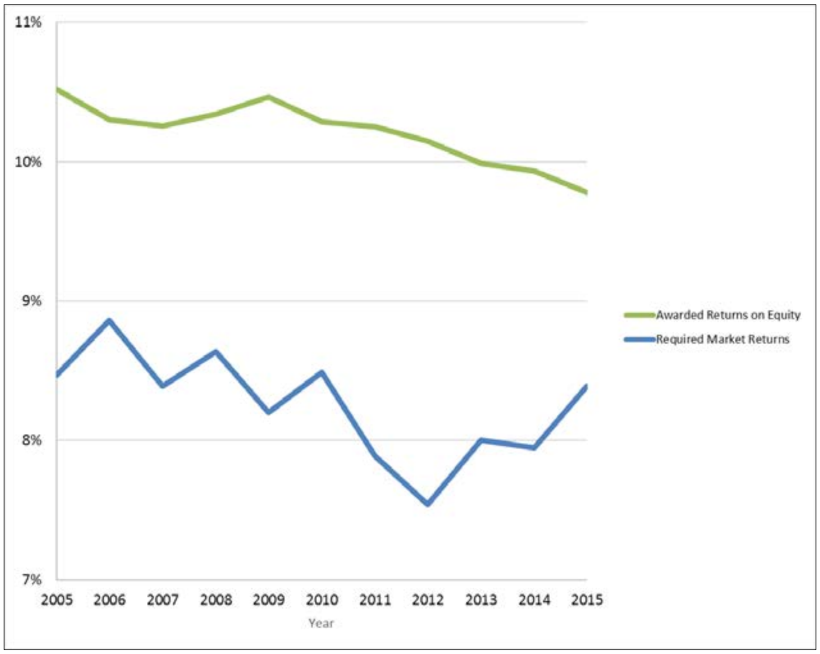
Figure 2:
Awarded Returns on Equity vs. Required Market Returns (2005 – 2015)[10]



The gap between the average awarded returns and utility cost of equity (which is below the bottom line showing required market returns), has resulted in an excess of ratepayer wealth being transferred to utility shareholders and the IRS for at least 10 years. This is likely due, in part, to the fact that many years ago (in the 1990s) interest rates were much higher, with average required market return around 12 percent. In that environment, the cost of equity for low-risk utility stocks might have been about 9 percent. Since that time, however, interest rates have dramatically declined among other economic changes, and it is clear that awarded returns have failed to keep pace with decreasing equity costs.

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[10] *See* Exh. DJG-4 at 14.

It is not hard to see why this trend of inflating awarded returns has occurred in the past. Because awarded returns have at times been based in part on a comparison with other awarded returns, the average awarded returns effectively fail to adapt to true market conditions. Once utility companies and regulatory commissions become accustomed to awarding rates of return higher than market conditions actually require, this trend becomes difficult to reverse. The fact is, utility stocks are *less risky* than the average stock in the market. As such, the required returns (cost of equity) on utility stocks should be less than the average required returns on the market. However, that is often not the case. What we have seen instead is a disconnect from the market-based cost of equity. For these reasons, the Commission should strive to move the awarded return to a level more closely aligned with the Company's actual, market-derived cost of capital while keeping in mind the following principles:

1. <u>Risk is the most important factor when determining the awarded return. The awarded return should be commensurate with those on investments of corresponding risk.</u>

The legal standards articulated in *Hope* and *Bluefield* demonstrate that the Court understands one of the most basic, fundamental concepts in financial theory: the more (less) risk an investor assumes, the more (less) return the investor requires. Since utility stocks are very low risk, the return required by equity investors should be relatively low. I have used financial models in this case to closely estimate the Company's cost of equity, and these financial models account for risk. The public utility industry is one of the least risky industries in the entire country. This is not surprising due to the presence of stable revenues, captive customers, the consistent demand for utility service, and operations that are essentially supported by the state. This means that, in the long run, the profits realized in riskier industries should be higher than the profits realized in the utility industry. To the extent awarded returns for utilities remain comparatively higher than the returns for companies in riskier industries, this is further evidence of the disconnect resulting from the regulatory process, rather than financial or market drivers.

2. <u>The awarded return should be sufficient to assure financial soundness under efficient management.</u>

Because awarded returns in the regulatory environment have not closely tracked market-based trends and commensurate risk, utility companies have been able to remain more than financially sound, perhaps in spite of management efficiencies. In fact, the transfer of wealth from ratepayers to shareholders has been so far removed from actual cost-based drivers, that even under relatively inefficient management a utility could remain financially sound. Therefore, regulatory commissions should strive to set the awarded return to a regulated utility at a level based on accurate market conditions, to promote prudent and efficient management and minimize economic waste.

IV. GENERAL CONCEPTS AND METHODOLOGY

Q: Discuss your general approach in estimating the cost of equity in this case.

A: While a competitive firm must estimate its own cost of capital to assess the profitability capital projects, regulators should determine a utility’s cost of capital to establish a fair rate of return. The legal standards set forth above do not include specific guidelines regarding the specific models that must be used to estimate the cost of equity. Over the years, however, regulatory commissions have consistently relied on several models. The models I have employed in this case have been widely used and accepted in regulatory proceedings for many years. These models include the Discounted Cash Flow Model

(DCF) and the Capital Asset Pricing Model (CAPM). The specific inputs and calculations for these models are described in more detail below.

Q: Explain why you used multiple models to estimate the cost of equity.

A: The models used to estimate the cost of equity attempt to measure the required return of equity investors by estimating a number of different inputs. It is preferable to use multiple models because the results of any one model may contain a degree of inconsistency, especially depending on the reliability of the inputs used at the time of conducting the model. By using multiple models, the analyst can compare the results of the models and look for outlying results and inconsistencies. Likewise, if multiple models produce a similar result, it may indicate a more narrow range for the cost of equity estimate.[11]

V. THE PROXY GROUP

Q: Explain the benefits of choosing a proxy group of companies in conducting cost of capital analyses.

A: The cost of equity models in this case can be used to estimate the cost of capital of any individual, publicly-traded company. There are advantages, however, to conducting cost of capital analysis on a "proxy group" of companies that are comparable to the target company. First, it is better to assess the financial soundness of a utility by comparing it a group of other financially sound utilities. Second, using a proxy group provides more reliability and confidence in the overall results because there is a larger sample size.

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[11] Exh. No. DJG-3, Morin at 28.

Finally, the use of a proxy group is often a pure necessity when the target company is a subsidiary that is not publicly traded, as is the case with Avista. This is because the financial models used in this case require information from publicly-traded firms, such as stock prices and dividends.

Q: Describe the proxy group you selected.

A: In this case, each utility company within my proxy group was also used in Mr. McKenzie's proxy group. There could be reasonable arguments made for the inclusion or exclusion of particular companies in a proxy group, but for all intents and purposes, the cost of equity estimates in rate cases are influenced far more by the assumptions and inputs to the various financial models than the composition of the proxy groups. A summary of the proxy group appears in my Exhibit DJG-4 at page 2.

VI. RISK AND RETURN CONCEPTS

Q: Discuss the general relationship between risk and return.

A: As discussed above, risk is among the most important factors for the Commission to consider when determining the allowed return. In order to comply with this standard, it is necessary to understand the relationship between risk and return. There is a direct relationship between risk and return: the more (or less) risk an investor assumes, the larger (or smaller) return the investor will demand. There are two primary types of risk that affect equity investors: firm-specific risk and market risk. Firm-specific risk affects individual firms, while market risk affects all companies in the market to varying degrees.

Q: Discuss the differences between firm-specific risk and market risk.

A: Firm-specific risk affects individual companies, rather than the entire market. For example, a competitive firm might overestimate customer demand for a new product, resulting in reduced sales revenue. This is an example of project risk.[12] There are several other types of firm-specific risks, including: (1) financial risk – the risk that equity investors of leveraged firms face as residual claimants on earnings; (2) default risk – the risk that a firm will default on its debt securities; and (3) business risk – which encompasses all other operating and managerial factors that may result in investors realizing more or less than their expected return in that particular company. While firm-specific risk affects individual companies, market risk affects all companies in the market to varying degrees. Examples of market risk include interest rate risk, inflation risk, and the risk of major socio-economic events. When there are changes in these risk factors, they affect all firms in the market to some extent.[13]

Q: Is it possible for investors to mitigate or eliminate firm-specific risk?

A: Yes. One of the fundamental concepts in finance is that firm-specific risk can be eliminated through diversification.[14] If someone irrationally invested all of their funds in one firm, they would be exposed to all of the firm-specific risk and the market risk inherent in that single firm. Rational investors, however, are risk-averse and seek to eliminate risk they can control. Investors can eliminate firm-specific risk by simply adding more stocks to their portfolio through a process called "diversification." There

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[12] Exh. DJG-6 (Aswath Damodaran, *Investment Valuation: Tools and Techniques for Determining the Value of Any Asset* 62-63 (3rd ed., John Wiley & Sons, Inc. 2012)).

[13] Exh. DJG-7 (Zvi Bodie, Alex Kane & Alan J. Marcus, *Essentials of Investments* 149 (9th ed., McGraw-Hill/Irwin 2013)).

[14] Exh. No. DJG-8 (John R. Graham, Scott B. Smart & William L. Megginson, *Corporate Finance: Linking Theory to What Companies Do* 179-80 (3rd ed., South Western Cengage Learning 2010)).

are two reasons why diversification eliminates firm-specific risk. First, each stock in a diversified portfolio represents a much smaller percentage of the overall portfolio than it would in a portfolio of just one or a few stocks. Thus, any firm-specific action that changes the stock price of one stock in the diversified portfolio will have only a small impact on the entire portfolio.[15] For example, an investor who had his or his entire portfolio invested in Enron stock at the beginning of 2001 would have lost the entire investment by the end of the year, as a result of exposure to the firm-specific risk of Enron's imprudent management. On the other hand, a rational, diversified investor who owned every stock in the S&P 500 would have actually earned a positive return over the same period of time.

The second reason why diversification eliminates firm-specific risk is that the effects of firm-specific actions on stock prices can be either positive or negative for each stock. Thus, in large portfolios, the net effect of these positive and negative firm-specific risk factors will be essentially zero and will not affect the value of the overall portfolio.[16] Firm-specific risk is also called "diversifiable risk" due to the fact that it can be easily eliminated through diversification.

Q: Is the assumption of firm-specific risk rewarded by the market through higher returns?

A: No. Because investors eliminate firm-specific risk through diversification, they know they cannot expect a higher return for assuming the firm-specific risk in any one

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[15] Exh. No. DJG-6, Damodaran at 64.

[16] *Id*.

company. Thus, the risks associated with an individual firm's operations, as well as managerial risk and default risk are not rewarded by the market. In fact, firm-specific risk is also called "unrewarded" risk for this reason. Market risk, on the other hand, cannot be eliminated through diversification. Market risks, such as interest rate risk and inflation risk, affect all stocks in the market to different degrees. Because market risk cannot be eliminated through diversification, investors who assume higher levels of market risk also expect higher returns. Market risk is also called "systematic risk." Scholars agree: "If investors can cheaply eliminate some risks through diversification, then we should not expect a security to earn higher returns for risks that can be eliminated through diversification. Investors can expect compensation only for bearing systematic risk (i.e., risk that cannot be diversified away)."[17]

These important concepts are illustrated in the figure below.

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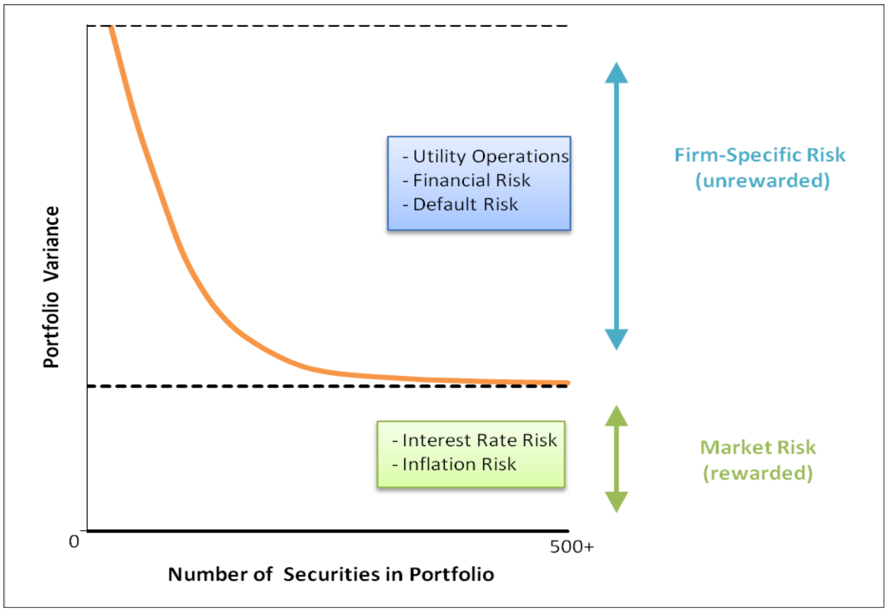
[17] Exh. DJG-8, Graham, Smart & Megginson at 180 (emphasis added).

**Figure 3:
Effects of Portfolio Diversification**



This figure shows that as stocks are added to a portfolio, the amount of firm-specific risk is reduced until it is essentially eliminated. No matter how many stocks are added, however, there remains a certain level of fixed market risk. The level of market risk will vary from firm to firm. Market risk is the only type of risk that is rewarded by the market, and is thus the primary type of risk the Commission should consider when determining the allowed return.

Q: Describe how market risk is measured.

A: Investors who want to eliminate firm-specific risk must hold a fully diversified portfolio. To determine the amount of risk that a single stock adds to the overall market portfolio, investors measure the covariance between a single stock and the market portfolio. The result of this calculation is called “beta.”[18] Beta represents the sensitivity of a given

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[18] *Id*. at 180-81.

security to the market as a whole. The market portfolio of all stocks has a beta equal to one. Stocks with betas greater than one are relatively more sensitive to market risk than the average stock. For example, if the market increases (decreases) by 1.0 percent, a stock with a beta of 1.5 will, on average, increase (decrease) by 1.5 percent. In contrast, stocks with betas of less than one are less sensitive to market risk. For example, if the market increases (decreases) by 1.0 percent, a stock with a beta of 0.5 will, on average, only increase (decrease) by 0.5 percent. Thus, stocks with low betas are relatively insulated from market conditions. The beta term is used in the Capital Asset Pricing Model to estimate the required return on equity, which is discussed in more detail later.

Q: Please describe the level of risk typically associated with of public utilities.

A: Recall that although market risk affects all firms in the market, it affects different firms to varying degrees. Firms with high betas are affected more than firms with low betas, which is why firms with high betas are riskier. Stocks with betas greater than one are generally known as "cyclical stocks." Firms in cyclical industries are sensitive to recurring patterns of recession and recovery known as the "business cycle."[19] Thus, cyclical firms are exposed to a greater level of market risk. Securities with betas less than one, other the other hand, are known as "defensive stocks." Companies in defensive industries, such as public utility companies, "will have low betas and performance that is comparatively unaffected by overall market conditions."[20] The figure below compares the betas of several industries and illustrates that the utility industry is one of the least

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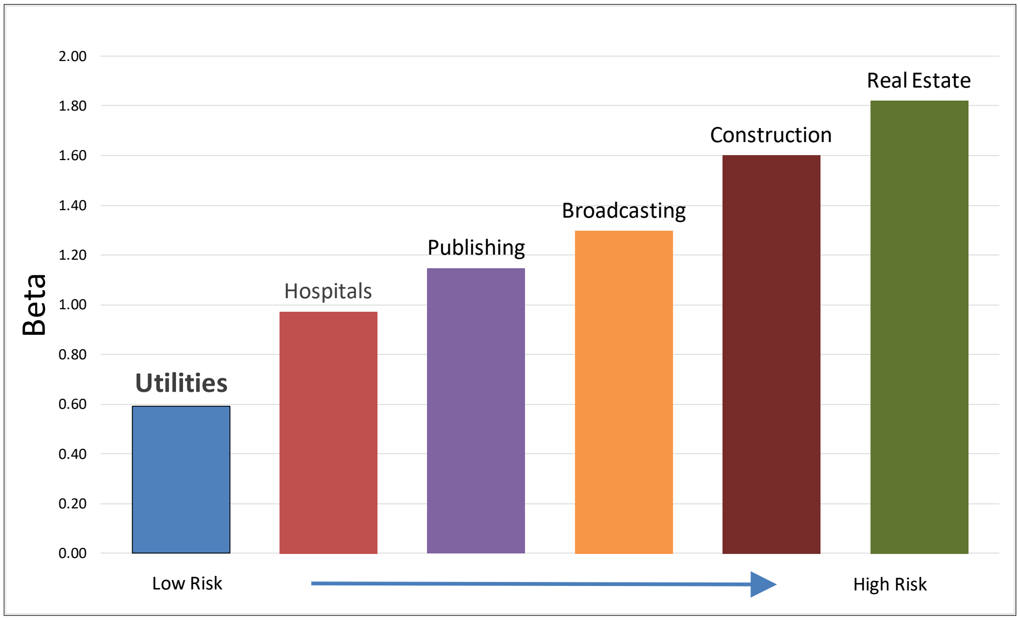
[19] Exh. DJG-7, Bodie, Kane & Marcus at 382.

[20] *Id*. at 383.

risky industries in the U.S. market.[21]

Figure 4: Beta by Industry



The fact that utilities are defensive firms that are exposed to little market risk is beneficial to society. When the business cycle enters a recession, consumers can be assured that their utility companies will be able to maintain normal business operations, and utility investors can be confident that utility stock prices will not widely fluctuate. Thus, because utilities are defensive firms that experience little market risk and are relatively insulated from market conditions, this fact should also be appropriately reflected in the Commission's awarded rate of return.

Q: Does this generally mean that investors in firms with low betas require a smaller return than the average required return on the market?

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[21] Exh. DJG-9, Betas by Sector (US) at http://pages.stern.nyu.edu/~adamodar/. The exact beta calculations are not as important as illustrating the well-known fact that utilities are very low-risk companies. The fact that the utility industry is one of the lowest risk industries in the country should not change from year to year.

A: Yes. This is the basic concept of the risk and return doctrine: The more (less) risk an investor assumes, the larger (smaller) return the investor will demand. So, if a particular stock is less risky than the market average, then an investor in that stock will require a smaller return than the average return on the market. Since utilities are low-risk companies with low betas, the required return (i.e., cost of capital) for utilities should be lower than the required return on the overall market.

Q: Are there other reasons Commission-awarded returns on equity have exceeded the required market returns for at least the last ten years?

A: Although it is indisputable that the true required return on utility stocks is less than the required return on the overall market, commission-awarded returns on equity have often exceeded market returns over the past ten years.[22] In addition to other factors discussed above, many awarded returns arise as the result of settlements. Settled returns are generally higher than market-based cost of capital because utilities may make concessions with other issues in a rate case in exchange for obtaining a higher awarded return. When awarded returns exceed the cost of equity, it results in an inappropriate transfer of wealth from ratepayers to shareholders and the federal government. Moving the allowed return closer to the Company's cost of equity in this case will comply with the requisite legal standards, track more closely with market conditions, allow the Company to remain financially healthy, and reduce the burden on ratepayers.

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[22] *See* Exh. DJG-4 at 14.

VII. DISCOUNTED CASH FLOW ANALYSIS

Q: Describe the Discounted Cash Flow model.

A: The Discounted Cash Flow ("DCF") Model is based on a fundamental financial model called the "dividend discount model," which maintains that the value of a security is equal to the present value of the future cash flows it generates. Cash flows from common stock are paid to investors in the form of dividends. There are several variations of the DCF Model. In its most general form, the DCF Model is expressed as follows:[23]

Equation 2:
General Discounted Cash Flow

$$P_0 = \frac{D_1}{(1+k)} + \frac{D_2}{(1+k)^2} + \cdots + \frac{D_n}{(1+k)^n}$$

where: P_0 = *current stock price*
$D_1 \ldots D_n$ = *expected future dividends*
k = *discount rate / required return*

The General DCF Model would require an estimation of an infinite stream of dividends. Since this would be impractical, analysts use more feasible variations of the General DCF Model, which are discussed further below.

Q: Describe the assumptions underlying all DCF Models.

A: Yes. The DCF Models rely on the following four assumptions:[24]

> 1. Investors evaluate common stocks in the classical valuation framework; that is, they trade securities rationally at prices reflecting their perceptions of value;

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[23] Exh. DJG-7, Bodie, Kane & Marcus at 410.

[24] Exh. DJG-3, Morin at 252.

2. Investors discount the expected cash flows at the same rate (K) in every future period;

3. The K obtained from the DCF equation corresponds to that specific stream of future cash flows alone; and

4. Dividends, rather than earnings, constitute the source of value.

Q: Describe the Constant Growth DCF Model.

A: The General DCF can be rearranged to make it more practical for estimating the cost of equity. Regulators typically rely on some variation of the Constant Growth DCF Model, which is expressed as follows:

Equation 3:
Constant Growth Discounted Cash Flow

$$K = \frac{D_1}{P_0} + g$$

where: K = *discount rate / required return on equity*
D_1 = *expected dividend per share one year from now*
P_0 = *current stock price*
g = *expected growth rate of future dividends*

Unlike the General DCF Model, the Constant Growth DCF Model solves directly for the required return (K). In addition, by assuming that dividends grow at a constant rate, the dividend stream from the General DCF Model may be essentially substituted with a term representing the expected constant growth rate of future dividends (g). The Constant Growth DCF Model may be considered in two parts. The first part is the dividend yield (D_1/P_0), and the second part is the growth rate (g). In other words, the required return in the DCF Model is equivalent to the dividend yield plus the growth rate.

Q: Does utilization of the Constant Growth DCF Model require additional assumptions?

A: Yes. In addition to the four assumptions listed above, the Constant Growth DCF Model relies on four additional assumptions as follows:[25]

1. The discount rate (K) must exceed the growth rate (g);
2. The dividend growth rate (g) is constant in every year to infinity;
3. Investors require the same return (K) in every year; and
4. There is no external financing; that is, growth is provided only by the retention of earnings.

Since the growth rate is assumed to be constant, it is important not to use growth rates that are unreasonably high. In fact, the constant growth rate estimate for a regulated utility with a defined service territory should not exceed the growth rate for the economy in which it operates.

Q: Describe the Quarterly Approximation DCF Model.

A: The basic form of the Constant Growth DCF Model described above is sometimes referred to as the "Annual" DCF Model. This is because the model assumes an annual dividend payment to be paid at the end of every year, as well as an increase in dividends once each year. In reality, however, most utilities pay dividends on a quarterly basis. The Constant Growth DCF equation may be modified to reflect the assumption that investors receive successive quarterly dividends and reinvest them throughout the year at the discount rate. This variation is called the Quarterly Approximation DCF Model.[26]

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[25] Exh. DJG-3, Morin at 254-56.
[26] Exh. DJG-3, Morin at 348.

Equation 4:
Quarterly Approximation Discounted Cash Flow

$$K = \left[\frac{d_0(1+g)^{1/4}}{P_0} + (1+g)^{1/4}\right]^4 - 1$$

where: K = *discount rate / required return*
d_0 = *current quarterly dividend per share*
P_0 = *stock price*
g = *expected growth rate of future dividends*

The Quarterly Approximation DCF Model assumes that dividends are paid quarterly and that each dividend is constant for four consecutive quarters. All else held constant, this model actually results in the highest cost of equity estimate for the utility in comparison to other DCF Models because it accounts for the quarterly compounding of dividends. There are several other variations of the Constant Growth (or Annual) DCF Model, including a Semi-Annual DCF Model which is used by the Federal Energy Regulatory Commission (FERC). These models, along with the Quarterly Approximation DCF Model, have been accepted in regulatory proceedings as useful tools for estimating the cost of equity. For this case, I have chosen to use the Quarterly Approximation DCF Model described above.

Q: Describe the inputs to the DCF Model.

A: There are three primary inputs in the DCF Model: (1) stock price (P_0); (2) dividend (d_0); and (3) growth rate (g). The stock prices and dividends are known inputs based on recorded data, while the growth rate projection must be estimated. I will discuss each of these inputs in turn.

A. Stock Price

$$\left[K = \frac{D_1}{\boldsymbol{P_0}} + g\right]$$

Q: Describe how you determined the stock price input of the DCF Model.

A: For the stock price (P_0), I used a 30-day average of stock prices for each company in the proxy group.[27] Analysts sometimes rely on average stock prices for longer periods (e.g., 60, 90, or 180 days). According to the efficient market hypothesis, however, markets reflect all relevant information available at a particular time, and prices adjust instantaneously to the arrival of new information.[28] Past stock prices, in essence, reflect outdated information. The DCF Model used in utility rate cases is a derivation of the dividend discount model, which is used to determine the current value of an asset. Thus, according to the dividend discount model and the efficient market hypothesis, the value for the "P_0" term in the DCF Model should technically be the current stock price, rather than an average.

Q: Explain why you used a 30-day average for the current stock price input.

A: Using a short-term average of stock prices for the current stock price input adheres to market efficiency principles which avoiding any irregularities that may arise from using a single current stock price. In the context of a utility rate proceeding there is a significant length of time from when an application is filed and responsive testimony is due.

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[27] *See* Exh. DJG-4 at 3.

[28] Exh. DJG-10 (Eugene F. Fama, *Efficient Capital Markets: A Review of Theory and Empirical Work*, Vol. 25, No. 2 The Journal of Finance 383 (1970)); Exh. DJG-8, Graham, Smart & Megginson at 357. The efficient market hypothesis was formally presented by Eugene Fama in 1970, and is a cornerstone of modern financial theory and practice.

Choosing a current stock price for one particular day during that time could raise a separate issue concerning which day was chosen to be used in the analysis. In addition, a single stock price on a particular day may be unusually high or low. It is arguably ill-advised to use a single stock price in a model that is ultimately used to set rates for several years, especially if a stock is experiencing some volatility. Thus, it is preferable to use a short-term average of stock prices, which represents a good balance between adhering to well-established concepts of market efficiency while avoiding any irregularities that may arise from using a single stock price on a given day. The stock prices I used in my DCF analysis are based on 30-day averages of adjusted closing stock prices for each company in the proxy group.[29]

B. Dividend

$$\left[K = \frac{\mathbf{D_1}}{P_0} + g\right]$$

Q: Describe how you determined the dividend input of the DCF Model.

A: The dividend term in the Quarterly Approximation DCF Model is the current quarterly dividend per share. I obtained the quarterly dividend paid in the fourth quarter of 2016 for each proxy company.[30] The Quarterly Approximation DCF Model assumes that the company increases its dividend payments each quarter. Thus, the model assumes that each quarterly dividend is greater than the previous one by $(1 + g)^{0.25}$. This expression

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[29] Exh. DJG-4 at 3. Adjusted closing prices, rather than actual closing prices, are ideal for analyzing historical stock prices. The adjusted price provides an accurate representation of the firm's equity value beyond the mere market price because it accounts for stock splits and dividends.

[30] Nasdaq Dividend History, http://www.nasdaq.com/quotes/dividend-history.aspx.

could be describe as the dividend quarterly growth rate, where the term “g” is the growth rate and the exponential term “0.25” signifies one quarter of the year.

Q: Does the Quarterly Approximation DCF Model result in the highest cost of equity relative to other DCF Models, all else held constant?

A: Yes. The DCF Model I employed in this case results in a higher DCF cost of equity estimate than the annual or semi-annual DCF Models due to the quarterly compounding of dividends inherent in the model.

C. Growth Rate

$$\left[K = \frac{\mathrm{D}_1}{P_0} + \boldsymbol{g}\right]$$

Q: Explain the importance of the growth rate input in the DCF Model.

A: The most critical input in the DCF Model is the growth rate. Unlike the stock price and dividend inputs, the growth rate must be estimated. As a result, the growth rate is often the most contentious DCF input in utility rate cases. The DCF model used in this case is based on the constant growth valuation model. As stated above, one of the inherent assumptions of this model is that dividends grow at a constant rate forever. Thus, the growth rate term in the constant growth DCF model is often called the “constant,” “stable,” or “terminal” growth rate. For young, high-growth firms, estimating the growth rate to be used in the model can be especially difficult. For mature, low-growth firms such as utilities, however, estimating the terminal growth rate is more straightforward, as discussed further below.

Q: Is it widely accepted that the terminal growth rate cannot exceed the growth rate of the economy, especially for a regulated utility company?

A: Yes. A fundamental concept in finance is that no firm can grow forever at a rate higher than the growth rate of the economy in which it operates.[31] Thus, the terminal growth rate used in the DCF Model should not exceed the aggregate economic growth rate. This is especially true when the DCF Model is conducted on public utilities because these firms usually have defined service territories beyond which they cannot grow. As stated by Dr. Damodaran: "If a firm is a purely domestic company, either because of internal constraints . . . or external constraints (such as those imposed by a government), the growth rate in the domestic economy will be the limiting value."[32] In fact, it is reasonable to assume that a regulated utility would grow at a rate that is <u>less</u> than the U.S. economic growth rate. Unlike competitive firms, which might increase their growth by launching a new product line, franchising, or expanding into new and developing markets, public utilities cannot do any of these things to grow. Gross domestic product (GDP) is one of the most widely-used measures of economic production, and is used to measure aggregate economic growth. According to the Congressional Budget Office's Budget Outlook, the long-term forecast for nominal U.S. GDP growth is 4.1 percent, which includes an inflation rate of two percent.[33] For mature companies in mature industries, such as utility companies, the terminal growth rate will likely fall between the

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[31] Exh. DJG-6, Damodaran at 306.

[32] *Id*.

[33] Exh. DJG-11 (Congressional Budget Office Long-Term Budget Outlook, https://www.cbo.gov/publication/51580).

expected rate of inflation and the expected rate of nominal GDP growth. Thus, Avista's terminal growth rate is between two percent and 4.1 percent.

Q: Is it reasonable to assume that the terminal growth rate will not exceed the risk-free rate?

A: Yes. In the long term, the risk-free rate will converge on the growth rate of the economy. For this reason, financial analysts often use the risk-free rate for the terminal growth rate value in the DCF model.[34] I discuss the risk-free rate in further detail later in this testimony.

Q: Why it is important when analyzing utility growth rates to consider the qualitative aspects of growth in addition to the quantitative aspects?

A: When analyzing growth rates for any firm, there are several quantitative methods and various growth determinants that can be used in the analysis. These can include both historical and projected analyses of revenue, operating income, net income, earnings, dividends, and other determinants.[35] While it may be important to consider one or more of these quantitative growth determinants, it may be even more important to consider qualitative aspects of growth when analyzing a regulated utility. This is because a utility's growth in dividends or earnings is going to be primarily driven by the return on equity awarded by the regulator. This creates a circular reference problem. In other words, if a regulator awards a higher ROE than the market anticipated, this could lead to higher growth rate estimates from analysts; if those same estimates are used in the DCF

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[34] Exh. DJG-6, Damodaran at 307.

[35] Exh. DJG-6, Damodaran at 271-302.

Model in the next rate case, it could lead to a higher awarded ROE; and the cycle continues. Therefore, it is important to begin the analysis with this simple qualitative question: How is this utility going to grow in the future? If this question were asked of a competitive firm, there could be a number of answers depending on the line of business, such as launching a new product line, engaging in mergers and acquisitions, franchising, rebranding to target a new demographic, expanding into developing markets, etc. Regulated utilities, however, cannot engage in these potential growth opportunities.

Q: Summarize the various terminal growth rate estimates you discussed.

A: For Avista, there are four different growth forecasts that could be used for the terminal growth rate in the DCF model: 1) nominal GDP; 2) inflation; and 3) the risk-free rate.

Figure 5:
Terminal Growth Rates

Growth Determinant	Rate
Nominal GDP	4.10%
Inflation	2.00%
Risk Free Rate	2.80%
Average	**2.97%**

It would not be unreasonable to use any of these rates by itself for Avista's terminal growth rate. For the long-term growth rate in my DCF model I selected the highest growth rate from this list, which is the forecasted nominal GDP growth of 4.1 percent. This growth rate estimate is likely high because it assumes that the growth rate of a mature, low-growth utility company with a defined service territory will match the growth rate of the entire U.S. economy over the long run. As a result, my final DCF cost of equity estimate is toward the higher end of the reasonable range.

Q: Describe the final results of your DCF Model.

A: I used the Quarterly Approximation DCF Model discussed above to estimate Avista's cost of equity capital. I obtained an average of reported dividends and stock prices from the proxy group, and I used a very reasonable terminal growth rate estimate for Avista. My DCF cost of equity estimate for Avista is 7.7 percent, as expressed in the following equation:[36]

Equation 5:
DCF Results

$$\mathbf{7.2}\% = \left[\frac{\$0.48(1+4.1\%)^{1/4}}{\$64.84} + (1+4.1\%)^{1/4}\right]^4 - 1$$

As noted above, this estimate is likely at the higher end of the appropriate range due to the fact that my growth rate estimate exceeds the Company's own load growth forecast.

Q: Mr. McKenzie's DCF Model yielded much higher results. Did you find specific problems with his analysis regarding the DCF Models?

A: Yes. Mr. McKenzie's DCF Model produced cost of equity results as high as 10.3 percent. The results of Mr. McKenzie's DCF Model are unreasonably high primarily due to his extremely inflated growth rate estimates. Mr. McKenzie incorporates long-term growth rates as high as 10.4 percent in his DCF estimate. Mr. McKenzie's growth rate assumptions are patently unreasonable. For example, Mr. McKenzie's relies on a long-

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[36] Exh. DJG-4 at 4, 5, and 6.

term growth rate for Black Hills Corp. of 10.4 percent. This means that Mr. McKenzie assumes that Black Hills Corp. will grow at a rate of 10.4 percent per year, every year going forward. However, the real, qualitative growth rate of any public utility is primarily limited by the population and load growth in its defined service territory. Yet, Mr. McKenzie assumes that Black Hills Corp. is going to grow at a rate nearly three times the projected long-term growth rate of the entire U.S. economy, which has access to markets across the planet. Because of this unreasonable assumption, Mr. McKenzie's DCF cost of equity estimates are far overstated.

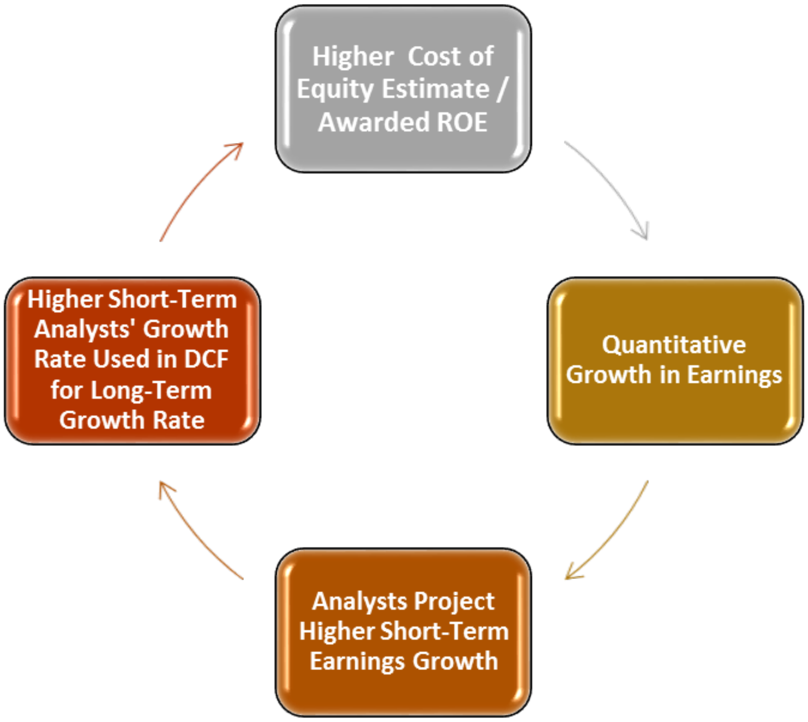
Q: Are there any other problems with Mr. McKenzie's growth rate estimates?

A: Yes. Mr. McKenzie, like most utility ROE witnesses, relies on the projected growth rates published by Zack's, Value Line, and other commercial services. However, these growth rate estimates are <u>short-term</u> estimates, whereas the growth rate input in the DCF model requires a <u>long-term</u> growth rate estimate.

Q: Please discuss the other way in which analysts' earnings growth projections do not provide indications of fair, qualitative growth for regulated utilities.

A: The fact that growth rate projections published by commercial analysts, such as Value Line, are merely short-term growth projections invalidates them for use in any constant growth DCF Model by definition. However, there is another problem with relying on these growth rate projections for the DCF Model in determining a fair rate of return for a regulated utility. If we give undue weight to commercial analysts' projections for utilities' earnings growth, it will not provide an accurate reflection of real, qualitative growth because a utility's earnings are heavily influenced by the ultimate figure that all of this analysis is supposed to help us estimate: the awarded return on equity. This creates a circular reference problem. In other words, if a regulator awards an ROE that is above market-based cost of capital (which is the case here), this could lead to higher growth rate projections from analysts. If these same inflated growth estimates are used in the DCF Model (and they often are by utility witnesses), it could lead to a higher awarded ROEs; and the cycle continues, as illustrated in the following figure:

Figure 6:
The "Circular Reference" Problem



Therefore, it is not advisable to simply consider a quantitative historical or projected growth rate in utility earnings, as this practice will not provide a reliable or accurate indication of real utility growth. Since the growth input in the DCF Model is a long-term growth rate, it should be bound by the U.S. economic growth rate as measured by GDP.

Q: Did Mr. McKenzie consistently apply the results of his own DCF Model?

A: No. Mr. McKenzie simply eliminated many of the lower DCF results produced by his own methodology.[37] This appears to be nothing more than an arbitrary tactic to skew the results of his DCF average to a higher number. If Mr. McKenzie had simply accepted the results of his own methodology (even with the unreasonably high growth rates) his average DCF results from Exhibit AMM-6 would be as follows:

Value Line:	8.4%
IBES:	8.6%
Zacks:	9.0%
S&P:	8.8%
Average:	**8.7%**

While a cost of equity of estimate of 8.7 percent for the Company is high, it is much more reasonable than the skewed results presented by Mr. McKenzie.

VIII. CAPITAL ASSET PRICING MODEL ANALYSIS

Q: Describe the Capital Asset Pricing Model.

A: The Capital Asset Pricing Model (CAPM) is a market-based model founded on the principle that investors demand higher returns for incurring additional risk.[38] The CAPM estimates this required return.

Q: What assumptions are inherent in the CAPM?

A: The CAPM relies on the following assumptions:

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[37] *See* McKenzie, Exh. AMM-6.

[38] Exh. DJG-12 (William F. Sharpe, *A Simplified Model for Portfolio Analysis* 277-93 (Management Science IX 1963)); Exh. DJG-8, Graham, Smart & Megginson at 208.

1. Investors are rational, risk-averse, and strive to maximize profit and terminal wealth;
2. Investors make choices on the basis of risk and return. Return is measured by the mean returns expected from a portfolio of assets; risk is measured by the variance of these portfolio returns;
3. Investors have homogenous expectations of risk and return;
4. Investors have identical time horizons;
5. Information is freely and simultaneously available to investors.
6. There is a risk-free asset, and investors can borrow and lend unlimited amounts at the risk-free rate;
7. There are no taxes, transaction costs, restrictions on selling short, or other market imperfections; and,
8. Total asset quality is fixed, and all assets are marketable and divisible.[39]

While some of these assumptions may appear to be restrictive, they do not outweigh the inherent value of the model. The CAPM has been widely used by firms, analysts, and regulators for decades to estimate the cost of equity capital.

Q: Is the CAPM approach consistent with the legal standards set forth by the U.S. Supreme Court?

A: Yes. Our courts have recognized that "the amount of <u>risk</u> in the business is a most important factor" in determining the allowed rate of return,[40] and that "the return to the equity owner should be commensurate with returns on investments in other enterprises having corresponding <u>risks</u>."[41] The CAPM is a useful model because it directly considers

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[39] *See id*.
[40] *Wilcox*, 212 U.S. at 48 (emphasis added).
[41] *Hope Nat. Gas Co*., 320 U.S. at 603 (emphasis added).

the amount of risk inherent in a business. It is arguably the strongest of the models usually presented in rate cases because unlike the DCF Model, the CAPM directly measures the most important component of a fair rate of return analysis: Risk.

Q: Describe the CAPM equation.

A: The basic CAPM equation is expressed as follows:

Equation 6:
Capital Asset Pricing Model

$$K = R_F + \beta_i(R_M - R_F)$$

where: K = *required return*
R_F = *risk-free rate*
β = *beta coefficient of asset i*
R_M = *required return on the overall market*

There are essentially three terms within the CAPM equation that are required to calculate the required return (K): (1) the risk-free rate (R_F); (2) the beta coefficient (β); and (3) the equity risk premium (R_M – R_F), which is the required return on the overall market less the risk-free rate. Each term is discussed in more detail below, along with the inputs I used for each term.

A. The Risk-Free Rate

$$\left[K = \boldsymbol{R_F} + \beta_i(R_M - R_F)\right]$$

Q: Explain the risk-free rate.

A: The first term in the CAPM is the risk-free rate (R_F). The risk-free rate is simply the level of return investors can achieve without assuming any risk. The risk-free rate represents the bare minimum return that any investor would require on a risky asset.

Even though no investment is technically void of risk, investors often use U.S. Treasury securities to represent the risk-free rate because they accept that those securities essentially contain no default risk. The Treasury issues securities with different maturities, including short-term Treasury Bills, intermediate-term Treasury Notes, and long-term Treasury Bonds.

Q: Is it preferable to use the yield on long-term Treasury bonds for the risk-free rate in the CAPM?

A: Yes. In valuing an asset, investors estimate cash flows over long periods of time. Common stock is viewed as a long-term investment, and the cash flows from dividends are assumed to last indefinitely. Thus, short-term Treasury bill yields are rarely used in the CAPM to represent the risk-free rate. Short-term rates are subject to greater volatility and can thus lead to unreliable estimates. Instead, long-term Treasury bonds are usually used to represent the risk-free rate in the CAPM.[42] I considered a 30-day average of daily Treasury yield curve rates on 30-year Treasury bonds in my risk-free rate estimate, which resulted in a risk-free rate of 2.81 percent.[43]

B. The Beta Coefficient

$$\left[K = R_F + \boldsymbol{\beta_i}(R_M - R_F)\right]$$

Q: Describe the beta coefficient.

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[42] Exh. DJG-3, Morin at 150.

[43] Exh. DJG-4 at 7.

A: As discussed above, beta represents the sensitivity of a given security to movements in the overall market. The CAPM states that in efficient capital markets, the expected risk premium on each investment is proportional to its beta. Recall that a security with a beta greater (less) than one is more (less) risky than the market portfolio. A stock's beta equals the covariance of the asset's returns with the returns on a market portfolio, divided by the portfolio's variance, as expressed in the following formula:[44]

Equation 7:
Beta

$$\beta_i = \frac{\sigma_{im}}{\sigma_m^2}$$

where: β_i = *beta of asset i*
σ_{im} = *covariance of asset i returns with market portfolio returns*
$\sigma^2{}_m$ = *variance of market portfolio*

Typically, an index such as the S&P 500 Index is used as proxy for the market portfolio. The historical betas for publicly traded firms are published by several commercial sources.[45] Beta may also be calculated through a linear regression analysis, which provides additional statistical information about the relationship between a single stock and the market portfolio. Also, as discussed above, beta represents the sensitivity of a given security to the market as a whole. The market portfolio of all stocks has a beta equal to one. Stocks with betas greater than one are relatively more sensitive to market risk than the average stock. For example, if the market increases (decreases) by 1.0 percent, a stock with a beta of 1.5 will, on average, increase (decrease) by 1.5 percent. In

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[44] Exh. DJG-8, Graham, Smart & Megginson at 180-81.

[45] E.g., Value Line, Bloomberg, and Merrill Lynch.

contrast, stocks with betas of less than one are less sensitive to market risk. For example, if the market increases (decreases) by 1.0 percent, a stock with a beta of 0.5 will, on average, only increase (decrease) by 0.5 percent.

Q: Describe the source for the betas you used in your CAPM analysis.

A: I used betas recently published by Value Line Investment Survey.[46] The beta for each proxy company was less than 1.0, and the average beta for the proxy group is 0.71. Thus, we have an objective measure to prove the well-known concept that utility stocks are less risky than the average stock in the market, which has a beta of 1.0.

C. The Equity Risk Premium

$$\left[K = R_F + \beta_i\left(\boldsymbol{R_M - R_F}\right)\right]$$

Q: Describe the equity risk premium.

A: The final term of the CAPM is the equity risk premium ("ERP"), which is the required return on the market portfolio less the risk-free rate (R_M – R_F). In other words, the ERP is the level of return investors expect above the risk-free rate in exchange for investing in risky securities. Many experts would agree that "the single most important variable for making investment decisions is the equity risk premium."[47] Likewise, the ERP is arguably the single most important factor in estimating the cost of capital in this matter. There are three basic methods to estimate the ERP: (1) calculating a historical average; (2) taking a survey of experts; and (3) calculating the implied equity risk premium. I

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[46] Exh. DJG-4 at 8.

[47] Exh. DJG-13 (Elroy Dimson, Paul Marsh & Mike Staunton, *Triumph of the Optimists: 101 Years of Global Investment Returns* 4 (Princeton University Press 2002)).

incorporated each one of these methods in determining the ERP used in my CAPM analysis. I will discuss each method in turn.

1. Historical Average

Q: Describe the historical equity risk premium.

A: The historical ERP may be calculated by simply taking the difference between returns on stocks and returns on government bonds over a certain period of time. Ibbotson, one of the most widely cited source for the historical ERP in the U.S.,[48] reports both the geometric mean and arithmetic mean for the returns of stocks and government bonds in its annual yearbooks.[49] Many practitioners rely on the historical ERP as an estimate for the forward-looking ERP because it is easy to obtain. However, there are disadvantages to relying on the historical ERP as an indication of the current ERP.

Q: What are the limitations of relying solely on a historical average to estimate the current or forward-looking ERP?

A: Many investors use the historic ERP because it is convenient and easy to calculate. What matters in the CAPM model, however, is not the actual risk premium from the past, but rather the current and forward-looking risk premium.[50] Some investors may think that a historic ERP provides some indication of what the prospective risk premium is, but there is empirical evidence to suggest the prospective, forward-looking ERP is actually lower than the historical ERP. In a landmark publication on risk premiums around the world,

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[48] *Id*. at 173.

[49] Exh. DJG-14 (2015 Ibbotson Stocks, Bonds, Bills, and Inflation Classic Yearbook 91 (Morningstar 2015)).

[50] Exh. DJG-8, Graham, Smart & Megginson at 330.

Triumph of the Optimists, the authors suggest through extensive empirical research that the prospective ERP is lower than the historical ERP.[51] This is due in large part to what is known as "survivorship bias" or "success bias" – a tendency for failed companies to be excluded from historical indices.[52] From their extensive analysis, the authors make the following conclusion regarding the prospective ERP: "The result is a forward-looking, geometric mean risk premium for the United States . . . of around 2½ to 4 percent and an arithmetic mean risk premium . . . that falls within a range from a little below 4 to a little above 5 percent."[53] Indeed, these results are lower than many reported historical risk premiums. Other noted experts agree:

> The historical risk premium obtained by looking at U.S. data is biased upwards because of survivor bias . . . . The true premium, it is argued, is much lower. This view is backed up by a study of large equity markets over the twentieth century (*Triumph of the Optimists*), which concluded that the historical risk premium is closer to 4%.[54]

Regardless of the variations in historic ERP estimates, many scholars and practitioners agree that simply relying on a historic ERP to estimate the risk premium going forward is not ideal. Fortunately, "a naïve reliance on long-run historical averages is not the only approach for estimating the expected risk premium."[55]

2. Expert Surveys

Q: Describe the expert survey approach to estimating the ERP.

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[51] Exh. DJG-13, Dimson, Marsh & Staunton 53.

[52] *Id*. at 34.

[53] *Id*. at 194.

[54] Exh. DJG-15 (Aswath Damodaran, *Equity Risk Premiums: Determinants, Estimation and Implications – The 2015 Edition* 17 (New York University 2015)).

[55] Exh. DJG-8, Graham, Smart & Megginson at 330.

A: As its name implies, the expert survey approach to estimating the ERP involves conducting a survey of experts including professors, analysts, chief financial officers and other executives around the country and asking them what they think the ERP is. Graham and Harvey have performed such a survey every year since 1996. In their 2016 survey, they found that experts around the country believe that the current risk premium is only 4.0 percent.[56] The IESE Business School conducts a similar expert survey, and recently reported an average ERP of 5.7 percent.[57] It should be noted that ERP values assumed by Mr. McKenzie are as high as 8.7 percent, which is a substantial departure from these survey results.

3. Implied Equity Risk Premium

Q: Describe the implied equity risk premium.

A: The third method of estimating the ERP is arguably the best. The implied ERP relies on the stable growth model proposed by Gordon, often called the "Gordon Growth Model," which is a basic stock valuation model widely used in finance for many years:[58]

Equation 8:
Gordon Growth Model

$$P_0 = \frac{D_1}{K - g}$$

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[56] Exh. DJG-16 (John R. Graham and Campbell R. Harvey, *The Equity Risk Premium in 2016*, at 3 (Fuqua School of Business, Duke University 2014)), copy available at https://papers.ssrn.com/sol3/papers.cfm?abstract_id=2816603.

[57] Exh. DJG-17 (Pablo Fernandez, Vitaly Pershin & Isabel F. Acin, *Market Risk Premium used in 171 Countries in 2016: A Survey with 6,932 Answers*, at 3 (IESE Business School 2015)), copy available at **https://papers.ssrn.com/sol3/papers.cfm?abstract_id=2954142**

[58] Exh. DJG-18 (Myron J. Gordon and Eli Shapiro, *Capital Equipment Analysis: The Required Rate of Profit* 102-10 (Management Science Vol. 3, No. 1 Oct. 1956)).

where: P_0 = *current value of stock*
D_1 = *value of next year's dividend*
K = *cost of equity capital / discount rate*
g = *constant growth rate in perpetuity for dividends*

This model is similar to the Constant Growth DCF Model presented in Equation 3 above ($K=D_1/P_0+g$). In fact, the underlying concept in both models is the same: The current value of an asset is equal to the present value of its future cash flows. Instead of using this model to determine the discount rate of one company, we can use it to determine the discount rate for the entire market by substituting the inputs of the model. Specifically, instead of using the current stock price (P_0), we will use the current value of the S&P 500 (V_{500}). Instead of using the dividends of a single firm, we will consider the dividends paid by the entire market. Additionally, we should consider potential dividends. In other words, stock buybacks should be considered in addition to paid dividends, as stock buybacks represent another way for the firm to transfer free cash flow to shareholders. Focusing on dividends alone without considering stock buybacks could understate the cash flow component of the model, and ultimately understate the implied ERP. The market dividend yield plus the market buyback yield gives us the gross cash yield to use as our cash flow in the numerator of the discount model. This gross cash yield is increased each year over the next five years by the growth rate. These cash flows must be discounted to determine their present value. The discount rate in each denominator is the risk-free rate (R_F) plus the discount rate (K). The following formula shows how the implied return is calculated. Since the current value of the S&P is known, we can solve for K: The implied market return.[59]

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[59] *See* Exh. DJG-4 at 9.

Equation 9:
Implied Market Return

$$V_{500} = \frac{CY_1(1+g)^1}{(1+R_F+K)^1} + \frac{CY_2(1+g)^2}{(1+R_F+K)^2} + \cdots + \frac{CY_5(1+g)^5 + TV}{(1+R_F+K)^5}$$

where: V_{500} = *current value of index (S&P 500)*
$CY_{1\text{-}5}$ = *average cash yield over last five years (includes dividends and buybacks)*
g = *compound growth rate in earnings over last five years*
R_F = *risk-free rate*
K = *implied market return (this is what we are solving for)*
TV = *terminal value* $= CY_5\,(1+R_F)\,/\,K$

The discount rate is called the "implied" return here because it is based on the current value of the index as well as the value of free cash flow to investors projected over the next five years. Thus, based on these inputs, the market is "implying" the expected return. After solving for the implied market return (K), we simply subtract the risk-free rate from it to arrive at the implied ERP.

Equation 10:
Implied Equity Risk Premium

$$Implied\ Expected\ Market\ Return - R_F = Implied\ ERP$$

Q: Discuss the results of your implied ERP calculation.

A: After collecting data for the index value, operating earnings, dividends, and buybacks for the S&P 500 over the past six years, I calculated the dividend yield, buyback yield, and gross cash yield for each year. I also calculated the compound annual growth rate (g) from operating earnings. I used these inputs, along with the risk-free rate and current value of the index to calculate a current expected return on the entire market of 8.09 percent. I subtracted the risk-free rate to arrive at the implied equity risk premium of 4.9 percent. Dr. Damodaran, one of the world's leading experts on the ERP, promotes the implied ERP method discussed above. He calculates monthly and annual implied ERPs with this method and publishes his results. Dr. Damodaran's average ERP estimate for October 2017 was 5.2 percent.[60]

Q: Discuss the results of your final ERP estimate.

A: For the final ERP estimate I used in my CAPM analysis, I averaged the results of the ERP surveys along with Dr. Damodaran's published ERP and my implied ERP calculation.[61] The results are presented in the following figure:

Figure 7:
Equity Risk Premium Results

IESE Business School Survey	5.7%
Graham & Harvey Survey	4.0%
Duff & Phelps Report	5.0%
Damodaran	5.2%
Garrett	4.9%
Highest	**5.7%**

While it would be reasonable to select any one of these ERP estimates, or the average of these estimates, I selected the <u>highest</u> ERP estimate of 5.7 percent for my CAPM in the interest of conservatism. The survey results published in the IESE report are markedly higher than the other sources I reviewed for the ERP, and in my opinion, an ERP estimate

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[60] *Implied Equity Risk Premium Update*, Damodaran Online, (last visited Oct. 27, 2017) http://pages.stern.nyu.edu/~adamodar/.

[61] Exh. DJG-4 at 10.

of 5.7 percent is likely overestimated given current market conditions. All else held constant, a higher ERP will result in a higher CAPM cost of equity estimate.

Q: Explain the final results of your CAPM analysis.

A: Using the inputs for the risk-free rate, beta coefficient, and equity risk premium discussed above, I calculated the CAPM cost of equity for each proxy company. Using the same CAPM equation presented above, the results of my CAPM analysis are expressed as follows:[62]

Equation 11:
CAPM Results

$$\mathbf{6.8}\% = 2.81\% + 0.71(5.7\%)$$

The CAPM suggests that Avista's cost of equity capital is about 6.8%. The CAPM may be displayed graphically through what is known as the Security Market Line ("SML"). The following figure shows the expected return (cost of equity) on the y-axis, and the average beta for the proxy group on the x-axis. The SML intercepts the y-axis at the level of the risk-free rate. The slope of the SML is the equity risk premium.

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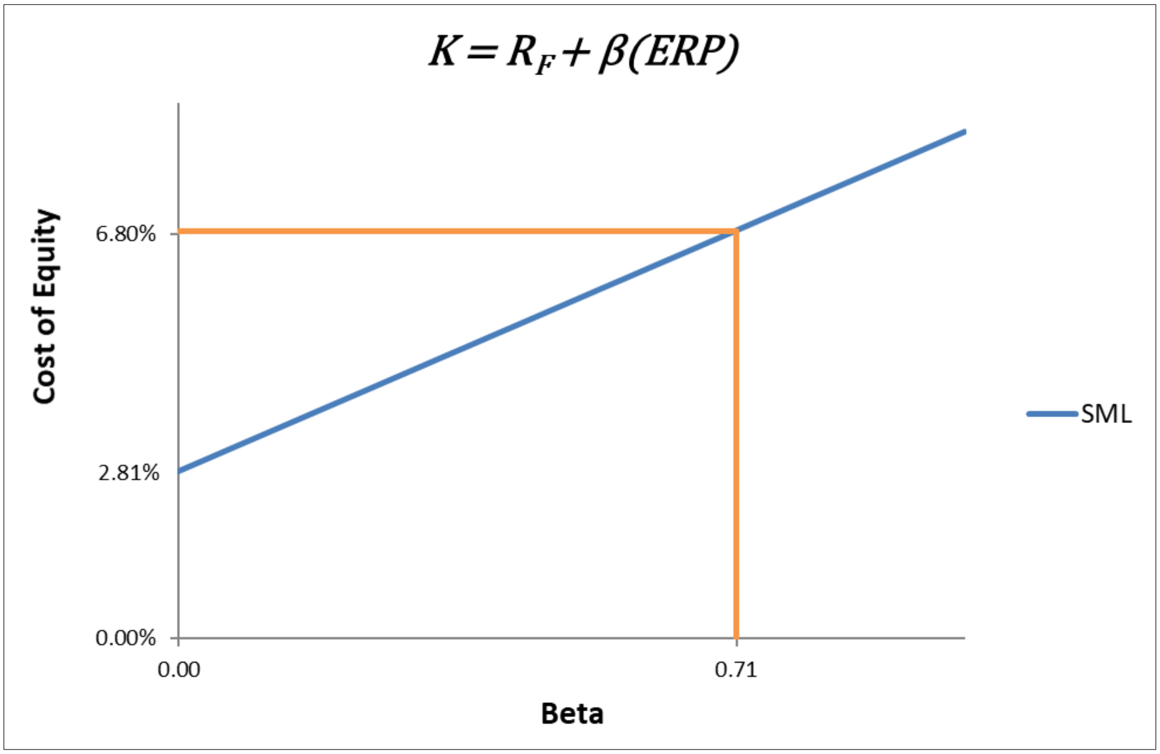
[62] Exh. DJG-4 at 11.

Figure 8:
CAPM Graph



The SML provides the required rate of return that will compensate investors for the beta risk of that investment. Thus, at an average beta of 0.71 for the proxy group, the estimated cost of equity for Avista is 6.8 percent.

Q: Mr. McKenzie's CAPM analysis yields considerably higher results. Did you find specific problems with Mr. McKenzie's CAPM assumptions and inputs?

A: Yes. Mr. McKenzie's cost of equity estimates through his CAPM analysis are as high as 11.85 percent. This is primarily due to overestimation of the equity risk premium.

Q: Did Mr. McKenzie rely on a realistic measure for the equity risk premium?

A: No. The ERP is one of three inputs in the CAPM equation, and it is one of the most single important factors for estimating the cost of equity in this case. As discussed above, I used two widely-accepted methods for estimating the ERP, including consulting expert surveys and calculating the implied ERP based on aggregate market data. In contrast, Mr. McKenzie essentially conducted a DCF analysis on nearly every company in the S&P 500. This means that Mr. McKenzie made 500 separate growth rate inputs for each company in his market portfolio. If his growth inputs were reasonable, then the model could theoretically produce reasonable results. Instead, however, many of Mr. McKenzie's growth rate inputs were not realistic. For example, Mr. McKenzie relied on a <u>long-term</u> annual growth input for American International Group of 46.7 percent.[63] This means that Mr. McKenzie is indicating he believes that American International Group could grows its earnings / dividends by 46.7 percent per year, every year, over the long run. Recall that, as a general rule, the long-term growth rate for any U.S. company cannot exceed long-term growth in GDP, which is projected at about four percent. This means that Mr. McKenzie's long-term growth estimate for this company is over 10 times anything that could be considered realistic. Many of Mr. McKenzie's other growth rate estimates are similarly overstated. This causes his estimates for the ERP and CAPM cost of equity to be overstated as well.

Q: What is the impact of Mr. McKenzie's flawed ERP estimate?

A: Mr. McKenzie's overestimated ERP is considerably higher than the range of ERPs utilized by firms and analysts across the country. Because the ERP is not firm-specific, there are fairly standardized ERP levels that are widely recognized by several prominent national expert surveys. For example, the IESE Business School expert survey reports an average ERP of 5.7 percent, which is markedly higher than other objective sources, yet

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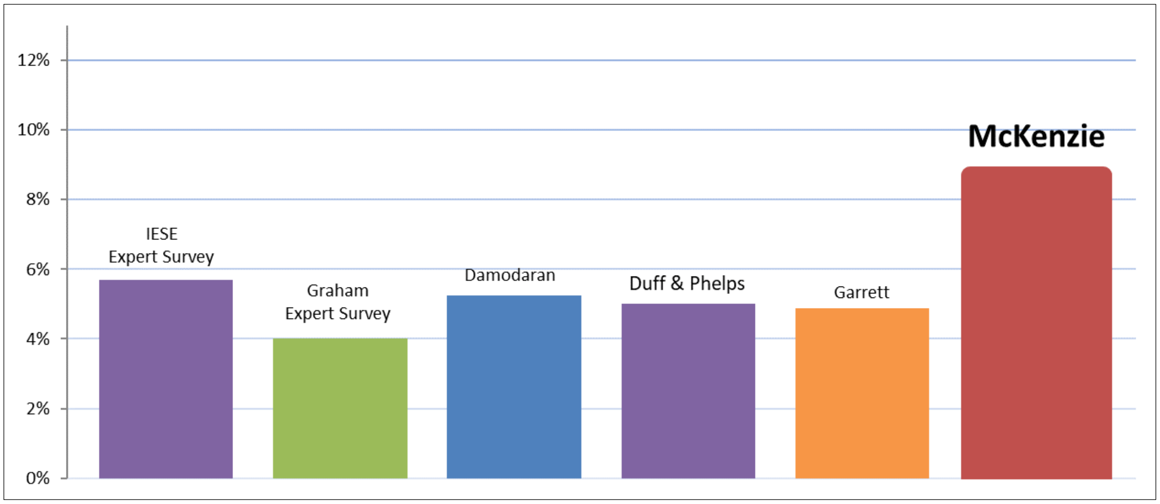
[63] Exh. DJG-21, Workpaper of Adrian M. McKenzie, WP-59.xlsm, Tab 2017 04 Market DCF.

still significantly lower than Mr. McKenzie's estimate. The following chart illustrates that Mr. McKenzie's ERP estimate is far out of line with objective sources and estimates:

Figure 9:
Equity Risk Premium Comparison



When compared with these well-established ERP benchmarks, it is clear that Mr. McKenzie's ERP estimate is not within the range of reasonableness. As a result, his CAPM cost of equity estimates are overstated.

Q: Did you also review Mr. McKenzie's Risk Premium Model?

A: Yes. Before I discuss Mr. McKenzie's risk premium model, I will reiterate that the CAPM itself is a "risk premium" model. In short, it takes the bare minimum return any investor would require for buying a stock (the risk-free rate), then adds a premium to compensate the investor for the extra risk he or he assumes by buying a stock rather than a riskless U.S. Treasury security. The CAPM has been utilized by companies around the world for decades for the same purpose we are using it in this case – to estimate cost of equity. When reasonable inputs are used in the CAPM, this model tends to produce cost of equity results for utility companies that are much lower than the excessive returns demanded by shareholders. Thus, utility witnesses often downplay the Nobel-Prize-winning CAPM and instead promote their own various risk-premium models.

In stark contrast to the CAPM, the risk premium models relied upon by utility witnesses are not market-based, and therefore have no value in helping us estimate the market-based cost of equity. Unlike the CAPM, which is found in almost every comprehensive financial textbook, the risk premium models used by utility witnesses are typically only found in texts written by other utility witnesses. Specifically, these risk premium models attempt to create an inappropriate link between market-based factors, such as interest rates, with awarded returns on equity. Inevitably, this type of model is used to justify a cost of equity that is much higher than one that would be dictated by market forces. In this case, Mr. McKenzie's risk premium model is no different.

Mr. McKenzie's risk premium model considers a comparison between awarded ROEs and bond yields, even though these two factors are not remotely connected. The legal standards governing this issue indicate that the awarded return on equity should be based on the cost of equity.[64] In turn, the cost of equity, as estimated through the CAPM, is driven by interest rates. Thus, the idea that the awarded ROE should be based on interest rates is already built into the CAPM, but <u>only if</u> regulators base the awarded ROE on the true cost of equity, which is about 7.0 percent in this case. Unfortunately, it is clear that for many years, awarded returns for utilities have escalated far above market-based cost of equity computations. Giving undue consideration to Mr. McKenzie's "risk

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[64] *Fed. Power Comm'n v. Hope Nat. Gas Co.*, 320 U.S. 591, 603 (1944) (emphasis added).

premium" model would only serve to perpetuate this trend, which has resulted in a significant wealth shift from ratepayers to shareholders for many years.

Q: Is it necessary to pursue modeling beyond the CAPM in order to assess risk premiums when determining an appropriately set cost of equity in this case?

A: No. Mr. McKenzie's risk premium model is not only inappropriate as presented, but it is also unnecessary. The CAPM already has a built-in risk premium factor known as the equity risk premium (ERP). Not only is the ERP a crucial factor in the CAPM, but many would agree that the ERP is "the single most important variable for making investment decisions. . . ."[65] Specifically, the ERP is the expected return on the market less the risk-free rate. In other words, the ERP is a function of market-driven forces. Unlike the risk premium presented in Mr. McKenzie's testimony, the ERP cannot be influenced by the decisions of a public utility commission. For that matter, it cannot be materially influenced by the decisions of any single company. Thus, the ERP has no material connection with the returns awarded to public utility companies in rate cases. This point is further strengthened by the expert surveys. Recall that the expert surveys ask thousands of experts across the country about the current ERP. When these experts are asked about the sources they relied on in giving their ERP estimate, it is not surprising that they make no mention of commission-awarded returns.[66] Moreover, many awarded returns arise out of settlements, which means that in complete contrast to the ERP, they

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[65] Exh. DJG-13, Dimson, Marsh & Staunton at 4.

[66] In the IESE Business School's 2014 survey, some of the respondents indicated which books, papers, and other sources they used as a reference to justify the equity risk premium that they used. The most cited references were Dr. Damodaran, Ibbotson, Duff & Phelps, Graham-Harvey, Bloomberg, Grabowski, Siegel, and other sources. Of course, there was no mention of commission-awarded returns.

are not reflective of market-driven forces. For all of these reasons, it is completely inappropriate to consider commission-awarded returns in any risk premium analysis. Thus, the Commission should disregard Mr. McKenzie's risk premium analysis.

IX. OTHER COST OF EQUITY ISSUES

Q: Are there any other issues raised in Mr. McKenzie's testimony to which you would like to respond?

A: Yes, in his direct testimony Mr. McKenzie raises several other issues in his testimony: (1) firm-specific risks; (2) size premium; (3) flotation costs; and (4) non-utility DCF Model. I will discuss each issue in turn.

A. Firm-Specific Risks

Q: Do you agree that the Company's firm-specific risk factors cited by Mr. McKenzie materially influence its cost of equity?

A: No. Recall that there are two primary types of risk: market risk, which affects all firms to varying degrees, and firm-specific risk, which affects individual firms. Mr. McKenzie suggests that certain firm-specific factors should have an increasing effect on the cost of equity, such as operating risks.[67] As discussed above, however, it is a well-known concept in finance that firm-specific risks are unrewarded by the market. This is because investors can easily eliminate firm-specific risks through portfolio diversification. Therefore, the Company's few and relatively small firm-specific business risks, while perhaps relevant to other issues in the rate case, have no meaningful effect on the cost of

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[67] *See generally* Direct Testimony of Adrien M. McKenzie, Exh. AMM-1T at 9-12.

equity estimate. Rather, it is market risk that is rewarded by the market, and this concept is thoroughly addressed in my CAPM analysis discussed above.

B. Size Premium

Q: Does a Company's relative size warrant a premium addition to the cost of equity estimate?

A: No. Mr. McKenzie suggests that Avista's cost of equity should be further inflated due to its relatively small size. Specifically, the average increase to Mr. McKenzie's CAPM due to his size adjustments is 70 basis points.[68] Utility cost of capital witnesses often refer to this as a "size premium." The size premium refers to the idea that the additional risk associated with smaller firms is not fully accounted for in their betas. The "size effect" phenomenon arose from a 1981 study conducted by Banz, which found that "in the 1936 – 1975 period, the common stock of small firms had, on average, higher risk-adjusted returns than the common stock of large firms."[69] According to Ibbotson, Banz's size effect study was "[o]ne of the most remarkable discoveries of modern finance."[70] Perhaps there was some merit to this idea at the time, but the size effect phenomenon was short lived. Banz's 1981 publication generated much interest in the size effect, and

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[68] *See* McKenzie, Exh. AMM-9.

[69] Exh. DJG-19 (Rolf W. Banz, *The Relationship Between Return and Market Value of Common Stocks* 3-18 (Journal of Financial Economics 9 (1981)).

[70] Exh. DJG-14 (2015 Ibbotson Stocks, Bonds, Bills, and Inflation Classic Yearbook 99 (Morningstar 2015)).

spurred the launch of significant new small-cap[71] investment funds. However, this "honeymoon period lasted for approximately two years." [72]

After 1983, U.S. small-cap stocks actually underperformed relative to large cap stocks. In other words, the size effect essentially reversed. In *Triumph of the Optimists,* the authors conducted an extensive empirical study of the size effect phenomenon around the world. They found that after the size effect phenomenon was discovered in 1981, it disappeared within a few years:

> It is clear . . . that there was a global reversal of the size effect in virtually every country, with the size premium not just disappearing but going into reverse. Researchers around the world universally fell victim to Murphy's Law, with the very effect they were documenting – and inventing explanations for – promptly reversing itself shortly after their studies were published.[73]

In other words, the authors assert that the very discovery of the size effect phenomenon likely caused its own demise. The authors ultimately concluded that it is "inappropriate to use the term 'size effect' to imply that we should automatically expect there to be a small-cap premium," Yet this is exactly what utility witnesses often do in attempting to artificially inflate the cost of equity with a size premium.

Other prominent sources have agreed that the size premium is no longer a relevant phenomenon. According to Ibbotson:

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[71] The term "small cap" refers to companies with an overall value of outstanding shares that is small relative to the rest of the market.

[72] Exh. DJG-13, Dimson, Marsh & Staunton at 131

[73] *Id*. at 133.

> The unpredictability of small-cap returns has given rise to another argument against the existence of a size premium: that markets have changed so that the size premium no longer exists. As evidence, one might observe the last 20 years of market data to see that the performance of large-cap stocks was basically equal to that of small cap stocks. In fact, large-cap stocks have outperformed small-cap stocks in five of the last 10 years.[74]

In addition to the studies discussed above, other scholars have concluded similar results. According to Kalesnik and Beck:

> Today, more than 30 years after the initial publication of Banz's paper, the empirical evidence is extremely weak even before adjusting for possible biases. . . . The U.S. long-term size premium is driven by the extreme outliers, which occurred three-quarters of a century ago. . . . Finally, adjusting for biases . . . makes the size premium vanish. If the size premium were discovered today, rather than in the 1980s, it would be challenging to even publish a paper documenting that small stocks outperform large ones.[75]

For all of these reasons, the Commission should reject the arbitrary size premium proposed by the Company.

C. Flotation Costs

Q: Please define "flotation costs," as referenced in Mr. McKenzie's testimony.

A: "Flotation costs" generally refer to the underwriter's compensation for the services it provides in connection with the securities offering.

Q: Describe Mr. McKenzie's opinion on this issue.

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[74] Exh. DJG-14, Ibbotson at 112.

[75] Exh. DJG-20, Vitali Kalesnik and Noah Beck, *Busting the Myth About Size* (Research Affiliates 2014), available at https://www.researchaffiliates.com/Our%20Ideas/Insights/Fundamentals/Pages/284_Busting_the_Myth_About_Size.aspx.

A: Mr. McKenzie suggests that flotation costs should have an increasing effect on the cost of equity of 10 basis points.[76]

Q: Do you agree with Mr. McKenzie's theory?

A: No. When companies issue equity securities, they typically hire at least one investment bank as an underwriter for the securities. The Commission should not allow recovery of flotation costs for the following reasons:

1. Flotation costs are not actual "out-of-pocket" costs.

Unlike the Company's other operating expenses, Avista does not actually pay for flotation costs. Instead, the underwriters used to facilitate the equity issuance are compensated through an "underwriting spread." An underwriting spread is the difference between the price at which the underwriter purchases the shares from the firm, and the price at which the underwriter sells the shares to investors.[77] Thus, Avista has not experienced any out-of-pocket flotation costs, and if it has, those costs should be included in the Company's expense schedules.

2. The market already accounts for flotation costs.

When an underwriter markets a firm's securities to investors, the investors are well aware of the underwriter's fees. In other words, the investors know that a portion of the price they are paying for the shares does not go directly to the company, but instead goes to compensate the underwriter for its services. In fact, federal law requires that the

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[76] *See* McKenzie, Exh. AMM-1T at 45:14-18.

[77] Exh. DJG-8, Graham, Smart & Megginson at 509.

underwriter's compensation be disclosed on the front page of the prospectus.[78] Thus, investors have already considered and accounted for flotation costs when making their decision to purchase shares at the quoted price. There is no need for the Company's shareholders to receive additional compensation to account for costs they have already considered and agreed to.

We see similar compensation structures in other kinds of business transactions. For example, a homeowner may hire a realtor and sell a home for $100,000. After the realtor takes a six percent commission, the seller nets $94,000. The buyer and seller agreed to the transaction notwithstanding the realtor's commission. Obviously, it would be unreasonable for the buyer or seller to demand additional funds from anyone after the deal is done to reimburse them for the realtor's fees. Likewise, investors of competitive firms do not expect additional compensation for flotation costs. Thus, it would not be appropriate for a commission standing in the place of competition to award a utility's investors with this additional compensation.

3. It is inappropriate to add any additional basis points to a cost of equity proposal that is already far above the true required return.

For the reasons discussed above, flotation costs should be disallowed from a technical standpoint; they should also be disallowed from a practical standpoint. Avista is asking this Commission to award it a cost of equity that is well over 200 basis points

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[78] *See* Regulation S-K, 17 C.F.R. § 229.501(b)(3) (requiring that the underwriter's discounts and commissions be disclosed on the outside cover page of the prospectus). A prospectus is a legal document that provides details about an investment offering.

above its true cost of equity. Under these circumstances, it is especially inappropriate to suggest that the effect of flotation costs should be considered in any way.

D. Non-Utility DCF Model

Q: Do you agree with the results of Mr. McKenzie's non-utility DCF Model?

A: No. Mr. McKenzie also conducted a DCF analysis on a group of non-utility firms. Cost of capital witnesses routinely conduct their analyses on a group of "proxy" companies that include regulated utilities. This practice likely stems from "corresponding risk" standard set forth by the *Hope* Court. That is, the risk inherent in the equity of competitive firms is simply not comparable to the risk inherent in the equity of regulated utilities. This is because the regulated utility industry is essentially the least risky industry in the entire country, as shown in Figure 4, discussed above. The beta term is used in the CAPM as an objective way to determine the impacts of market risk on an individual firm. Beta is calculated through linear regression analysis that considers the correlation between the returns on an individual stock with the returns on the market portfolio (i.e., all stocks). The betas for regulated utilities are decisively and consistently lower than the betas of competitive firms, which means that the stocks of regulated utilities are less risky than the stocks of competitive firms.

This is why cost of capital witnesses routinely conduct their DCF, CAPM, and comparable earnings analyses on a "proxy" group of <u>regulated utilities</u>, not unregulated, competitive firms – because competitive firms are simply not comparable to regulated utilities in terms of their risk profiles. Thus, Mr. McKenzie's analysis is based on a faulty premise and provides no accurate, fair, or reasonable indication of Avista's cost of equity in this case.

X. COST OF EQUITY SUMMARY

Q: Mr. Garrett, please summarize the results of the DCF and CAPM cost of equity models you presented in testimony.

A: The following table shows the cost of equity results from each of the models I employed in this case.

Figure 10:
Cost of Equity Summary[79]

Model	Cost of Equity
Discounted Cash Flow Model	7.2%
Capital Asset Pricing Model	6.8%
Average	**7.0%**

The average cost of equity of the DCF Model and the CAPM is 7.0 percent. Furthermore, it is noteworthy that these two models produced comparable results, especially considering the fact that the inputs for the two modes are completely different. Again, the DCF Model considers stock price, dividends, and a long-term growth rate. The CAPM considers the risk-free rate, beta, and the equity risk premium. These inputs

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[79] Exh. DJG-4 at 12.

are relatively unrelated to each other, and yet the models produced similar results.[80] This fact further highlights the validity of these two models, which have been relied upon by executives, analysts, academics, and regulators for decades to value companies and estimate cost of equity.

Q: What do you recommend for the awarded return on equity?

A: The Commission should strive to award a return on equity that reflects the market-based cost of equity. However, the awarded return must also consider broader ratemaking principles and be reasonable under the circumstances. The results of the financial models presented in this case indicate a cost of equity estimate of 7.0 percent. In the interest of achieving a gradual movement toward the appropriate market-based cost of equity, I recommend the Commission in this case adopt an awarded return on equity of nine percent, which is the midpoint in a range of reasonableness of 8.75 percent to 9.25 percent.

XI. CAPITAL STRUCTURE

Q: Describe, in general, the concept of a company's "capital structure."

A: "Capital structure" refers to the way a firm finances its overall operations through external sources. The primary sources of long-term, external financing are debt capital and equity capital. Debt capital usually comes in the form of contractual bond issues that require the firm make payments, while equity capital represents an ownership interest in the form of stock. Because a firm cannot pay dividends on common stock until it

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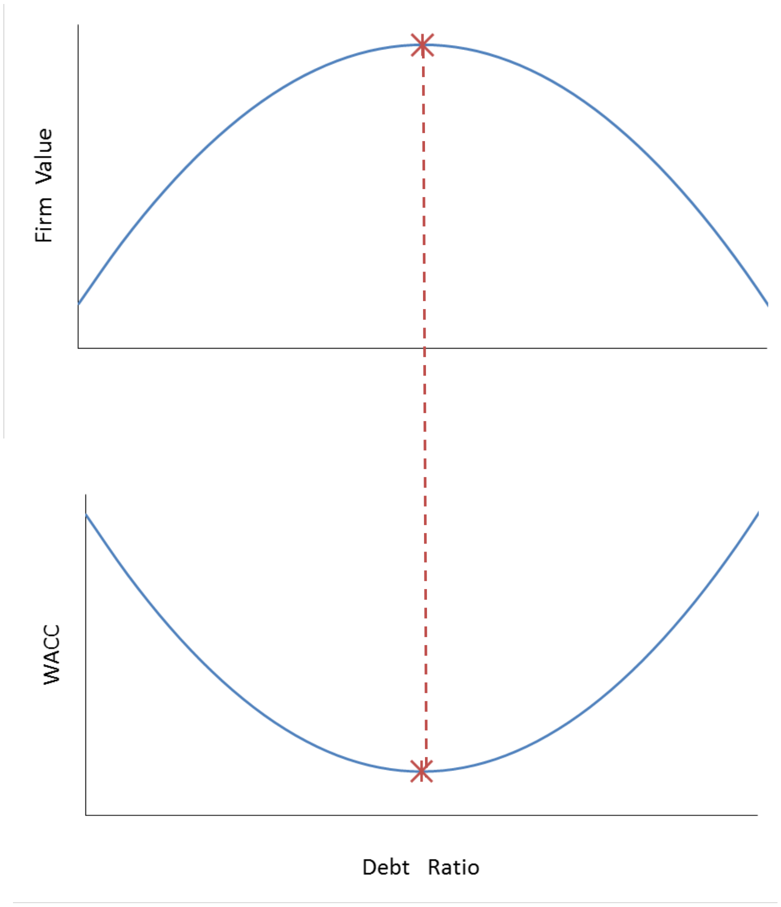
[80] These results also highlight the fact that the growth rate used in my DCF Model, nominal U.S. GDP growth, is a relatively high growth rate estimate for a utility company. Using a growth rate closer to the risk-free rate or Avista's projected load growth would have made the results of the DCF Model even closer to the CAPM.

satisfies its debt obligations to bondholders, stockholders are referred to as "residual claimants." The fact that stockholders have a lower priority to claims on company assets increases their risk and required return relative to bondholders. Thus, equity capital has a higher cost than debt capital. Firms can reduce their weighted average cost of capital (WACC) by recapitalizing and increasing their debt financing. In addition, because interest expense is tax deductible, increasing debt also adds value to the firm by reducing the firm's tax obligation.

Q: Is it true that by increasing debt, competitive firms can add value and reduce their WACC?

A: Yes. A competitive firm can add value by increasing debt. After a certain point, however, the marginal cost of additional debt outweighs its marginal benefit. This is because the more debt the firm uses, the higher interest expense it must pay, and the likelihood of loss increases. This increases the risk of recovery for both bondholders and shareholders, causing both groups of investors to demand a greater return on their investment. Thus, if debt financing is too high, the firm's WACC will increase instead of decrease. The following figure illustrates these concepts.

Figure 11: Optimal Debt Ratio



As shown in this figure, a competitive firm's value is maximized when the WACC is minimized. In both of these graphs, the debt ratio [D/(D+E)] is shown on the x-axis. By increasing its debt ratio, a competitive firm can minimize its WACC and maximize its value. At a certain point, however, the benefits of increasing debt do not outweigh the costs of the additional risks to both bondholders and shareholders, as each type of investor will demand higher returns for the additional risk they have assumed.[81]

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[81] Exh. DJG-8, Graham, Smart & Megginson at 440-41.

Q: Does the rate base rate of return model effectively incentivize utilities to operate at the optimal capital structure?

A: No. While it is true that competitive firms maximize their value by minimizing their WACC, this is not the case for regulated utilities. Under the rate base rate of return model, a higher WACC results in higher rates, all else held constant. The basic revenue requirement equation is as follows:

Equation 12:
Revenue Requirement for Regulated Utilities

$$RR = O + d + T + \boldsymbol{r}(A - D)$$

where: RR = *revenue requirement*
O = *operating expenses*
d = *depreciation expense*
T = *corporate tax*
$\boldsymbol{r}$ = ***weighted average cost of capital (WACC)***
A = *plant investments*
D = *accumulated depreciation*

As shown in this equation, utilities can increase their revenue requirement by <u>increasing</u> their WACC, not by minimizing it. Thus, because there is no incentive for a regulated utility to minimize its WACC, a Commission standing in the place of competition must ensure that the regulated utility is operating at the lowest reasonable WACC.

Q: Do you believe that, generally speaking, utilities can afford to have higher debt levels than other industries?

A: Yes. Because regulated utilities have large amounts of fixed assets, stable earnings, and low risk relative to other industries, they can afford to have higher debt ratios (or "leverage"). As aptly stated by Dr. Damodaran:

> Since financial leverage multiplies the underlying business risk, it stands to reason that firms that have high business risk should be reluctant to take on financial leverage. It also stands to reason that firms that operate in stable businesses should be much more willing to take on financial leverage. <u>Utilities</u>, for instance, have historically had high debt ratios but have not had high betas, mostly because their underlying businesses have been stable and fairly predictable.[82]

Note in the passage above that the author explicitly contrasts utilities with firms that have high underlying business risk. Because utilities have low levels risk and operate a stable business, they should generally operate with relatively high levels of debt to achieve their optimal capital structure. There are objective methods available to estimate the optimal capital structure, as discussed further below.

Q: Is it appropriate to solely consider the capital structures of the proxy group in assessing a prudent capital structure?

A: No. Utility witnesses often argue that regulators should consider only the capital structures of other regulated utilities in assessing the proper capital structure. This type of analysis is oversimplified and insufficient for three important reasons:

1. <u>Utilities do not have a financial incentive to operate at the optimal capital structure</u>. Under the rate base rate of return model, utilities do not have a natural financial incentive to minimize their cost of capital; in fact, they have a financial incentive to do the opposite. Competitive firms, in contrast, can maximize their value by minimizing their cost of capital. Competitive firms minimize their cost of capital by including a sufficient amount of debt in their capital structures. Simply comparing the debt ratios of other regulated utilities will not indicate an appropriate capital structure for the Company.

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[82] Exh. DJG-6, Damodaran at 196 (emphasis added).

Rather, it is likely to justify debt ratios that are far too low. It is the Commission's role to act as a surrogate for competition and thereby ensure that the capital structure of a regulated monopoly is similar to what would be appropriate in a <u>competitive</u> environment, not a regulated environment. This cannot be accomplished by simply looking at the capital structures of other regulated utilities or the target utility's test-year capital structure.

2. <u>The optimal capital structure is unique to each firm</u>.

As discussed further below, the optimal capital structure for a firm is dependent on several unique financial metrics for that firm. The other companies in the proxy group have different financial metrics than the target utility, and thus have different optimal capital structures. An objective analysis should be performed using the financial metrics of the target utility in order to estimate its unique optimal capital structure.

3. <u>The capital structures of the proxy group may not have been approved by their regulatory commissions</u>.

The actual capital structure of any utility falls within the realm of managerial discretion. Regulatory commissions, however, have a duty to impute a proper capital structure if the company's actual capital structure is inappropriate. Thus, the actual capital structures of other utilities may have been deemed inappropriate by their own commission. For all of the foregoing reasons, simply comparing the capital structures of other regulated utilities has no place in a proper capital structure analysis.

Q: Describe an objective approach to estimating a firm's optimal capital structure.

A: My analysis of the optimal capital structure includes objective methods to measure the effects of increasing debt on both the cost of debt and cost of equity. I will discuss the effects of increasing the debt ratio on each type of security separately.

Cost of Debt

As discussed above, increasing the debt ratio will increase the cost of debt. To objectively measure how much the cost of debt increases, I considered the spreads above the risk-free rate for various levels of bond ratings and interest coverage ratios. The following table shows increasing interest rates for debt based on different bond rating levels.

Figure 12:
Bond Rating Spreads

Ratings Table

Coverage Ratio	Bond Rating	Spread	Interest Rate
> 8.5	Aaa/AAA	0.75%	3.52%
6.5 - 8.49	Aa2/AA	1.00%	3.77%
5.5 - 6.49	A1/A+	1.10%	3.87%
4.25 - 5.49	A2/A	1.25%	4.02%
3.0 - 4.24	A3/A-	1.75%	4.52%
2.5 - 2.99	Baa2/BBB	2.25%	5.02%
2.25 - 2.49	Ba1/BB+	3.25%	6.02%
2.0 - 2.249	Ba2/BB	4.25%	7.02%
1.75 - 1.99	B1/B+	5.50%	8.27%
1.5 - 1.74	B2/B	6.50%	9.27%
1.25 - 1.49	B3/B-	7.50%	10.27%
0.8 - 1.249	Caa/CCC	9.00%	11.77%

As shown in this table, the spreads over the risk-free rate gradually increase as bond ratings fall.[83] The spread is added to the risk-free rate to obtain the interest rates shown in the far right column. This concept is somewhat comparable to the interest rate a mortgage lender would charge a borrower. The mortgage lender's advertised rate is usually the lowest rate, or the "prime" rate, which is available to borrowers with stellar credit scores. As credit scores decrease, however, the offered interest rate will increase. The bond ratings in this figure are based on various levels of interest coverage ratios shown in the far left column. The interest coverage ratio, as its name implies, is a metric used by financial analysts to gauge a firm's ability to pay its interest expense from its available earnings before interest and taxes (EBIT). (Likewise, the mortgage lender would consider the borrower's personal income-debt ratio). The formula for the interest coverage ratio is as follows:

Equation 13:
Interest Coverage Ratio

$$\frac{Earnings\ before\ Interest\ and\ Taxes}{Interest\ Expense}$$

As the debt ratio rises, the interest coverage ratio falls, the bond ratings increase, and the cost of debt increases. Now that we have an objective way of measuring how increasing the debt ratio affects the cost of debt, we need to measure how increasing the debt ratio affects the cost of equity.

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[83] The link between interest coverage ratios and ratings was developed by looking at all rated companies in the U.S. The default spreads are obtained from traded bonds. The spreads are added to the risk-free rate to obtain the interest rates in the table. http://pages.stern.nyu.edu/~adamodar/New_Home_Page/datafile/ratings.htm.

Cost of Equity

As with the cost of debt, increasing the debt ratio also increases the cost of equity. To objectively measure how much the cost of equity increases, I first calculated the Company's unlevered beta. The unlevered beta is determined by the assets owned by the firm, and removes the effects of financial leverage. As leverage increases, equity investors bear increasing amounts of risk, leading to higher betas. Before the effects of financial leverage can be accounted for, however, the effects of leverage must first be removed, which is accomplished through the unlevered beta equation:[84]

Equation 14:
Unlevered Beta

$$\beta_U = \frac{\beta_L}{\left[1 + (1 - T_c)\left(\frac{D}{E}\right)\right]}$$

where: β_U = *unlevered beta (or "asset" beta)*
β_L = *average levered beta of proxy group*
T_C = *corporate tax rate*
D = *book value of debt*
E = *book value of equity*

Using this equation, the beta for the firm can be unlevered, and then "re-levered" based on various debt ratios (by rearranging this equation to solve for β_L). So, by using the Bond Rating Spreads table and the unlevered beta equation, the costs of both debt and equity can be increased in correspondence with increasing the debt ratio, until the ideal capital structure is found: where the weighted average cost of capital is minimized.

Q: Describe Avista's optimal capital structure.

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[84] Exh. DJG-6, Damodaran at 197. This formula was originally developed by Hamada in 1972.

A: The Company proposes a debt ratio of 50 percent in this case. I analyzed the Company's optimal capital structure based on the approach discussed above. The following table presents different levels of Avista's weighted average cost of capital (WACC) based on increasing debt ratios.

Figure 13:
Avista's WACC at Various Debt Ratios

Debt Ratio	Levered Beta	True Cost of Equity	Awarded ROE	Coverage Ratio	After-tax Debt Cost	Optimal WACC	WACC at 9.9% ROE
0%	0.426	5.24%	9.90%	∞	2.22%	5.24%	9.90%
20%	0.495	5.63%	9.90%	7.74	2.35%	4.97%	8.39%
30%	0.545	5.91%	9.90%	5.16	2.54%	4.90%	7.69%
40%	0.611	6.29%	9.90%	3.87	2.64%	4.83%	7.00%
50%	0.703	6.82%	9.90%	3.10	2.64%	4.73%	6.27%
60%	0.842	7.61%	9.90%	2.58	2.87%	4.76%	5.68%
70%	1.073	8.92%	9.90%	2.21	3.45%	5.09%	5.39%

In the figure above, the column on the far left shows increasing levels of debt ratios. At a debt ratio of zero percent, the utility's beta is completely unlevered. As the debt ratio in the far left column increases, both the cost of equity and the cost of debt increase; however, the weighted average cost of capital decreases. This table indicates that if we rely on the true cost of equity (about seven percent), the Company's proposed debt ratio of only 50 percent appears reasonable. However, the Company is requesting a 9.9 percent awarded return. If this request is factored into the equation (in the fourth column from the left), then the weighted average cost of equity is minimized at a much higher debt ratio – near 70 percent. I am not suggesting that the Commission should impute a capital structure consisting of 70 percent debt. However, as with its other costs, the Company has a duty to seek the lowest reasonable capital costs, which includes the weighted average cost of capital. In that regard, the Company's request of a 9.9 percent awarded ROE <u>and</u> a debt ratio of only 50 percent is patently unreasonable. Even at an awarded ROE of nine percent (my recommendation), the Company's optimal debt ratio is likely around 60 percent.

Q: Is your opinion based in part on the fact that hundreds of competitive firms around the country utilize high debt ratios in order to maximize profits?

A: Yes. In fact, there are currently more than 1,000 firms across the country with debt ratios of 60 percent or greater, with an average debt ratio of 68 percent, as shown in the following figure:[85]

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[85] Exh. DJG-4 at 16.

Figure 14:
Industries with Debt Ratios of 60% or Greater

Industry	Number of Fimrs	Debt Ratio
Advertising	44	73%
Air Transport	20	57%
Auto & Truck	19	74%
Bank (Money Center)	9	67%
Beverage (Soft)	43	64%
Broadcasting	29	68%
Brokerage & Investment Banking	42	77%
Cable TV	19	69%
Coal & Related Energy	38	69%
Hospitals/Healthcare Facilities	58	66%
Hotel/Gaming	73	61%
Office Equipment & Services	24	67%
Packaging & Container	25	63%
Paper/Forest Products	20	74%
R.E.I.T.	221	64%
Restaurant/Dining	83	61%
Retail (Automotive)	26	70%
Retail (Building Supply)	5	67%
Retail (Distributors)	83	60%
Telecom (Wireless)	19	61%
Telecom. Services	65	65%
Tobacco	20	85%
Trucking	26	74%
Total / Average	**1011**	**68%**

Many of the industries shown here, like public utilities, are generally well-established industries with large amounts of capital assets. These shareholders of these industries demand higher debt ratios in order to maximize their profits. There are several notable industries that are relatively comparable to public utilities in some ways. For example, the Cable TV industry has an average debt ratio of about 69 percent. Likewise, the telecommunication services industry has a debt ratio of 65 percent.

Q. What is your recommendation regarding the Company's capital structure?

A. Despite evidence presented above indicating that the Company's most prudent debt ratio could be as high as 60%, I recommend the Commission adopt the Company's test year debt ratio of 51.5%. Avista has requested a hypothetical debt ratio of only 50% debt. The Company's request is primarily based on the debt ratios of other regulated utilities. As discussed above, it is not appropriate to base the analysis of a prudent capital structure on the actual capital structures of other utilities because (1) utilities do not have an incentive to operate with capital structures that result in minimized capital costs; (2) the optimal capital structure is unique to each firm; and (3) the capital structures of the proxy group many not have been approved by their respective commissions. Finally, my analysis presented above reveals that a debt ratio as low as 50% would require a much lower corresponding ROE than is being proposed in this case. For all of these reasons, the only hypothetical capital structure in this case that would be appropriate is one that consists of more debt, not less debt. Therefore, the Commission should accept the Company's test year capital structure consisting of 51.5% debt and 48.5% equity.

XII. CONCLUSION AND RECOMMENDATION

Q: Summarize the key points of your testimony.

A: The key points of my testimony are summarized as follows:

1. The legal standards governing this issue are clear that the awarded rate of return should be based on the Company's cost of capital.

2. When the awarded rate of return exceeds the actual cost of capital, it results in an inappropriate transfer of excess wealth from customers to shareholders.

3. The models I used in this case indicate the Company's cost of equity is about 7.0%. However, under prudent ratemaking principles, the Commission should award Avista's shareholders with a return on equity of 9.0%, which is within a reasonable range of 8.75% - 9.25%. Although we must move awarded returns

toward true cost of equity, we should do so gradually rather than abruptly to avoid volatility within the industry.

Q: Does this conclude your testimony?

A: Yes, including any exhibits, appendices, and other items attached hereto.

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EDUCATION

University of Oklahoma	Norman, OK
Master of Business Administration	2014
Areas of Concentration: Finance, Energy	
University of Oklahoma College of Law	Norman, OK
Juris Doctor	2007
Member, American Indian Law Review	
University of Oklahoma	Norman, OK
Bachelor of Business Administration	2003
Major: Finance	

PROFESSIONAL DESIGNATIONS

Society of Depreciation Professionals
Certified Depreciation Professional (CDP)

Society of Utility and Regulatory Financial Analysts
Certified Rate of Return Analyst (CRRA)

The Mediation Institute
Certified Civil / Commercial & Employment Mediator

WORK EXPERIENCE

Resolve Utility Consulting PLLC	Oklahoma City, OK
Managing Member	2016 – Present

Provide expert analysis and testimony specializing in depreciation and cost of capital issues for clients in utility regulatory proceedings.

Oklahoma Corporation Commission	Oklahoma City, OK
Public Utility Regulatory Analyst	2012 – 2016
Assistant General Counsel	2011 – 2012

Represented commission staff in utility regulatory proceedings and provided legal opinions to commissioners. Provided expert analysis and testimony in depreciation, cost of capital, incentive compensation, payroll and other issues.

Perebus Counsel, PLLC Oklahoma City, OK
Managing Member 2009 – 2011
Represented clients in the areas of family law, estate planning, debt negotiations, business organization, and utility regulation.

Moricoli & Schovanec, P.C. Oklahoma City, OK
Associate Attorney 2007 – 2009
Represented clients in the areas of contracts, oil and gas, business structures and estate administration.

TEACHING EXPERIENCE

University of Oklahoma Norman, OK
Adjunct Instructor – "Conflict Resolution" 2014 – Present
Adjunct Instructor – "Ethics in Leadership"

Rose State College Midwest City, OK
Adjunct Instructor – "Legal Research" 2013 – 2015
Adjunct Instructor – "Oil & Gas Law"

PUBLICATIONS

American Indian Law Review Norman, OK
"Vine of the Dead: Reviving Equal Protection Rites for Religious Drug Use" 2006
(31 Am. Indian L. Rev. 143)

VOLUNTEER EXPERIENCE

Calm Waters Oklahoma City, OK
Board Member 2015 – Present
Participate in management of operations, attend meetings, review performance, compensation, and financial records. Assist in fundraising events.

Group Facilitator & Fundraiser 2014 – Present
Facilitate group meetings designed to help children and families cope with divorce and tragic events. Assist in fundraising events.

St. Jude Children's Research Hospital Oklahoma City, OK
Oklahoma Fundraising Committee 2008 – 2010
Raised money for charity by organizing local fundraising events.

PROFESSIONAL ASSOCIATIONS

Oklahoma Bar Association 2007 – Present

Society of Depreciation Professionals 2014 – Present
Board Member – President 2017
Participate in management of operations, attend meetings, review performance, organize presentation agenda.

Society of Utility Regulatory Financial Analysts 2014 – Present

SELECTED CONTINUING PROFESSIONAL EDUCATION

Society of Depreciation Professionals — Austin, TX
"Life and Net Salvage Analysis" — 2015
Extensive instruction on utility depreciation, including actuarial and simulation life analysis modes, gross salvage, cost of removal, life cycle analysis, and technology forecasting.

Society of Depreciation Professionals — New Orleans, LA
"Introduction to Depreciation" and "Extended Training" — 2014
Extensive instruction on utility depreciation, including average lives and net salvage.

Society of Utility and Regulatory Financial Analysts — Indianapolis, IN
46th Financial Forum. "The Regulatory Compact: Is it Still Relevant?" — 2014
Forum discussions on current issues.

New Mexico State University, Center for Public Utilities — Santa Fe, NM
Current Issues 2012, "The Santa Fe Conference" — 2012
Forum discussions on various current issues in utility regulation.

Michigan State University, Institute of Public Utilities — Clearwater, FL
"39th Eastern NARUC Utility Rate School" — 2011
One-week, hands-on training emphasizing the fundamentals of the utility ratemaking process.

New Mexico State University, Center for Public Utilities — Albuquerque, NM
"The Basics: Practical Regulatory Training for the Changing Electric Industries" — 2010
One-week, hands-on training designed to provide a solid foundation in core areas of utility ratemaking.

The Mediation Institute — Oklahoma City, OK
"Civil / Commercial & Employment Mediation Training" — 2009
Extensive instruction and mock mediations designed to build foundations in conducting mediations in civil matters.

Utility Regulatory Proceedings

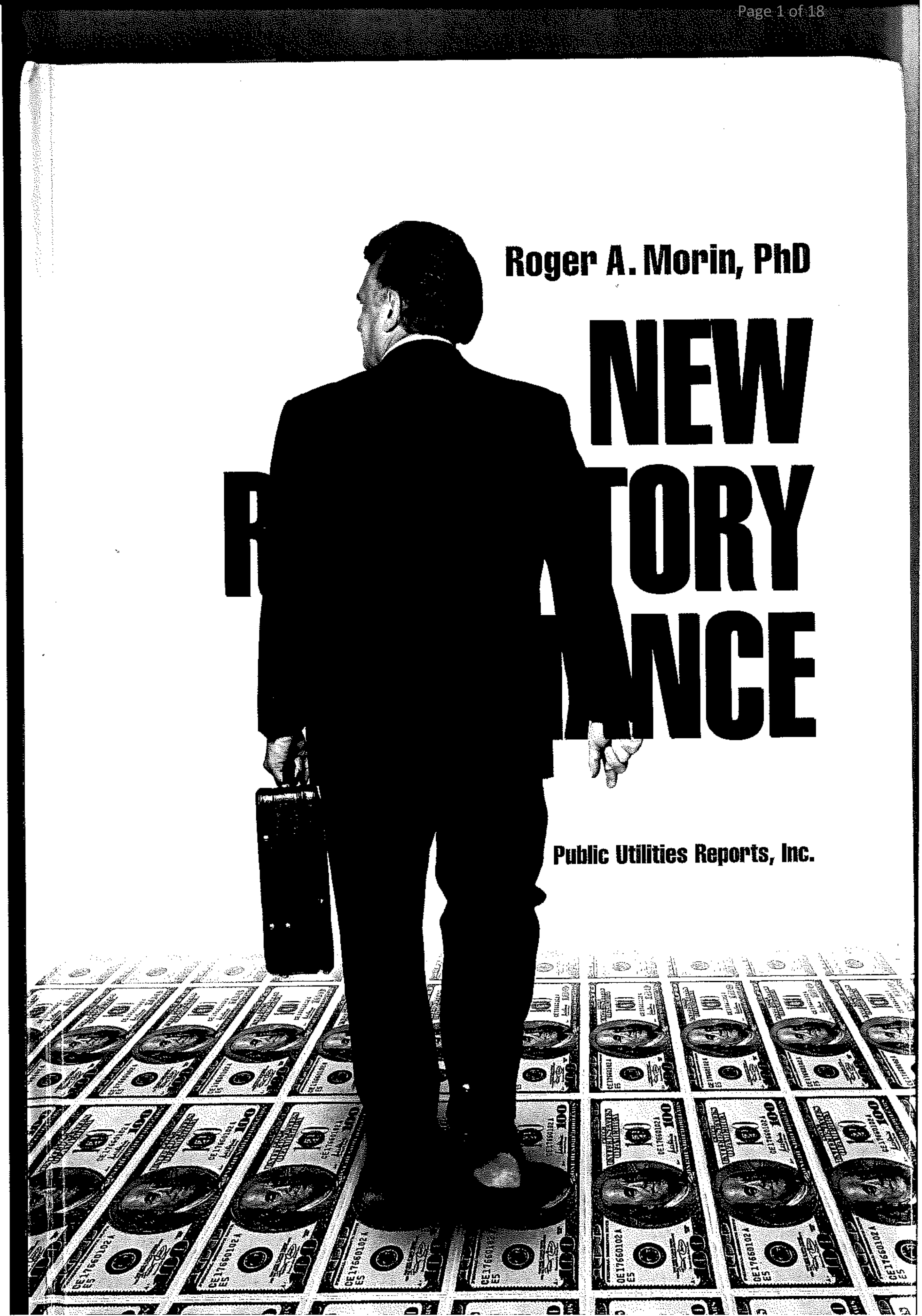
State	Regulatory Agency / Company-Applicant	Docket Number	Testimony / Analysis Issues	Type	Date
TX	Railroad Commission of Texas Atmos Pipeline - Texas	GUD 10580	Depreciation rates, depreciation grouping procedure	Prefiled	3/22/2017
TX	Public Utility Commission of Texas Sharyland Utility Co.	PUC 45414	Depreciation rates, simulated and actuarial analysis	Prefiled	2/28/2017
OK	Oklahoma Corporation Commission Empire District Electric Co.	PUD 201600468	Cost of capital, depreciation rates, terminal salvage, lifespans	Prefiled	3/13/2017
TX	Railroad Commission of Texas CenterPoint Energy Texas Gas	GUD 10567	Depreciation rates, simulated and actuarial analysis	Prefiled	2/21/2017
AR	Arkansas Public Service Commission Oklahoma Gas & Electric Co.	160-159-GU	Cost of capital, depreciation rates, terminal salvage, lifespans	Prefiled	1/31/2017
FL	Florida Public Service Commission Peoples Gas	160-159-GU	Depreciation rates	Report	11/4/2016
AZ	Arizona Corporation Commission Arizona Public Service Co.	E-01345A-16-0036	Cost of capital, depreciation rates, terminal salvage, lifespans	Pre-filed	12/28/2016
NV	Nevada Public Utilities Commission Sierra Pacific Power Co.	16-06008	Depreciation rates, terminal salvage, lifespans, theoretical reserve	Pre-filed	9/23/2016
OK	Oklahoma Corporation Commission Oklahoma Gas & Electric Co.	PUD 201500273	Cost of capital, depreciation rates, terminal salvage, lifespans	Pre-filed Live	3/21/2016 5/3/2016
OK	Oklahoma Corporation Commission Public Service Co. of Oklahoma	PUD 201500208	Cost of capital, depreciation rates, terminal salvage, lifespans	Pre-filed Live	10/14/2015 12/8/2015
OK	Oklahoma Corporation Commission Oklahoma Natural Gas Co.	PUD 201500213	Cost of capital and depreciation rates	Pre-filed	10/19/2015

Utility Regulatory Proceedings

State	Regulatory Agency / Company-Applicant	Docket Number	Testimony / Analysis Issues	Type	Date
OK	Oklahoma Corporation Commission Oak Hills Water System	PUD 201500123	Cost of capital and depreciation rates	Pre-filed Live	7/8/2015 8/14/2015
OK	Oklahoma Corporation Commission CenterPoint Energy Oklahoma Gas	PUD 201400227	Fuel prudence review and fuel adjustment clause	Pre-filed Live	11/3/2014 2/10/2015
OK	Oklahoma Corporation Commission Public Service Co. of Oklahoma	PUD 201400233	Certificate of authority to issue new debt securities	Pre-filed Live	9/12/2014 9/25/2014
OK	Oklahoma Corporation Commission Empire District Electric Co.	PUD 201400226	Fuel prudence review and fuel adjustment clause	Pre-filed Live	12/9/2014 1/22/2015
OK	Oklahoma Corporation Commission Fort Cobb Fuel Authority	PUD 201400219	Fuel prudence review and fuel adjustment clause	Pre-filed Live	 1/29/2015
OK	Oklahoma Corporation Commission Fort Cobb Fuel Authority	PUD 201400140	Outside services, legislative advocacy, payroll expense, and insurance expense	Pre-filed	12/16/2014
OK	Oklahoma Corporation Commission Public Service Co. of Oklahoma	PUD 201300201	Authorization of standby and supplemental tariff	Pre-filed Live	12/9/2013 12/19/2013
OK	Oklahoma Corporation Commission Fort Cobb Fuel Authority	PUD 201300134	Fuel prudence review and fuel adjustment clause	Pre-filed Live	10/23/2013 1/30/2014
OK	Oklahoma Corporation Commission Empire District Electric Co.	PUD 201300131	Fuel prudence review and fuel adjustment clause	Pre-filed Live	11/21/2013 12/19/2013
OK	Oklahoma Corporation Commission CenterPoint Energy Oklahoma Gas	PUD 201300127	Fuel prudence review and fuel adjustment clause	Pre-filed Live	10/21/2013 1/23/2014
OK	Oklahoma Corporation Commission	PUD 201200185	Gas transportation contract extension	Pre-filed	9/20/2012

Utility Regulatory Proceedings

State	Regulatory Agency / Company-Applicant	Docket Number	Testimony / Analysis Issues	Type	Date
	Oklahoma Gas & Electric Co.			Live	10/9/2012
OK	Oklahoma Corporation Commission Empire District Electric Co.	PUD 201200170	Fuel prudence review and fuel adjustment clause	Pre-filed Live	10/31/2012 12/13/2012
OK	Oklahoma Corporation Commission Oklahoma Gas & Electric Co.	PUD 201200169	Fuel prudence review and fuel adjustment clause	Pre-filed Live	12/19/2012 4/4/2013



Roger A. Morin, PhD

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Public Utilities Reports, Inc.

supply side of capital markets, but also by reference to the demand side of the capital markets.

The demand side viewpoint recognizes that regulated utilities are private corporations with shareholders-owners, and that management's principal responsibility is to maximize their well-being, as measured by stock price. Thus, only those investment decisions that maximize the price of the stock should be undertaken. A utility company will continue to invest in real physical assets if the return on these investments exceeds or equals its cost of capital. The cost of capital is the minimum rate of return that must be earned on assets to justify their acquisition, and the regulator must set the allowed return so that optimal investment rates are obtained, and that no other investment rate would result in a higher share price.

In this context, the cost of capital is the expected earnings on the utility's investments that are required in order for the value of the previously invested capital to remain unchanged. If new capital does not earn its price or required rate of return, the value of existing equity has to make up the difference. If the new capital earns a return greater than its price, existing shareholders will participate in the difference. The converse is true as well. If earnings on the investment of capital meet the required rate of return, existing shareholders will neither gain nor lose.

Cost of Capital = Required Rate of Return
= Required Earnings / Capital Invested

1.8 The Allowed Rate of Return and Cost of Capital

The regulator should set the allowed rate of return equal to the cost of capital so that the utility can achieve the optimal rate of investment at the minimum price to the ratepayers. This can be demonstrated as follows.

In Example 1-2 shown earlier, a utility with a rate base of $900 million was considered, financed 60% by debt and 40% by equity. The cost of capital was estimated at 8.2%. Now, suppose the regulator sets the allowed return at 6% instead. To service the claims of both the bondholders and shareholders, earnings over costs should amount to $73.8 million, that is, 8.2% × $900 million.

If the utility is allowed a return of only 6% on a rate base of $900 million, earnings of only $54.0 million are produced. While the earnings are sufficient to cover the interest payments of $37.8 million ($900 × .60 × 7%) to the bondholders who have a prior claim on earnings, they are not enough to cover the claims of shareholders in the amount of $36 million ($900 × .40 ×

10%). The stock price has to fall to a level such that an investor who purchases the stock after the price reduction will just obtain his opportunity cost. If the utility nevertheless undertakes mandatory capital investments that are allowed to earn 6%, while the cost of the funds is 8.2%, the inevitable result is a reduction in stock price and a wealth transfer from shareholders to ratepayers.

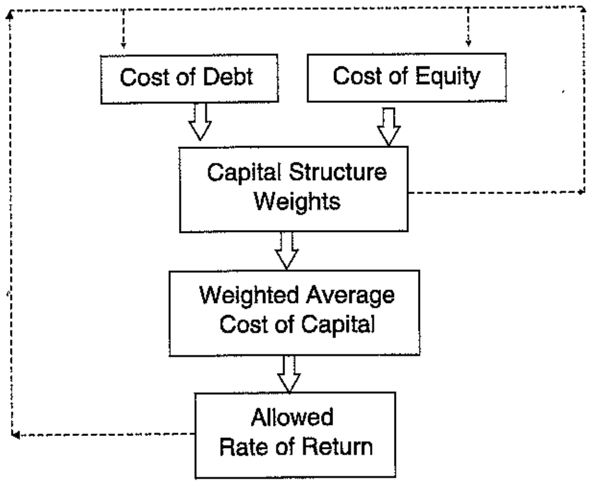
Conversely, if the allowed rate of return is greater than the cost of capital, capital investments are undertaken and investors' opportunity costs are more than achieved. Any excess earnings over and above those required to service debt capital accrue to the equity holders, and the stock price increases. In this case, the wealth transfer occurs from ratepayers to shareholders.

Investments are undertaken by the utility with no wealth transfer between ratepayers and shareholders only if the allowed rate of return is set equal to the cost of capital. In this case, the expected earnings generated from investments are just sufficient to service the claims of the debt and equity holders, no more no less. Setting the allowed return equal to the cost of capital is the only policy that will produce optimal investment rates at the minimum price to the ratepayer.

1.9 Determining the Cost of Capital

The general procedure that has evolved for determining the allowed rate of return is schematically depicted in Figure 1-6. The cost of debt and common equity are first determined separately, then weighted by the proportions of

FIGURE 1-6
SCHEMATIC REPRESENTATION OF COST OF CAPITAL DETERMINATION



enunciated in the *Bluefield* and *Hope* cases. Some of the techniques treat risk explicitly and directly as a separate variable in the model; others treat risk implicitly and indirectly as somehow subsumed in security prices. These techniques are summarized in Figure 1-7.

1.10 The Use of Multiple Methods in Cost of Equity Determination

The court cases discussed previously indicated that there are no specific rules or infallible models for determining a fair rate of return. It is dangerous and inappropriate to rely on only one methodology in determining the cost of equity. The results from only one method are likely to contain a high degree of measurement error. The regulator's hands should not be bound to one methodology of estimating equity costs, nor should the regulator ignore relevant evidence and back itself into a corner. For instance, by relying solely on the DCF model at a time when the fundamental assumptions underlying the DCF model are tenuous, a regulatory body greatly limits its flexibility and increases the risk of authorizing unreasonable rates of return. The same is true for any one specific model.

There are four generic methodologies available to measure the cost of equity: DCF, Risk Premium, and CAPM, which are market-oriented, and Comparable Earnings, which is accounting-oriented. Each generic market-based methodology in turn contains several variants.

When measuring equity costs, which essentially deals with the measurement of investor expectations, no one single methodology provides a foolproof panacea. Each methodology requires the exercise of considerable judgment on the reasonableness of the assumptions underlying the methodology and on the reasonableness of the proxies used to validate the theory. It follows that more than one methodology should be employed in arriving at a judgment on the cost of equity and that these methodologies should be applied across a series of comparable risk companies. More on this issue in Chapter 15.

The concept of cost of capital described in this chapter can be succinctly summarized as follows: A regulated utility should be entitled to a return that allows it to raise the necessary capital to meet service demand without cost to existing shareholders. This return is the weighted average of the embedded cost of debt and preferred capital, and a return on the common equity capital equal to the currently required return on equity. The two principal problems in implementing the approach are the determination of the appropriate set of capital structure weights and the estimation of the required return on equity. The optimal capital structure issue is treated in Chapters 16, 17, and 18.

these two years excluded. It is clear from this example that a long time period is required to accurately estimate the equity risk premium. The shorter 30-year period places too much emphasis on the poor market performances of 1973–1974. In fact, the equity risk premium recovers significantly in more recent periods once the years 1973 and 1974 are truncated from the analysis, as seen in the rolling 20-year and 10-year Ibbotson data.

Some analysts employ a rolling average approach. For example, the analyst arbitrarily assumes a given time frame over which the equity risk premium should be calculated, say 30 years, and calculates a 30-year equity risk premium for all time periods from 1926 to the present. There is a premium for 1926–1955, 1927–1956, and so on to the present. The successive premiums are averaged to arrive at the eventual equity risk premium. This approach is highly suspect because it overweighs the middle years. In the example, the year 1926 appears in one 30-year average, 1927 in two 30-year averages, etc. Yet, the most current (and relevant) time period only appears once. The middle periods are given an inordinate amount of weight using this approach. The other fallacy of the approach is that it assumes that a 30-year period is an appropriate historical window over which to estimate the equity risk premium. This assumption is highly arbitrary.

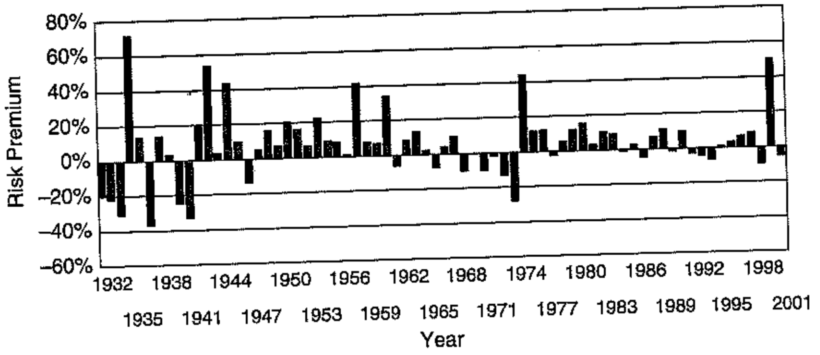
While forward-looking risk premiums based on expected returns are preferable, historical return studies over long periods still provide a useful guide for the future. This is because over long periods, investors' expectations are eventually revised to match historical realizations, as market prices adjust to match anticipated and actual investment results. Otherwise, investors would never commit investment capital. In the long run, the difference between expected and realized risk premiums will decline because short-run periods during which investors earn a lower risk premium than they expect are offset by short-run periods during which investors earn a higher risk premium than they expect. Second, the investors' current expectations concerning the amount by which the return on equity will exceed the bond yield will be strongly influenced by historical differences in returns to bond and stock investors. For these reasons, we can estimate investors' current expected returns from an equity investment from knowledge of current bond yields and past differences between returns on stocks and bonds.

Computational Issues: Arithmetic vs Geometric Average

The second problem in relying on historical return results is the method of averaging historical returns, that is, whether to use the ordinary average (arithmetic mean) or the geometric mean return. Because valuation is forward-looking, the appropriate average is the one that most accurately approximates the expected future rate of return. The best estimate of expected returns over a given future holding period is the arithmetic average. Only arithmetic means are correct for forecasting purposes and for estimating the cost of capital. There is no theoretical or empirical justification for the use of geometric mean rates of returns as a measure of the appropriate discount rate in computing the cost of capital or in computing present values. There is no dispute in academic circles as to whether the arithmetic or geometric average should be used for purposes of computing the cost of capital. The arithmetic mean should always be used in calculating the present value of a cash flow stream. Appendix A contains a comprehensive discussion of this issue, including the underlying theory, empirical evidence, and formal demonstrations.

Drawn from an actual rate case, the implementation of the historical Risk Premium approach is illustrated in Example 4-1 for the electric utility industry. Over the long term, realized utility equity risk premiums were 5.6% above Treasury bond yields for electric utilities.

FIGURE 4-2
EQUITY RISK PREMIUM
Electric Utilities 1931–2002



EXAMPLE 4-1

As a proxy for the risk premium applicable to the electric utility industry, a historical risk premium for the electric utility industry is estimated with an annual time series analysis applied to the industry as a whole, using *Moody's Electric Utility Index* as an industry proxy. The analysis is depicted in Figure 4-2. The risk premium is estimated by computing the actual return on equity capital for Moody's Index for each year, using the actual stock prices and dividends of the index, and then subtracting the long-term government bond return for that year. Dividend yields and stock prices on the index are obtained from *Moody's*

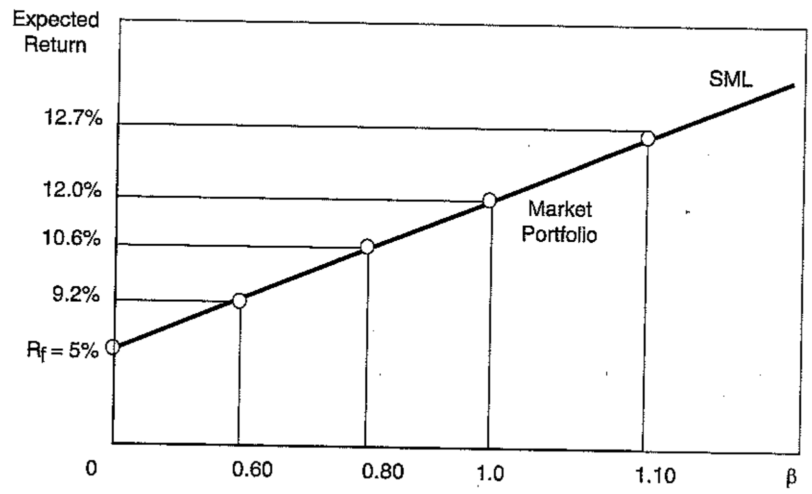
(continued next page)

is therefore the return necessary to attract capital to investments of a given risk, taking into account the soundness criterion of *Bluefield.*

5.3 CAPM Application

At first glance, the CAPM appears simple in application. Numerical values of the CAPM's three input parameters, R_F, beta, and the market risk premium $(R_M - R_F)$ are estimated and inserted into the CAPM formula to produce the cost of equity estimate, or used in reading the cost of equity directly from the SML. A numerical example is shown in Figure 5-3.

FIGURE 5-3
CAPM ESTIMATES OF UTILITY COST OF CAPITAL: ILLUSTRATION



Assuming a 5% risk-free rate, and a 12% market return, that is, a market risk premium of 7%, the cost of equity estimates for three companies are 9.2%, 10.6%, and 12.7%, respectively, corresponding to their respective betas of 0.60, 0.80, and 1.10.

Despite the CAPM's conceptual appeal and mechanistic simplicity, operationalizing the CAPM to estimate a fair return on equity presents several practical difficulties. From the start, the model itself is a prospective, forward-looking model. To stress this point, the following equation restates the CAPM formula with expectational operators attached to each input variable:

$$E(K) = E(R_F) + E(\beta) \times [E(R_M) - E(R_F)] \qquad (5\text{-}2)$$

historical risk premium approach assumes that the average realized return is an appropriate surrogate for expected return, or, in other words, that investor expectations are realized. However, realized returns can be substantially different from prospective returns anticipated by investors, especially when measured over short time periods.

The prospective (forecast) approach examines the returns expected from investments in common equities and bonds. The risk premium is simply the difference between the expected returns on stocks and bonds. The prospective approach is subject to the inevitable measurement errors involved in computing expected returns.

Therefore, a regulatory body should rely on the results of both historical and prospective studies in arriving at an appropriate risk premium, data permitting. Each proxy for the expected risk premium brings information to the judgment process from a different light. Neither proxy is without blemish, each has advantages and shortcomings. Historical risk premium data are available and verifiable, but may no longer be applicable if structural shifts have occurred. Prospective risk premium data may be more relevant since they encompass both history and current changes, but are nevertheless imperfect proxies. Giving equal weight to the historical risk premium and the prospective risk premium forecast represents a compromise between the certainty of the past and its possible irrelevance versus the greater relevance of the forecast and its possible estimation error.[13]

Faced with this myriad, and often conflicting, evidence on the magnitude of the risk premium, a regulator might very well be confused about the correct market risk premium. The author's opinion is that a range of 5% to 8% is reasonable for the United States with a slight preference for the upper end of the range.

As in the case of the beta estimate and risk-free rate estimate, a sensitivity analysis of possible CAPM cost of capital estimates should be conducted for a specified utility using a reasonable range of estimates for the market return. See Figure 5-6 for an illustration.

The range of cost of capital estimates obtained using a separate range for each of the three input variables to the CAPM, beta, risk-free rate, and market

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[13] A survey of professional practices published in 1998 by Bruner, Eades, Harris, and Higgins (1998) found that 71% of textbooks/tradebooks used a historical arithmetic mean as the market risk premium and 60% of financial advisors used either a market risk premium of 7.0–7.4% (similar to the arithmetic mean) or a long-term arithmetic mean. For corporations, there was no single method that represented a consensus.

1. That investors, in fact, evaluate common stocks in the classical valuation framework, and trade securities rationally at prices reflecting their perceptions of value. Given the universality and pervasiveness of the classical valuation framework in investment education and in the professional investment community, this assumption is plausible.
2. That investors discount the expected cash flows at the same rate K in every future period. In other words, a flat yield curve is assumed. If K varies over time, there is no single required return rate, and practical estimates of the required return must be considered as weighted averages of $\{K_1, K_2, K_3 \ldots K_n\}$. Since each of the 1-period return requirements can be thought of as an interest rate plus a risk premium, the required return to a multiple time horizon can be viewed as an average interest rate plus an average risk premium. More complex discounting models that incorporate these varying ''yield curve effects'' are available, but are of limited practical usefulness.
3. That the K obtained from the fundamental DCF equation corresponds to that specific stream of future cash flows alone, and no other. There may be alternate company policies (dividend payout, capital structure) that would generate the same future cash flows, but these policies may alter the risk of the cash flow stream, and hence modify the investor's required return, K.
4. That dividends, rather than earnings, constitute the source of value. The rationale for computing the value of common stock from dividends is that the only cash values ever received by investors are dividends. This does not mean that earnings are unimportant for they provide the basis for paying dividends.

Focusing on the present value of expected earnings can be misleading. It is earnings net of any investment required to produce the earnings that are of interest, and not earnings alone. For example, a company expects earnings per share of $1.00 per year; but to sustain the stream of future earnings, the company needs to invest in real assets at the rate of $1.00 per share each year. Since an amount equal to each year's earnings must be channeled into new asset investment, no sustainable dividend payout, hence value, is possible. In general, even for a non-dividend-paying company, earnings will eventually outrun the firm's need for additional asset investment, creating the capacity to pay dividends.

The finance literature has produced three general approaches to determine value, each involving discounting three different streams of money: (1) the present value of expected dividends, (2) the present value of expected earnings net of required investment, and (3) the present value of the cash flows produced by assets. All three approaches are equivalent, provided they are properly formulated.[2]

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[2] The equivalence between the three approaches is demonstrated in several financial texts. See for example Morin (2002) and Francis (2000).

8.3 The Standard DCF Model

The general common stock valuation model embodied in Equation 8-5 is not very operational, since it requires an estimation of an infinite stream of dividends. But by assigning a particular configuration to the dividend stream, a more practical formula can be derived. A formal derivation of the standard DCF model is provided in Appendix 8-A. Basically, assuming that dividends grow at a constant rate forever, that is,

$$D_t = D_0(1 + g)^t \tag{8-6}$$

Where g = expected dividend per share growth
D_0 = current dividend per share
D_1 = expected dividend per share one year from now

and substituting these values of future dividends per share into Equation 8-5, the familiar reduced form of the general dividend valuation model is obtained:

$$P_0 = \frac{D_1}{K - g} \tag{8-7}$$

In words, this fundamental equation states that the market price of a share of common stock is the value of next year's expected dividend discounted at the market's required return net of the effect of growth. Solving the equation for K, the cost of equity capital, the standard DCF formulation widely used in regulatory proceedings is obtained:

$$K = \frac{D_1}{P_0} + g \tag{8-8}$$

This formula states that under certain simplifying assumptions discussed below, which investors frequently make, the equity investor's expected return, K, can be envisaged as the sum of an expected dividend yield, (D_1/P_0), plus the expected growth rate of future dividends, g. Investors set the equity price so as to obtain an appropriate return consistent with the risk of the investment and with the return forgone in investments of comparable risk. The basic idea of the standard DCF approach to estimating the cost of equity capital is to infer K from the observed share price and from an estimate of investors' expected future dividends. The principal appeal of the approach is its simplicity and its correspondence with the intuitive notion of dividends plus capital appreciation as a measure of investors' total expected return. The assumptions underlying the model are discussed in detail below. Essentially, a constant average growth trend for both dividends and earnings, a stable dividend payout and capital structure policy, and a discount rate in excess of the expected growth rate are assumed. A simple example will illustrate the standard DCF model, sometimes referred to as the ''annual'' or ''single-period'' DCF model.

EXAMPLE 8-1

We have the following market data for Utility X:

current dividend per share = \$1.62
current stock price = \$25.00
expected dividend growth = 4%

From Equation 8-8, the standard DCF model produces a cost of equity of:

$$
\begin{aligned}
K &= D_1 / P_0 + g \\
&= D_0(1+g) / P_0 + g \\
&= \$1.62\ (1.04) / \$25 + .04 \\
&= 6.7\% + 4.0\% = 10.7\%
\end{aligned}
$$

Note that next year's expected dividend is the current spot dividend increased by the expected growth rate in dividends. In general, implementation of the approach requires finding D_0 and P_0 from readily available sources of market data; the growth rate, g, can be estimated using several techniques. One way is to rely on analysts' long-term growth forecasts. Chapter 9 will discuss the application of the DCF formulation in detail.

Standard DCF Model Assumptions

The assumptions underlying the standard DCF model have been the source of controversy, confusion, and misunderstanding in rate hearings. This section clarifies these assumptions.

Theories are simplifications of reality and the models articulated from theories are necessarily abstractions from and simplifications of the existing world so as to facilitate understanding and explanation of the real world. The DCF model is no exception to the rule. The assumptions of the standard DCF model are as follows:

Assumption #1. The four assumptions discussed earlier in conjunction with the general classical theory of security valuation still remain in force.

Assumption #2. The discount rate, K, must exceed the growth rate, g. In other words, the standard DCF model does not apply to growth stocks. In Equation 8-7, it is clear that as g approaches K, the denominator gets progressively smaller, and the price of the stock infinitely large. If g exceeds K, the price becomes negative, an implausible situation. In the derivation of the standard

DCF equation (8-7) from the general stock valuation equation (8-5), it was necessary to assume g is less than K in order for the series of terms to converge toward a finite number. With this assumption, the present value of steadily growing dividends becomes smaller as the discounting effect of K in the denominator more than offsets the effect of such growth in the numerator.

This assumption is realistic for most public utilities. Investors require a return commensurate with the amount of risk assumed, and this return likely exceeds the expected growth rate in dividends for most public utilities. Although it is possible that a firm could sustain very high growth rates for a few years, no firm could double or triple its earnings and dividends indefinitely.

Assumption #3. The dividend growth rate is constant in every year to infinity. This assumption is not as problematic as it appears. It is not necessary that g be constant year after year to make the model valid. The growth rate may vary randomly around some average expected value. Random variations around trend are perfectly acceptable, as long as the mean expected growth is constant. The growth rate must be ''expectationally constant,'' to use formal statistical jargon. This assumption greatly simplifies the model without detracting from its usefulness.

If investors expect growth patterns to prevail in the future other than constant infinite growth, more complex DCF models are available. For example, investors may expect dividends to grow at a relatively modest pace for the first 5 years and to resume a higher normal steady-state course thereafter, or conversely. The general valuation framework of Equation 8-5 can handle such situations. The ''non-constant growth'' model presented later in the chapter is a popular version of the DCF model.

It should be pointed out that the standard DCF model does not require infinite holding periods to remain valid. It simply assumes that the stock will be yielding the same rate of return at the time of sale as it is currently yielding. Example 8-2 illustrates this point.

Another way of stating this assumption is that the DCF model assumes that market price grows at the same rate as dividends. Although g has been specified in the model to be the expected rate of growth in dividends, it is also implicitly the expected rate of increase in stock price (expected capital gain) as well as the expected growth rate in earnings per share. This can be seen from Equation 8-7, which in period 1 would give:

$$P_1 = D_2 / (K - g)$$

But $D_2 = D_1(1 + g)$ and $P_0 = D_1 / (K - g)$

so that $P_1 = D_1(1 + g) / (K - g) = P_0(1 + g)$

yield must be adjusted for the flotation cost allowance by dividing it by (1 − f), where f is the flotation cost factor.[6]

$$K = D_1/P_0\,(1 - f) + g \qquad (9\text{-}4)$$

9.3 Growth Estimates: Historical Growth

The principal difficulty in calculating the required return by the DCF approach is in ascertaining the growth rate that investors are currently expecting. While there is no infallible method for assessing what the growth rate is precisely, an explicit assumption about its magnitude cannot be avoided. Estimating the growth component is the most difficult and controversial step in implementing DCF since it is a quantity that lies buried in the minds of investors. Three general approaches to estimating expected growth can be used, each with its own strengths and blemishes:

1. historical growth rates
2. analysts' forecasts
3. sustainable growth rates

This section describes the historical growth approach while the next two sections address the other two approaches.

Historical growth rates in dividends, earnings, and book value are often used as proxies for investor expectations in DCF analysis. Investors are certainly influenced to some extent by historical growth rates in formulating their future growth expectations. In addition, these historical indicators are widely used by analysts, investors, and expert witnesses in regulatory proceedings, at least as a starting point in their company analyses. Professional certified financial analysts are also well-versed in the use of historical growth indicators. To wit, the calculation of historical growth rates is normally one of the first steps in security analysis. Historical indicators are also used extensively in scholarly research. There exists a vast literature in empirical finance designed to evaluate the use of historical financial information as surrogates for expected values. This literature is discussed in the next section.

When using historical growth rates in a regulatory environment, a convenient starting point is to focus on the utility in question, and to assume that its growth profile is relatively stable and predictable. Under circumstances of stability, it is reasonable to examine past growth trends in earnings, dividends,

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[6] The conceptual and empirical support for the flotation cost adjustment is fully discussed in Chapter 10.

and book values as proxies for investor expectations. The fundamental assumption is made that investors arrive at their expected growth forecast by simply extrapolating past history. In other words, historical growth rates influence investor anticipations of long-run growth rates.

In using historical growth rates, three decisions must be made: (1) which historical data series is most relevant; (2) over what past period; and (3) which computational method is most appropriate.

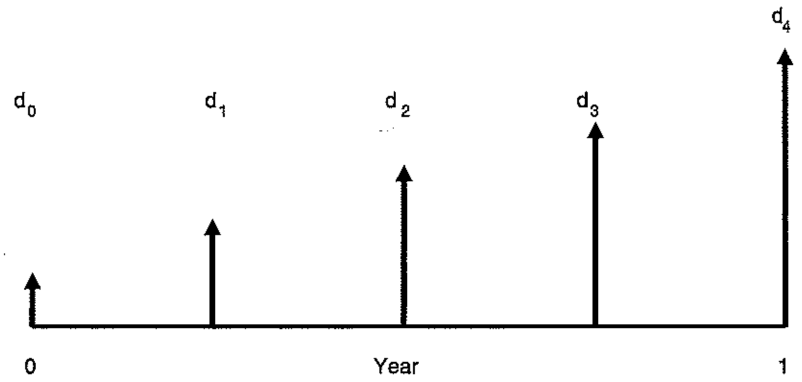
Historical Series

DCF proponents have variously based their historical computations on earnings per share, dividends per share, and book value per share. Of the three possible growth rate measures, growth in dividends per share is likely to be preferable, at least conceptually. DCF theory states clearly that it is expected future cash flows in the form of dividends that constitute investment value.

However, since the ability to pay dividends stems from a company's ability to generate earnings, growth in earnings per share can be expected to strongly influence the market's dividend growth expectations. After all, dividend growth can only be sustained if there is growth in earnings. It is the expectation of earnings growth that is the principal driver of stock prices. On the down side, using earnings growth as a surrogate for expected dividend growth can be problematic since historical earnings per share are frequently more volatile than dividends per share. Past growth rates of earnings per share tend to be very volatile and can sometimes lead to unreasonable results, such as negative growth rates. For example, in the 1990s and early 2000s, electric and gas company earnings growth rates were unstable and volatile, and such growth rates could not reasonably be expected to continue. Historically based DCF estimates of the cost of equity were downward-biased by the anemic historical growth rates of earnings and dividends in those years of major restructuring efforts, writeoffs, mergers and acquisitions, and shrinking profitability in the passage from a regulated monopoly to a competitive industry.

The relative stability of earnings and dividends is displayed in Figure 9-1 for The Southern Company. Under normal circumstances, dividend growth rates are not nearly as affected by year-to-year inconsistencies in accounting procedures as are earnings growth rates, and they are not as likely to be distorted by an unusually poor or bad year. Dividend growth is more stable than earnings growth because dividends reflect normalized long-term earnings rather than transitory earnings, because investors value stable dividends, and because companies are reluctant to cut dividends because of the information effect of dividend payments.

FIGURE 11-3
QUARTERLY DCF CONSTANT GROWTH



$$d_1 = d_0(1 + g)^{0.25}$$

$$d_2 = d_0(1 + g)^{0.50}$$

$$d_3 = d_0(1 + g)^{0.75}$$

$$d_4 = d_0(1 + g)^{1.00}$$

is computationally laborious. The following quarterly DCF model is a useful approximation and is far less laborious, although it does require the assumption that the company increases its dividend payments each quarter. The model assumes that each quarterly dividend differs from the previous one by $(1 + g)^{0.25}$, where g is the growth rate and the term 0.25 denotes one quarter of the year. Figure 11-3 shows the assumed dividend pattern. If it is assumed that dividends grow at a constant rate of g% every quarter starting from a base of d_0, the current quarterly rate, the company's stock price is given by:

$$P_0 = \sum_{n=1}^{4} \frac{d_0(1 + g)^{n/4}}{(1 + K)^{n/4}} + g$$

Which simplifies to:

$$P_0 = \frac{d_0(1 + g)^{1/4}}{(1 + K)^{1/4} - (1 + g)^{1/4}}$$

Solving the above equation for K, the simplified DCF formula for estimating the cost of equity under quarterly dividend payments emerges as Equation 11-4.

$$K = \left[\frac{d_0(1 + g)^{1/4}}{P_0} + (1 + g)^{1/4}\right] - 1 \qquad (11\text{-}4)$$

volume of trading on public exchanges, and a ceiling on the amount of dividend cuts over a past period.

In defining a population of comparable-risk companies, care must be taken not to include other utilities in the sample, since the rate of return on other utilities depends on the allowed rate of return. The historical book return on equity for regulated firms is not determined by competitive forces but instead reflects the past actions of regulatory commissions. It would be circular to set a fair return based on the past actions of other regulators, much like observing a series of duplicate images in multiple mirrors. The rates of return earned by other regulated utilities may very well have been reasonable under historical conditions, but they are still subject to tests of reasonableness under current and prospective conditions.

Time Period

The cost of capital of a company refers to the expected long-run earnings level of other firms with similar risk. But a company's achieved earnings in any given year are likely to exceed or be less than their long-run average. Such deviations from expectations occur at the macroeconomic level as well. At the peak of the business cycle, firms generally earn more than their cost of capital, while at the trough the reverse is typical. Aggregating returns over a large number of comparable-risk unregulated firms averages the abnormally high and low rates of profitability in any given year. Furthermore, to dampen cyclical aberrations and remove the effects of cyclical peaks and troughs in profitability, an average over several time periods should be employed. The time period should include at least one full business cycle that is representative of prospective economic conditions for the next cycle. Such cyclical variations can be gauged by the official turning points in the U.S. business cycle, reported in *Business Conditions Digest.*

Averaging achieved returns over a full business cycle can serve as a reasonable compromise between the dual objectives of being representative of current economic conditions and of smoothing out cyclical fluctuations in earnings on unregulated firms. Some analysts confine their return study to the most recent time period. The most serious flaw of this approach is that historical returns on equity vary from year to year, responding to the cyclical forces of recession and expansion and to economic, industry-specific and company-specific trends. The most recent period is not likely to mirror expectations and be representative of prospective business conditions. Moreover, in the short run, reported book profitability frequently moves in the opposite direction to interest rates and to investors' required returns. For example, a period of disinflation and falling interest rates will increase company earnings and earned equity returns, while investors' return requirements are falling, and conversely.

Chapter 16
Weighted Average Cost of Capital

Traditionally, the allowed rate of return in regulatory hearings is calculated as the weighted average of the cost of each individual component of the capital structure weighted by its book value. This is illustrated in Table 16-1, where the capital structure, expressed as percent of book value, consists of 40% debt, 10% preferred stock, and 50% common stock, with individual cost rates of 8%, 6%, and 12%, respectively.

The estimated allowed rate of return of 9.8%, also known as the *weighted average cost of capital* (''WACC''), is then applied to the book value of the rate base to determine the total revenue requirements (costs of service) needed to service the capital employed by the utility.

Knowledge of the 9.8% allowed rate of return on total capital is not enough to determine the total cost of capital to the ratepayers, however, for it ignores the tax burden. Assuming a 50% tax rate, in order to provide a $1 return to the bondholders, the utility requires only $1 of revenue. But it takes $2 of pre-tax revenue to provide a $1 return to the preferred and common equity holders because the utility must pay corporate income taxes. Returning to the above example, if the rate base is $100 and the tax rate 50%, to provide a return of $3.20 on the bondholders' $40 investment, the utility requires $3.20 of pre-tax revenues. But to provide a return of $0.60 + $6.00 = $6.60 to the preferred and common equity holders' $60 investment, the regulatory commission must allow a profit of 2 x $6.60 = $13.20. From the ratepayers' viewpoint, the total cost of capital inclusive of taxes is $3.20 + $13.20 = $16.40, or 16.4%. The computation is shown on Table 16-2.

An alternate and equivalent computational procedure, shown in Table 16-3, is to express the cost of debt directly on an after-tax basis, and then compute the after-tax weighted average cost of capital (''ATWACC'').

TABLE 16-1
ILLUSTRATIVE COST OF CAPITAL CALCULATION

Source	$$ Amount	% Weight	% Cost	Weighted Cost
Debt	$40	40%	8%	3.2%
Preferred	$10	10%	6%	0.6%
Equity	$50	50%	12%	6.0%
				9.8%

TABLE 16-2
ILLUSTRATIVE COST OF CAPITAL CALCULATION

Source	$$ Amount	% Weight	% Cost	Weighted Cost	Tax Factor	Capital Cost Including Tax
Debt	$40	40%	8%	3.2%	1.0	3.2%
Preferred	$10	10%	6%	0.6%	2.0	1.2%
Equity	$50	50%	12%	6.0%	2.0	12.0%
				9.8%		**16.4%**

Note: The tax factor is 1/(1 − tax rate): with a 50% corporate tax rate 1/(1 − 0.50) = 1/0.50 = 2.0

TABLE 16-3
ILLUSTRATIVE COST OF CAPITAL CALCULATION
Alternate Version

Source	% Weight	Return	After-tax Cost	Weighted Cost
Debt	40.0%	8.0%	4.0%	1.6%
Preferred	10.0%	6.0%	6.0%	0.6%
Equity	50.0%	12.0%	12.0%	6.0%
				8.2%

The resulting ATWACC is then multiplied by the tax factor to obtain directly the cost of capital inclusive of taxes. Going back to the above example, the after-tax cost of debt is 8% (1 − T) = 8% (1 − .50) = 4%, where T is the tax rate. The weighted cost of debt is then 1.6%, for a total WACC of 8.2%, instead of the 9.8% shown above. The pre-tax cost of capital is then simply the post-tax figure of 8.2% multiplied by the tax factor of 2, or 16.4%, the same figure obtained with the first procedure. Appendix 16-A shows that the dollar revenue requirement is the same whether the tax shield from debt financing is treated implicitly by multiplying the cost of debt by (1 − T) or explicitly as a separate line item in computing the revenue requirement.

More generally, if K_d and K_e are the costs of debt and equity, and W_d and W_e are, respectively, the weights of debt and equity to the total value of capital, the weighted average cost of capital, K, can be expressed as:

$$K = K_d W_d + K_e W_e \qquad (16\text{-}1)$$

Several issues regarding the WACC arise in regulatory proceedings, particularly with regard to the optimal set of weights W_d and W_e. Section 16.1

Exhibit DJG-4

Weighted Average Awarded Return Recommendation

Source	**Capital Structure**	**Cost Rates**	**Weighted Cost**
Long-term Debt	51.50%	5.62%	2.89%
Common Equity	48.50%	9.00% 8.75% 9.25%	4.37% 4.24% 4.49%

Recommended Range for Awarded Rate of Return

7.14%	**7.26%**	**7.38%**
LOW	**RECOMMENDED**	**HIGH**

Company	Ticker	[1] Market Cap. ($ millions)	[2] Market Category	[3] Moody's Ratings	[4] Value Line Safety Rank	[5] Financial Strength	[6] Value Line Region
ALLETE	ALE	4,000	Mid Cap	A3	2	A	Central
Ameren Corp.	AEE	15,000	Large Cap	Baa1	2	A	Central
Avangrid, Inc.	AGR	15,000	Large Cap	Baa1	2	B++	East
Avista Corp.	AVA	3,300	Mid Cap	Baa1	2	A	West
Black Hills	BKH	3,700	Mid Cap	Baa2	2	A	West
CMS Energy Corp.	CMS	14,000	Large Cap	Baa1	2	B++	Central
Dominion Energy	D	50,000	Large Cap	Baa2	2	B++	East
DTE Energy Co.	DTE	20,000	Large Cap	Baa1	2	B++	Central
Edison International	EIX	25,000	Large Cap	A3	2	A	West
El Paso Electric	EE	2,100	Mid Cap	Baa1	2	B++	West
Exelon Corp.	EXC	37,000	Large Cap	Baa2	3	B++	East
Hawaiian Elec.	HE	3,500	Mid Cap	Baa2	2	A	West
IDACORP, Inc.	IDA	4,200	Mid Cap	Baa1	2	A	West
NorthWestern Corp.	NWE	2,900	Mid Cap	Baa1	3	B+	West
Otter Tail Corp.	OTTR	1,700	Small Cap	A3	2	A	Central
PG&E Corp.	PCG	34,000	Large Cap	A3	2	B++	West
Portland General	POR	4,000	Mid Cap	A3	2	B++	West
Sempra Energy	SRE	28,000	Large Cap	Baa1	2	A	West

[1], [4], [5], [6] Value Line Investment Survey
[2] Large Cap > $10 billion; Mid Cap > $2 billion; Small Cap > $200 million
[3] Bond ratings

Ticker	^GSPC	ALE	AEE	AGR	AVA	BKH	CMS	D	DTE	EIX	EE	EXC	HE	IDA	NWE	OTTR	PCG	POR	SRE
30-day Average	2517	78.05	59.24	47.53	51.79	68.89	47.24	77.90	109.38	78.64	55.91	37.98	33.84	89.36	58.27	43.75	67.52	46.30	115.60
Standard Deviation	29.4	0.63	0.82	0.59	0.34	0.91	0.80	1.00	1.56	1.20	1.14	0.71	0.39	1.04	0.79	1.10	4.03	0.61	1.93
09/06/17	2466	77.63	59.04	47.50	51.68	69.80	48.18	77.99	110.85	79.00	54.71	37.55	33.48	89.72	59.17	42.05	69.14	46.82	116.37
09/07/17	2465	78.65	59.45	48.04	52.02	69.93	48.46	78.25	111.03	79.61	55.16	38.03	33.60	90.83	59.46	42.65	69.76	47.25	118.13
09/08/17	2461	79.14	59.66	48.37	51.74	70.22	48.66	79.24	112.01	80.18	55.61	38.17	34.13	91.40	59.71	42.80	70.09	47.52	118.32
09/11/17	2488	79.14	60.47	48.62	51.56	70.83	49.10	79.92	112.67	80.92	56.01	38.47	34.42	91.88	60.11	43.20	71.00	47.84	119.21
09/12/17	2496	77.75	59.45	47.69	51.39	69.50	48.08	78.18	110.47	80.18	54.96	37.88	33.86	89.87	58.82	42.10	69.61	46.89	117.50
09/13/17	2498	77.56	58.89	47.22	51.33	69.48	47.50	78.07	109.41	80.42	54.67	37.80	33.74	89.50	58.70	42.15	69.01	46.54	117.29
09/14/17	2496	78.20	59.56	47.42	51.39	69.82	47.89	79.17	110.76	80.47	55.15	38.10	33.82	89.96	59.23	42.60	69.54	46.74	118.25
09/15/17	2500	78.41	59.81	47.51	51.64	69.63	47.87	79.50	111.45	80.64	55.15	37.61	33.72	90.06	58.96	43.00	69.73	46.62	118.45
09/18/17	2504	77.65	59.25	47.02	51.39	68.95	47.63	78.55	110.53	79.99	54.80	37.40	33.49	89.18	58.41	42.90	69.11	46.02	116.83
09/19/17	2507	77.68	59.25	46.77	51.53	69.00	47.39	78.33	110.18	79.16	54.85	37.28	33.37	89.11	58.47	43.05	68.74	46.23	116.75
09/20/17	2508	77.29	58.74	46.49	51.43	68.51	47.07	77.62	108.88	78.54	54.40	36.91	33.37	88.60	58.34	42.95	68.51	45.93	115.76
09/21/17	2501	77.15	58.81	46.56	51.30	68.73	47.06	77.30	109.02	78.11	54.90	37.15	33.56	88.52	58.64	43.00	68.64	45.96	115.91
09/22/17	2502	76.91	58.51	46.50	51.40	68.60	46.47	77.04	108.38	77.74	54.65	37.12	33.48	87.75	57.75	42.80	68.36	45.58	115.94
09/25/17	2497	77.56	59.38	47.10	51.53	69.23	47.10	78.02	109.74	78.72	55.30	37.60	33.80	88.67	57.87	43.15	68.75	45.95	116.88
09/26/17	2497	77.42	59.14	47.14	51.49	68.96	46.94	78.21	109.53	78.60	55.20	37.17	33.70	88.41	57.78	43.35	68.50	46.06	116.26
09/27/17	2507	77.70	57.92	46.75	51.54	69.07	46.23	76.78	107.24	77.54	55.30	37.15	33.55	88.35	57.42	44.05	67.44	45.90	114.50
09/28/17	2510	77.84	58.36	47.19	51.62	69.33	46.35	77.03	107.66	77.57	55.60	37.40	33.67	88.58	57.50	44.30	68.05	46.11	114.58
09/29/17	2519	77.29	57.84	47.42	51.77	68.87	46.32	76.93	107.36	77.17	55.25	37.67	33.37	87.93	56.94	43.35	68.09	45.64	114.13
10/02/17	2529	77.82	58.39	47.54	52.20	68.62	46.42	76.74	107.37	77.00	56.05	38.14	33.64	88.78	57.27	44.45	68.13	46.10	113.98
10/03/17	2535	77.55	57.93	47.48	52.18	69.08	45.97	75.95	106.60	77.01	56.15	37.84	33.64	88.42	57.30	44.35	68.31	46.12	113.87
10/04/17	2538	78.49	58.55	47.94	52.16	69.73	46.49	76.87	107.55	77.85	56.70	38.19	33.82	89.15	57.66	44.75	69.20	46.48	114.56
10/05/17	2552	78.49	58.84	48.31	52.05	69.15	46.53	76.81	107.75	77.68	56.75	38.35	33.78	88.69	57.73	44.85	69.07	46.31	112.02
10/06/17	2549	78.31	58.78	47.64	52.02	67.08	46.30	76.81	107.68	77.62	56.85	38.15	33.90	88.44	57.37	44.80	68.84	46.07	111.95
10/09/17	2545	78.08	59.15	47.38	52.20	67.09	46.48	76.64	107.93	77.55	57.05	38.04	33.80	88.38	57.44	44.40	68.65	46.05	112.63
10/10/17	2551	78.81	59.86	47.88	52.35	68.04	47.00	77.74	109.29	78.54	57.65	38.30	34.07	89.25	57.91	45.20	69.19	46.58	113.81
10/11/17	2555	79.06	59.78	48.25	52.24	68.00	47.35	78.01	109.70	78.42	57.85	38.46	34.28	89.54	58.00	45.25	69.15	46.70	114.94
10/12/17	2551	78.81	60.34	48.39	52.23	68.42	47.72	78.63	110.67	79.66	58.20	38.94	34.35	90.67	58.66	45.45	64.50	46.59	115.96
10/13/17	2553	78.38	60.41	47.99	52.23	67.67	47.54	78.62	109.89	77.99	58.10	39.24	34.33	90.34	58.44	45.30	57.72	46.25	114.87
10/16/17	2558	78.01	60.64	47.78	52.01	67.69	47.41	79.03	109.63	77.64	57.00	39.67	34.67	90.18	58.50	45.10	53.43	44.77	114.46
10/17/17	2559	78.78	60.95	48.11	51.99	67.60	47.81	78.96	110.16	77.60	57.15	39.60	34.89	90.59	58.63	45.25	57.44	45.50	114.04

All prices are adjusted closing prices reported by Yahoo! Finance, http://finance.yahoo.com

Company	Ticker	[1] Dividend	[2] Stock Price	[3] Dividend Yield
ALLETE	ALE	0.535	78.05	0.69%
Ameren Corp.	AEE	0.440	59.24	0.74%
Avangrid, Inc.	AGR	0.432	47.53	0.91%
Avista Corp.	AVA	0.357	51.79	0.69%
Black Hills	BKH	0.445	68.89	0.65%
CMS Energy Corp.	CMS	0.333	47.24	0.70%
Dominion Energy	D	0.755	77.90	0.97%
DTE Energy Co.	DTE	0.825	109.38	0.75%
Edison International	EIX	0.543	78.64	0.69%
El Paso Electric	EE	0.335	55.91	0.60%
Exelon Corp.	EXC	0.327	37.98	0.86%
Hawaiian Elec.	HE	0.310	33.84	0.92%
IDACORP, Inc.	IDA	0.550	89.36	0.62%
NorthWestern Corp.	NWE	0.525	58.27	0.90%
Otter Tail Corp.	OTTR	0.320	43.75	0.73%
PG&E Corp.	PCG	0.530	67.52	0.78%
Portland General	POR	0.340	46.30	0.73%
Sempra Energy	SRE	0.822	115.60	0.71%
Average		**$0.48**	**$64.84**	**0.76%**

[1] Third quarter 2017 dividends per share. Nasdaq.com

[2] Average stock price from DJG stock price exhibit.

[3] = [1] / [2]

Growth Determinant	Rate	
Nominal GDP	4.10%	[1]
Inflation	2.00%	[2]
Risk Free Rate	2.80%	[3]
Average	**2.97%**	

[1], [2] CBO Long-Term Budget Outlook 2016 - 2046
[3] From DJG risk-free rate exhibit

Dockets UE-170485 and UG-170486
Exhibit DJG-4
Final DCF Result

[1]	[2]	[3]	[4]
Dividend (d_0)	Stock Price (P_0)	Growth Rate (g)	**DCF Result**
$0.48	$64.84	4.10%	**7.2%**

[1] Average proxy dividend from DJG dividend exhibit

[2] Average proxy stock price from DJG dividend exhibit

[3] Highest growth rate from DJG growth determinant exhibit

[4] Quarterly DCF Approximation = $[d_0(1 + g)^{0.25}/P_0 + (1 + g)^{0.25}]^4 - 1$

Date	Rate
09/05/17	2.69%
09/06/17	2.72%
09/07/17	2.66%
09/08/17	2.67%
09/11/17	2.75%
09/12/17	2.78%
09/13/17	2.79%
09/14/17	2.77%
09/15/17	2.77%
09/18/17	2.80%
09/19/17	2.81%
09/20/17	2.82%
09/21/17	2.80%
09/22/17	2.80%
09/25/17	2.76%
09/26/17	2.78%
09/27/17	2.86%
09/28/17	2.87%
09/29/17	2.86%
10/02/17	2.87%
10/03/17	2.87%
10/04/17	2.87%
10/05/17	2.89%
10/06/17	2.91%
10/10/17	2.88%
10/11/17	2.88%
10/12/17	2.86%
10/13/17	2.81%
10/16/17	2.82%
10/17/17	2.80%
Average	**2.81%**

*Daily Treasury Yield Curve Rates on 30-year T-bonds, http://www.treasury.gov/resources-center/data-chart-center/interest-rates/.

Company	Ticker	Beta
ALLETE	ALE	0.75
Ameren Corp.	AEE	0.65
Avangrid, Inc.	AGR	NMF
Avista Corp.	AVA	0.70
Black Hills	BKH	0.85
CMS Energy Corp.	CMS	0.65
Dominion Energy	D	0.65
DTE Energy Co.	DTE	0.65
Edison International	EIX	0.60
El Paso Electric	EE	0.75
Exelon Corp.	EXC	0.70
Hawaiian Elec.	HE	0.70
IDACORP, Inc.	IDA	0.70
NorthWestern Corp.	NWE	0.65
Otter Tail Corp.	OTTR	0.90
PG&E Corp.	PCG	0.65
Portland General	POR	0.70
Sempra Energy	SRE	0.80
Average		0.71

*Betas from Value Line Investment Survey

Year	[1] Index Value	[2] Operating Earnings	[3] Dividends	[4] Buybacks	[5] Earnings Yield	[6] Dividend Yield	[7] Buyback Yield	[8] Gross Cash Yield
2011	11,385	877	240	405	7.70%	2.11%	3.56%	5.67%
2012	12,742	870	281	399	6.83%	2.20%	3.13%	5.33%
2013	16,495	956	312	476	5.80%	1.89%	2.88%	4.77%
2014	18,245	1,004	350	553	5.50%	1.92%	3.03%	4.95%
2015	17,900	885	382	572	4.95%	2.14%	3.20%	5.33%
2015	19,268	920	397	536	4.77%	2.06%	2.78%	4.85%

Cash Yield	5.15%	[9]
Growth Rate	0.96%	[10]
Risk-free Rate	2.81%	[11]
Current Index Value	2,517	[12]

Year	[13] 1	[14] 2	[15] 3	[16] 4	[17] 5
Expected Dividends	131	132	133	135	136
Expected Terminal Value					2868
Present Value	122	114	107	100	2074
Intrinsic Index Value	2517	[18]			
Required Return on Market	7.68%	[19]			
Implied Equity Risk Premium	**4.88%**	[20]			

[1-4] S&P Quarterly Press Releases, data found at https://us.spindices.com/indices/equity/sp-500 (additional info tab) (all dollar figures are in $ billions)

[1] Market value of S&P 500

[5] = [2] / [1]

[6] = [3] / [1]

[7] = [4] / [1]

[8] = [6] + [7]

[9] = Average of [8]

[10] = Compund annual growth rate of [2] = (end value / beginning value)$^{1/4}$-1

[11] Risk-free rate from DJG risk-free rate exhibit

[12] 30-day average of closing index prices from DJG stock price exhibit

[13-16] Expected dividends = [9]*[12]*(1+[10])^{n} ; Present value = expected dividend / (1+[11]+[19])^{n}

[17] Expected terminal value = expected dividend * (1+[11]) / [19] ; Present value = (expected dividend + expected terminal value) / (1+[11]+[19])^{n}

[18] = Sum([13-17]) present values.

[19] = [20] + [11]

[20] Internal rate of return calculation setting [18] equal to [12] and solving for the discount rate

IESE Business School Survey	5.7%	[1]
Graham & Harvey Survey	4.0%	[2]
Duff & Phelps Report	5.0%	[3]
Damodaran	5.2%	[4]
Garrett	4.9%	[5]
Highest	**5.7%**	

[1] IESE Business School Survey

[2] Graham and Harvey Survey

[3] Duff & Phelps Client Alert 2016

[4] Highest ERP est., http://pages.stern.nyu.edu/~adamodar/

[5] From DJG implied ERP exhibit

Company	Ticker	[1] Risk-Free Rate	[2] Value Line Beta	[3] Risk Premium	[4] CAPM Results
ALLETE	ALE	2.81%	0.750	5.70%	7.1%
Ameren Corp.	AEE	2.81%	0.650	5.70%	6.5%
Avangrid, Inc.	AGR	2.81%	NMF	5.70%	NA
Avista Corp.	AVA	2.81%	0.700	5.70%	6.8%
Black Hills	BKH	2.81%	0.850	5.70%	7.7%
CMS Energy Corp.	CMS	2.81%	0.650	5.70%	6.5%
Dominion Energy	D	2.81%	0.650	5.70%	6.5%
DTE Energy Co.	DTE	2.81%	0.650	5.70%	6.5%
Edison International	EIX	2.81%	0.600	5.70%	6.2%
El Paso Electric	EE	2.81%	0.750	5.70%	7.1%
Exelon Corp.	EXC	2.81%	0.700	5.70%	6.8%
Hawaiian Elec.	HE	2.81%	0.700	5.70%	6.8%
IDACORP, Inc.	IDA	2.81%	0.700	5.70%	6.8%
NorthWestern Corp.	NWE	2.81%	0.650	5.70%	6.5%
Otter Tail Corp.	OTTR	2.81%	0.900	5.70%	7.9%
PG&E Corp.	PCG	2.81%	0.650	5.70%	6.5%
Portland General	POR	2.81%	0.700	5.70%	6.8%
Sempra Energy	SRE	2.81%	0.800	5.70%	7.4%
Average			0.709		**6.8%**

[1] From DJG risk-free rate exhibit
[2] From DJG beta exhibit
[3] From DJG equity risk premium exhibit
[6] = [1] + [2] * [3]

Model	**Cost of Equity**
Discounted Cash Flow Model	7.2%
Capital Asset Pricing Model	6.8%
Average	**7.0%**

Source	Estimate	
IESE Survey	8.5%	[1]
Graham Harvey Survey	6.8%	[2]
Damodaran	8.0%	[3]
Garrett	7.7%	[4]
Average	7.8%	

[1] Average reported ERP + riskfree rate

[2] Average reported ERP + risk-free rate

[3] Recent highest reported ERP + risk-free rate

[4] From implied ERP exhibit herein

Awarded Returns vs. Market Cost of Equity (2005-2016)

Quarter	[1] Cases Filed	[2] Average Awarded ROE
2005.1	4	10.55%
2005.2	12	10.13%
2005.3	8	10.84%
2005.4	10	10.57%
2006.1	11	10.38%
2006.2	18	10.39%
2006.3	7	10.06%
2006.4	12	10.38%
2007.1	11	10.30%
2007.2	16	10.27%
2007.3	8	10.02%
2007.4	11	10.44%
2008.1	7	10.15%
2008.2	8	10.41%
2008.3	21	10.42%
2008.4	6	10.38%
2009.1	13	10.31%
2009.2	22	10.55%
2009.3	17	10.46%
2009.4	14	10.54%
2010.1	16	10.45%
2010.2	19	10.12%
2010.3	12	10.27%
2010.4	8	10.30%
2011.1	8	10.35%
2011.2	15	10.24%
2011.3	17	10.13%
2011.4	10	10.29%
2012.1	17	10.84%
2012.2	16	9.92%
2012.3	8	9.78%
2012.4	12	10.05%
2013.1	19	10.23%
2013.2	16	9.77%
2013.3	4	10.06%
2013.4	7	9.90%
2014.1	9	10.23%
2014.2	25	9.83%
2014.3	8	9.89%
2014.4	16	9.78%
2015.1	10	10.37%
2015.2	21	9.73%
2015.3	6	9.40%
2015.4	11	9.62%
2016.1	14	10.26%
2016.2	27	9.57%
2016.3	12	9.76%
2016.4	17	9.57%

Year	[3] Annual Market Return	
2005	4.83%	
2006	15.61%	
2007	5.48%	
2008	-36.55%	
2009	25.94%	
2010	14.82%	
2011	2.10%	
2012	15.89%	
2013	32.15%	
2014	13.25%	
2015	1.38%	
2016	11.74%	
Average		
Arithmetic	8.89%	[4]
Geometric	7.39%	[5]
Average Return on All Stocks	**8.1%**	[6]
Average Utility Awarded ROE	**10.2%**	[7]

Year	[8] Market Cost of Equity
2005	8.47%
2006	8.86%
2007	8.39%
2008	8.64%
2009	8.20%
2010	8.49%
2011	7.89%
2012	7.54%
2013	8.00%
2014	7.95%
2015	8.39%
2016	8.14%
Average	**8.25%**

[1] Edison Electric Institute Financial Update. Number of cases filed in each quarter.
[2] Edison Electric Institute Financial Update. Average awarded utility ROE each quarter.
[3] Historical stock returns. NYU Stern School of Business. http://pages.stern.nyu.edu/~adamodar/.
[4] = Average of [3]
[5] = Geometric mean of [3]
[6] = Average ([4],[5])
[7] = Average of [2]
[8] Annual required market returns. NYU Stern School of Business. http://pages.stern.nyu.edu/~adamodar/ (adding risk-free rate to implied ERP)

Inputs

EBIT	289,803	[1]
Interest Expense	87,130	[2]
Book Debt	1,682,004	[3]
Book Equity	1,648,727	[4]
Debt / Capital	50.50%	[5]
Debt / Equity	102%	[6]
Debt Cost	5.62%	[7]
Tax Rate	35%	[8]
Unlevered Beta	0.43	[9]
Risk-free Rate	2.81%	[10]
Equity Risk Premium	5.70%	[11]
Coverage Ratio	3.33	[12]
Bond Rating	Baa1	[13]

Ratings Table

[14] Coverage Ratio	[15] Bond Rating	[16] Spread	[17] Interest Rate
8.5 - 10.00	Aaa/AAA	0.60%	3.41%
6.5 - 8.49	Aa2/AA	0.80%	3.61%
5.5 - 6.49	A1/A+	1.00%	3.81%
4.25 - 5.49	A2/A	1.10%	3.91%
3.0 - 4.24	A3/A-	1.25%	4.06%
2.5 - 2.99	Baa2/BBB	1.60%	4.41%
2.25 - 2.49	Ba1/BB+	2.50%	5.31%
2.0 - 2.24	Ba2/BB	3.00%	5.81%
1.75 - 1.99	B1/B+	3.75%	6.56%
1.5 - 1.74	B2/B	4.50%	7.31%
1.25 - 1.49	B3/B-	5.50%	8.31%
0.8 - 1.24	Caa/CCC	6.50%	9.31%

Optimal Capital Structure Calculation

[18] Debt Ratio	[19] D/E Ratio	[20] Levered Beta	[21] True Cost of Equity	[22] Awarded ROE	[23] Debt Level	[24] Interest Expense	[25] Coverage Ratio	[26] Pre-tax Debt Cost	[27] After-tax Debt Cost	[28] Optimal WACC	[29] WACC at 9.9% ROE
0%	0%	0.426	5.24%	9.90%	0	0	∞	3.41%	2.22%	5.24%	9.90%
20%	25%	0.495	5.63%	9.90%	666,146	37,437	7.74	3.61%	2.35%	4.97%	8.39%
30%	43%	0.545	5.91%	9.90%	999,219	56,156	5.16	3.91%	2.54%	4.90%	7.69%
40%	67%	0.611	6.29%	9.90%	1,332,292	74,875	3.87	4.06%	2.64%	4.83%	7.00%
50%	100%	0.703	6.82%	9.90%	1,665,366	93,594	3.10	4.06%	2.64%	4.73%	6.27%
60%	150%	0.842	7.61%	9.90%	1,998,439	112,312	2.58	4.41%	2.87%	4.76%	5.68%
70%	233%	1.073	8.92%	9.90%	2,331,512	131,031	2.21	5.31%	3.45%	5.09%	5.39%

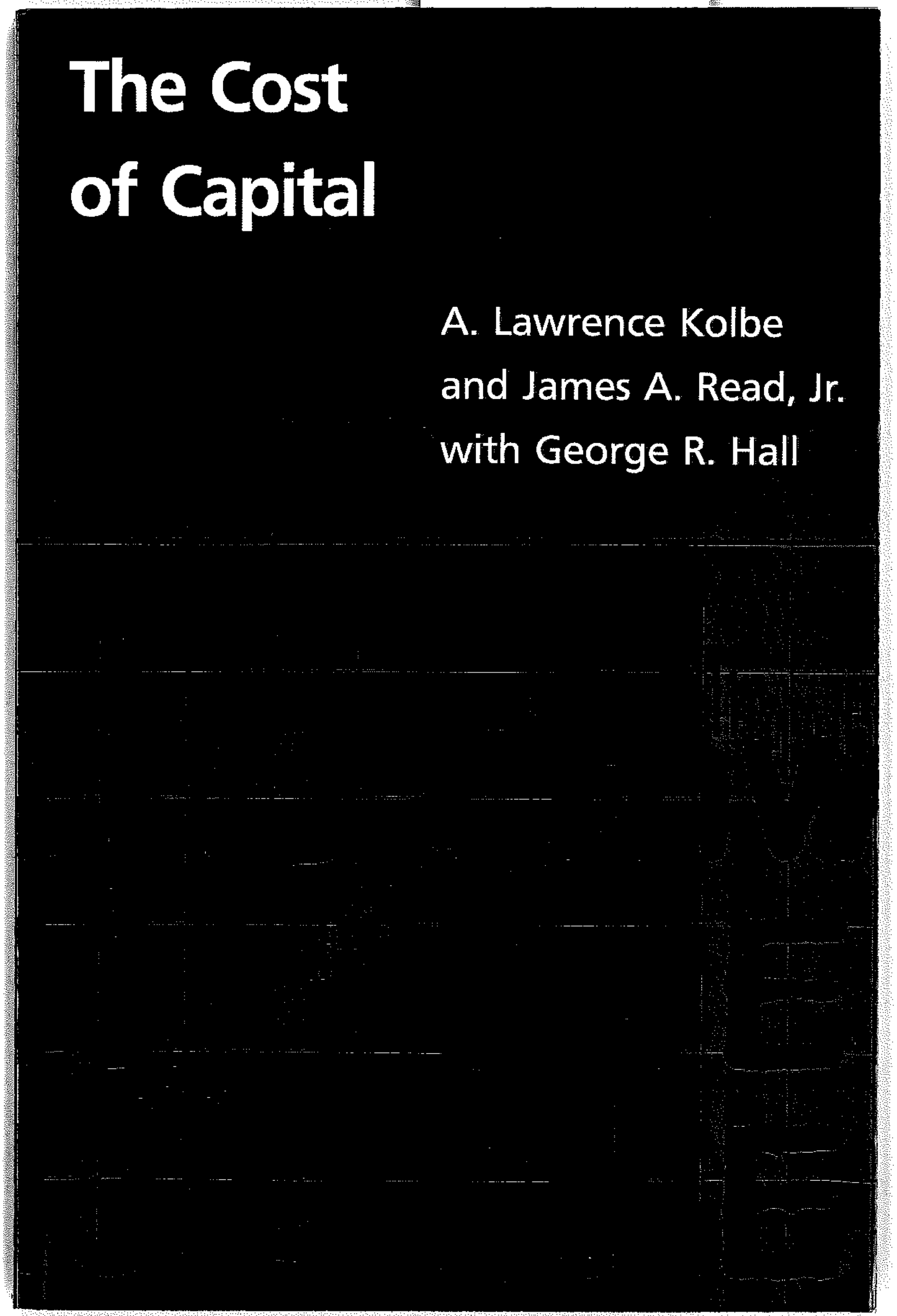
[1], [2] Q-AG-01-002 AG-1-2 Attachment (11) (000's)
[3], [4] Company Schedule DPH-31 (000's)
[5] = [3] / ([3] + [4])
[6] = [3] / [4]
[7] Company Schedule DPH-31
[8] Estimated corporate tax rate
[9] Average beta / (1+(1 - [8])*[6])
[10] From DJG risk-free rate exhibit
[11] From DJG equity risk premium exhibit
[12] = [1] / [2]
[13] Company bond rating
[14] Ranges of coverage ratios
[15] Moody's / S&P bond ratings
[16] NYU spread over risk-free rate
[17] = [16] + [10] = est. debt cost
[18] = debt / total capital
[19] = [18] / (1 - [18])
[20] = [9] * (1 + (1 - [8]) * [6]
[21] = [10] + [20] * [11]
[22] Recommended awarded ROE
[23] = [18] * ([3] + [4]); (000's)
[24] = [22] * [7]; (000's)
[25] = [1] / [23]
[26] Debt cost given coverage ratio per Ratings Table
[27] = [25] * (1 - [8])
[28] = ([18] * [26]) + ((1 - [18]) * [21])
[29] = ([18] * [26]) + ((1 - [18]) * [22])

Industry	Number of Fimrs	Debt Ratio
Advertising	41	87%
Hospitals/Healthcare Facilities	38	84%
Broadcasting	30	83%
Restaurant/Dining	86	82%
Tobacco	22	80%
Coal & Related Energy	38	79%
Brokerage & Investment Banking	45	76%
Retail (Building Supply)	6	75%
Retail (Automotive)	25	73%
Auto & Truck	15	73%
Trucking	30	73%
Packaging & Container	26	66%
Bank (Money Center)	10	66%
Beverage (Soft)	36	66%
Office Equipment & Services	24	65%
Telecom. Services	67	64%
Retail (Distributors)	88	62%
Power	68	62%
Hotel/Gaming	69	61%
Telecom (Wireless)	17	61%
R.E.I.T.	238	60%
Food Wholesalers	16	60%
Retail (Grocery and Food)	14	59%
Real Estate (Operations & Services)	54	59%
Transportation	17	59%
Chemical (Basic)	45	58%
Construction Supplies	51	58%
Environmental & Waste Services	89	57%
Farming/Agriculture	37	56%
Business & Consumer Services	165	56%
Air Transport	18	56%
Green & Renewable Energy	25	55%
Computer Services	117	54%
Oil/Gas Distribution	78	54%
Utility (Water)	22	54%
Cable TV	14	53%
Steel	38	53%
Rubber& Tires	4	52%
Drugs (Biotechnology)	426	52%
Chemical (Specialty)	100	52%
Recreation	66	51%
Software (System & Application)	236	51%
Metals & Mining	97	51%
Beverage (Alcoholic)	25	51%
Information Services	64	51%
Household Products	129	51%
Chemical (Diversified)	8	50%
Aerospace/Defense	96	50%
Building Materials	41	50%
Oil/Gas (Production and Exploration)	330	50%
Investments & Asset Management	156	49%
Auto Parts	63	48%
Total / Average	**3660**	**61%**

http://pages.stern.nyu.edu/~adamodar/New_Home_Page/datafile/dbtfund.htm

Company	Ticker	Debt Ratio
ALLETE	ALE	42%
Ameren Corp.	AEE	48%
Avangrid, Inc.	AGR	23%
Avista Corp.	AVA	51%
Black Hills	BKH	67%
CMS Energy Corp.	CMS	67%
Dominion Energy	D	67%
DTE Energy Co.	DTE	56%
Edison International	EIX	42%
El Paso Electric	EE	53%
Exelon Corp.	EXC	56%
Hawaiian Elec.	HE	42%
IDACORP, Inc.	IDA	45%
NorthWestern Corp.	NWE	52%
Otter Tail Corp.	OTTR	43%
PG&E Corp.	PCG	47%
Portland General	POR	48%
Sempra Energy	SRE	53%
Average		50%

Debt ratios from Value Line Investment Survey



The Cost of Capital

A. Lawrence Kolbe
and James A. Read, Jr.
with George R. Hall

and Discounted Cash Flow (the techniques mentioned here are defined and evaluated in chapter 3). Since these methods look only at firms in a single "risk class," they do not require that the analyst estimate the entire risk-return line shown in figure 2.1; they focus directly on the vertical axis.

To use the second strategy, the analyst must examine (at least implicitly) both measures of the stock's risk and the current position of the market line. Methods that require explicit risk measurement include the Capital Asset Pricing Model and the Risk Positioning techniques. These methods first position the firm on the horizontal axis of figure 2.1, and then (again, at least implicitly) use an estimate of the risk-return line to find the proper level for the cost of capital on the vertical axis.

The advantages of one strategy are the disadvantages of the other. The first strategy avoids the need for an estimate of the market line but requires that the evidence used must be from investments of comparable risk. This immediately excludes data from other firms of differing risk. More subtly, it can exclude data on the firm whose cost of capital is now being estimated, if either its risk or the market line has changed since the evidence was collected.

This focus on estimation strategies may be premature. If the reader does not accept that the cost of capital as just defined is the right target for regulators, the general approaches to cost of capital estimation may be of little interest. The remainder of the chapter uses two approaches to develop the reasons that the cost of capital is indeed the appropriate allowed rate of return for a regulated company's investors.[7]

2. Why the Allowed Rate of Return Should Equal the Cost of Capital

Law

The United States Supreme Court has established that investors in companies subject to rate regulation must be allowed *an opportunity* to earn returns sufficient to attract capital and comparable to those they would expect in the unregulated sector for bearing the same degree of risk. The *Bluefield* and *Hope* cases provide the seminal decisions.[8]

The *Hope* test is the basic criterion for a legally sufficient rate of return on equity. The court stated:

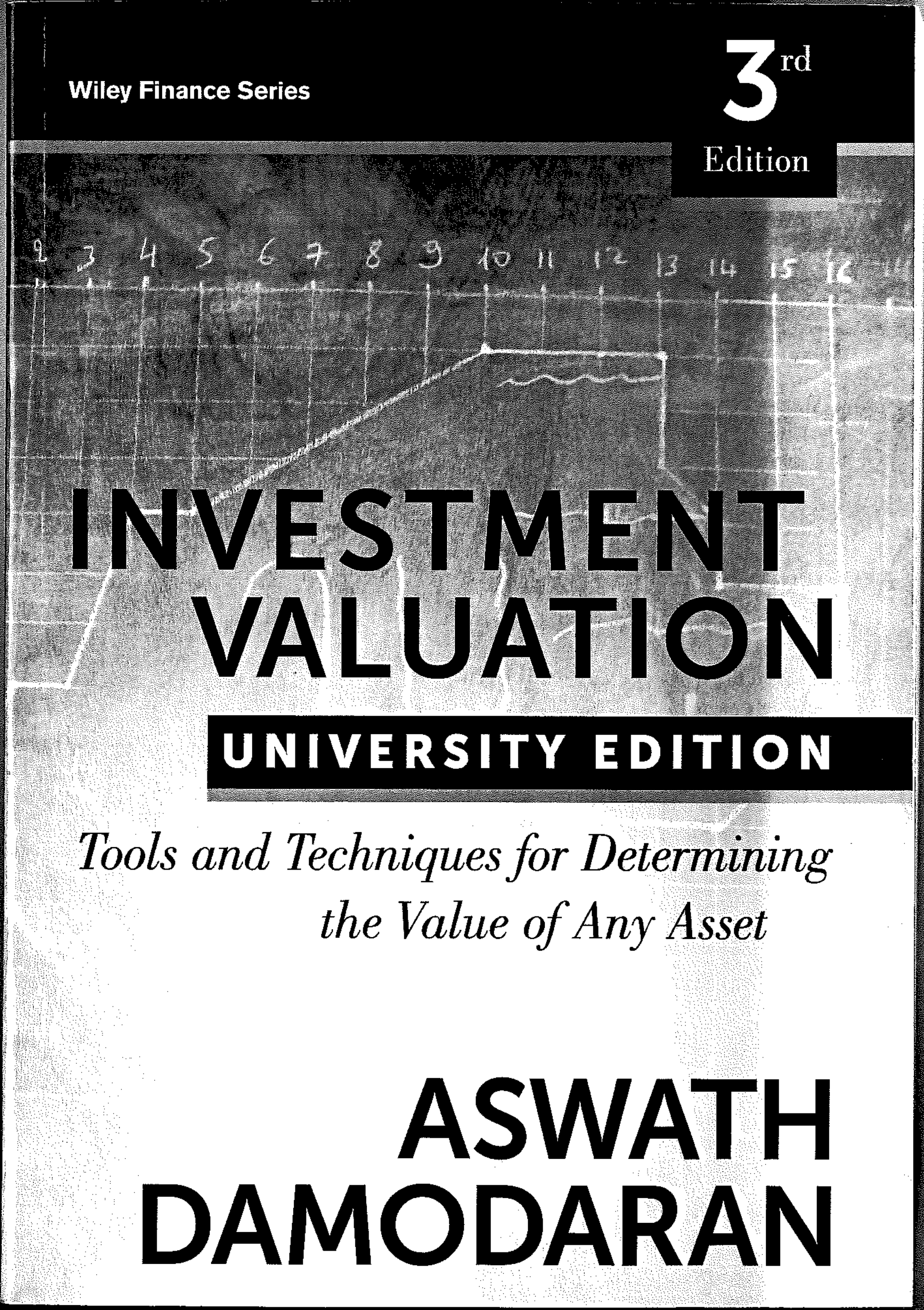
> The rate-making process under the act, i.e., the fixing of "just and reasonable" rates, involves a balancing of the investor and the consumer interests. Thus we stated in the *Natural Gas Pipeline Co.* case that "regulation does not insure that the business shall produce net revenues." 315 U.S. p. 590. But such considerations aside, the investor interest has a legitimate concern with the financial integrity of the company whose rates are being regulated. From the investor or company point of view it is important that there be enough revenue not only for operating expenses but also for the capital costs of the business. These include service on the debt and dividends on the stock . . . . By that standard the return to the equity owner should be commensurate with returns on investments in other enterprises having corresponding risks. That return, moreover, should be sufficient to assure confidence in the financial integrity of the enterprise, so as to maintain its credit and to attract capital.[9]

Since by definition the cost of capital of a regulated firm represents precisely the expected return that investors could anticipate from other investments while bearing no more and no less risk, and since investors will not provide capital unless the investment is expected to yield its opportunity cost of capital, the correspondence of the definition of the cost of capital with the court's definition of legally required earnings appears clear. *Hope* refers to both "commensurate" earnings and the attraction of capital. These two approaches are harmonized when the allowed rate of return is set equal to the cost of capital.

Hope is sometimes cited for the proposition that some specific method of establishing the rate of return on equity is the only legally permissible technique. However, *Hope* states clearly that it is the "end result" of the regulation process that determines legality, not the specific technique used to calculate rate of return. All the standard cost-of-capital estimation techniques can meet the requisite legal tests; it is the way they are applied that is important.

Despite the obvious correspondence between the precepts of *Hope* and the financial concept of the cost of capital, public utility statutes and the applicable case law give no detailed prescription for what constitutes a "just and reasonable" rate of return on equity. In the absence of detailed guidelines from legislatures or the higher courts, various general judicial concepts about rate setting have been developed and applied by courts. The key concepts are:

1. Balance: the establishment of a just and reasonable rate involves a balancing of the investor and consumer interests.



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When return distributions are normal, the characteristics of any investment can be measured with two variables—the expected return, which represents the opportunity in the investment, and the standard deviation or variance, which represents the danger. In this scenario, a rational investor, faced with a choice between two investments with the same standard deviation but different expected returns, will always pick the one with the higher expected return.

In the more general case, where distributions are neither symmetric nor normal, it is still conceivable that investors will choose between investments on the basis of only the expected return and the variance, if they possess utility functions that allow them to do so.[1] It is far more likely, however, that they prefer positive skewed distributions to negatively skewed ones, and distributions with a lower likelihood of jumps (lower kurtosis) over those with a higher likelihood of jumps (higher kurtosis). In this world, investors will trade off the good (higher expected returns and more positive skewness) against the bad (higher variance and kurtosis) in making investments.

In closing, it should be noted that the expected returns and variances that we run into in practice are almost always estimated using past returns rather than future returns. The assumption made when using historical variances is that past return distributions are good indicators of future return distributions. When this assumption is violated, as is the case when the asset's characteristics have changed significantly over time, the historical estimates may not be good measures of risk.

***optvar.xls:* This is a dataset on the Web that summarizes standard deviations and variances of stocks in various sectors in the United States.**

Diversifiable and Nondiversifiable Risk

Although there are many reasons why actual returns may differ from expected returns, we can group the reasons into two categories: firm-specific and marketwide. The risks that arise from firm-specific actions affect one or a few investments, while the risks arising from marketwide reasons affect many or all investments. This distinction is critical to the way we assess risk in finance.

Components of Risk When an investor buys stock or takes an equity position in a firm, he or she is exposed to many risks. Some risk may affect only one or a few firms, and this risk is categorized as firm-specific risk. Within this category, we would consider a wide range of risks, starting with the risk that a firm may have misjudged the demand for a product from its customers; we call this project risk. For instance, consider

---

[1] A utility function is a way of summarizing investor preferences into a generic term called "utility" on the basis of some choice variables. In this case, for instance, the investors' utility or satisfaction is stated as a function of wealth. By doing so, we effectively can answer questions such as, Will investors be twice as happy if they have twice as much wealth? Does each marginal increase in wealth lead to less additional utility than the prior marginal increase? In one specific form of this function, the quadratic utility function, the entire utility of an investor can be compressed into the expected wealth measure and the standard deviation in that wealth.

Boeing's investment in a Super Jumbo jet. This investment is based on the assumption that airlines want a larger airplane and are willing to pay a high price for it. If Boeing has misjudged this demand, it will clearly have an impact on Boeing's earnings and value, but it should not have a significant effect on other firms in the market. The risk could also arise from competitors proving to be stronger or weaker than anticipated, called competitive risk. For instance, assume that Boeing and Airbus are competing for an order from Qantas, the Australian airline. The possibility that Airbus may win the bid is a potential source of risk to Boeing and perhaps some of its suppliers, but again, few other firms will be affected by it. Similarly, Disney recently launched magazines aimed at teenage girls, hoping to capitalize on the success of its TV shows. Whether it succeeds is clearly important to Disney and its competitors, but it is unlikely to have an impact on the rest of the market. In fact, risk measures can be extended to include risks that may affect an entire sector but are restricted to that sector; we call this sector risk. For instance, a cut in the defense budget in the United States will adversely affect all firms in the defense business, including Boeing, but there should be no significant impact on other sectors. What is common across the three risks described—project, competitive, and sector risk—is that they affect only a small subset of firms.

There is another group of risks that is much more pervasive and affects many if not all investments. For instance, when interest rates increase, all investments are negatively affected, albeit to different degrees. Similarly, when the economy weakens, all firms feel the effects, though cyclical firms (such as automobiles, steel, and housing) may feel it more. We term this risk market risk.

Finally, there are risks that fall in a gray area, depending on how many assets they affect. For instance, when the dollar strengthens against other currencies, it has a significant impact on the earnings and values of firms with international operations. If most firms in the market have significant international operations, it could well be categorized as market risk. If only a few do, it would be closer to firm-specific risk. Figure 4.4 summarizes the spectrum of firm-specific and market risks.

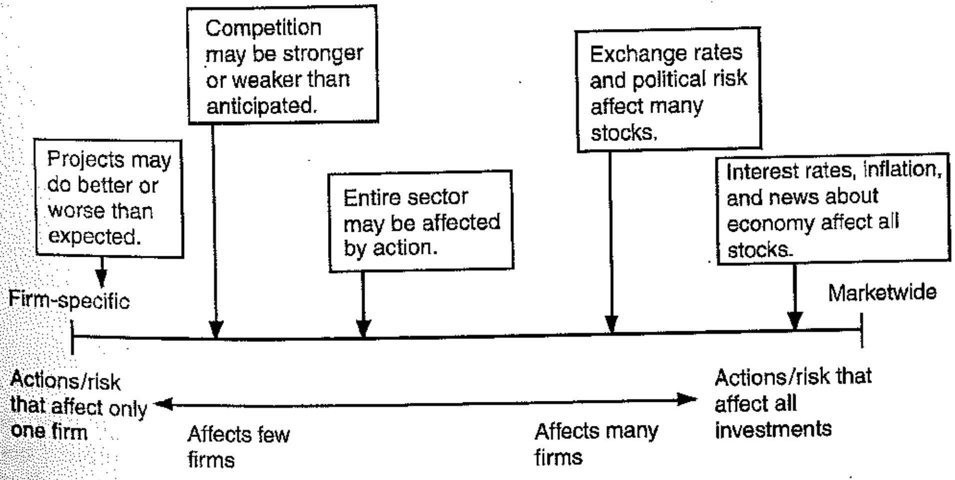


FIGURE 4.4 Breakdown of Risk

Why Diversification Reduces or Eliminates Firm-Specific Risk: An Intuitive Explanation As an investor, you could invest all your portfolio in one asset. If you do so, you are exposed to both firm-specific and market risk. If, however, you expand your portfolio to include other assets or stocks, you are diversifying, and by doing so you can reduce your exposure to firm-specific risk. There are two reasons why diversification reduces or, at the limit, eliminates firm-specific risk. The first is that each investment in a diversified portfolio is a much smaller percentage of that portfolio than would be the case if you were not diversified. Any action that increases or decreases the value of only that investment or a small group of investments will have only a small impact on your overall portfolio, whereas undiversified investors are much more exposed to changes in the values of the investments in their portfolios. The second reason is that the effects of firm-specific actions on the prices of individual assets in a portfolio can be either positive or negative for each asset for any period. Thus, in very large portfolios this risk will average out to zero and will not affect the overall value of the portfolio.

In contrast, the effects of marketwide movements are likely to be in the same direction for most or all investments in a portfolio, though some assets may be affected more than others. For instance, other things being equal, an increase in interest rates will lower the values of most assets in a portfolio. Being more diversified does not eliminate this risk.

A Statistical Analysis of Diversification-Reducing Risk The effects of diversification on risk can be illustrated fairly dramatically by examining the effects of increasing the number of assets in a portfolio on portfolio variance. The variance in a portfolio is partially determined by the variances of the individual assets in the portfolio and partially by how they move together; the latter is measured statistically with a correlation coefficient or the covariance across investments in the portfolio. It is the covariance term that provides an insight into why diversification will reduce risk and by how much.

Consider a portfolio of two assets. Asset A has an expected return of μ_A and a variance in returns of σ^2_A, while asset B has an expected return of μ_B and a variance in returns of σ^2_B. The correlation in returns between the two assets, which measures how the assets move together, is ρ_{AB}. The expected returns and variances of a two-asset portfolio can be written as a function of these inputs and the proportion of the portfolio going to each asset.

$$\mu_{\text{portfolio}} = w_A \mu_A + (1 - w_A)\mu_B$$
$$\sigma^2_{\text{portfolio}} = w_A^2 \sigma_A^2 + (1 - w_A)^2 \sigma_B^2 + 2w_A(1 - w_A)\rho_{AB}\sigma_A\sigma_B$$

where w_A = Proportion of the portfolio in asset A

The last term in the variance formulation is sometimes written in terms of the covariance in returns between the two assets, which is:

$$\sigma_{AB} = \rho_{AB}\sigma_A\sigma_B$$

The savings that accrue from diversification are a function of the correlation coefficient. Other things remaining equal, the higher the correlation in returns between the two assets, the smaller are the potential benefits from diversification. It is rationale presented by those who use shorter periods is that the risk aversion of the average investor is likely to change over time, and that using a shorter time period provides a more updated estimate. This has to be offset against a cost associated with using shorter time periods, which is the greater noise in the risk premium estimate. In fact, given the annual standard deviation in stock prices[7] between 1926 and 2010 of 20 percent, the standard error[8] associated with the risk premium estimate can be estimated for different estimation periods in Table 7.2.

Note that to get reasonable standard errors, we need very long time periods of historical returns. Conversely, the standard errors from 10-year and 20-year estimates are likely to be almost as large as or larger than the actual risk premium estimated. This cost of using shorter time periods seems, in our view, to overwhelm any advantages associated with getting a more updated premium.

2. *Choice of risk-free security.* The Ibbotson database reports returns on both Treasury bills (T-bills) and Treasury bonds (T-bonds), and the risk premium for stocks can be estimated relative to each. Given that the yield curve in the United States has been upward-sloping for most of the past seven decades, the risk premium is larger when estimated relative to shorter-term government securities (such as Treasury bills). *The risk-free rate chosen in computing the premium has to be consistent with the risk-free rate used to compute expected returns.* Thus, if the Treasury bill rate is used as the risk-free rate, the premium has to be the premium earned by stocks over that rate. If the Treasury bond rate is used as the risk-free rate, the premium has to be estimated relative to that rate. For the most part, in corporate finance and valuation, the risk-free rate will be a long-term default-free Treasury (government) bond rate and not a Treasury bill rate. Thus, the risk premium used should be the premium earned by stocks over Treasury bonds.

3. *Arithmetic and geometric averages.* The final sticking point when it comes to estimating historical premiums relates to how the average returns on stocks, Treasury bonds, and Treasury bills are computed. The arithmetic average return measures the simple mean of the series of annual returns, whereas the geometric

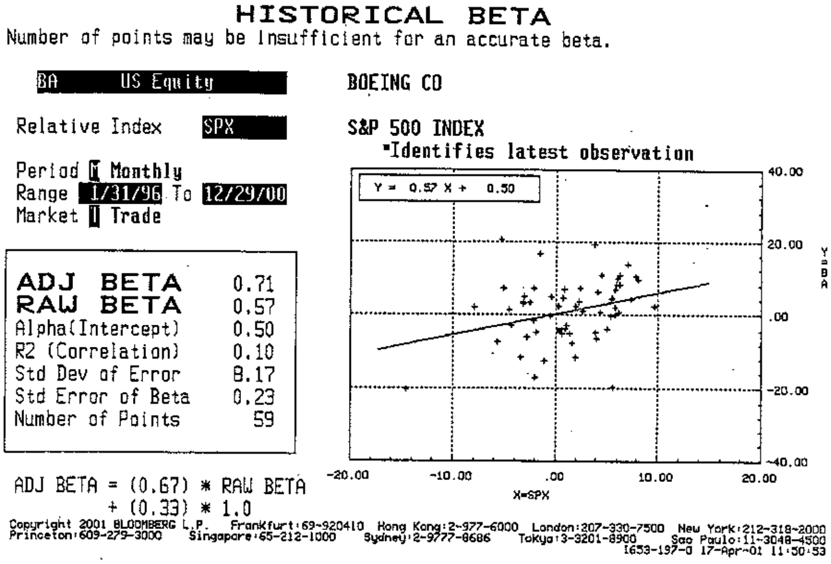
TABLE 7.2 Standard Errors in Risk Premium Estimates

Estimation Period	Standard Error of Risk Premium Estimate
5 years	$20\%/\sqrt{5} = 8.94\%$
10 years	$20\%/\sqrt{10} = 6.32\%$
25 years	$20\%/\sqrt{25} = 4.00\%$
50 years	$20\%/\sqrt{50} = 2.83\%$

---

[7] For the historical data on stock returns, bond returns, and bill returns, check under "Updated Data" in www.stern.nyu.edu/~adamodar.

[8] These estimates of the standard error are probably understated, because they are based on the assumption that annual returns are uncorrelated over time. There is substantial empirical evidence that returns are correlated over time, which would make this standard error estimate much larger.



HISTORICAL BETA

Number of points may be Insufficient for an accurate beta.

BA US Equity

BOEING CO

Relative Index SPX

S&P 500 INDEX

•Identifies latest observation

Period M Monthly
Range 1/31/96 To 12/29/00
Market T Trade

ADJ BETA	0.71
RAW BETA	0.57
Alpha(Intercept)	0.50
R2 (Correlation)	0.10
Std Dev of Error	8.17
Std Error of Beta	0.23
Number of Points	59

ADJ BETA = (0.67) * RAW BETA
+ (0.33) * 1.0

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Princeton:609-279-3000 Singapore:65-212-1000 Sydney:2-9777-8686 Tokyo:3-3201-8900 Sao Paulo:11-3048-4500
1653-197-0 17-Apr-01 11:50:53

FIGURE 8.2 Beta Estimate for Boeing

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from Bloomberg for Boeing, using the same period as our regression (January 1996 to December 2000).

While the time period used is identical to the one used in our earlier regression, there are subtle differences between this regression and the one in Figure 8.1. First, Bloomberg uses price appreciation in the stock and the market index in estimating betas and ignores dividends.[4] The fact that dividends are ignored does not make much difference for a company like Boeing, but it could make a difference for a company that either pays no dividends or pays significantly higher dividends than the market. This explains the mild differences in the intercept (0.50% versus 0.54%) and the beta (0.57 versus 0.56).

Second, Bloomberg also computes what it calls an adjusted beta, which is estimated as follows:

$$\text{Adjusted beta} = \text{Raw beta}(0.67) + 1.00(0.33)$$

These weights (0.67 and 0.33) do not vary across stocks, and this process pushes all estimated betas toward 1. Most services employ similar procedures to adjust betas toward 1. In doing so, they are drawing on empirical evidence that suggests that the betas for most companies, over time, tend to move toward the average beta, which is 1. This may be explained by the fact that firms get more diversified in their product mix and client base as they get larger. While we agree with the notion that betas move toward 1 over time, the weighting process used by most services strikes us as arbitrary and not particularly useful.

---

[4] This is done purely for computational convenience.

The unlevered beta of a firm is determined by the nature of its products and services (cyclicality, discretionary nature) and its operating leverage. It is often also referred to as the asset beta, since it is determined by the assets owned by the firm. Thus, the levered beta, which is also the beta for an equity investment in a firm, is determined both by the riskiness of the business it operates in and by the amount of financial leverage risk it has taken on.

Since financial leverage multiplies the underlying business risk, it stands to reason that firms that have high business risk should be reluctant to take on financial leverage. It also stands to reason that firms that operate in stable businesses should be much more willing to take on financial leverage. Utilities, for instance, have historically had high debt ratios but have not had high betas, mostly because their underlying businesses have been stable and fairly predictable.

ILLUSTRATION 8.3: Effects of Leverage on Betas: Boeing

From the regression for the period from 1996 to 2000, Boeing had a historical beta of 0.56. Since this regression uses stock prices of Boeing over this period, we began by estimating the average debt-to-equity ratio between 1996 and 2000, using market values for debt and equity.

$$\text{Average debt-to-equity ratio between 1996 and 2000} = 15.56\%$$

The beta over the 1996–2000 period reflects this average leverage. To estimate the unlevered beta over the period, a marginal tax rate of 35% is used:

$$\begin{aligned}\text{Unlevered beta} &= \text{Current beta}/[1 + (1 - \text{Tax rate})(\text{Average debt/Equity})]\\ &= 0.56/[1 + (1 - 0.35)(0.1556)] = 0.51\end{aligned}$$

The unlevered beta for Boeing over the 1996–2000 period is 0.51. The levered beta at different levels of debt can then be estimated:

$$\text{Levered beta} = \text{Unlevered beta} \times [1 + (1 - \text{Tax rate})(\text{Debt/Equity})]$$

For instance, if Boeing were to increase its debt equity ratio to 10%, its equity beta will be:

$$\text{Levered beta (@10\% D/E)} = 0.51 \times [1 + (1 - 0.35)(0.10)] = 0.543$$

If the debt equity ratio were raised to 25%, the equity beta would be:

$$\text{Levered beta (@25\% D/E)} = 0.51 \times [1 + (1 - 0.35)(0.25)] = 0.59$$

The following table summarizes the beta estimates for different levels of financial leverage ranging from 0% to 90% debt.

Debt to Capital	*Debt/Equity Ratio*	*Beta*	*Effect of Leverage*
0%	0.00%	0.51	0.00
10%	11.11%	0.55	0.04
20%	25.00%	0.59	0.08
30%	42.86%	0.65	0.14
40%	66.67%	0.73	0.22
50%	100.00%	0.84	0.33
60%	150.00%	1.00	0.50
70%	233.33%	1.28	0.77
80%	400.00%	1.83	1.32
90%	900.00%	3.48	2.98

As Boeing's financial leverage increases, the beta increases concurrently.

levbeta.xls. **This spreadsheet allows you to estimate the unlevered beta for a firm and compute the betas as a function of the leverage of the firm.**

Bottom-Up Betas Breaking down betas into their business risk and financial leverage components provides us with an alternative way of estimating betas, in which we do not need past prices on an individual firm or asset to estimate its beta.

To develop this alternative approach, we need to introduce an additional property of betas that proves invaluable. The beta of two assets put together is a weighted average of the individual asset betas, with the weights based on market value. Consequently, the beta for a firm is a weighted average of the betas of all the different businesses it is in. We can estimate the beta for a firm in five steps:

Step 1: Identify the business or businesses the firm operates in.

Step 2: Find other publicly traded firms in each business and obtain their regression betas, which we use to compute an average beta for the firms.

Step 3: Estimate the average unlevered beta for the business by unlevering the average (or median) beta for the firms by their average (or median) debt-to-equity ratio. Alternatively, we could estimate the unlevered beta for each firm and then compute the average of the unlevered betas. The first approach is preferable because unlevering an erroneous regression beta is likely to compound the error.

$$\text{Unlevered beta}_{\text{business}} = \text{Beta}_{\text{comparable firms}}/[1 + (1 - t)(\text{D/E ratio}_{\text{comparable firms}})]$$

Step 4: Estimate an unlevered beta for the firm being analyzed, taking a weighted average of the unlevered betas for the businesses it operates in, using the proportion of firm value derived from each business as the weights. If values are not available, use operating income or revenues as weights. This weighted average is called the bottom-up unlevered beta.

$$\text{Unlevered beta}_{\text{firm}} = \sum_{j=1}^{j=k}\left(\text{Unlevered beta}_j \times \text{Value weight}_j\right)$$

where the firm is assumed to operating in k different businesses.

Step 5: Finally, estimate the current market values of debt and equity at the firm and use this debt-to-equity ratio to estimate a levered beta.

The betas estimated using this processs are called bottom-up betas.

The Case for Bottom-Up Betas At first sight, the use of bottom-up betas may seem to leave us exposed to all of the problems noted with regression betas. After

3. *Extent of disagreement between analysts.* While consensus earnings growth rates are useful in valuation, the extent of disagreement between analysts measured by the standard deviation in growth predictions is also a useful measure of the reliability of the consensus forecasts. Givoly and Lakonsihok (1984) found that the dispersion of earnings is correlated with other measures of risk such as beta and is a good predictor of expected returns.

4. *Quality of analysts following the stock.* This is the hardest of the variables to quantify. One measure of quality is the size of the forecast error made by analysts following a stock, relative to models that use only historical data—the smaller this relative error, the larger the weight that should be attached to analyst forecasts. Another measure is the effect on stock prices of analyst revisions—the more informative the forecasts, the greater the effect on stock prices. There are some who argue that the focus on consensus forecasts misses the point that some analysts are better than others in predicting earnings, and that their forecasts should be isolated from the rest and weighted more.

Analyst forecasts may be useful in coming up with a predicted growth rate for a firm, but there is a danger to blindly following consensus forecasts. Analysts often make significant errors in forecasting earnings, partly because they depend on the same data sources (which might have been erroneous or misleading) and partly because they sometimes overlook significant shifts in the fundamental characteristics of the firm. The secret to successful valuation often lies in discovering inconsistencies between analysts' forecasts of growth and a firm's fundamentals. The next section examines this relationship in more detail.

FUNDAMENTAL DETERMINANTS OF GROWTH

With both historical and analyst estimates, growth is an exogenous variable that affects value but is divorced from the operating details of the firm. The soundest way of incorporating growth into value is to make it endogenous i.e., tie in more closely to the actions that a business takes to create and sustain that growth. This section begins by considering the relationship between fundamentals and growth in equity income, and then moves on to look at the determinants of growth in operating income.

Growth in Equity Earnings

When estimating cash flows to equity, we usually begin with estimates of net income, if we are valuing equity in the aggregate, or earnings per share, if we are valuing equity per share. This section begins by presenting the fundamentals that determine expected growth in earnings per share and then move on to consider a more expanded version of the model that looks at growth in net income.

Growth in Earnings per Share The simplest relationship determining growth is one based on the retention ratio (percentage of earnings retained in the firm) and the return on equity on its projects. Firms that have higher retention ratios and earn higher returns on equity should have much higher growth rates in earnings per share than firms that do not share these characteristics. To establish this, note that:

$$g_t = (NI_t - NI_{t-1})/NI_{t-1}$$

where g_t = Growth rate in net income
NI_t = Net income in year t

Also note that the ROE in period t can be written as NI in period t divided by the Book value of equity in period t − 1. Given the definition of return on equity, the net income in year t − 1 can be written as:

$$NI_{t-1} = \text{Book value of equity}_{t-2} \times ROE_{t-1}$$

where ROE_{t-1} = Return on equity in year t − 1

The net income in year t can be written as:

$$NI_t = (\text{Book value of equity}_{t-2} + \text{Retained earnings}_{t-1}) \times ROE_t$$

Assuming that the return on equity is unchanged (i.e., $ROE_t = ROE_{t-1} = ROE$):

$$\begin{aligned} g_t &= \text{Retained earnings}_{t-1}/NI_{t-1} \times ROE \\ &= \text{Retention ratio} \times ROE \\ &= b \times ROE \end{aligned}$$

where b is the retention ratio. Note that the firm is not being allowed to raise equity by issuing new shares. Consequently, the growth rate in net income and the growth rate in earnings per share are the same in this formulation.

ILLUSTRATION 11.5: Growth in Earnings per Share

This illustration considers the expected growth rate in earnings based on the retention ratio and return on equity for three firms—Consolidated Edison, a regulated utility that provides power to New York City and its environs; Procter & Gamble, a leading brand-name consumer product firm; and Intel, the technology giant—in 2010. The following table summarizes the returns on equity, retention ratios, and expected growth rates in earnings for the three firms in 2010:

	Return on Equity	*Retention Ratio*	*Expected Growth Rate*
Consolidated Edison	9.79%	36.00%	3.52%
Procter & Gamble	20.09%	50.26%	10.10%
Intel	32.00%	70.00%	22.40%

Intel has the highest expected growth rate in earnings per share, assuming that it can maintain its current return on equity and retention ratio. Procter & Gamble also can be expected to post a healthy growth rate, notwithstanding the fact that it pays out more than 50% of its earnings as dividends because of its high return on equity. Con Ed, on the other hand, has a very low expected growth rate because its return on equity and retention ratio are anemic.

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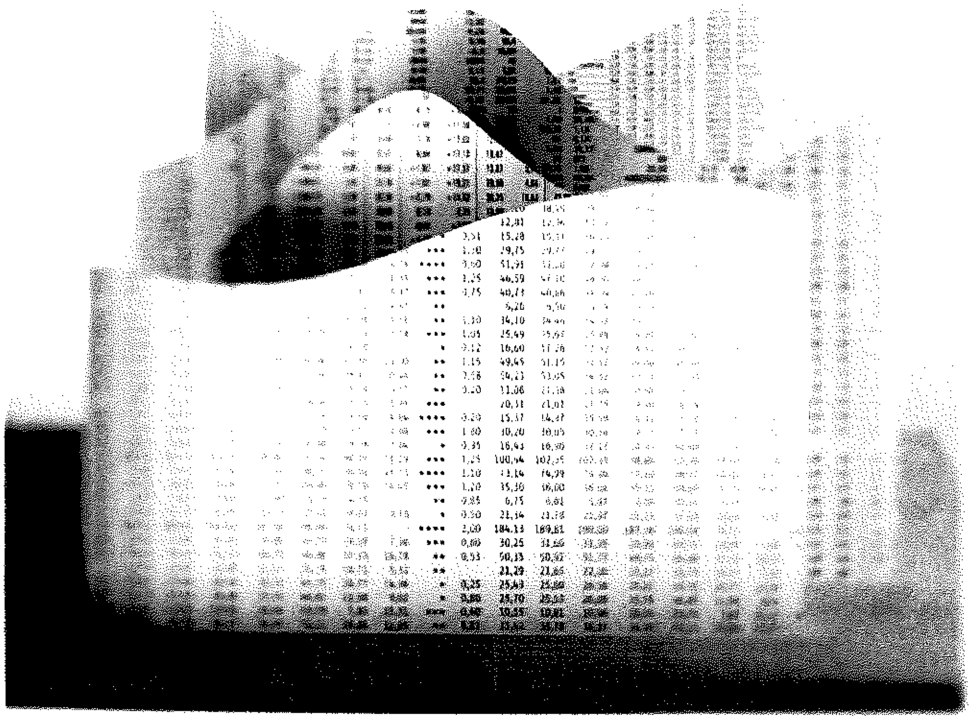
Essentials of Investments



Ninth Edition

6.1 DIVERSIFICATION AND PORTFOLIO RISK

Suppose you have in your risky portfolio only one stock, say, Dell Computers. What are the sources of risk affecting this "portfolio"?

We can identify two broad sources of uncertainty. The first is the risk from general economic conditions, such as business cycles, inflation, interest rates, exchange rates, and so forth. None of these macroeconomic factors can be predicted with certainty, and all affect Dell stock. Then you must add firm-specific influences, such as Dell's success in R&D, its management style and philosophy, and so on. Firm-specific factors are those that affect Dell without noticeably affecting other firms.

Now consider adding another security to the risky portfolio. If you invest half of your risky portfolio in ExxonMobil, leaving the other half in Dell, what happens to portfolio risk? Because the firm-specific influences on the two stocks differ (statistically speaking, the influences are independent), this strategy should reduce portfolio risk. For example, when oil prices fall, hurting ExxonMobil, computer prices might rise, helping Dell. The two effects are offsetting, which stabilizes portfolio return.

But why stop at only two stocks? Diversifying into many more securities continues to reduce exposure to firm-specific factors, so portfolio volatility should continue to fall. Ultimately, however, there is no way to avoid all risk. To the extent that virtually all securities are affected by common (risky) macroeconomic factors, we cannot eliminate exposure to general economic risk, no matter how many stocks we hold.

Figure 6.1 illustrates these concepts. When all risk is firm-specific, as in Figure 6.1A, diversification can reduce risk to low levels. With all risk sources independent, and with investment spread across many securities, exposure to any particular source of risk is negligible. This is an application of the law of large numbers. The reduction of risk to very low levels because of independent risk sources is called the *insurance principle.*

When a common source of risk affects all firms, however, even extensive diversification cannot eliminate all risk. In Figure 6.1B, portfolio standard deviation falls as the number of securities increases, but it is not reduced to zero. The risk that remains even after diversification is called **market risk,** risk that is attributable to marketwide risk sources. Other terms are **systematic risk** or **nondiversifiable risk.** The risk that *can* be eliminated by diversification is called **unique risk, firm-specific risk, nonsystematic risk,** or **diversifiable risk.**

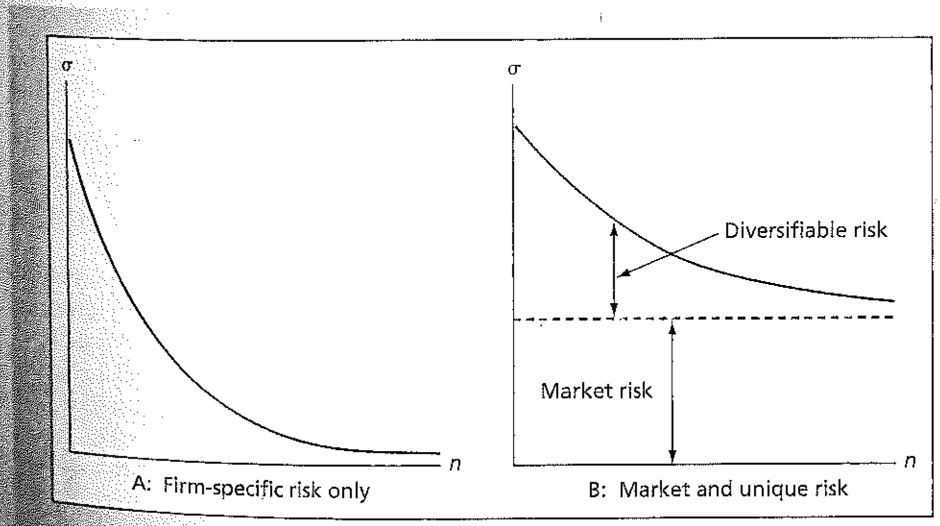
This analysis is borne out by empirical studies. Figure 6.2 shows the effect of portfolio diversification, using data on NYSE stocks. The figure shows the average standard deviations of equally weighted portfolios constructed by selecting stocks at random as a function of the number of stocks in the portfolio. On average, portfolio risk does fall with diversification, but

market risk, systematic risk, nondiversifiable risk

Risk factors common to the whole economy.

unique risk, firm-specific risk, nonsystematic risk, diversifiable risk

Risk that can be eliminated by diversification.



A: Firm-specific risk only

B: Market and unique risk

FIGURE 6.1

Portfolio risk as a function of the number of stocks in the portfolio the economy toward this goal. In contrast, supply-side policies treat the issue of the productive capacity of the economy. The goal is to create an environment in which workers and owners of capital have the maximum incentive and ability to produce and develop goods.

Supply-side economists also pay considerable attention to tax policy. While demand-siders look at the effect of taxes on consumption demand, supply-siders focus on incentives and marginal tax rates. They argue that lowering tax rates will elicit more investment and improve incentives to work, thereby enhancing economic growth. Some go so far as to claim that reductions in tax rates can lead to increases in tax revenues because the lower tax rates will cause the economy and the revenue tax base to grow by more than the tax rate is reduced.

CONCEPT check 12.3

Large tax cuts in 2001 were followed by relatively rapid growth in GDP. How would demand-side and supply-side economists differ in their interpretations of this phenomenon?

12.6 BUSINESS CYCLES

We've looked at the tools the government uses to fine-tune the economy, attempting to maintain low unemployment and low inflation. Despite these efforts, economies repeatedly seem to pass through good and bad times. One determinant of the broad asset allocation decision of many analysts is a forecast of whether the macroeconomy is improving or deteriorating. A forecast that differs from the market consensus can have a major impact on investment strategy.

The Business Cycle

business cycles

Recurring cycles of recession and recovery.

peak

The transition from the end of an expansion to the start of a contraction.

trough

The transition point between recession and recovery.

The economy recurrently experiences periods of expansion and contraction, although the length and depth of these cycles can be irregular. These recurring patterns of recession and recovery are called **business cycles.** Figure 12.4 presents graphs of several measures of production and output. The production series all show clear variation around a generally rising trend. The bottom graph of capacity utilization also evidences a clear cyclical (although irregular) pattern.

The transition points across cycles are called peaks and troughs, identified by the boundaries of the shaded areas of the graph. A **peak** is the transition from the end of an expansion to the start of a contraction. A **trough** occurs at the bottom of a recession just as the economy enters a recovery. The shaded areas in Figure 12.4 all represent periods of recession.

cyclical industries

Industries with above-average sensitivity to the state of the economy.

As the economy passes through different stages of the business cycle, the relative profitability of different industry groups might be expected to vary. For example, at a trough, just before the economy begins to recover from a recession, one would expect that **cyclical industries,** those with above-average sensitivity to the state of the economy, would tend to outperform other industries. Examples of cyclical industries are producers of durable goods, such as automobiles or washing machines. Because purchases of these goods can be deferred during a recession, sales are particularly sensitive to macroeconomic conditions. Other cyclical industries are producers of capital goods, that is, goods used by other firms to produce their own products. When demand is slack, few companies will be expanding and purchasing capital goods. Therefore, the capital goods industry bears the brunt of a slowdown but does well in an expansion.

defensive industries

Industries with below-average sensitivity to the state of the economy.

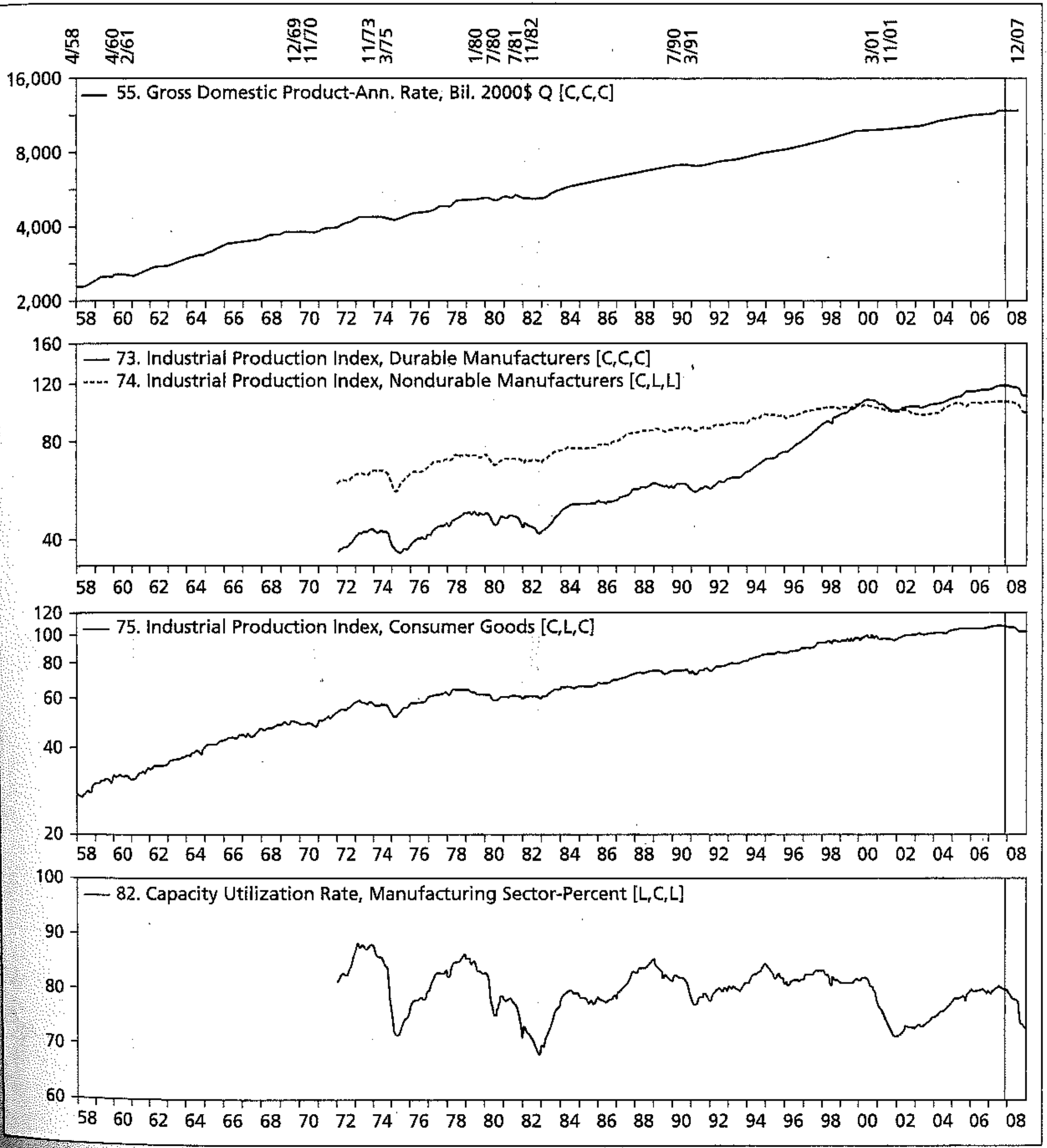
In contrast to cyclical firms, **defensive industries** have little sensitivity to the business cycle. These are industries that produce goods for which sales and profits are least sensitive to the state of the economy. Defensive industries include food producers and processors, pharmaceutical firms, and public utilities. These industries will outperform others when the economy enters a recession.

The cyclical/defensive classification corresponds well to the notion of systematic or market risk introduced in our discussion of portfolio theory. When perceptions about the health of the economy become more optimistic, for example, the prices of most stocks will increase as forecasts of profitability rise. Because the cyclical firms are most sensitive to such developments,

FIGURE 12.4

Cyclical indicators

Source: The Conference Board, *Business Cycle Indicators*, December 2008. Used with permission of The Conference Board, Inc.



their stock prices will rise the most. Thus, firms in cyclical industries will tend to have high-beta stocks. In general, then, stocks of cyclical firms will show the best results when economic news is positive, but they will also show the worst results when that news is bad. Conversely, defensive firms will have low betas and performance that is comparatively unaffected by overall market conditions.

If your assessments of the state of the business cycle were reliably more accurate than those of other investors, choosing between cyclical and defensive industries would be easy. You

5. *Turnarounds.* These are firms that are in bankruptcy or soon might be. If they can recover from what might appear to be imminent disaster, they can offer tremendous investment returns. A good example of this type of firm would be Chrysler in 1982, when it required a government guarantee on its debt to avoid bankruptcy. The stock price rose 15-fold in the next five years.
6. *Asset plays.* These are firms that have valuable assets not currently reflected in the stock price. For example, a company may own or be located on valuable real estate that is worth as much or more than the company's business enterprises. Sometimes the hidden asset can be tax-loss carryforwards. Other times the assets may be intangible. For example, a cable company might have a valuable list of cable subscribers. These assets do not immediately generate cash flow and so may be more easily overlooked by other analysts attempting to value the firm.

Industry Structure and Performance

The maturation of an industry involves regular changes in the firm's competitive environment. As a final topic, we examine the relationship between industry structure, competitive strategy, and profitability. Michael Porter (1980, 1985) has highlighted these five determinants of competition: threat of entry from new competitors, rivalry between existing competitors, price pressure from substitute products, the bargaining power of buyers, and the bargaining power of suppliers.

Threat of entry New entrants to an industry put pressure on price and profits. Even if a firm has not yet entered an industry, the potential for it to do so places pressure on prices, since high prices and profit margins will encourage entry by new competitors. Therefore, barriers to entry can be a key determinant of industry profitability. Barriers can take many forms. For example, existing firms may already have secure distribution channels for their products based on long-standing relationships with customers or suppliers that would be costly for a new entrant to duplicate. Brand loyalty also makes it difficult for new entrants to penetrate a market and gives firms more pricing discretion. Proprietary knowledge or patent protection also may give firms advantages in serving a market. Finally, an existing firm's experience in a market may give it cost advantages due to the learning that takes place over time.

Rivalry between existing competitors When there are several competitors in an industry, there will generally be more price competition and lower profit margins as competitors seek to expand their share of the market. Slow industry growth contributes to this competition since expansion must come at the expense of a rival's market share. High fixed costs also create pressure to reduce prices since fixed costs put greater pressure on firms to operate near full capacity. Industries producing relatively homogeneous goods also are subject to considerable price pressure since firms cannot compete on the basis of product differentiation.

Pressure from substitute products Substitute products mean that the industry faces competition from firms in related industries. For example, sugar producers compete with corn syrup producers. Wool producers compete with synthetic fiber producers. The availability of substitutes limits the prices that can be charged to customers.

Bargaining power of buyers If a buyer purchases a large fraction of an industry's output, it will have considerable bargaining power and can demand price concessions. For example, auto producers can put pressure on suppliers of auto parts. This reduces the profitability of the auto parts industry.

Bargaining power of suppliers If a supplier of a key input has monopolistic control over the product, it can demand higher prices for the good and squeeze profits out of the industry. One special case of this issue pertains to organized labor as a supplier of a key

$$V_0 = \frac{D_1}{1+k} + \frac{D_2}{(1+k)^2} + \cdots + \frac{D_H + P_H}{(1+k)^H} \quad \text{(13.2)}$$

Note the similarity between this formula and the bond valuation formula developed in Chapter 10. Each relates price to the present value of a stream of payments (coupons in the case of bonds, dividends in the case of stocks) and a final payment (the face value of the bond or the sales price of the stock). The key differences in the case of stocks are the uncertainty of dividends, the lack of a fixed maturity date, and the unknown sales price at the horizon date. Indeed, one can continue to substitute for price indefinitely to conclude

$$V_0 = \frac{D_1}{1+k} + \frac{D_2}{(1+k)^2} + \frac{D_3}{(1+k)^3} + \cdots \quad \text{(13.3)}$$

dividend discount model (DDM)

A formula for the intrinsic value of a firm equal to the present value of all expected future dividends.

Equation 13.3 states the stock price should equal the present value of all expected future dividends into perpetuity. This formula is called the **dividend discount model (DDM)** of stock prices.

It is tempting, but incorrect, to conclude from Equation 13.3 that the DDM focuses exclusively on dividends and ignores capital gains as a motive for investing in stock. Indeed, we assume explicitly in Equation 13.1 that capital gains (as reflected in the expected sales price, P_1) are part of the stock's value. At the same time, the price at which you can sell a stock in the future depends on dividend forecasts at that time.

The reason only dividends appear in Equation 13.3 is not that investors ignore capital gains. It is instead that those capital gains will be determined by dividend forecasts at the time the stock is sold. That is why in Equation 13.2 we can write the stock price as the present value of dividends plus sales price for *any* horizon date. P_H is the present value at time H of all dividends expected to be paid after the horizon date. That value is then discounted back to today, time 0. The DDM asserts that stock prices are determined ultimately by the cash flows accruing to stockholders, and those are dividends.

The Constant-Growth DDM

Equation 13.3 as it stands is still not very useful in valuing a stock because it requires dividend forecasts for every year into the indefinite future. To make the DDM practical, we need to introduce some simplifying assumptions. A useful and common first pass at the problem is to assume that dividends are trending upward at a stable growth rate that we will call g. For example, if $g = .05$ and the most recently paid dividend was $D_0 = 3.81$, expected future dividends are

$$\begin{aligned}
D_1 &= D_0(1+g) = 3.81 \times 1.05 = 4.00 \\
D_2 &= D_0(1+g)^2 = 3.81 \times (1.05)^2 = 4.20 \\
D_3 &= D_0(1+g)^3 = 3.81 \times (1.05)^3 = 4.41 \text{ etc.}
\end{aligned}$$

Using these dividend forecasts in Equation 13.3, we solve for intrinsic value as

$$V_0 = \frac{D_0(1+g)}{1+k} + \frac{D_0(1+g)^2}{(1+k)^2} + \frac{D_0(1+g)^3}{(1+k)^3} + \cdots$$

This equation can be simplified to

$$V_0 = \frac{D_0(1+g)}{k-g} = \frac{D_1}{k-g} \quad \text{(13.4)}$$

Note in Equation 13.4 that we divide D_1 (not D_0) by $k - g$ to calculate intrinsic value. If the market capitalization rate for Steady State is 12%, we can use Equation 13.4 to show that the intrinsic value of a share of Steady State stock is

CONCEPT check 13.3

a. Calculate the price of a firm with a plowback ratio of .60 if its ROE is 20%. Current earnings, E_1, will be $5 per share, and $k = 12.5\%$.

b. What if ROE is 10%, which is less than the market capitalization rate? Compare the firm's price in this instance to that of a firm with the same ROE and E_1 but a plowback ratio of $b = 0$.

Life Cycles and Multistage Growth Models

As useful as the constant-growth DDM formula is, you need to remember that it is based on a simplifying assumption, namely, that the dividend growth rate will be constant forever. In fact, firms typically pass through life cycles with very different dividend profiles in different phases. In early years, there are ample opportunities for profitable reinvestment in the company. Payout ratios are low, and growth is correspondingly rapid. In later years, the firm matures, production capacity is sufficient to meet market demand, competitors enter the market, and attractive opportunities for reinvestment may become harder to find. In this mature phase, the firm may choose to increase the dividend payout ratio, rather than retain earnings. The dividend level increases, but thereafter it grows at a slower rate because the company has fewer growth opportunities.

Table 13.2 illustrates this profile. It gives Value Line's forecasts of return on capital, dividend payout ratio, and projected three-year growth rate in earnings per share of a sample of the firms included in the computer software and services industry versus those of East Coast electric utilities. (We compare return on capital rather than return on equity because the latter is affected by leverage, which tends to be far greater in the electric utility industry than in the software industry. Return on capital measures operating income per dollar of total long-term

TABLE 13.2 Financial ratios in two industries

	Return on Capital	Payout Ratio	Growth Rate 2012–2015
Computer Software			
Adobe Systems	13.0%	0.0%	15.4%
Cognizant	19.0	0.0	21.0
Compuware	16.5	0.0	18.6
Intuit	21.0	21.0	13.3
Microsoft	31.5	30.0	10.2
Oracle	20.0	14.0	10.3
Red Hat	14.0	0.0	17.9
Parametric Tech	15.5	0.0	9.6
SAP	18.5	28.0	6.7
Median	18.5%	0.0%	13.3%
Electric Utilities			
Central Hudson G&E	6.0%	67.0%	2.6%
Central Vermont	6.0	54.0	1.9
Consolidated Edison	6.0	63.0	2.7
Duke Energy	5.5	65.0	4.4
Northeast Utilities	6.5	47.0	6.3
NStar	9.0	60.0	8.4
Pennsylvania Power (PPL Corp.)	7.0	55.0	3.6
Public Services Enter.	6.5	45.0	8.4
United Illuminating	5.0	73.0	2.2
Median	6.0%	60.0%	3.6%

Source: From *Value Line Investment Survey*, November and December 2011. Reprinted with permission of Value Line Investment Survey © 2012 Value Line, Inc. All Rights Reserved Worldwide. "Value Line" is a registered trademark of Value Line Inc.

financing, regardless of whether the source of the capital supplied is debt or equity. We will return to this issue in the next chapter.)

By and large, software firms have attractive investment opportunities. The median return on capital of these firms is forecast to be 18.5%, and the firms have responded with quite high plowback ratios. Most of these firms pay no dividends at all. The high returns on capital and high plowback ratios result in rapid growth. The median growth rate of earnings per share in this group is projected at 13.3%.

In contrast, the electric utilities are more representative of mature firms. Their median return on capital is lower, 6%; dividend payout is higher, 60%; and average growth rate is lower, 3.6%. We conclude that the higher payouts of the electric utilities reflect their more limited opportunities to reinvest earnings at attractive rates of return.

To value companies with temporarily high growth, analysts use a multistage version of the dividend discount model. Dividends in the early high-growth period are forecast and their combined present value is calculated. Then, once the firm is projected to settle down to a steady growth phase, the constant-growth DDM is applied to value the remaining stream of dividends.

We can illustrate this with a real-life example using a **two-stage DDM.** Figure 13.2 is a *Value Line Investment Survey* report on Honda Motor Co. Some of Honda's relevant information of the end of 2011 is highlighted.

two-stage DDM
Dividend discount model in which dividend growth is assumed to level off only at some future date.

Honda's beta appears at the circled A, its recent stock price at the B, the per-share dividend payments at the C, the ROE (referred to as "return on shareholder equity") at the D, and the dividend payout ratio (referred to as "all dividends to net profits") at the E.[4] The rows ending at C, D, and E are historical time series. The boldfaced italicized entries under 2012 are estimates for that year. Similarly, the entries in the far right column (labeled 14–16) are forecasts for some time between 2014 and 2016, which we will take to be 2015.

Value Line provides explicit dividend forecasts over the relative short term, with dividends rising from $.72 in 2012 to $1 in 2015. We can obtain dividend inputs for this initial period by using the explicit forecasts for 2012–2015 and linear interpolation for the years between:

2012	$ .72
2013	$ .81
2014	$ .90
2015	$1.00

Now let us assume the dividend growth rate will be steady beyond 2015. What is a reasonable guess for that steady-state growth rate? Value Line forecasts a dividend payout ratio of .25 and an ROE of 10%, implying long-term growth will be

$$g = \text{ROE} \times b = 10\% \times (1 - .25) = 7.5\%$$

Our estimate of Honda's intrinsic value using an investment horizon of 2015 is therefore obtained from Equation 13.2, which we restate here:

$$\begin{aligned} V_{2011} &= \frac{D_{2012}}{(1+k)} + \frac{D_{2013}}{(1+k)^2} + \frac{D_{2014}}{(1+k)^3} + \frac{D_{2015} + P_{2015}}{(1+k)^4} \\ &= \frac{.72}{(1+k)} + \frac{.81}{(1+k)^2} + \frac{.90}{(1+k)^3} + \frac{1.00 + P_{2015}}{(1+k)^4} \end{aligned}$$

Here, P_{2015} represents the forecast price at which we can sell our shares of Honda at the end of 2015, when dividends enter their constant-growth phase. That price, according to the constant-growth DDM, should be

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[4] Because Honda is a Japanese firm, Americans would hold its shares via ADRs, or American Depository Receipts. ADRs are not shares of the firm but are *claims* to shares of the underlying foreign stock that are then traded in U.S. security markets. Value Line notes that each Honda ADR is a claim on one common share, but in other cases, each ADR may represent a claim to either multiple shares or even fractional shares.

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equal to $\overline{\sigma}^2$ and that, across any pair of stocks (say, stock i and stock j), the average covariance is $\overline{\sigma}_{ij}$. Then the portfolio variance equation can be written as shown at the bottom of Figure 5.7.

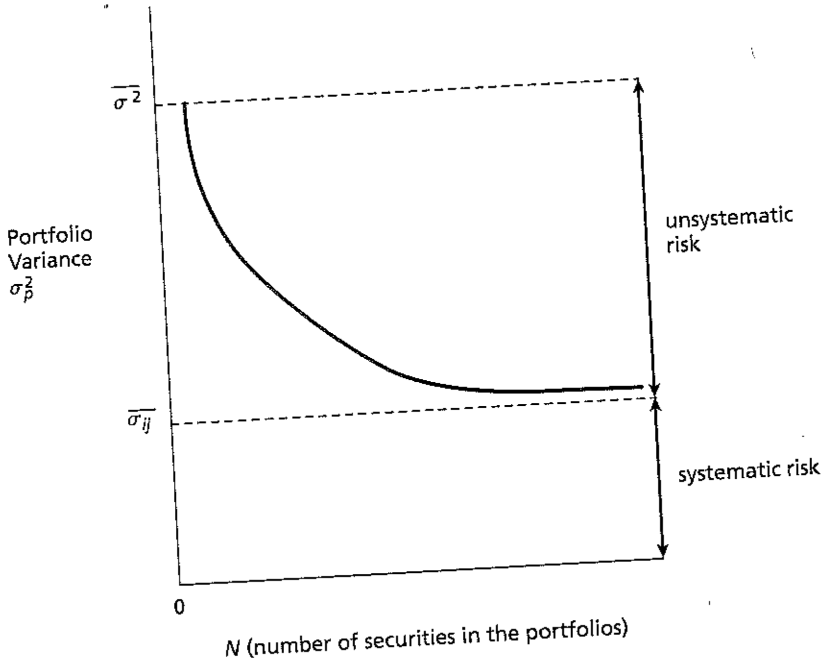
As the number N of stocks in the portfolio becomes very large, the variance term $N(1/N)^2\,\overline{\sigma}^2$ approaches zero. This means the average variance of individual stocks has no impact on portfolio variance. As N increases, the second term in the equation converges to $\overline{\sigma}_{ij}$, indicating that what really determines the risk of a large portfolio is the average covariance between all pairs of securities. A large portfolio consisting of securities that are, on average, only weakly correlated with each other will have a lower variance than a portfolio that consists of highly correlated securities.

Figure 5.8 plots the relationship between the number of securities in a portfolio and the portfolio's variance given by this equation. For investors, the figure contains both good and bad news. The good news is that as the number of securities in the portfolio increases, the portfolio's variance declines. Given the proliferation of low-cost mutual funds available today, investors can construct portfolios containing hundreds of securities, thereby reducing the variance of their investment portfolio to some degree. The bad news is that the marginal risk reduction benefit of adding more securities to the portfolio decreases as the number of securities in the portfolio increases. Not even a very well diversified portfolio can eliminate all risk.

FIGURE 5.8

Effect of Diversification on Portfolio Variance

Adding more securities to a portfolio lowers the portfolio's volatility, but the incremental benefit of adding more securities declines as the number of securities rises.



Because some risks systematically affect almost all securities, there is a limit to the risk reduction achievable by adding more securities to a portfolio. The average covariance term $\overline{\sigma}_{ij}$ represents this limit. No matter how diversified a portfolio becomes, its variance cannot fall below the average covariance of securities in the portfolio. Financial economists give this type of risk special names: undiversifiable risk, **systematic risk**, or market

JOB INTERVIEW QUESTION

What is the difference between systematic and unsystematic risk?

risk. Similarly, the risk that diversification eliminates is called diversifiable risk, **unsystematic risk**, idiosyncratic risk, or unique risk.

In real-world terms, what exactly is systematic risk? This is a difficult question to answer, and we explore it in more depth in the next chapter. For now, we just say that systematic risks are those that are common across all types of securities. Fluctuations in gross domestic product, inflation, oil prices, or interest rates can be thought of as systematic risks, and so might certain political factors. For example, the legal system governing investors and markets in a given country can influence systematic risk because that system determines the level of protection given to minority shareholders, creditors, and ordinary investors. When investors perceive that the legal system protects their interests, their willingness to trade and invest in securities increases and so the returns they require for bearing risk decline.

Smart Ethics Video

Utpal Bhattacharya, Indiana University

"The cost of equity goes up if insider trading laws are not enforced."

See the entire interview at **SMARTFinance**

If investors can cheaply eliminate some risks through diversification, then we should not expect a security to earn higher returns for risks that can be eliminated through diversification. Investors can expect compensation only for bearing systematic risk (i.e., risk that cannot be diversified away). Refer back to the Example following Equation 5.4, which showed that the average return on Merck stock was about the same over 10 years as the average return on the S&P 500 *even though* Merck stock was much more volatile than the index. An undiversified investor who held only Merck stock had to bear twice as much volatility as an investor who owned the S&P 500, even though both investors earned the same reward. This is not to say that Merck was or is a bad investment. The point is that holding Merck (or any other individual company's stock) *in isolation* is a poor investment strategy. Undiversified portfolios are generally suboptimal because they expose investors to unsystematic risk without offering higher returns.

Measuring the Systematic Risk of an Individual Security

The previous section demonstrated two important facts. First, the formula for portfolio variance shows that each security contributes to a portfolio's risk through two channels, the security's own variance and its covariance with all other securities in the portfolio. In diversified portfolios, only the second channel matters. This implies that an individual stock's variance may be a poor measure of its risk. The variance of a stock captures its total volatility, some of which is unsystematic and some of which is systematic. Second, because diversification eliminates unsystematic risk, the market provides no reward for bearing it. As a consequence, though we still expect to see a positive relationship in the market between risk and return, we can no longer be confident that a positive relationship will exist between returns on an individual asset and its variance. Again, a stock's variance captures both its systematic and unsystematic fluctuations, but only the systematic component should be correlated with returns.

We need a new measure for an individual asset's risk, one that captures only the systematic component of its volatility. Remember, the primary contribution to portfolio risk from a single asset comes from its covariance with all the other assets in the portfolio. Imagine that an investor holds a fully diversified portfolio—literally, a portfolio containing every asset available in the economy. How would this investor determine the contribution of a single security to the portfolio's risk? One way to do that would be to measure the covariance between a single asset and the portfolio. Recall, though, the difficulty that nonstandard units cause for interpreting covariance calculations. A standardized measure would be preferable, and finance theory gives us just such a measure in the concept of *beta*:

$$\beta_i = \frac{\sigma_{im}}{\sigma_m^2} \qquad \text{(Eq. 5.1}$$

The **beta** of asset *i* (β_i) equals the covariance of the asset's returns with the returns on the overall portfolio, divided by the portfolio's variance. As you will see in the next chapter, the portfolio we refer to here is known as "the market portfolio," a value-weighted portfolio of all available assets.[14] A security's beta gives us a standardized measure of its covariance with all other assets, or a measure of its systematic risk. If the market rewards only systematic risk and if beta captures the systematic risk of an individual asset, then we should observe a positive relationship between values of beta and returns in the market.

Observe that the formula for an asset's beta closely resembles that of the correlation coefficient:

$$\beta_i = \frac{\sigma_{im}}{(\sigma_m)(\sigma_m)};$$

$$\rho_i = \frac{\sigma_{im}}{(\sigma_i)(\sigma_m)}$$

The equations are identical except in one respect: The denominator of the correlation coefficient multiplies the standard deviations of the asset and the market, whereas the denominator of the beta formula squares the standard deviation of the market. This small adjustment to the denominator makes the interpretation of beta a little different from that of the correlation coefficient. First, unlike ρ, beta has no maximum or minimum value. Second, beta indicates how much the individual asset's return moves, on average, when the market moves by 1%. For example, if a stock has a beta of 1.5, then, when the market return increases by 1%, the stock return will (on average) increase by 1.5%.

EXAMPLE

Now that we understand the beta measure of a stock's risk, how does it compare to the measure we started with, standard deviation? Comparing the monthly returns on each of the four stocks listed in Table 5.3 to returns on the overall stock market, suppose you calculate the following statistics:

Stock	Covariance with Market
Mead	0.0031
Boise	0.0026
Nike	0.0011
Arrow	−0.0003

If the variance of market returns were 0.0028, then the betas of the four stocks would be as follows:

Mead 1.11 Boise 0.93 Nike 0.39 Arrow −0.11

These betas contain several surprises. First, based on comparison of the standard deviations of each stock in Table 5.3, we concluded that Nike was the riskiest security. Comparing the betas, however, suggests that Nike is less risky than either Mead or Boise Cascade. Recall

(continued)

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[14] The modifier "value-weighted" means that the fraction invested in a particular security is equal to that security's total market value as a percentage of the market value of all securities. For example, if the total market value of all securities in the market is $10 trillion and if the total market value of a certain company's stock equals $100 billion, then the fraction of that stock in a value-weighted portfolio would be 0.01, or 1%.

6.4 THE CAPITAL ASSET PRICING MODEL (CAPM)

The Security Market Line

The basic CAPM was developed almost simultaneously during the mid-1960s by William Sharpe (1964), John Lintner, and Jan Mossin (1966); it was quickly embraced by academic researchers and, in time, by practitioners as well. The reason for the CAPM's widespread acceptance is not hard to understand—for the first time, researchers and practitioners had a model that generated specific predictions about the risk–return characteristics of individual assets, and this relation was driven by how each asset *covaries* with the market portfolio.

The formal development of the CAPM requires several assumptions about investors and markets. Rather than present a detailed list of these assumptions, we present the logic of the CAPM as it flows from the material we have covered so far.

1. Investors are risk averse and require higher returns on riskier investments.
2. Because investors can diversify, they care only about the systematic (or undiversifiable) risk of any investment.
3. The market offers no reward for bearing unsystematic risk because it can be diversified away.
4. Some portfolios are better than others. Portfolios that maximize expected return for any level of risk are efficient portfolios.
5. If investors can borrow and lend at the risk-free rate, then there exists a single risky portfolio that dominates all others. Only portfolios consisting of the risk-free asset and the optimal risky portfolio are efficient.
6. If investors have homogeneous expectations then they will agree on the composition of the optimal portfolio. In equilibrium, the optimal portfolio will be the market portfolio.
7. The central insight of the CAPM is that if all investors hold the market portfolio then—when evaluating the risk of any specific asset—they will be concerned with the covariance of that asset with the overall market. The implication is that any measure of an asset's systematic risk exposure must capture how it covaries with the rest of the market. An asset's beta provides a quantitative measure of this risk, and therefore the CAPM predicts a positive, linear relationship between expected return and beta. In the CAPM, beta risk (or market risk) is the only risk that is priced.

The **capital asset pricing model (CAPM)** indicates that the expected return on a specific asset, $E(R_i)$, equals the risk-free rate plus a premium that depends on the asset's beta, β_i, and the expected risk premium on the market portfolio, $E(R_m) - R_f$:

$$E(R_i) = R_f + \beta_i\left[E(R_m) - R_f\right] \quad \text{(Eq. [illegible])}$$

Recall that beta measures an asset's correlation with a broader portfolio—in this case, the market portfolio. The higher the beta of a security, the greater the security's exposure to systematic risk and the higher the expected return it must offer investors. Although there are three variables (R_f, β_i, and $E(R_m)$) on the right-hand side of the CAPM equation, only beta changes from one security to the next. For that reason, analysts classify the CAPM as a **single-factor model**, meaning that just one variable explains differences in returns across securities.

JOB INTERVIEW QUESTION

How would you estimate the expected return of a stock?

Figure 6.8 plots the CAPM equation on a diagram with the expected return on the y-axis and beta on the x-axis. The intercept of this line is R_f, and its slope is $E(R_m) - R_f$. According to the CAPM, the equilibrium expected returns of all securities must plot on

In the CAPM, you recall, the market portfolio is a value-weighted combination assets in the economy. At present, we are unaware of any market index that attem incorporate every type of asset. When using the CAPM, most practitioners and ac ics use the returns on a broad-based stock index as a proxy for the true market port Accordingly, rather than try to estimate the expected risk premium on the market po lio, analysts usually focus on the expected equity risk premium: the difference in exp returns between a well-diversified portfolio of common stocks and a risk-free asse as a U.S. Treasury bill.

Since 1900, the average real return on stocks outpaced the average real return o Treasury bills by about 5.4% per year. But in the CAPM, what matters is not the equity risk premium from the past but rather the expected equity risk premium lo forward. Though many analysts trust the historical evidence and simply plug in a close to 6% for the term $E(R_m - R_f)$, a naive reliance on long-run historical averages the only approach for estimating the expected risk premium. Using an unbiased es is important because an error in the risk premium translates directly into an err project's discount rate and thus in its NPV.

One variable that analysts can use to obtain a forward-looking estimate of th premium is the market's aggregate earnings yield, which is the reciprocal of the pr earnings ratios. For example, to calculate the earnings yield for the S&P 500, add earnings of all 500 companies and divide by the aggregate market value of these Corporate earnings fluctuate with the business cycle, so analysts usually try to out, or *normalize*, these temporary effects before using the earnings yield to estim risk premium. In the United States, the long-run average value of the earnings about 7%, a little less than the average real return on stocks. It should come as no that the earnings yield is closely related to the real return on stocks. After all, sto resent a claim on corporate earnings.

A second forward-looking method for estimating the equity risk premium dividend growth model. Recall that this model calculates the present value of a p dividend stream growing at a constant rate, g:

$$P_0 = \frac{D_1}{r - g}$$

Rearranging this equation shows that the required return on the stock equals th the dividend yield and the dividend growth rate:

$$r = \frac{D_1}{P_0} + g$$

To use this model when estimating the equity risk premium, we must thi equation in aggregate, macroeconomic terms. In other words, r represents required return on the stock market rather than the required return on stock. The ratio D_1/P_0 represents the aggregate dividend yield, and g repre (real) growth rate of aggregate dividends. From 1872 to 1950, the expected premium derived from this model almost exactly matched the actual risk measured using average historical returns (a little more than 4%). From 195 however, the average real return on equities was much higher than predict dividend growth model.[8]

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[8] The opposite has been true since 2000: real equity returns have been *lower* on average than the di model would predict.

efficiency is more important because efficient capital markets incorporate all relevant information into financial asset prices, which in turn helps ensure that promising investments receive funding.

The concept of efficient capital markets is one of the most influential contributions that financial economics has made to modern economic thought. The **efficient markets hypothesis (EMH)**, as formally presented by Eugene Fama in 1970, has revolutionized financial thought, practice, and regulation. The EMH asserts that financial asset prices fully reflect all available information. What do we mean by "all available information"? The answer to this question varies, and we discuss three distinct versions of the efficient markets hypothesis.

The Three Forms of Market Efficiency

The EMH presents three increasingly stringent definitions of efficiency based on the information that market prices reflect: weak-form, semistrong-form, and strong-form efficiency.

Weak-Form Efficiency In markets characterized by **weak-form efficiency**, asset prices incorporate all information from the historical record—that is, all information about price trends or repeating patterns that occurred in the past. This implies that trading strategies based on analyses of historical pricing trends or relationships cannot consistently yield market-beating returns.

Prices in a weak-form efficient market will be unpredictable and will change only in response to the arrival of new information. In technical terms, this means that prices follow a **random walk**: they wander aimlessly, with no connection to past price changes and no tendency to return to a mean value over time.

Semistrong-Form Efficiency The second form of market efficiency, **semistrong-form efficiency**, asserts that asset prices incorporate *all publicly available information.* The key point about this form of efficiency is that the prices need only reflect information from *public* sources (e.g., newspapers, press releases, computer databases).

There is both a "stock" and a "flow" aspect to the information processing capabilities of semistrong-form efficient markets: First, the *level* of asset prices should correctly reflect all pertinent historical, current, and predictable future information that investors can obtain from public sources. Second, asset prices should *change* fully and instantaneously in response to relevant new information.

Strong-Form Efficiency In markets characterized by **strong-form efficiency**, asset prices reflect *all* information, both public and private. This extreme form of market efficiency implies that important company-specific information will be fully incorporated in asset prices with the very first trade after the information is generated.

In strong-form efficient markets, most insider trading would be unprofitable and there would be no benefit to ferreting out information on publicly traded companies. Any data morsel so obtained would already be reflected in stock and bond prices.

Table 10.1 on page 358 describes the three forms of market efficiency and summarizes the key implications of each form.

Malcolm Baker, Harvard University

"There is increasing evidence that stock markets aren't perfectly efficient."

See the entire interview at **SMARTFinance**

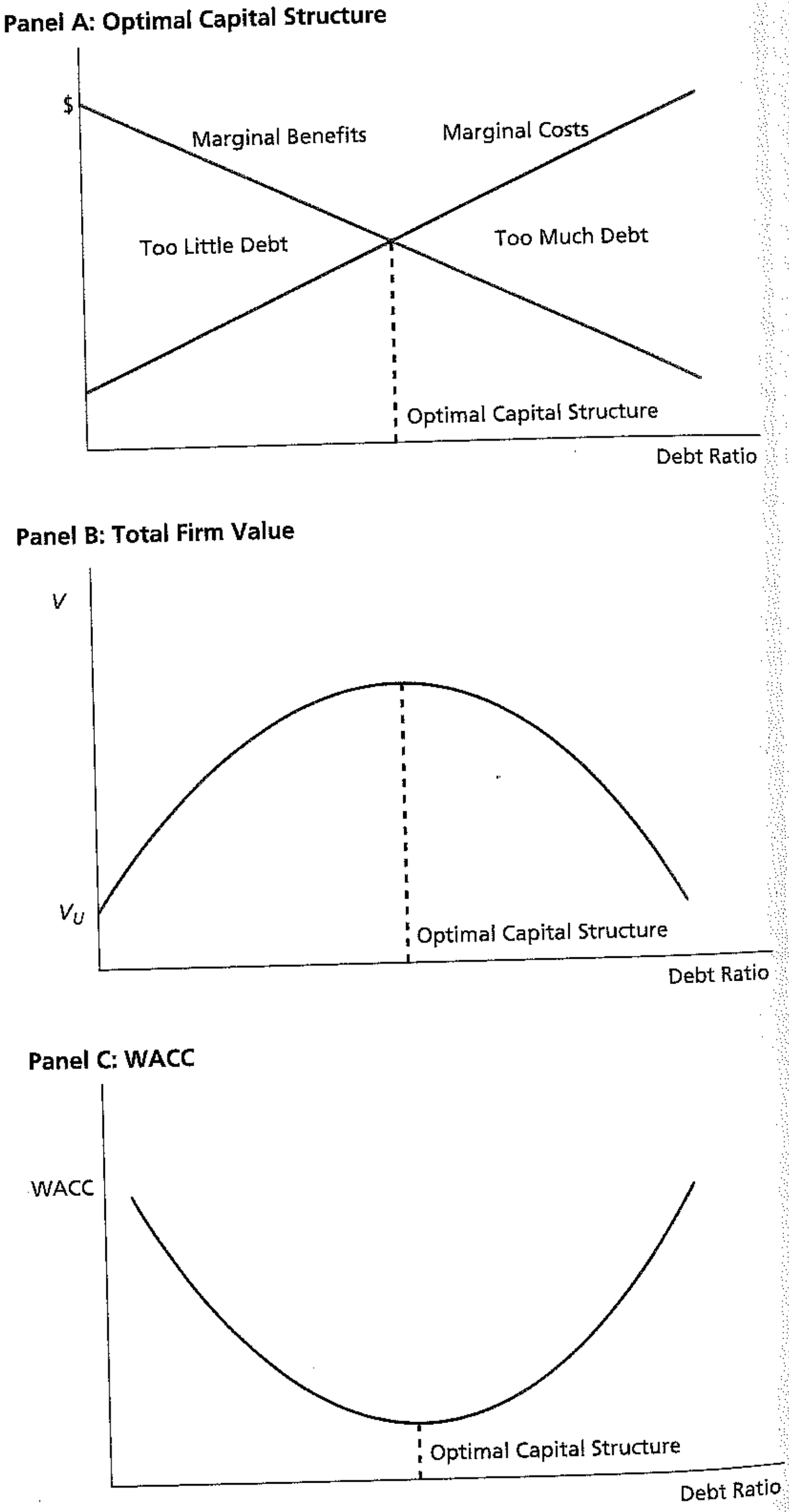
Does Empirical Evidence Support Market Efficiency?

Ultimately, whether financial markets are efficient is an empirical question. For more than a quarter of a century, the efficient market hypothesis enjoyed overwhelming support among financial economists. However, in recent years a large body of empirical evidence challenging the EMH has caused many former "true believers" to take a fresh look at the efficiency question. It also seems likely that the paralysis and near-collapse of global

FIGURE 13.2

Weighing Debt's Benefits and Costs to Find Optimal Capital Structure

The optimal amount of debt occurs where the marginal cost and marginal benefit curves intersect. At that point, total firm value is at its peak, and the weighted average cost of capital (WACC) is at its minimum.



combination of equity and debt, with the debt interest sheltering cash flows from taxes. Even so, most firms do not finance their activities exclusively with debt. This suggests that managers see debt as having costs that at some point offset debt's tax advantages. Based on observing what companies actually do, the optimal capital structure for most firms is apparently one that contains some debt, but not too much.

How do managers trade-off the benefits and costs of debt to establish a target capital structure that maximizes firm value? Figure 13.2 offers a conceptual answer to this question. The blue line in Panel A shows that the marginal benefit of borrowing an additional dollar falls as the firm's overall debt ratio rises. The red line indicates that costs associated with using debt rise as leverage increases. We will explain in the next section why marginal benefits fall and marginal costs rise as debt increases, but for now you can just take the benefits and costs in Figure 13.2 as given. As in any cost–benefit analysis, the optimum occurs when marginal benefits and marginal costs are equal. Therefore, a manager facing these cost and benefit curves would choose a debt level where the two curves intersect. To the left of that point, the firm has too little debt in the sense that marginal benefits exceed marginal costs, so adding more debt would increase firm value. At higher debt levels, debt's marginal costs exceed its benefits, so adding leverage decreases firm value.

John Graham, Duke University

"Corporate managers trade off the benefits of debt and the costs of debt."

See the entire interview at **SMARTFinance**

Panel B shows the relation between total firm value and leverage. If a firm has no debt, its value equals *Vu*. From that point, if the firm adds debt to its capital structure, its value begins to rise. At some point, firm value reaches a peak, and from that point, adding more debt decreases the value of the firm. The graph shows that, at the same point where the marginal benefit and cost curves in Panel A intersect, firm value reaches its peak. The point at which firm value begins to fall as leverage rises is exactly when debt's marginal costs first exceed its marginal benefits.

At the end of this chapter, we will demonstrate how to find the optimal debt ratio. But how much difference does finding the right capital structure really make in the overall value of the firm? In a recent study, van Binsbergen, Graham, and Yang (2008) estimate that, for the average firm, appropriate debt choices can increase firm value by about 5%. In some companies, like the one described at the beginning of Chapter 14, the increase in value may be 10% or more.

Panel C of Figure 13.2 demonstrates how a firm's weighted-average-cost of capital (WACC) changes as leverage rises. Here, the relation is U-shaped. A firm with no leverage can reduce its WACC by substituting debt for equity, but, eventually, the firm reaches a point where further increases in debt cause the WACC to increase. Naturally, managers want to find the debt ratio that minimizes the cost of capital because doing so maximizes firm value. Therefore, the optimum point in Panel C is the same optimum debt ratio in Panels A and B.

In the next section, we explore in more detail why debt's marginal benefits fall and its marginal costs rise as a firm uses more debt in its capital structure. To begin, we revisit the tax advantage of debt, taking into account some important features of the tax code that we have neglected thus far.

Concept Review Questions

1. How large would the costs of debt have to be in order to justify a firm's decision to operate with 100% equity?
2. If a firm is operating well below its optimum debt level, then what market forces might prompt it to use more debt?

The Investment Banker's Role in Equity Issues

Co-head of M&A
Sachs in the

at the
of industry and
markets."

entire interview at
Finance

We now turn to the services that investment banks provide to issuing companies, with particular attention to U.S. practices.[1] We focus on common stock offerings, though the procedures for selling bonds and preferred stocks are similar. Investment banks play several different roles throughout the securities offering process, and this section describes the evolution of these roles over the course of an issue. We also describe how issuers compensate IBs for the services they provide.

Although firms can issue stock without the assistance of investment bankers, in practice almost all firms hire IBs to help issue equity. Firms can choose an investment banker in one of two ways. The most common approach is a **negotiated offer**, where the issuing firm negotiates the terms of the offer directly with one investment bank. Alternatively, in a **competitively bid offer**, the firm announces the terms of its intended equity sale and then investment banks bid for the business. Intuition suggests that competitive bidding should be cheaper, but the empirical evidence is mixed. One clear sign that competitive offers are not better and cheaper is that the vast majority of equity sales are negotiated. If the costs of negotiated deals were truly higher, then why would so many firms choose that approach?

Firms issuing securities often hire more than one investment bank. In these cases, one of the banks is usually named the **lead underwriter**, or *book-runner*, while the other leading banks are called **co-managers**. Chen and Ritter (2000) argue that firms often prefer to issue securities with several co-managers because doing so increases the number of stock analysts that will follow the firm after the offering. Firms believe that a larger analyst following leads to greater liquidity and higher stock values. Cliff and Denis (2004) verify the importance of attracting the coverage of top-rated analysts by showing that issuing firms willingly allow their IPO share price to be set low enough to attract excess demand and high trading volume, since this will indirectly compensate the underwriters' star analysts.

Investment bankers sell equity under two types of contracts. In a **best-efforts offering**, the investment bank merely promises to give its best effort to sell the firm's securities at the agreed-upon price but makes no guarantee about the ultimate success of the offering. If there is insufficient demand, the firm withdraws the issue from the market. Best-efforts offerings are most commonly used for small, high-risk companies, and the IB receives a commission based on the number of shares sold.

In contrast, in a **firm-commitment offering** the investment bank agrees to underwrite the issue, meaning that the bank guarantees (**underwrites**) the offering price. The IB actually purchases the shares from the firm and resells them to investors. This arrangement requires the investment bank to bear the risk of inadequate demand for the issuer's shares, but banks mitigate this risk in two ways. First, the lead underwriter forms an **underwriting syndicate** consisting of many investment banks. These banks collectively purchase the firm's shares and market them, thus spreading the risk across the syndicate. Second, underwriters go to great lengths to determine the demand for a new issue before it comes to market, and they generally set the issue's *offer price* and take possession of the securities no more than a day or two before the issue date. These steps help ensure that the investment bank faces only a small risk of being unable to sell the shares that it underwrites.

In firm-commitment offerings, investment banks receive compensation for their services via the **underwriting spread**, the difference between the price at which the banks purchase shares from firms (the **net price**) and the price at which they sell the shares to institutional and individual investors (the **offer price**). In some offerings, underwriters receive additional compensation in the form of warrants that grant the right to buy shares

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[1] Ljungqvist, Jenkinson, and Wilhelm (2003) and DeGeorge, Derrien, and Womack (2007) document an increasing tendency for security issues around the world to conform to U.S. standards.

Date updated:	*5-Jan-17*	
Created by:	Aswath Damodaran, adamodar@stern.nyu.edu	
What is this data?	Total Beta (beta for completely undiversified investor)	US companies
Home Page:	http://www.damodaran.com	
Data website:	http://www.stern.nyu.edu/~adamodar/New_Home_Page/data.html	
Companies in each industry:	http://www.stern.nyu.edu/~adamodar/pc/datasets/indname.xls	
Variable definitions:	http://www.stern.nyu.edu/~adamodar/New_Home_Page/datafile/variable.htm	

Industry Name	***Number of firms***	***Average Unlevered Beta***	***Average Levered Beta***	***age correlation with the m***	***Total Unlevered Beta***	***Total Levered Beta***
Advertising	41	0.91	1.36	18.37%	4.95	7.42
Aerospace/Defense	96	0.94	1.07	30.35%	3.09	3.54
Air Transport	18	0.76	1.12	33.71%	2.26	3.33
Apparel	58	0.71	0.88	23.90%	2.95	3.68
Auto & Truck	15	0.38	0.85	34.42%	1.10	2.46
Auto Parts	63	0.94	1.12	26.78%	3.50	4.19
Bank (Money Center)	10	0.41	0.86	44.03%	0.92	1.95
Banks (Regional)	645	0.36	0.47	26.86%	1.36	1.76
Beverage (Alcoholic)	25	0.71	0.79	22.04%	3.22	3.60
Beverage (Soft)	36	0.78	0.91	17.32%	4.49	5.26
Broadcasting	30	0.70	1.22	42.36%	1.65	2.88
Brokerage & Investment Banking	45	0.42	1.08	36.21%	1.16	2.97
Building Materials	41	0.87	1.01	37.50%	2.32	2.69
Business & Consumer Services	165	0.85	1.07	26.37%	3.22	4.06
Cable TV	14	0.82	1.12	37.74%	2.17	2.96
Chemical (Basic)	45	0.68	1.00	25.94%	2.61	3.86
Chemical (Diversified)	8	1.22	1.52	46.54%	2.62	3.26
Chemical (Specialty)	100	0.98	1.20	32.00%	3.07	3.74
Coal & Related Energy	38	0.61	1.36	17.63%	3.43	7.73
Computer Services	117	0.83	0.99	25.93%	3.20	3.80
Computers/Peripherals	55	0.94	1.06	24.58%	3.84	4.29
Construction Supplies	51	1.02	1.31	39.31%	2.60	3.34
Diversified	24	0.63	0.76	41.66%	1.51	1.84
Drugs (Biotechnology)	426	1.25	1.40	22.02%	5.66	6.37
Drugs (Pharmaceutical)	164	0.93	1.02	19.66%	4.71	5.17
Education	36	1.05	1.23	23.69%	4.42	5.17
Electrical Equipment	119	1.04	1.14	23.42%	4.42	4.88
Electronics (Consumer & Office)	24	0.97	1.08	21.07%	4.61	5.14
Electronics (General)	164	0.83	0.86	23.34%	3.54	3.70
Engineering/Construction	48	1.01	1.18	36.14%	2.79	3.27
Entertainment	79	0.97	1.20	18.79%	5.14	6.40
Environmental & Waste Services	89	0.63	0.85	18.59%	3.38	4.58
Farming/Agriculture	37	0.62	0.92	21.50%	2.86	4.28
Financial Svcs. (Non-bank & Insurance)	258	0.07	0.65	25.96%	0.26	2.52
Food Processing	87	0.63	0.75	24.81%	2.54	3.04
Food Wholesalers	16	0.93	1.20	25.24%	3.68	4.77
Furn/Home Furnishings	30	0.69	0.84	29.72%	2.33	2.81
Green & Renewable Energy	25	0.47	1.14	15.99%	2.93	7.13

Healthcare Products	254	0.92	1.04	21.71%	4.25	4.77
Healthcare Support Services	121	0.82	0.94	23.91%	3.42	3.93
Heathcare Information and Technology	125	0.83	0.95	23.14%	3.59	4.11
Homebuilding	33	0.77	1.08	40.94%	1.88	2.63
Hospitals/Healthcare Facilities	38	0.45	1.10	26.59%	1.67	4.13
Hotel/Gaming	69	0.67	0.96	29.66%	2.25	3.24
Household Products	129	0.69	0.80	18.61%	3.70	4.28
Information Services	64	0.87	0.98	41.74%	2.08	2.35
Insurance (General)	19	0.71	0.90	52.06%	1.37	1.74
Insurance (Life)	22	0.80	1.03	47.22%	1.70	2.19
Insurance (Prop/Cas.)	50	0.70	0.83	41.52%	1.69	2.00
Investments & Asset Management	156	0.68	0.90	31.15%	2.17	2.88
Machinery	127	0.93	1.06	37.89%	2.46	2.80
Metals & Mining	97	0.89	1.30	14.78%	6.01	8.82
Office Equipment & Services	24	1.09	1.49	35.76%	3.06	4.17
Oil/Gas (Integrated)	7	0.95	1.08	40.13%	2.36	2.69
Oil/Gas (Production and Exploration)	330	0.99	1.38	19.43%	5.11	7.09
Oil/Gas Distribution	78	0.69	1.20	35.63%	1.93	3.35
Oilfield Svcs/Equip.	148	1.11	1.37	26.20%	4.24	5.21
Packaging & Container	26	0.60	0.84	40.79%	1.48	2.06
Paper/Forest Products	23	0.83	1.12	26.74%	3.09	4.18
Power	68	0.33	0.54	27.97%	1.17	1.95
Precious Metals	109	1.10	1.25	10.75%	10.27	11.64
Publishing & Newspapers	37	0.96	1.32	31.33%	3.07	4.21
R.E.I.T.	238	0.41	0.72	38.78%	1.07	1.87
Real Estate (Development)	18	0.47	0.68	21.44%	2.19	3.19
Real Estate (General/Diversified)	11	1.09	1.27	33.87%	3.23	3.75
Real Estate (Operations & Services)	54	0.62	0.99	17.87%	3.45	5.55
Recreation	66	0.76	0.92	24.31%	3.12	3.79
Reinsurance	3	0.65	0.75	49.51%	1.31	1.51
Restaurant/Dining	86	0.61	0.77	29.02%	2.10	2.65
Retail (Automotive)	25	0.63	0.91	37.27%	1.69	2.45
Retail (Building Supply)	6	1.12	1.30	48.01%	2.34	2.70
Retail (Distributors)	88	0.77	1.10	30.13%	2.54	3.67
Retail (General)	19	0.82	1.05	37.73%	2.18	2.78
Retail (Grocery and Food)	14	0.46	0.69	27.87%	1.66	2.48
Retail (Online)	57	1.17	1.23	23.73%	4.94	5.17
Retail (Special Lines)	108	0.76	1.02	30.50%	2.48	3.35
Rubber& Tires	4	0.89	1.35	40.55%	2.19	3.32
Semiconductor	80	1.11	1.20	37.81%	2.94	3.16
Semiconductor Equip	45	1.10	1.10	33.63%	3.26	3.26
Shipbuilding & Marine	11	0.85	1.20	22.47%	3.80	5.35
Shoe	10	0.83	0.85	31.50%	2.63	2.70
Software (Entertainment)	13	0.96	0.98	19.34%	4.99	5.07
Software (Internet)	297	1.12	1.13	20.06%	5.57	5.65
Software (System & Application)	236	0.99	1.13	26.39%	3.75	4.27

Steel	38	1.19	1.60	31.72%	3.76	5.04
Telecom (Wireless)	17	0.58	1.12	28.50%	2.04	3.92
Telecom. Equipment	107	0.86	0.99	26.14%	3.30	3.78
Telecom. Services	67	0.68	1.04	23.73%	2.85	4.39
Tobacco	22	1.13	1.28	20.34%	5.57	6.29
Transportation	17	0.83	1.01	42.25%	1.97	2.40
Transportation (Railroads)	7	0.66	0.79	42.18%	1.56	1.86
Trucking	30	0.76	1.21	37.64%	2.01	3.20
Utility (General)	18	0.25	0.38	27.33%	0.92	1.38
Utility (Water)	22	0.47	0.65	24.48%	1.93	2.65
Total Market	**7330**	**0.65**	**1.00**	**26.67%**	**2.45**	**3.75**
Total Market (without financials)	**6100**	**0.85**	**1.08**	**26.30%**	**3.22**	**4.11**

American Finance Association

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SESSION TOPIC: STOCK MARKET PRICE BEHAVIOR

SESSION CHAIRMAN: BURTON G. MALKIEL

EFFICIENT CAPITAL MARKETS: A REVIEW OF THEORY AND EMPIRICAL WORK*

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I. INTRODUCTION

THE PRIMARY ROLE of the capital market is allocation of ownership of the economy's capital stock. In general terms, the ideal is a market in which prices provide accurate signals for resource allocation: that is, a market in which firms can make production-investment decisions, and investors can choose among the securities that represent ownership of firms' activities under the assumption that security prices at any time "fully reflect" all available information. A market in which prices always "fully reflect" available information is called "efficient."

This paper reviews the theoretical and empirical literature on the efficient markets model. After a discussion of the theory, empirical work concerned with the adjustment of security prices to three relevant information subsets is considered. First, *weak form* tests, in which the information set is just historical prices, are discussed. Then *semi-strong form* tests, in which the concern is whether prices efficiently adjust to other information that is obviously publicly available (e.g., announcements of annual earnings, stock splits, etc.) are considered. Finally, *strong form* tests concerned with whether given investors or groups have monopolistic access to any information relevant for price formation are reviewed.[1] We shall conclude that, with but a few exceptions, the efficient markets model stands up well.

Though we proceed from theory to empirical work, to keep the proper historical perspective we should note to a large extent the empirical work in this area preceded the development of the theory. The theory is presented first here in order to more easily judge which of the empirical results are most relevant from the viewpoint of the theory. The empirical work itself, however, will then be reviewed in more or less historical sequence.

Finally, the perceptive reader will surely recognize instances in this paper where relevant studies are not specifically discussed. In such cases my apologies should be taken for granted. The area is so bountiful that some such injustices are unavoidable. But the primary goal here will have been accomplished if a coherent picture of the main lines of the work on efficient markets is presented, along with an accurate picture of the current state of the arts.

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[1] The distinction between weak and strong form tests was first suggested by Harry Roberts.

II. The Theory of Efficient Markets

A. *Expected Return or "Fair Game" Models*

The definitional statement that in an efficient market prices "fully reflect" available information is so general that it has no empirically testable implications. To make the model testable, the process of price formation must be specified in more detail. In essence we must define somewhat more exactly what is meant by the term "fully reflect."

One possibility would be to posit that equilibrium prices (or expected returns) on securities are generated as in the "two parameter" Sharpe [40]-Lintner [24, 25] world. In general, however, the theoretical models and especially the empirical tests of capital market efficiency have not been this specific. Most of the available work is based only on the assumption that the conditions of market equilibrium can (somehow) be stated in terms of expected returns. In general terms, like the two parameter model such theories would posit that conditional on some relevant information set, the equilibrium expected return on a security is a function of its "risk." And different theories would differ primarily in how "risk" is defined.

All members of the class of such "expected return theories" can, however, be described notationally as follows:

$$E(\tilde{p}_{j,t+1}|\Phi_t) = [1 + E(\tilde{r}_{j,t+1}|\Phi_t)]p_{jt}, \tag{1}$$

where E is the expected value operator; p_{jt} is the price of security j at time t; $p_{j,t+1}$ is its price at t + 1 (with reinvestment of any intermediate cash income from the security); $r_{j,t+1}$ is the one-period percentage return $(p_{j,t+1} - p_{jt})/p_{jt}$; Φ_t is a general symbol for whatever set of information is assumed to be "fully reflected" in the price at t; and the tildes indicate that $p_{j,t+1}$ and $r_{j,t+1}$ are random variables at t.

The value of the equilibrium expected return $E(\tilde{r}_{j,t+1}|\Phi_t)$ projected on the basis of the information Φ_t would be determined from the particular expected return theory at hand. The conditional expectation notation of (1) is meant to imply, however, that whatever expected return model is assumed to apply, the information in Φ_t is fully utilized in determining equilibrium expected returns. And this is the sense in which Φ_t is "fully reflected" in the formation of the price p_{jt}.

But we should note right off that, simple as it is, the assumption that the conditions of market equilibrium can be stated in terms of expected returns elevates the purely mathematical concept of expected value to a status not necessarily implied by the general notion of market efficiency. The expected value is just one of many possible summary measures of a distribution of returns, and market efficiency per se (i.e., the general notion that prices "fully reflect" available information) does not imbue it with any special importance. Thus, the results of tests based on this assumption depend to some extent on its validity as well as on the efficiency of the market. But some such assumption is the unavoidable price one must pay to give the theory of efficient markets empirical content.

The assumptions that the conditions of market equilibrium can be stated in terms of expected returns and that equilibrium expected returns are formed on the basis of (and thus "fully reflect") the information set Φ_t have a major empirical implication—they rule out the possibility of trading systems based only on information in Φ_t that have expected profits or returns in excess of equilibrium expected profits or returns. Thus let

$$x_{j,t+1} = p_{j,t+1} - E(p_{j,t+1}|\Phi_t). \tag{2}$$

Then

$$E(\tilde{x}_{j,t+1}|\Phi_t) = 0 \tag{3}$$

which, *by definition,* says that the sequence $\{x_{jt}\}$ is a "fair game" with respect to the information sequence $\{\phi_t\}$. Or, equivalently, let

$$z_{j,t+1} = r_{j,t+1} - E(\tilde{r}_{j,t+1}|\Phi_t), \tag{4}$$

then

$$E(\tilde{z}_{j,t+1}|\Phi_t) = 0, \tag{5}$$

so that the sequence $\{z_{jt}\}$ is also a "fair game" with respect to the information sequence $\{\Phi\}$.

In economic terms, $x_{j,t+1}$ is the excess market value of security j at time $t + 1$: it is the difference between the observed price and the expected value of the price that was projected at t on the basis of the information Φ_t. And similarly, $z_{j,t+1}$ is the return at $t + 1$ in excess of the equilibrium expected return projected at t. Let

$$\alpha(\Phi_t) = [\alpha_1(\Phi_t), \alpha_2(\Phi_t), \ldots, \alpha_n(\Phi_t)]$$

be any trading system based on Φ_t which tells the investor the amounts $\alpha_j(\Phi_t)$ of funds available at t that are to be invested in each of the n available securities. The total excess market value at $t + 1$ that will be generated by such a system is

$$V_{t+1} = \sum_{j=1}^{n} \alpha_j(\Phi_t)[r_{j,t+1} - E(\tilde{r}_{j,t+1}|\Phi_t)],$$

which, from the "fair game" property of (5) has expectation,

$$E(\tilde{V}_{t+1}|\Phi_t) = \sum_{j=1}^{n} \alpha_j(\Phi_t)E(\tilde{z}_{j,t+1}|\Phi_t) = 0.$$

The expected return or "fair game" efficient markets model[2] has other important testable implications, but these are better saved for the later discussion of the empirical work. Now we turn to two special cases of the model, the submartingale and the random walk, that (as we shall see later) play an important role in the empirical literature.

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[2] Though we shall sometimes refer to the model summarized by (1) as the "fair game" model, keep in mind that the "fair game" properties of the model are *implications* of the assumptions that (i) the conditions of market equilibrium can be stated in terms of expected returns, and (ii) the information Φ_t is fully utilized by the market in forming equilibrium expected returns and thus current prices.

The role of "fair game" models in the theory of efficient markets was first recognized and studied rigorously by Mandelbrot [27] and Samuelson [38]. Their work will be discussed in more detail later.

B. *The Submartingale Model*

Suppose we assume in (1) that for all t and Φ_t

$$E(\tilde{p}_{j,t+1}|\Phi_t) \geqslant p_{jt}, \quad \text{or equivalently,} \quad E(\tilde{r}_{j,t+1}|\Phi_t) \geqslant 0. \tag{6}$$

This is a statement that the price sequence $\{p_{jt}\}$ for security j follows a submartingale with respect to the information sequence $\{\Phi_t\}$, which is to say nothing more than that the expected value of next period's price, as projected on the basis of the information Φ_t, is equal to or greater than the current price. If (6) holds as an equality (so that expected returns and price changes are zero), then the price sequence follows a martingale.

A submartingale in prices has one important empirical implication. Consider the set of "one security and cash" mechanical trading rules by which we mean systems that concentrate on individual securities and that define the conditions under which the investor would hold a given security, sell it short, or simply hold cash at any time t. Then the assumption of (6) that expected returns conditional on Φ_t are non-negative directly implies that such trading rules based only on the information in Φ_t cannot have greater expected profits than a policy of always buying-and-holding the security during the future period in question. Tests of such rules will be an important part of the empirical evidence on the efficient markets model.[3]

C. *The Random Walk Model*

In the early treatments of the efficient markets model, the statement that the current price of a security "fully reflects" available information was assumed to imply that successive price changes (or more usually, successive one-period returns) are independent. In addition, it was usually assumed that successive changes (or returns) are identically distributed. Together the two hypotheses constitute the random walk model. Formally, the model says

$$f(r_{j,t+1}|\Phi_t) = f(r_{j,t+1}), \tag{7}$$

which is the usual statement that the conditional and marginal probability distributions of an independent random variable are identical. In addition, the density function f must be the same for all t.[4]

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[3] Note that the expected profitability of "one security and cash" trading systems vis-à-vis buy-and-hold is not ruled out by the general expected return or "fair game" efficient markets model. The latter rules out systems with expected profits in excess of equilibrium expected returns, but since in principle it allows equilibrium expected returns to be negative, holding cash (which always has zero actual and thus expected return) may have higher expected return than holding some security.

And negative equilibrium expected returns for some securities are quite possible. For example, in the Sharpe [40]-Lintner [24, 25] model (which is in turn a natural extension of the portfolio models of Markowitz [30] and Tobin [43]) the equilibrium expected return on a security depends on the extent to which the dispersion in the security's return distribution is related to dispersion in the returns on all other securities. A security whose returns on average move opposite to the general market is particularly valuable in reducing dispersion of portfolio returns, and so its equilibrium expected return may well be negative.

[4] The terminology is loose. Prices will only follow a random walk if price changes are independent, identically distributed; and even then we should say "random walk with drift" since expected price changes can be non-zero. If one-period returns are independent, identically distributed, prices will not follow a random walk since the distribution of price changes will depend

Expression (7) of course says much more than the general expected return model summarized by (1). For example, if we restrict (1) by assuming that the expected return on security j is constant over time, then we have

$$E(\tilde{r}_{j,t+1}|\Phi_t) = E(\tilde{r}_{j,t+1}). \tag{8}$$

This says that the mean of the distribution of $r_{j,t+1}$ is independent of the information available at t, Φ_t, whereas the random walk model of (7) in addition says that the entire distribution is independent of Φ_t.[5]

We argue later that it is best to regard the random walk model as an extension of the general expected return or "fair game" efficient markets model in the sense of making a more detailed statement about the economic environment. The "fair game" model just says that the conditions of market equilibrium can be stated in terms of expected returns, and thus it says little about the details of the stochastic process generating returns. A random walk arises within the context of such a model when the environment is (fortuitously) such that the evolution of investor tastes and the process generating new information combine to produce equilibria in which return distributions repeat themselves through time.

Thus it is not surprising that empirical tests of the "random walk" model that are in fact tests of "fair game" properties are more strongly in support of the model than tests of the additional (and, from the viewpoint of expected return market efficiency, superfluous) pure independence assumption. (But it is perhaps equally surprising that, as we shall soon see, the evidence against the independence of returns over time is as weak as it is.)

D. *Market Conditions Consistent with Efficiency*

Before turning to the empirical work, however, a few words about the market conditions that might help or hinder efficient adjustment of prices to information are in order. First, it is easy to determine *sufficient* conditions for capital market efficiency. For example, consider a market in which (i) there are no transactions costs in trading securities, (ii) all available information is costlessly available to all market participants, and (iii) all agree on the implications of current information for the current price and distributions of future prices of each security. In such a market, the current price of a security obviously "fully reflects" all available information.

But a frictionless market in which all information is freely available and investors agree on its implications is, of course, not descriptive of markets met in practice. Fortunately, these conditions are sufficient for market efficiency, but not necessary. For example, as long as transactors take account of all

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on the price level. But though rigorous terminology is usually desirable, our loose use of terms should not cause confusion; and our usage follows that of the efficient markets literature.

Note also that in the random walk literature, the information set Φ_t in (7) is usually assumed to include only the past return history, $r_{j,t}, r_{j,t-1}, \cdots$

[5] The random walk model does not say, however, that past information is of no value in *assessing* distributions of future returns. Indeed since return distributions are assumed to be stationary through time, past returns are the best source of such information. The random walk model does say, however, that the *sequence* (or the order) of the past returns is of no consequence in assessing distributions of future returns.

available information, even large transactions costs that inhibit the flow of transactions do not in themselves imply that when transactions do take place, prices will not "fully reflect" available information. Similarly (and speaking, as above, somewhat loosely), the market may be efficient if "sufficient numbers" of investors have ready access to available information. And disagreement among investors about the implications of given information does not in itself imply market inefficiency unless there are investors who can consistently make better evaluations of available information than are implicit in market prices.

But though transactions costs, information that is not freely available to all investors, and disagreement among investors about the implications of given information are not necessarily sources of market inefficiency, they are potential sources. And all three exist to some extent in real world markets. Measuring their effects on the process of price formation is, of course, the major goal of empirical work in this area.

III. THE EVIDENCE

All the empirical research on the theory of efficient markets has been concerned with whether prices "fully reflect" particular subsets of available information. Historically, the empirical work evolved more or less as follows. The initial studies were concerned with what we call *weak form* tests in which the information subset of interest is just past price (or return) histories. Most of the results here come from the random walk literature. When extensive tests seemed to support the efficiency hypothesis at this level, attention was turned to *semi-strong form* tests in which the concern is the speed of price adjustment to other obviously publicly available information (e.g., announcements of stock splits, annual reports, new security issues, etc.). Finally, *strong form* tests in which the concern is whether any investor or groups (e.g., managements of mutual funds) have monopolistic access to any information relevant for the formation of prices have recently appeared. We review the empirical research in more or less this historical sequence.

First, however, we should note that what we have called *the* efficient markets model in the discussions of earlier sections is the hypothesis that security prices at any point in time "fully reflect" *all* available information. Though we shall argue that the model stands up rather well to the data, it is obviously an extreme null hypothesis. And, like any other extreme null hyposthesis, we do not expect it to be literally true. The categorization of the tests into weak, semi-strong, and strong form will serve the useful purpose of allowing us to pinpoint the level of information at which the hypothesis breaks down. And we shall contend that there is no important evidence against the hypothesis in the weak and semi-strong form tests (i.e., prices seem to efficiently adjust to obviously publicly available information), and only limited evidence against the hypothesis in the strong form tests (i.e., monopolistic access to information about prices does not seem to be a prevalent phenomenon in the investment community).

A. *Weak Form Tests of the Efficient Markets Model*

1. Random Walks and Fair Games: A Little Historical Background

As noted earlier, all of the empirical work on efficient markets can be considered within the context of the general expected return or "fair game" model, and much of the evidence bears directly on the special submartingale expected return model of (6). Indeed, in the early literature, discussions of the efficient markets model were phrased in terms of the even more special random walk model, though we shall argue that most of the early authors were in fact concerned with more general versions of the "fair game" model.

Some of the confusion in the early random walk writings is understandable. Research on security prices did not begin with the development of a theory of price formation which was then subjected to empirical tests. Rather, the impetus for the development of a theory came from the accumulation of evidence in the middle 1950's and early 1960's that the behavior of common stock and other speculative prices could be well approximated by a random walk. Faced with the evidence, economists felt compelled to offer some rationalization. What resulted was a theory of efficient markets stated in terms of random walks, but usually implying some more general "fair game" model.

It was not until the work of Samuelson [38] and Mandelbrot [27] in 1965 and 1966 that the role of "fair game" expected return models in the theory of efficient markets and the relationships between these models and the theory of random walks were rigorously studied.[6] And these papers came somewhat after the major empirical work on random walks. In the earlier work, "theoretical" discussions, though usually intuitively appealing, were always lacking in rigor and often either vague or *ad hoc*. In short, until the Mandelbrot-Samuelson models appeared, there existed a large body of empirical results in search of a rigorous theory.

Thus, though his contributions were ignored for sixty years, the first statement and test of the random walk model was that of Bachelier [3] in 1900. But his "fundamental principle" for the behavior of prices was that speculation should be a "fair game"; in particular, the expected profits to the speculator should be zero. With the benefit of the modern theory of stochastic processes, we know now that the process implied by this fundamental principle is a martingale.

After Bachelier, research on the behavior of security prices lagged until the

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[6] Basing their analyses on futures contracts in commodity markets, Mandelbrot and Samuelson show that if the price of such a contract at time t is the expected value at t (given information Φ_t) of the spot price at the termination of the contract, then the futures price will follow a martingale with respect to the information sequence $\{\Phi_t\}$; that is, the expected price change from period to period will be zero, and the price changes will be a "fair game." If the equilibrium expected return is not assumed to be zero, our more general "fair game" model, summarized by (1), is obtained.

But though the Mandelbrot-Samuelson approach certainly illuminates the process of price formation in commodity markets, we have seen that "fair game" expected return models can be derived in much simpler fashion. In particular, (1) is just a formalization of the assumptions that the conditions of market equilibrium can be stated in terms of expected returns and that the information Φ_t is used in forming market prices at t.

coming of the computer. In 1953 Kendall [21] examined the behavior of weekly changes in nineteen indices of British industrial share prices and in spot prices for cotton (New York) and wheat (Chicago). After extensive analysis of serial correlations, he suggests, in quite graphic terms:

> The series looks like a wandering one, almost as if once a week the Demon of Chance drew a random number from a symetrical population of fixed dispersion and added it to the current price to determine the next week's price [21, p. 13].

Kendall's conclusion had in fact been suggested earlier by Working [47], though his suggestion lacked the force provided by Kendall's empirical results. And the implications of the conclusion for stock market research and financial analysis were later underlined by Roberts [36].

But the suggestion by Kendall, Working, and Roberts that series of speculative prices may be well described by random walks was based on observation. None of these authors attempted to provide much economic rationale for the hypothesis, and, indeed, Kendall felt that economists would generally reject it. Osborne [33] suggested market conditions, similar to those assumed by Bachelier, that would lead to a random walk. But in his model, independence of successive price changes derives from the assumption that the decisions of investors in an individual security are independent from transaction to transaction—which is little in the way of an economic model.

Whenever economists (prior to Mandelbrot and Samuelson) tried to provide economic justification for the random walk, their arguments usually implied a "fair game." For example, Alexander [8, p. 200] states:

> If one were to start out with the assumption that a stock or commodity speculation is a "fair game" with equal expectation of gain or loss or, more accurately, with an expectation of zero gain, one would be well on the way to picturing the behavior of speculative prices as a random walk.

There is an awareness here that the "fair game" assumption is not sufficient to lead to a random walk, but Alexander never expands on the comment. Similarly, Cootner [8, p. 232] states:

> If any substantial group of buyers thought prices were too low, their buying would force up the prices. The reverse would be true for sellers. Except for appreciation due to earnings retention, the conditional expectation of tomorrow's price, given today's price, is today's price.
>
> In such a world, the only price changes that would occur are those that result from new information. Since there is no reason to expect that information to be non-random in appearance, the period-to-period price changes of a stock should be random movements, statistically independent of one another.

Though somewhat imprecise, the last sentence of the first paragraph seems to point to a "fair game" model rather than a random walk.[7] In this light, the second paragraph can be viewed as an attempt to describe environmental conditions that would reduce a "fair game" to a random walk. But the specification imposed on the information generating process is insufficient for this purpose; one would, for example, also have to say something about investor

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[7] The appropriate conditioning statement would be "Given the sequence of historical prices."

tastes. Finally, lest I be accused of criticizing others too severely for ambiguity, lack of rigor and incorrect conclusions,

> By contrast, the stock market trader has a much more practical criterion for judging what constitutes important dependence in successive price changes. For his purposes the random walk model is valid as long as knowledge of the past behavior of the series of price changes cannot be used to increase expected gains. More specifically, the independence assumption is an adequate description of reality as long as the actual degree of dependence in the series of price changes is not sufficient to allow the past history of the series to be used to predict the future in a way which makes expected profits greater than they would be under a naive buy-and hold model [10, p 35].

We know now, of course, that this last condition hardly requires a random walk. It will in fact be met by the submartingale model of (6).

But one should not be too hard on the theoretical efforts of the early empirical random walk literature. The arguments were usually appealing; where they fell short was in awareness of developments in the theory of stochastic processes. Moreover, we shall now see that most of the empirical evidence in the random walk literature can easily be interpreted as tests of more general expected return or "fair game" models.[8]

2. Tests of Market Efficiency in the Random Walk Literature

As discussed earlier, "fair game" models imply the "impossibility" of various sorts of trading systems. Some of the random walk literature has been concerned with testing the profitability of such systems. More of the literature has, however, been concerned with tests of serial covariances of returns. We shall now show that, like a random walk, the serial covariances of a "fair game" are zero, so that these tests are also relevant for the expected return models.

If x_t is a "fair game," its unconditional expectation is zero and its serial covariance can be written in general form as:

$$E(\tilde{x}_{t+\tau}\,\tilde{x}_t) = \int_{x_t} x_t E(\tilde{x}_{t+\tau}|x_t) f(x_t) dx_t,$$

where f indicates a density function. But if x_t is a "fair game,"

$$E(\tilde{x}_{t+\tau}|x_t) = 0.^9$$

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[8] Our brief historical review is meant only to provide perspective, and it is, of course, somewhat incomplete. For example, we have ignored the important contributions to the early random walk literature in studies of warrants and other options by Sprenkle, Kruizenga, Boness, and others. Much of this early work on options is summarized in [8].

[9] More generally, if the sequence $\{x_t\}$ is a fair game with respect to the information sequence $\{\Phi_t\}$, (i.e., $E(\tilde{x}_{t+1}|\Phi_t) = 0$ for all Φ_t); then x_t is a fair game with respect to any Φ'_t that is a subset of Φ_t (i.e., $E(\tilde{x}_{t+1}|\Phi'_t) = 0$ for all Φ'_t). To show this, let $\Phi_t = (\Phi'_t, \ \Phi''_t)$. Then, using Stieltjes integrals and the symbol F to denote cumulative distinction functions, the conditional expectation

$$E(\tilde{x}_{t+1}|\Phi'_t) = \int_{\Phi''_t}\int_{x_{t+1}} x_{t+1} dF(x_{t+1}, \Phi''_t|\Phi'_t) \quad = \int_{\Phi''_t}\left[\int_{x_{t+1}} x_{t+1} dF(x_{t+1}|\Phi'_t, \Phi''_t)\right] dF(\Phi''_t|\Phi'_t).$$

From this it follows that for all lags, the serial covariances between lagged values of a "fair game" variable are zero. Thus, observations of a "fair game" variable are linearly independent.[10]

But the "fair game" model does not necessarily imply that the serial covariances of *one-period returns* are zero. In the weak form tests of this model the "fair game" variable is

$$z_{j,t} = r_{j,t} - E(\tilde{r}_{j,t}|r_{j,t-1}, r_{j,t-2}, \ldots). \quad \text{(Cf. fn. 9)} \qquad (9)$$

But the covariance between, for example, r_{jt} and $r_{j,t+1}$ is

$$E([\tilde{r}_{j,t+1} - E(\tilde{r}_{j,t+1})]\ [\tilde{r}_{jt} - E(\tilde{r}_{jt})]) = \int_{r_{jt}} [r_{jt} - E(\tilde{r}_{jt})]\ [E(\tilde{r}_{j,t+1}|r_{jt}) - E(\tilde{r}_{j,t+1})]f(r_{jt})dr_{jt},$$

and (9) does not imply that $E(\tilde{r}_{j,t+1}|r_{jt}) = E(\tilde{r}_{j,t+1})$: In the "fair game" efficient markets model, the deviation of the return for $t + 1$ from its conditional expectation is a "fair game" variable, but the conditional expectation itself can depend on the return observed for t.[11]

In the random walk literature, this problem is not recognized, since it is assumed that the expected return (and indeed the entire distribution of returns) is stationary through time. In practice, this implies estimating serial covariances by taking cross products of deviations of observed returns from the overall sample mean return. It is somewhat fortuitous, then, that this procedure, which represents a rather gross approximation from the viewpoint of the general expected return efficient markets model, does not seem to greatly affect the results of the covariance tests, at least for common stocks.[12]

But the integral in brackets is just $E(\tilde{x}_{t+1}|\Phi_t)$ which by the "fair game" assumption is 0, so that

$$E(x_{t+1}|\Phi'_t) = 0 \text{ for all } \Phi'_t \subset \Phi_t.$$

[10] But though zero serial covariances are consistent with a "fair game," they do not imply such a process. A "fair game" also rules out many types of non linear dependence. Thus using arguments similar to those above, it can be shown that if x is a "fair game," $E(\tilde{x}_t\tilde{x}_{t+1} \cdots \tilde{x}_{t+\tau}) = 0$ for all τ, which is not implied by $E(\tilde{x}_t\tilde{x}_{t+\tau}) = 0$ for all τ. For example, consider a three-period case where x must be either ± 1. Suppose the process is $x_{t+2} = \text{sign}\ (x_t x_{t+1})$, i.e.,

x_t	x_{t+1}	$\rightarrow$	x_{t+2}
+	+	→	+
+	—	→	—
—	+	→	—
—	—	→	+.

If probabilities are uniformly distributed across events,

$$E(\tilde{x}_{t+2}|x_{t+1}) = E(\tilde{x}_{t+2}|x_t) = E(\tilde{x}_{t+1}|x_t) = E(\tilde{x}_{t+2}) = E(\tilde{x}_{t+1}) = E(\tilde{x}_t) = 0,$$

so that all pairwise serial covariances are zero. But the process is not a "fair game," since $E(\tilde{x}_{t+2}|x_{t+1}, x_t) \neq 0$, and knowledge of (x_{t+1}, x_t) can be used as the basis of a simple "system" with positive expected profit.

[11] For example, suppose the level of one-period returns follows a martingale so that

$$E(\tilde{r}_{j,t+1}|r_{jt}, r_{j,t-1} \ldots) = r_{jt}.$$

Then covariances between successive returns will be nonzero (though in this special case first differences of returns will be uncorrelated).

[12] The reason is probably that for stocks, changes in equilibrium expected returns for the

TABLE 1 (from [10])
First-order Serial Correlation Coefficients for One-, Four-, Nine-, and Sixteen-Day Changes in Log_e Price

	Differencing Interval (Days)			
Stock	One	Four	Nine	Sixteen
Allied Chemical	.017	.029	−.091	−.118
Alcoa	.118*	.095	−.112	−.044
American Can	−.087*	−.124*	−.060	.031
A. T. & T.	−.039	−.010	−.009	−.003
American Tobacco	.111*	−.175*	.033	.007
Anaconda	.067*	−.068	−.125	.202
Bethlehem Steel	.013	−.122	−.148	.112
Chrysler	.012	.060	−.026	.040
Du Pont	.013	.069	−.043	−.055
Eastman Kodak	.025	−.006	−.053	−.023
General Electric	.011	.020	−.004	.000
General Foods	.061*	−.005	−.140	−.098
General Motors	−.004	−.128*	.009	−.028
Goodyear	−.123*	.001	−.037	.033
International Harvester	−.017	−.068	−.244*	.116
International Nickel	.096*	.038	.124	.041
International Paper	.046	.060	−.004	−.010
Johns Manville	.006	−.068	−.002	.002
Owens Illinois	−.021	−.006	.003	−.022
Procter & Gamble	.099*	−.006	.098	.076
Sears	.097*	−.070	−.113	.041
Standard Oil (Calif.)	.025	−.143*	−.046	.040
Standard Oil (N.J.)	.008	−.109	−.082	−.121
Swift & Co.	−.004	−.072	.118	−.197
Texaco	.094*	−.053	−.047	−.178
Union Carbide	.107*	.049	−.101	.124
United Aircraft	.014	−.190*	−.192*	−.040
U.S. Steel	.040	−.006	−.056	.236*
Westinghouse	−.027	−.097	−.137	.067
Woolworth	.028	−.033	−.112	.040

* Coefficient is twice its computed standard error.

For example, Table 1 (taken from [10]) shows the serial correlations between successive changes in the natural log of price for each of the thirty stocks of the Dow Jones Industrial Average, for time periods that vary slightly from stock to stock, but usually run from about the end of 1957 to September 26, 1962. The serial correlations of successive changes in $\log_e$ price are shown for differencing intervals of one, four, nine, and sixteen days.[13]

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common differencing intervals of a day, a week, or a month, are trivial relative to other sources of variation in returns. Later, when we consider Roll's work [37], we shall see that this is not true for one week returns on U.S. Government Treasury Bills.

[13] The use of changes in $\log_e$ price as the measure of return is common in the random walk literature. It can be justified in several ways. But for current purposes, it is sufficient to note that for price changes less than fifteen per cent, the change in $\log_e$ price is approximately the percentage price change or one-period return. And for differencing intervals shorter than one month, returns in excess of fifteen per cent are unusual. Thus [10] reports that for the data of Table 1, tests carried out on percentage or one-period returns yielded results essentially identical to the tests based on changes in $\log_e$ price.

The results in Table 1 are typical of those reported by others for tests based on serial covariances. (Cf. Kendall [21], Moore [31], Alexander [1], and the results of Granger and Morgenstern [17] and Godfrey, Granger and Morgenstern [16] obtained by means of spectral analysis.) Specifically, there is no evidence of substantial linear dependence between lagged price changes or returns. In absolute terms the measured serial correlations are always close to zero.

Looking hard, though, one can probably find evidence of statistically "significant" linear dependence in Table 1 (and again this is true of results reported by others). For the daily returns eleven of the serial correlations are more than twice their computed standard errors, and twenty-two out of thirty are positive. On the other hand, twenty-one and twenty-four of the coefficients for the four and nine day differences are negative. But with samples of the size underlying Table 1 ($N = 1200$-1700 observations per stock on a daily basis) statistically "significant" deviations from zero covariance are not necessarily a basis for rejecting the efficient markets model. For the results in Table 1, the standard errors of the serial correlations were approximated as $(1/(N-1))^{1/2}$, which for the daily data implies that a correlation as small as .06 is more than twice its standard error. But a coefficient this size implies that a linear relationship with the lagged price change can be used to explain about .36% of the variation in the current price change, which is probably insignificant from an economic viewpoint. In particular, it is unlikely that the small absolute levels of serial correlation that are always observed can be used as the basis of substantially profitable trading systems.[14]

It is, of course, difficult to judge what degree of serial correlation would imply the existence of trading rules with substantial expected profits. (And indeed we shall soon have to be a little more precise about what is implied by "substantial" profits.) Moreover, zero serial covariances are consistent with a "fair game" model, but as noted earlier (fn. 10), there are types of nonlinear dependence that imply the existence of profitable trading systems, and yet do not imply nonzero serial covariances. Thus, for many reasons it is desirable to directly test the profitability of various trading rules.

The first major evidence on trading rules was Alexander's [1, 2]. He tests a variety of systems, but the most thoroughly examined can be decribed as follows: If the price of a security moves up at least y%, buy and hold the security until its price moves down at least y% from a subsequent high, at which time simultaneously sell and go short. The short position is maintained until the price rises at least y% above a subsequent low, at which time one covers the short position and buys. Moves less than y% in either direction are

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[14] Given the evidence of Kendall [21], Mandelbrot [28], Fama [10] and others that large price changes occur much more frequently than would be expected if the generating process were Gaussian, the expression $(1/(N-1))^{1/2}$ understates the sampling dispersion of the serial correlation coefficient, and thus leads to an overstatement of significance levels. In addition, the fact that sample serial correlations are predominantly of one sign or the other is not in itself evidence of linear dependence. If, as the work of King [23] and Blume [7] indicates, there is a market factor whose behavior affects the returns on all securities, the sample behavior of this market factor may lead to a predominance of signs of one type in the serial correlations for individual securities, even though the population serial correlations for both the market factor and the returns on individual securities are zero. For a more extensive analysis of these issues see [10].

ignored. Such a system is called a y% filter. It is obviously a "one security and cash" trading rule, so that the results it produces are relevant for the submartingale expected return model of (6).

After extensive tests using daily data on price indices from 1897 to 1959 and filters from one to fifty per cent, and after correcting some incorrect presumptions in the initial results of [1] (see fn. 25), in his final paper on the subject, Alexander concludes:

> In fact, at this point I should advise any reader who is interested only in practical results, and who is not a floor trader and so must pay commissions, to turn to other sources on how to beat buy and hold. The rest of this article is devoted principally to a theoretical consideration of whether the observed results are consistent with a random walk hypothesis [8], p. 351).

Later in the paper Alexander concludes that there is some evidence in his results against the independence assumption of the random walk model. But market efficiency does not require a random walk, and from the viewpoint of the submartingale model of (6), the conclusion that the filters cannot beat buy-and-hold is support for the efficient markets hypothesis. Further support is provided by Fama and Blume [13] who compare the profitability of various filters to buy-and-hold for the individual stocks of the Dow-Jones Industrial Average. (The data are those underlying Table 1.)

But again, looking hard one can find evidence in the filter tests of both Alexander and Fama-Blume that is inconsistent with the submartingale efficient markets model, if that model is interpreted in a strict sense. In particular, the results for very small filters (1 per cent in Alexander's tests and .5, 1.0, and 1.5 per cent in the tests of Fama-Blume) indicate that it is possible to devise trading schemes based on very short-term (preferably intra-day but at most daily) price swings that will on average outperform buy-and-hold. The average profits on individual transactions from such schemes are miniscule, but they generate transactions so frequently that over longer periods and ignoring commissions they outperform buy-and-hold by a substantial margin. These results are evidence of persistence or positive dependence in very short-term price movements. And, interestingly, this is consistent with the evidence for slight positive linear dependence in successive daily price changes produced by the serial correlations.[15]

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[15] Though strictly speaking, such tests of pure independence are not directly relevant for expected return models, it is interesting that the conclusion that very short-term swings in prices persist slightly longer than would be expected under the martingale hypothesis is also supported by the results of non-parametric runs tests applied to the daily data of Table 1. (See [10], Tables 12-15.) For the daily price changes, the actual number of runs of price changes of the same sign is less than the expected number for 26 out of 30 stocks. Moreover, of the eight stocks for which the actual number of runs is more than two standard errors less than the expected number, five of the same stocks have positive daily, first order serial correlations in Table 1 that are more than twice their standard errors. But in both cases the statistical "significance" of the results is largely a reflection of the large sample sizes. Just as the serial correlations are small in absolute terms (the average is .026), the differences between the expected and actual number of runs on average are only three per cent of the total expected number.

On the other hand, it is also interesting that the runs tests do not support the suggestion of slight negative dependence in four and nine day changes that appeared in the serial correlations. In the runs tests such negative dependence would appear as a tendency for the actual number of runs to exceed the expected number. In fact, for the four and nine day price changes, for 17 and

But when one takes account of even the minimum trading costs that would be generated by small filters, their advantage over buy-and-hold disappears. For example, even a floor trader (i.e., a person who owns a seat) on the New York Stock Exchange must pay clearinghouse fees on his trades that amount to about .1 per cent per turnaround transaction (i.e., sales plus purchase). Fama-Blume show that because small filters produce such frequent trades, these minimum trading costs are sufficient to wipe out their advantage over buy-and-hold.

Thus the filter tests, like the serial correlations, produce empirically noticeable departures from the strict implications of the efficient markets model. But, in spite of any statistical significance they might have, from an economic viewpoint the departures are so small that it seems hardly justified to use them to declare the market inefficient.

3. Other Tests of Independence in the Random Walk Literature

It is probably best to regard the random walk model as a special case of the more general expected return model in the sense of making a more detailed specification of the economic environment. That is, the basic model of market equilibrium is the "fair game" expected return model, with a random walk arising when additional environmental conditions are such that distributions of one-period returns repeat themselves through time. From this viewpoint violations of the pure independence assumption of the random walk model are to be expected. But when judged relative to the benchmark provided by the random walk model, these violations can provide insights into the nature of the market environment.

For example, one departure from the pure independence assumption of the random walk model has been noted by Osborne [34], Fama ([10], Table 17 and Figure 8), and others. In particular, large daily price changes tend to be followed by large daily changes. The signs of the successor changes are apparently random, however, which indicates that the phenomenon represents a denial of the random walk model but not of the market efficiency hypothesis. Nevertheless, it is interesting to speculate why the phenomenon might arise. It may be that when important new information comes into the market it cannot always be immediately evaluated precisely. Thus, sometimes the initial price will overadjust to the information, and other times it will underadjust. But since the evidence indicates that the price changes on days following the initial large change are random in sign, the initial large change at least represents an unbiased adjustment to the ultimate price effects of the information, and this is sufficient for the expected return efficient markets model.

Niederhoffer and Osborne [32] document two departures from complete randomness in common stock price changes from transaction to transaction. First, their data indicate that reversals (pairs of consecutive price changes of opposite sign) are from two to three times as likely as continuations (pairs of consecutive price changes of the same sign). Second, a continuation is

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18 of the 30 stocks in Table 1 the actual number of runs is less than the expected number. Indeed, runs tests in general show no consistent evidence of dependence for any differencing interval longer than a day, which seems especially pertinent in light of the comments in footnote 14.

slightly more frequent after a preceding continuation than after a reversal. That is, let (+|++) indicate the occurrence of a positive price change, given two preceding positive changes. Then the events (+|++) and (−|−−) are slightly more frequent than (+|+−) or (−|−+).[16]

Niederhoffer and Osborne offer explanations for these phenomena based on the market structure of the New York Stock Exchange (N.Y.S.E.). In particular, there are three major types of orders that an investor might place in a given stock: (a) buy limit (buy at a specified price or lower), (b) sell limit (sell at a specified price or higher), and (c) buy or sell at market (at the lowest selling or highest buying price of another investor). A book of unexecuted limit orders in a given stock is kept by the specialist in that stock on the floor of the exchange. Unexecuted sell limit orders are, of course, at higher prices than unexecuted buy limit orders. On both exchanges, the smallest non-zero price change allowed is ⅛ point.

Suppose now that there is more than one unexecuted sell limit order at the lowest price of any such order. A transaction at this price (initiated by an order to buy at market[17]) can only be followed either by a transaction at the same price (if the next market order is to buy) or by a transaction at a lower price (if the next market order is to sell). Consecutive price increases can usually only occur when consecutive market orders to buy exhaust the sell limit orders at a given price.[18] In short, the excessive tendency toward reversal for consecutive non-zero price changes could result from bunching of unexecuted buy and sell limit orders.

The tendency for the events (+|++) and (−|−−) to occur slightly more frequently than (+|+−) and (−|−+) requires a more involved explanation which we shall not attempt to reproduce in full here. In brief, Niederhoffer and Osborne contend that the higher frequency of (+|++) relative to (+|+−) arises from a tendency for limit orders "to be concentrated at integers (26, 43), halves (26½, 43½), quarters and odd eighths in descending order of preference."[19] The frequency of the event (+|++), which usually requires that sell limit orders be exhausted at at least two consecutively higher prices (the last of which is relatively more frequently at an odd eighth), more heavily reflects the absence of sell limit orders at odd eighths than the event (+|+−), which usually implies that sell limit orders at only one price have been exhausted and so more or less reflects the average bunching of limit orders at all eighths.

But though Niederhoffer and Osborne present convincing evidence of sta-

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[16] On a transaction to transaction basis, positive and negative price changes are about equally likely. Thus, under the assumption that price changes are random, any pair of non-zero changes should be as likely as any other, and likewise for triplets of consecutive non-zero changes.

[17] A buy limit order for a price equal to or greater than the lowest available sell limit price is effectively an order to buy at market, and is treated as such by the broker.

[18] The exception is when there is a gap of more than ⅛ between the highest unexecuted buy limit and the lowest unexecuted sell limit order, so that market orders (and new limit orders) can be crossed at intermediate prices.

[19] Their empirical documentation for this claim is a few samples of specialists' books for selected days, plus the observation [34] that actual trading prices, at least for volatile high priced stocks, seem to be concentrated at integers, halves, quarters and odd eighths in descending order.

tistically significant departures from independence in price changes from transaction to transaction, and though their analysis of their findings presents interesting insights into the process of market making on the major exchanges, the types of dependence uncovered do not imply market inefficiency. The best documented source of dependence, the tendency toward excessive reversals in pairs of non-zero price changes, seems to be a direct result of the ability of investors to place limit orders as well as orders at market, and this negative dependence in itself does not imply the existence of profitable trading rules. Similarly, the apparent tendency for observed transactions (and, by implication, limit orders) to be concentrated at integers, halves, even eighths and odd eighths in descending order is an interesting fact about investor behavior, but in itself is not a basis on which to conclude that the market is inefficient.[20]

The Niederhoffer-Osborne analysis of market making does, however, point clearly to the existence of market inefficiency, but with respect to strong form tests of the efficient markets model. In particular, the list of unexecuted buy and sell limit orders in the specialist's book is important information about the likely future behavior of prices, and this information is only available to the specialist. When the specialist is asked for a quote, he gives the prices and can give the quantities of the highest buy limit and lowest sell limit orders on his book, but he is prevented by law from divulging the book's full contents. The interested reader can easily imagine situations where the structure of limit orders in the book could be used as the basis of a profitable trading rule.[21] But the record seems to speak for itself:

> It should not be assumed that these transactions undertaken by the specialist, and in which he is involved as buyer or seller in 24 per cent of all market volume, are necessarily a burden to him. Typically, the specialist sells above his last purchase on 83 per cent of all his sales, and buys below his last sale on 81 per cent of all his purchases ([32], p. 908).

Thus it seems that the specialist has monopoly power over an important block of information, and, not unexpectedly, uses his monopoly to turn a profit. And this, of course, is evidence of market inefficiency in the strong form sense. The important economic question, of course, is whether the market making

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[20] Niederhoffer and Osborne offer little to refute this conclusion. For example ([32], p. 914):

> Although the specific properties reported in this study have a significance from a statistical point of view, the reader may well ask whether or not they are helpful in a practical sense. Certain trading rules emerge as a result of our analysis. One is that limit and stop orders should be placed at odd eights, preferably at $\frac{7}{8}$ for sell orders and at $\frac{1}{8}$ for buy orders. Another is to buy when a stock advances through a barrier and to sell when it sinks through a barrier.

The first "trading rule" tells the investor to resist his innate inclination to place orders at integers, but rather to place sell orders $\frac{1}{8}$ below an integer and buy orders $\frac{1}{8}$ above. Successful execution of the orders is then more likely, since the congestion of orders that occur at integers is avoided. But the cost of this success is apparent. The second "trading rule" seems no more promising, if indeed it can even be translated into a concrete prescription for action.

[21] See, for example, ([32], p. 908). But it is unlikely that anyone but the specialist could earn substantial profits from knowledge of the structure of unexecuted limit orders on the book. The specialist makes trading profits by engaging in many transactions, each of which has a small average profit; but for any other trader, including those with seats on the exchange, these profits would be eaten up by commissions to the specialist.

function of the specialist could be fulfilled more economically by some non-monopolistic mechanism.[22]

4. Distributional Evidence

At this date the weight of the empirical evidence is such that economists would generally agree that whatever dependence exists in series of historical returns cannot be used to make profitable predictions of the future. Indeed, for returns that cover periods of a day or longer, there is little in the evidence that would cause rejection of the stronger random walk model, at least as a good first approximation.

Rather, the last burning issue of the random walk literature has centered on the nature of the distribution of price changes (which, we should note immediately, is an important issue for the efficient markets hypothesis since the nature of the distribution affects both the types of statistical tools relevant for testing the hypothesis and the interpretation of any results obtained). A model implying normally distributed price changes was first proposed by Bachelier [3], who assumed that price changes from transaction to transaction are independent, identically distributed random variables with finite variances. If transactions are fairly uniformly spread across time, and if the number of transactions per day, week, or month is very large, then the Central Limit Theorem leads us to expect that these price changes will have normal or Gaussian distributions.

Osborne [33], Moore [31], and Kendall [21] all thought their empirical evidence supported the normality hypothesis, but all observed high tails (i.e., higher proportions of large observations) in their data distributions vis-à-vis what would be expected if the distributions were normal. Drawing on these findings and some empirical work of his own, Mandelbrot [28] then suggested that these departures from normality could be explained by a more general form of the Bachelier model. In particular, if one does not assume that distributions of price changes from transaction to transaction necessarily have finite variances, then the limiting distributions for price changes over longer differencing intervals could be any member of the stable class, which includes the normal as a special case. Non-normal stable distributions have higher tails than the normal, and so can account for this empirically observed feature of distributions of price changes. After extensive testing (involving the data from the stocks in Table 1), Fama [10] concludes that non-normal stable distributions are a better description of distributions of daily returns on common stocks than the normal. This conclusion is also supported by the empirical work of Blume [7] on common stocks, and it has been extended to U.S. Government Treasury Bills by Roll [37].

Economists have, however, been reluctant to accept these results,[23] primar-

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[22] With modern computers, it is hard to believe that a more competitive and economical system would not be feasible. It does not seem technologically impossible to replace the entire floor of the N.Y.S.E. with a computer, fed by many remote consoles, that kept all the books now kept by the specialists, that could easily make the entire book on any stock available to anybody (so that interested individuals could then compete to "make a market" in a stock) and that carried out transactions automatically.

[23] Some have suggested that the long-tailed empirical distributions might result from processes

ily because of the wealth of statistical techniques available for dealing with normal variables and the relative paucity of such techniques for non-normal stable variables. But perhaps the biggest contribution of Mandelbrot's work has been to stimulate research on stable distributions and estimation procedures to be applied to stable variables. (See, for example, Wise [46], Fama and Roll [15], and Blattberg and Sargent [6], among others.) The advance of statistical sophistication (and the importance of examining distributional assumptions in testing the efficient markets model) is well illustrated in Roll [37], as compared, for example, with the early empirical work of Mandelbrot [28] and Fama [10].

5. "Fair Game" Models in the Treasury Bill Market

Roll's work is novel in other respects as well. Coming after the efficient markets models of Mandelbrot [27] and Samuelson [38], it is the first weak form empirical work that is consciously in the "fair game" rather than the random walk tradition.

More important, as we saw earlier, the "fair game" properties of the general expected return models apply to

$$z_{jt} = r_{jt} - E(\tilde{r}_{jt}|\Phi_{t-1}). \tag{10}$$

For data on common stocks, tests of "fair game" (and random walk) properties seem to go well when the conditional expected return is estimated as the average return for the sample of data at hand. Apparently the variation in common stock returns about their expected values is so large relative to any changes in the expected values that the latter can safely be ignored. But, as Roll demonstrates, this result does not hold for Treasury Bills. Thus, to test the "fair game" model on Treasury Bills requires explicit economic theory for the evolution of expected returns through time.

Roll uses three existing theories of the term structure (the pure expectations hypothesis of Lutz [26] and two market segmentation hypotheses, one of which is the familiar "liquidity preference" hypothesis of Hicks [18] and Kessel [22]) for this purpose.[24] In his models r_{jt} is the rate observed from the term structure at period t for one week loans to commence at $t + j - 1$, and can be thought of as a "futures" rate. Thus $r_{j+1,\, t-1}$ is likewise the rate on

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that are mixtures of normal distributions with different variances. Press [35], for example, suggests a Poisson mixture of normals in which the resulting distributions of price changes have long tails but finite variances. On the other hand, Mandelbrot and Taylor [29] show that other mixtures of normals can still lead to non-normal stable distributions of price changes for finite differencing intervals.

If, as Press' model would imply, distributions of price changes are long-tailed but have finite variances, then distributions of price changes over longer and longer differencing intervals should be progressively closer to the normal. No such convergence to normality was observed in [10] (though admittedly the techniques used were somewhat rough). Rather, except for origin and scale, the distributions for longer differencing intervals seem to have the same "high-tailed" characteristics as distributins for shorter differencing intervals, which is as would be expected if the distributions are non-normal stable.

[24] As noted early in our discussions, all available tests of market efficiency are implicitly also tests of expected return models of market equilibrium. But Roll formulates explicitly the economic models underlying his estimates of expected returns, and emphasizes that he is simultaneously testing economic models of the term structure as well as market efficiency.

one week loans to commence at $t + j - 1$, but observed in this case at $t - 1$. Similarly, L_{jt} is the so-called "liquidity premium" in r_{jt}; that is

$$r_{jt} = E(\tilde{r}_{0,t+j-1} | \Phi_t) + L_{jt}.$$

In words, the one-week "futures" rate for period $t + j - 1$ observed from the term structure at t is the expectation at t of the "spot" rate for $t + j - 1$ plus a "liquidity premium" (which could, however, be positive or negative).

In all three theories of the term structure considered by Roll, the conditional expectation required in (10) is of the form

$$E(\tilde{r}_{j,t} | \Phi_{t-1}) = r_{j+1,t-1} + E(\tilde{L}_{jt} | \Phi_{t-1}) - L_{j+1,t-1}.$$

The three theories differ only in the values assigned to the "liquidity premiums." For example, in the "liquidity preference" hypothesis, investors must always be paid a positive premium for bearing interest rate uncertainty, so that the L_{jt} are always positive. By contrast, in the "pure expectations" hypothesis, all liquidity premiums are assumed to be zero, so that

$$E(\tilde{r}_{jt} | \Phi_{t-1}) = r_{j+1,t-1}.$$

After extensive testing, Roll concludes (i) that the two market segmentation hypotheses fit the data better than the pure expectations hypothesis, with perhaps a slight advantage for the "liquidity preference" hypothesis, and (ii) that as far as his tests are concerned, the market for Treasury Bills is effcient. Indeed, it is interesting that when the best fitting term structure model is used to estimate the conditional expected "futures" rate in (10), the resulting variable z_{jt} seems to be serially independent! It is also interesting that if he simply assumed that his data distributions were normal, Roll's results would not be so strongly in support of the efficient markets model. In this case taking account of the observed high tails of the data distributions substantially affected the interpretation of the results.[25]

6. Tests of a Multiple Security Expected Return Model

Though the weak form tests support the "fair game" efficient markets model, all of the evidence examined so far consists of what we might call "single security tests." That is, the price or return histories of individual securities are examined for evidence of dependence that might be used as the basis of a trading system for *that* security. We have not discussed tests of whether securities are "appropriately priced" vis-à-vis one another.

But to judge whether differences between average returns are "appropriate" an economic theory of equilibrium expected returns is required. At the moment, the only fully developed theory is that of Sharpe [40] and Lintner [24,

[25] The importance of distributional assumptions is also illustrated in Alexander's work on trading rules. In his initial tests of filter systems [1], Alexander assumed that purchases could always be executed exactly (rather than at least) y% above lows and sales exactly y% below highs. Mandelbrot [28] pointed out, however, that though this assumption would do little harm with normally distributed price changes (since price series are then essentially continuous), with non-normal stable distributions it would introduce substantial positive bias into the filter profits (since with such distributions price series will show many discontinuities). In his later tests [2], Alexander does indeed find that taking account of the discontinuities (i.e., the presence of large price changes) in his data substantially lowers the profitability of the filters.

25] referred to earlier. In this model (which is a direct outgrowth of the mean-standard deviation portfolio models of investor equilibrium of Markowitz [30] and Tobin [43]), the expected return on security j from time t to t + 1 is

$$E(\tilde{r}_{j,t+1}|\Phi_t) = r_{f,t+1} + \left[\frac{E(\tilde{r}_{m,t+1}|\Phi_t) - r_{f,t+1}}{\sigma(\tilde{r}_{m,t+1}|\Phi_t)}\right]\frac{\text{cov}\ (\tilde{r}_{j,t+1}, \tilde{r}_{m,t+1}|\Phi_t)}{\sigma(\tilde{r}_{m,t+1}|\Phi_t)}, \tag{11}$$

where $r_{f,t+1}$ is the return from t to t + 1 on an asset that is riskless in money terms; $r_{m,t+1}$ is the return on the "market portfolio" m (a portfolio of all investment assets with each weighted in proportion to the total market value of all its outstanding units); $\sigma^2(\tilde{r}_{m,t+1}|\Phi_t)$ is the variance of the return on m; cov $(\tilde{r}_{j,t+1}, \tilde{r}_{m,t+1}|\Phi_t)$ is the covariance between the returns on j and m; and the appearance of Φ_t indicates that the various expected returns, variance and covariance, could in principle depend on Φ_t. Though Sharpe and Lintner derive (11) as a one-period model, the result is given a multiperiod justification and interpretation in [11]. The model has also been extended in (12) to the case where the one-period returns could have stable distributions with infinite variances.

In words, (11) says that the expected one-period return on a security is the one-period riskless rate of interest $r_{f,t+1}$ plus a "risk premium" that is proportional to $\text{cov}(\tilde{r}_{j,t+1}, \tilde{r}_{m,t+1}|\Phi_t)/\sigma(\tilde{r}_{m,t+1}|\Phi_t)$. In the Sharpe-Lintner model each investor holds some combination of the riskless asset and the market portfolio, so that, given a mean-standard deviation framework, the risk of an individual asset can be measured by its contribution to the standard deviation of the return on the market portfolio. This contribution is in fact cov $(\tilde{r}_{j,t+1}, \tilde{r}_{m,t+1}|\Phi_t)/\sigma(\tilde{r}_{m,t+1}|\Phi_t)$.[26] The factor

$$[E(\tilde{r}_{m,t+1}|\Phi_t) - r_{f,t+1}]/\sigma(\tilde{r}_{m,t+1}|\Phi_t),$$

which is the same for all securities, is then regarded as the market price of risk.

Published empirical tests of the Sharpe-Lintner model are not yet available, though much work is in progress. There is some published work, however, which, though not directed at the Sharpe-Lintner model, is at least consistent with some of its implications. The stated goal of this work has been to determine the extent to which the returns on a given security are related to the returns on other securities. It started (again) with Kendall's [21] finding that though common stock price changes do not seem to be serially correlated, there is a high degree of cross-correlation between the *simultaneous* returns of different securities. This line of attack was continued by King [23] who (using factor analysis of a sample of monthly returns on sixty N.Y.S.E. stocks for the period 1926-60) found that on average about 50% of the variance of an individual stock's returns could be accounted for by a "market factor" which affects the returns on all stocks, with "industry factors" accounting for at most an additional 10% of the variance.

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[26] That is,

$$\sum_j \text{cov}\ (\tilde{r}_{j,t+1}, \tilde{r}_{m,t+1}|\Phi_t)/\sigma(\tilde{r}_{m,t+1}|\Phi_t) = \sigma(\tilde{r}_{m,t+1}|\Phi_t).$$

For our purposes, however, the work of Fama, Fisher, Jensen, and Roll [14] (henceforth FFJR) and the more extensive work of Blume [7] on monthly return data is more relevant. They test the following "market model," originally suggested by Markowitz [30]:

$$\tilde{r}_{j,t+1} = \alpha_j + \beta_j \, \tilde{r}_{M,t+1} + \tilde{u}_{j,t+1} \tag{12}$$

where $r_{j,t+1}$ is the rate of return on security j for month t, $r_{M,t+1}$ is the corresponding return on a market index M, α_j and β_j are parameters that can vary from security to security, and $u_{j,t+1}$ is a random disturbance. The tests of FFJR and subsequently those of Blume indicate that (12) is well specified as a linear regression model in that (i) the estimated parameters $\hat{\alpha}_j$ and $\hat{\beta}_j$ remain fairly constant over long periods of time (e.g., the entire post-World War II period in the case of Blume), (ii) $r_{M,t+1}$ and the estimated $\hat{u}_{j,t+1}$, are close to serially independent, and (iii) the $\hat{u}_{j,t+1}$ seem to be independent of $r_{M,t+1}$.

Thus the observed properties of the "market model" are consistent with the expected return efficient markets model, and, in addition, the "market model" tells us something about the process generating expected returns from security to security. In particular,

$$E(\tilde{r}_{j,t+1}) = \alpha_j + \beta_j E(\tilde{r}_{M,t+1}). \tag{13}$$

The question now is to what extent (13) is consistent with the Sharpe-Lintner expected return model summarized by (11). Rearranging (11) we obtain

$$E(\tilde{r}_{j,t+1}|\Phi_t) = \alpha_j(\Phi_t) + \beta_j(\Phi_t)E(\tilde{r}_{m,t+1}|\Phi_t), \tag{14}$$

where, noting that the riskless rate $r_{f,t+1}$ is itself part of the information set Φ_t, we have

$$\alpha_j(\Phi_t) = r_{f,t+1}[1 - \beta_j(\Phi_t)], \tag{15}$$

and

$$\beta_j(\Phi_t) = \frac{\text{cov}\,(\tilde{r}_{j,t+1}, \tilde{r}_{m,t+1}|\Phi_t)}{\sigma^2(\tilde{r}_{m,t+1}|\Phi_t)}. \tag{16}$$

With some simplifying assumptions, (14) can be reduced to (13). In particular, if the covariance and variance that determine $\beta_j(\Phi_t)$ in (16) are the same for all t and Φ_t, then $\beta_j(\Phi_t)$ in (16) corresponds to β_j in (12) and (13), and the least squares *estimate* of β_j in (12) is in fact just the ratio of the sample values of the covariance and variance in (16). If we also assume that $r_{f,t+1}$ is the same for all t, and that the behavior of the returns on the market portfolio m are closely approximated by the returns on some representative index M, we will have come a long way toward equating (13) and (11). Indeed, the only missing link is whether in the estimated parameters of (12)

$$\hat{\alpha}_j \cong r_f(1 - \hat{\beta}_j). \tag{17}$$

Neither FFJR nor Blume attack this question directly, though some of Blume's evidence is at least promising. In particular, the magnitudes of the estimated $\hat{\alpha}_j$ are roughly consistent with (17) in the sense that the estimates are always close to zero (as they should be with monthly return data).[27]

In a sense, though, in establishing the apparent empirical validity of the "market model" of (12), both too much and too little have been shown *vis-à-vis* the Sharpe-Lintner expected return model of (11). We know that during the post-World War II period one-month interest rates on riskless assets (e.g., government bills with one month to maturity) have not been constant. Thus, if expected security returns were generated by a version of the "market model" that is fully consistent with the Sharpe-Lintner model, we would, according to (15), expect to observe some non-stationarity in the estimates of α_j. On a monthly basis, however, variation through time in one-period riskless interest rates is probably trivial relative to variation in other factors affecting monthly common stock returns, so that more powerful statistical methods would be necessary to study the effects of changes in the riskless rate.

In any case, since the work of FFJR and Blume on the "market model" was not concerned with relating this model to the Sharpe-Lintner model, we can only say that the results for the former are somewhat consistent with the implications of the latter. But the results for the "market model" are, after all, just a statistical description of the return generating process, and they are probably somewhat consistent with other models of equilibrium expected returns. Thus the only way to generate strong empirical conclusions about the Sharpe-Lintner model is to test it directly. On the other hand, any alternative model of equilibrium expected returns must be somewhat consistent with the "market model," given the evidence in its support.

B. *Tests of Martingale Models of the Semi-strong Form*

In general, semi-strong form tests of efficient markets models are concerned with whether current prices "fully reflect" all obviously publicly available information. Each individual test, however, is concerned with the adjustment of security prices to one kind of information generating event (e.g., stock splits, announcements of financial reports by firms, new security issues, etc.). Thus each test only brings supporting evidence for the model, with the idea that by accumulating such evidence the validity of the model will be "established."

In fact, however, though the available evidence is in support of the efficient markets model, it is limited to a few major types of information generating events. The initial major work is apparently the study of stock splits by Fama,

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[27] With least squares applied to monthly return data, the estimate of α_j in (12) is

$$\hat{\alpha}_j = \bar{r}_{j,t} - \hat{\beta}_j \bar{r}_{M,t},$$

where the bars indicate sample mean returns. But, in fact, Blume applies the market model to the wealth relatives $R_{jt} = 1 + r_{jt}$ and $R_{Mt} = 1 + r_{Mt}$. This yields precisely the same estimate of β_j as least squares applied to (12), but the intercept is now

$$\hat{\alpha}'_j = \bar{R}_{jt} - \hat{\beta}_j \bar{R}_{Mt} = 1 + \bar{r}_{jt} - \hat{\beta}_j(1 + \bar{r}_{Mt}) = 1 - \hat{\beta}_j + \hat{\alpha}_j.$$

Thus what Blume in fact finds is that for almost all securities, $\hat{\alpha}'_j + \hat{\beta}_j \cong 1$, which implies that $\hat{\alpha}_j$ is close to 0.

Fisher, Jensen, and Roll (FFJR) [14], and all the subsequent studies summarized here are adaptations and extensions of the techniques developed in FFJR. Thus, this paper will first be reviewed in some detail, and then the other studies will be considered.

1. Splits and the Adjustment of Stock Prices to New Information

Since the only apparent result of a stock split is to multiply the number of shares per shareholder without increasing claims to real assets, splits in themselves are not necessarily sources of new information. The presumption of FFJR is that splits may often be associated with the appearance of more fundamentally important information. The idea is to examine security returns around split dates to see first if there is any "unusual" behavior, and, if so, to what extent it can be accounted for by relationships between splits and other more fundamental variables.

The approach of FFJR to the problem relies heavily on the "market model" of (12). In this model if a stock split is associated with abnormal behavior, this would be reflected in the estimated regression residuals for the months surrounding the split. For a given split, define month O as the month in which the effective date of a split occurs, month 1 as the month immediately following the split month, month -1 as the month preceding, etc. Now define the average residual over all split securities for month m (where for each security m is measured relative to the split month) as

$$u_m = \sum_{j=1}^{N} \frac{\hat{u}_{jm}}{N},$$

where $\hat{u}_{jm}$ is the sample regression residual for security j in month m and N is the number of splits. Next, define the cumulative average residual U_m as

$$U_m = \sum_{k=-29}^{m} u_k.$$

The average residual u_m can be interpreted as the average deviation (in month m relative to split months) of the returns of split stocks from their normal relationships with the market. Similarly, U_m can be interpreted as the cumulative deviation (from month -29 to month m). Finally, define u^+_m, u^-_m, U^+_m, and U^-_m as the average and cumulative average residuals for splits followed by "increased" (+) and "decreased" (−) dividends. An "increase" is a case where the percentage change in dividends on the split share in the year after the split is greater than the percentage change for the N.Y.S.E. as a whole, while a "decrease" is a case of relative dividend decline.

The essence of the results of FFJR are then summarized in Figure 1, which shows the cumulative average residuals U_m U^+_m, and U^-_m for $-29 \leqq m \leqq 30$. The sample includes all 940 stock splits on the N.Y.S.E. from 1927-59, where the exchange was at least five new shares for four old, and where the security was listed for at least twelve months before and after the split.

For all three dividend categories the cumulative average residuals rise in the 29 months prior to the split, and in fact the average residuals (not shown here) are uniformly positive. This cannot be attributed to the splitting process, since in only about ten per cent of the cases is the time between the announcement and effective dates of a split greater than four months. Rather, it seems that firms tend to split their shares during "abnormally" good times—that is, during periods when the prices of their shares have increased more than would

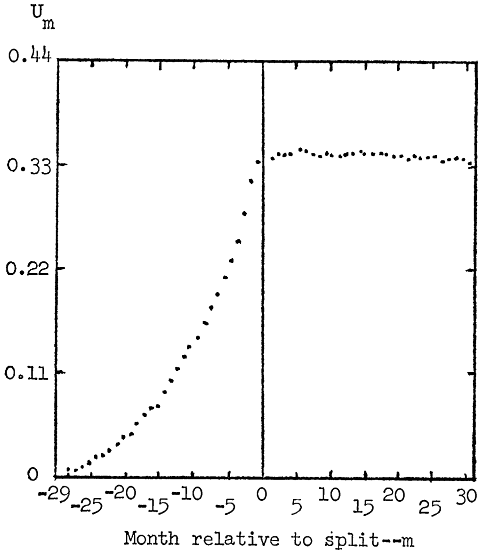

FIGURE 1a
Cumulative average residuals—all splits.

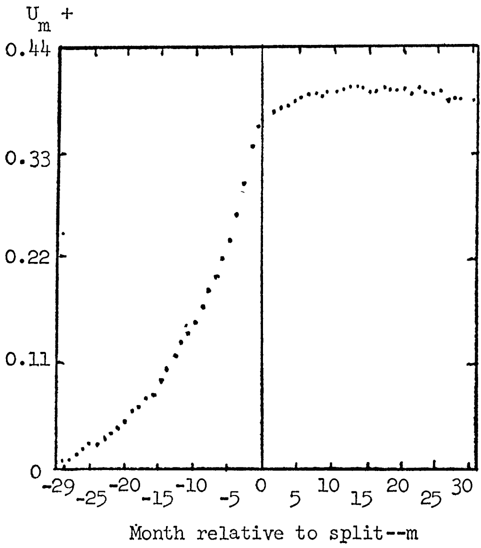

FIGURE 1b
Cumulative average residuals for dividend "increases."

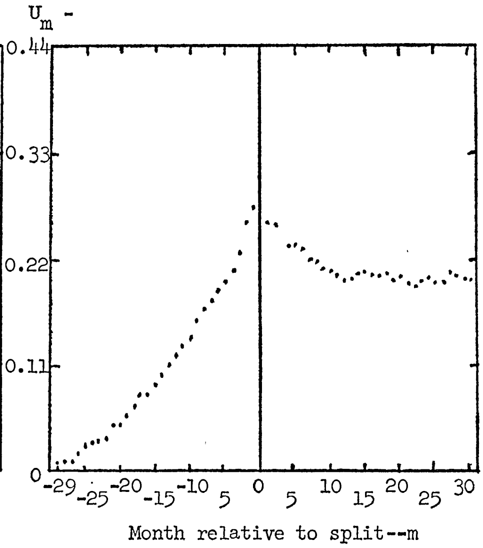

FIGURE 1c
Cumulative average residuals for dividend "decreases."

be implied by their normal relationships with general market prices, which itself probably reflects a sharp improvement, relative to the market, in the earnings prospects of these firms sometime during the years immediately preceding a split.[28]

After the split month there is almost no further movement in U_m, the cumulative average residual for all splits. This is striking, since 71.5 per cent (672 out of 940) of all splits experienced greater percentage dividend increases in the year after the split than the average for all securities on the N.Y.S.E. In light of this, FFJR suggest that when a split is announced the market interprets this (and correctly so) as a signal that the company's directors are probably confident that future earnings will be sufficient to maintain dividend payments at a higher level. Thus the large price increases in the months immediately preceding a split may be due to an alteration in expectations concerning the future earning potential of the firm, rather than to any intrinsic effects of the split itself.

If this hypothesis is correct, return behavior subsequent to splits should be substantially different for the cases where the dividend increase materializes than for the cases where it does not. FFJR argue that in fact the differences are in the directions that would be predicted. The fact that the cumulative average residuals for the "increased" dividends (Figure 1b) drift upward but only slightly in the year *after* the split is consistent with the hypothesis that when the split is *declared*, there is a price adjustment in anticipation of future dividend increases. But the behavior of the residuals for stock splits associated with "decreased" dividends offers even stronger evidence for the split hypothesis. The cumulative average residuals for these stocks (Figure 1c) rise in the few months before the split, but then fall dramatically in the few months after the split when the anticipated dividend increase is not forthcoming. When a year has passed after the split, the cumulative average residual has fallen to about where it was five months prior to the split, which is about the earliest time reliable information about a split is likely to reach the market. Thus by the time it becomes clear that the anticipated dividend increase is not forthcoming, the apparent effects of the split seem to have been wiped away, and the stock's returns have reverted to their normal relationship with market returns.

Finally, and most important, although the behavior of post-split returns will be very different depending on whether or not dividend "increases" occur, and in spite of the fact that a large majority of split securities do experience dividend "increases," when all splits are examined together (Figure 1a), subsequent to the split there is no net movement up or down in the cumulative

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[28] It is important to note, however, that as FFJR indicate, the persistent upward drift of the cumulative average residuals in the months preceding the split is not a phenomenon that could be used to increase expected trading profits. The reason is that the behavior of the average residuals is not representative of the behavior of the residuals for individual securities. In months prior to the split, successive sample residuals for individual securities seem to be independent. But in most cases, there are a few months in which the residuals are abnormally large and positive. The months of large residuals differ from security to security, however, and these differences in timing explain why the signs of the average residuals are uniformly positive for many months preceding the split.

average residuals. Thus, apparently the market makes unbiased forecasts of the implications of a split for future dividends, and these forecasts are fully reflected in the prices of the security by the end of the split month. After considerably more data analysis than can be summarized here, FFJR conclude that their results lend considerable support to the conclusion that the stock market is efficient, at least with respect to its abiliy to adjust to the information implicit in a split.

2. Other Studies of Public Announcements

Variants of the method of residual analysis developed in [14] have been used by others to study the effects of different kinds of public announcements, and all of these also support the efficient markets hypothesis.

Thus using data on 261 major firms for the period 1946-66, Ball and Brown [4] apply the method to study the effects of annual earnings announcements. They use the residuals from a time series regression of the annual earnings of a firm on the average earnings of all their firms to classify the firm's earnings for a given year as having "increased" or "decreased" relative to the market. Residuals from regressions of monthly common stock returns on an index of returns (i.e., the market model of (12)) are then used to compute cumulative average return residuals separately for the earnings that "increased," and those that "decreased." The cumulative average return residuals rise throughout the year in advance of the announcement for the earnings "increased" category, and fall for the earnings "decreased" category.[29] Ball and Brown [4, p. 175] conclude that in fact no more than about ten to fifteen percent of the information in the annual earnings announcement has not been anticipated by the month of the announcement.

On the macro level, Waud [45] has used the method of residual analysis to examine the effects of announcements of discount rate changes by Federal Reserve Banks. In this case the residuals are essentially just the deviations of the daily returns on the Standard and Poor's 500 Index from the average daily return. He finds evidence of a statistically significant "announcement effect" on stock returns for the first trading day following an announcement, but the magnitude of the adjustment is small, never exceeding .5%. More interesting from the viewpoint of the efficient markets hypothesis is his conclusion that, if anything, the market anticipates the announcements (or information is somehow leaked in advance). This conclusion is based on the non-random patterns of the signs of average return residuals on the days immediately preceding the announcement.

Further evidence in support of the efficient markets hypothesis is provided in the work of Scholes [39] on large secondary offerings of common stock (ie., large underwritten sales of existing common stocks by individuals and institutions) and on new issues of stock. He finds that on average secondary issues are associated with a decline of between one and two per cent in the cumulative average residual returns for the corresponding common stocks. Since the magnitude of the price adjustment is unrelated to the size of the

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[29] But the comment of footnote 28 is again relevant here.

issue, Scholes concludes that the adjustment is not due to "selling pressure" (as is commonly believed), but rather results from negative information implicit in the fact that somebody is trying to sell a large block of a firm's stock. Moreover, he presents evidence that the value of the information in a secondary depends to some extent on the vendor; somewhat as would be expected, by far the largest negative cumulative average residuals occur where the vendor is the corporation itself or one of its officers, with investment companies a distant second. But the identity of the vendor is not generally known at the time of the secondary, and corporate insiders need only report their transactions in their own company's stock to the S.E.C. within six days after a sale. By this time the market on average has fully adjusted to the information in the secondary, as indicated by the fact that the average residuals behave randomly thereafter.

Note, however, that though this is evidence that prices adjust efficiently to public information, it is also evidence that corporate insiders at least sometimes have important information about their firm that is not yet publicly known. Thus Scholes' evidence for secondary distributions provides support for the efficient markets model in the semi-strong form sense, but also some strong-form evidence against the model.

Though his results here are only preliminary, Scholes also reports on an application of the method of residual analysis to a sample of 696 new issues of common stock during the period 1926-66. As in the FFJR study of splits, the cumulative average residuals rise in the months preceding the new security offering (suggesting that new issues tend to come after favorable recent events)[30] but behave randomly in the months following the offering (indicating that whatever information is contained in the new issue is on average fully reflected in the price of the month of the offering).

In short, the available semi-strong form evidence on the effect of various sorts of public announcements on common stock returns is all consistent with the efficient markets model. The strong point of the evidence, however, is its consistency rather than its quantity; in fact, few different types of public information have been examined, though those treated are among the obviously most important. Moreover, as we shall now see, the amount of semi-strong form evidence is voluminous compared to the strong form tests that are available.

C. *Strong Form Tests of the Efficient Markets Models*

The strong form tests of the efficient markets model are concerned with whether all available information is fully reflected in prices in the sense that no individual has higher expected trading profits than others because he has monopolistic access to some information. We would not, of course, expect this model to be an exact description of reality, and indeed, the preceding discussions have already indicated the existence of contradictory evidence. In particular, Niederhoffer and Osborne [32] have pointed out that specialists on the N.Y.S.E. apparently use their monopolistic access to information concern-

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[30] Footnote 28 is again relevant here.

ing unfilled limit orders to generate monopoly profits, and Scholes' evidence [39] indicates that officers of corporations sometimes have monopolistic access to information about their firms.

Since we already have enough evidence to determine that the model is not strictly valid, we can now turn to other interesting questions. Specifically, how far down through the investment community do deviations from the model permeate? Does it pay for the average investor (or the average economist) to expend resources searching out little known information? Are such activities even generally profitable for various groups of market "professionals"? More generally, who are the people in the investment community that have access to "special information"?

Though this is a fascinating problem, only one group has been studied in any depth—the managements of open end mutual funds. Several studies are available (e.g., Sharpe [41, 42] and Treynor [44]), but the most thorough are Jensen's [19, 20], and our comments will be limited to his work. We shall first present the theoretical model underlying his tests, and then go on to his empirical results.

1. Theoretical Framework

In studying the performance of mutual funds the major goals are to determine (a) whether in general fund managers seem to have access to special information which allows them to generate "abnormal" expected returns, and (b) whether some funds are better at uncovering such special information than others. Since the criterion will simply be the ability of funds to produce higher returns than some norm with no attempt to determine what is responsible for the high returns, the "special information" that leads to high performance could be either keener insight into the implications of publicly available information than is implicit in market prices or monopolistic access to specific information. Thus the tests of the performance of the mutual fund industry are not strictly strong form tests of the efficient markets model.

The major theoretical (and practical) problem in using the mutual fund industry to test the efficient markets model is developing a "norm" against which performance can be judged. The norm must represent the results of an investment policy based on the assumption that prices fully reflect all available information. And if one believes that investors are generally risk averse and so on average must be compensated for any risks undertaken, then one has the problem of finding appropriate definitions of risk and evaluating each fund relative to a norm with its chosen level of risk.

Jensen uses the Sharpe [40]-Lintner [24, 25] model of equilibrium expected returns discussed above to derive a norm consistent with these goals. From (14)-(16), in this model the expected return on an asset or portfolio j from t to t + 1 is

$$E(\tilde{r}_{j,t+1}|\Phi_t) = r_{f,t+1}\,[1 - \beta_j(\Phi_t)] + E(\tilde{r}_{m,t+1}|\Phi_t)\beta_j(\Phi_t), \qquad (18)$$

where the various symbols are as defined in Section III. A. 6. But (18) is an *ex ante* relationship, and to evaluate performance an *ex post* norm is needed.

One way the latter can be obtained is to substitute the realized return on the market portfolio for the expected return in (18) with the result[31]

$$E(\tilde{r}_{j,t+1}|\Phi_t, r_{m,t+1}) = r_{f,t+1}\,[1 - \beta_j(\Phi_t)] + r_{m,t+1}\beta_j(\Phi_t). \qquad (19)$$

Geometrically, (19) says that within the context of the Sharpe-Lintner model, the expected return on j (given information Φ_t and the return $r_{m,t+1}$ on the market portfolio) is a linear function of its risk

$$\beta_j(\Phi_t) = \text{cov}\,(\tilde{r}_{j,t+1}, \tilde{r}_{m,t+1}|\Phi_t)/\sigma^2(\tilde{r}_{m,t+1}|\Phi_t),$$

as indicated in Figure 2. Assuming that the value of $\beta_j(\Phi_t)$ is somehow known, or can be reliably estimated, if j is a mutual fund, its *ex post* performance from t to t + 1 might now be evaluated by plotting its combination of realized return $r_{j,t+1}$ and risk in Figure 2. If (as for the point a) the combination falls above the expected return line (or, as it is more commonly called, the "market line"), it has done better than would be expected given its level of risk, while if (as for the point b) it falls below the line it has done worse.

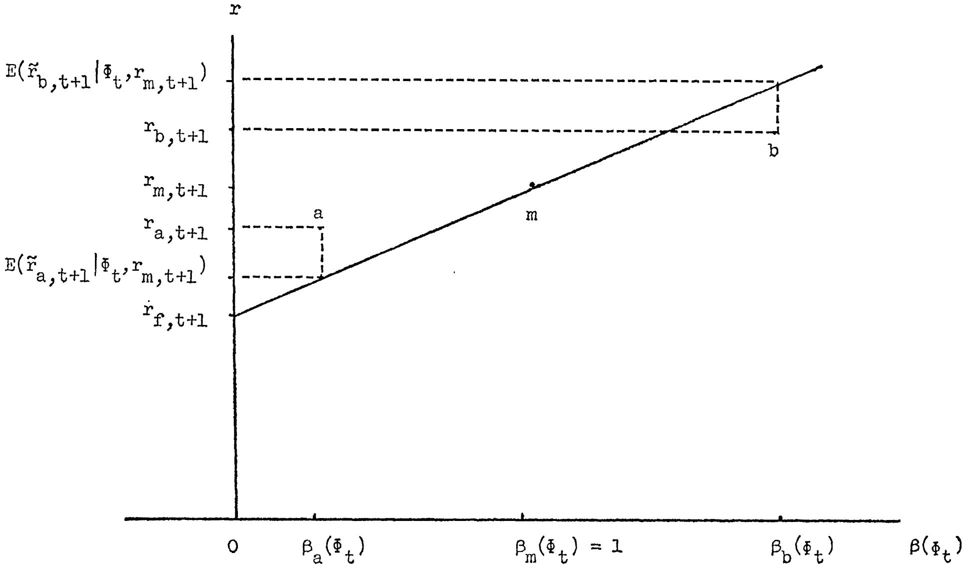

FIGURE 2
Performance Evaluation Graph

Alternatively, the market line shows the combinations of return and risk provided by portfolios that are simple mixtures of the riskless asset and the market portfolio m. The returns and risks for such portfolios (call them c) are

$$r_{c,t+1} = \alpha r_{f,t+1} + (1 - \alpha) r_{m,t+1}$$

$$\beta_c(\Phi_t) = \frac{\text{cov}\,(\tilde{r}_{c,t+1}, \tilde{r}_{m,t+1}|\Phi_t)}{\sigma^2(\tilde{r}_{m,t+1}|\Phi_t)} = \frac{\text{cov}\,((1-\alpha)\tilde{r}_{m,t+1}, \tilde{r}_{m,t+1}|\Phi_t)}{\sigma^2(\tilde{r}_{m,t+1}|\Phi_t)} = 1 - \alpha,$$

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[31] The assumption here is that the return $\tilde{r}_{j,t+1}$ is generated according to

$$\tilde{r}_{j,t+1} = r_{f,t+1}[1 - \beta_j(\Phi_t)] + r_{m,t+1}\beta_j(\Phi_t) + \tilde{u}_{j,t+1},$$

and

$$E(\tilde{u}_{j,t+1}|r_{m,t+1}) = 0 \text{ for all } r_{m,t+1}.$$

where α is the proportion of portfolio funds invested in the riskless asset. Thus, when $1 \geqslant \alpha \geqslant 0$ we obtain the combinations of return and risk along the market line from $r_{f,t+1}$ to m in Figure 2, while when $\alpha < 0$ (and under the assumption that investors can borrow at the same rate that they lend) we obtain the combinations of return and risk along the extension of the line through m. In this interpretation, the market line represents the results of a naive investment strategy, which the investor who thinks prices reflect all available information might follow. The performance of a mutual fund is then measured relative to this naive strategy.

2. Empirical Results

Jensen uses this risk-return framework to evaluate the performance of 115 mutual funds over the ten year period 1955-64. He argues at length for measuring return as the nominal ten year rate with continuous compounding (i.e., the natural log of the ratio of terminal wealth after ten years to initial wealth) and for using historical data on nominal one-year rates with continuous compounding to estimate risk. The Standard and Poor Index of 500 major common stocks is used as the proxy for the market portfolio.

The general question to be answered is whether mutual fund managements have any special insights or information which allows them to earn returns above the norm. But Jensen attacks the question on several levels. First, can the funds in general do well enough to compensate investors for loading charges, management fees, and other costs that might be avoided by simply choosing the combination of the riskless asset f and the market portfolio m with risk level comparable to that of the fund's actual portfolio? The answer seems to be an emphatic no. As far as net returns to investors are concerned, in 89 out of 115 cases, the fund's risk-return combination for the ten year period is below the market line for the period, and the average over all funds of the deviations of ten year returns from the market time is −14.6%. That is, on average the consumer's wealth after ten years of holding mutual funds is about fifteen per cent less than if he held the corresponding portfolios along the market line.

But the loading charge that an investor pays in buying into a fund is usually a pure salesman's commission that the fund itself never gets to invest. Thus one might ask whether, ignoring loading charges (i.e., assuming no such charges were paid by the investor), in general fund managements can earn returns sufficiently above the norm to cover all other expenses that are presumably more directly related to the management of the fund portfolios. Again, the answer seems to be no. Even when loading charges are ignored in computing returns, the risk-return combinations for 72 out of 115 funds are below the market line, and the average deviation of ten year returns from the market line is −8.9%.

Finally, as a somewhat stronger test of the efficient markets model, one would like to know if, ignoring all expenses, fund managements in general showed any ability to pick securities that outperformed the norm. Unfortunately, this question cannot be answered with precision for individual funds since, curiously, data on brokerage commissions are not published regularly.

But Jensen suggests the available evidence indicates that the answer to the question is again probably negative. Specifically, adding back all other published expenses of funds to their returns, the risk-return combinations for 58 out of 115 funds were below the market line, and the average deviation of ten year return from the line was —2.5%. But part of this result is due to the absence of a correction for brokerage commissions. Estimating these commissions from average portfolio turnover rates for all funds for the period 1953-58, and adding them back to returns for all funds increases the average deviation from the market line from —2.5% to .09%, which still is not indicative of the existence of special information among mutual fund managers.

But though mutual fund managers in general do not seem to have access to information not already fully reflected in prices, perhaps there are individual funds that consistently do better than the norm, and so provide at least some strong form evidence against the efficient markets model. If there are such funds, however, they escape Jensen's search. For example, for individual funds, returns above the norm in one subperiod do not seem to be associated with performance above the norm in other subperiods. And regardless of how returns are measured (i.e., net or gross of loading charges and other expenses), the number of funds with large positive deviations of returns from the market line of Figure 2 is less than the number that would be expected by chance with 115 funds under the assumption that fund managements have no special talents in predicting returns.[32]

Jensen argues that though his results apply to only one segment of the investment community, they are nevertheless striking evidence in favor of the efficient markets model:

> Although these results certainly do not imply that the strong form of the martingale hypothesis holds for all investors and for all time, they provide strong evidence in support of that hypothesis. One must realize that these analysts are extremely well endowed. Moreover, they operate in the securities markets every day and have wide-ranging contacts and associations in both the business and financial communities. Thus, the fact that they are apparently unable to forecast returns accurately enough to recover their research and transactions costs is a striking piece of evidence in favor of the strong form of the martingale hypothesis—at least as far as the extensive subset of information available to these analysts is concerned [20, p. 170].

IV. SUMMARY AND CONCLUSIONS

The preceding (rather lengthy) analysis can be summarized as follows. In general terms, the theory of efficient markets is concerned with whether prices at any point in time "fully reflect" available information. The theory only has empirical content, however, within the context of a more specific model of

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[32] On the other hand, there is some suggestion in Scholes' [39] work on secondary issues that mutual funds may occassionally have access to "special information." After corporate insiders, the next largest negative price changes occur when the secondary seller is an investment company (including mutual funds), though on average the price changes are much smaller (i.e., closer to 0) than when the seller is a corporate insider.

Moreover, Jensen's evidence itself, though not indicative of the existence of special information among mutual fund managers, is not sufficiently precise to conclude that such information never exists. This stronger conclusion would require exact data on unavoidable expenses (including brokerage commissions) of portfolio management incurred by funds.

market equilibrium, that is, a model that specifies the nature of market equilibrium when prices "fully reflect" available information. We have seen that all of the available empirical literature is implicitly or explicitly based on the assumption that the conditions of market equilibrium can be stated in terms of expected returns. This assumption is the basis of the expected return or "fair game" efficient markets models.

The empirical work itself can be divided into three categories depending on the nature of the information subset of interest. *Strong-form* tests are concerned with whether individual investors or groups have monopolistic access to any information relevant for price formation. One would not expect such an extreme model to be an exact description of the world, and it is probably best viewed as a benchmark against which the importance of deviations from market efficiency can be judged. In the less restrictive *semi-strong-form* tests the information subset of interest includes all obviously publicly available information, while in the *weak form* tests the information subset is just historical price or return sequences.

Weak form tests of the efficient market model are the most voluminous, and it seems fair to say that the results are strongly in support. Though statistically significant evidence for dependence in successive price changes or returns has been found, some of this is consistent with the "fair game" model and the rest does not appear to be sufficient to declare the market inefficient. Indeed, at least for price changes or returns covering a day or longer, there isn't much evidence against the "fair game" model's more ambitious offspring, the random walk.

Thus, there is consistent evidence of positive dependence in day-to-day price changes and returns on common stocks, and the dependence is of a form that can be used as the basis of marginally profitable trading rules. In Fama's data [10] the dependence shows up as serial correlations that are consistently positive but also consistently close to zero, and as a slight tendency for observed numbers of runs of positive and negative price changes to be less than the numbers that would be expected from a purely random process. More important, the dependence also shows up in the filter tests of Alexander [1, 2] and those of Fama and Blume [13] as a tendency for very small filters to produce profits in excess of buy-and-hold. But any systems (like the filters) that attempt to turn short-term dependence into trading profits of necessity generate so many transactions that their expected profits would be absorbed by even the minimum commissions (security handling fees) that floor traders on major exchanges must pay. Thus, using a less than completely strict interpretation of market efficiency, this positive dependence does not seem of sufficient importance to warrant rejection of the efficient markets model.

Evidence in contradiction of the "fair game" efficient markets model for price changes or returns covering periods longer than a single day is more difficult to find. Cootner [9], and Moore [31] report preponderantly negative (but again small) serial correlations in weekly common stock returns, and this result appears also in the four day returns analyzed by Fama [10]. But it does not appear in runs tests of [10], where, if anything, there is some slight indication of positive dependence, but actually not much evidence of any dependence at all. In any case, there is no indication that whatever dependence exists in weekly returns can be used as the basis of profitable trading rules.

Other existing evidence of dependence in returns provides interesting insights into the process of price formation in the stock market, but it is not relevant for testing the efficient markets model. For example, Fama [10] shows that large daily price changes tend to be folowed by large changes, but of unpredictable sign. This suggests that important information cannot be completely evaluated immediately, but that the initial first day's adjustment of prices to the information is unbiased, which is sufficient for the martingale model. More interesting and important, however, is the Niederhoffer-Osborne [32] finding of a tendency toward excessive reversals in common stock price changes from transaction to transaction. They explain this as a logical result of the mechanism whereby orders to buy and sell at market are matched against existing limit orders on the books of the specialist. Given the way this tendency toward excessive reversals arises, however, there seems to be no way it can be used as the basis of a profitable trading rule. As they rightly claim, their results are a strong refutation of the theory of random walks, at least as applied to price changes from transaction to transaction, but they do not constitute refutation of the economically more relevant "fair game" efficient markets model.

Semi-strong form tests, in which prices are assumed to fully reflect all obviously publicly available information, have also supported the efficient markets hypothesis. Thus Fama, Fisher, Jensen, and Roll [14] find that the information in stock splits concerning the firm's future dividend payments is on average fully reflected in the price of a split share at the time of the split. Ball and Brown [4] and Scholes [39] come to similar conclusions with respect to the information contained in (i) annual earning announcements by firms and (ii) new issues and large block secondary issues of common stock. Though only a few different types of information generating events are represented here, they are among the more important, and the results are probably indicative of what can be expected in future studies.

As noted earlier, the strong-form efficient markets model, in which prices are assumed to fully reflect all available information, is probably best viewed as a benchmark against which deviations from market efficiency (interpreted in its strictest sense) can be judged. Two such deviations have in fact been observed. First, Niederhoffer and Osborne [32] point out that specialists on major security exchanges have monopolistic access to information on unexecuted limit orders and they use this information to generate trading profits. This raises the question of whether the "market making" function of the specialist (if indeed this is a meaningful economic function) could not as effectively be carried out by some other mechanism that did not imply monopolistic access to information. Second, Scholes [39] finds that, not unexpectedly, corporate insiders often have monopolistic access to information about their firms.

At the moment, however, corporate insiders and specialists are the only two groups whose monopolistic access to information has been documented. There is no evidence that deviations from the strong form of the efficient markets model permeate down any further through the investment community. For the purposes of most investors the efficient markets model seems a good first (and second) approximation to reality.

In short, the evidence in support of the efficient markets model is extensive, and (somewhat uniquely in economics) contradictory evidence is sparse. Nevertheless, we certainly do not want to leave the impression that all issues are closed. The old saw, "much remains to be done," is relevant here as elsewhere. Indeed, as is often the case in successful scientific research, now that we know we've been in the past, we are able to pose and (hopefully) to answer an even more interesting set of questions for the future. In this case the most pressing field of future endeavor is the development and testing of models of market equilibrium under uncertainty. When the process generating equilibrium expected returns is better understood (and assuming that some expected return model turns out to be relevant), we will have a more substantial framework for more sophisticated intersecurity tests of market efficiency.

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APPENDIX

A

CBO's Projections of Economic and Demographic Trends

The long-term outlook for the federal budget as described in this report was developed on the basis of the Congressional Budget Office's projections for a host of economic and demographic trends for the next three decades. (Average values for 2016 to 2046, the period encompassed by CBO's extended baseline, are shown in Table A-1. A set of annual projections is included in the supplemental data for this report, available online at www.cbo.gov/publication/51580.)

CBO's Approach to Economic Projections

Through 2026, the economic projections presented in this volume are the same as those that CBO published in its January 2016 forecast (which underlies the agency's most recent 10-year budget projections).[1] For the years beyond 2026, CBO's projections generally reflect historical trends and projected demographic changes.

Comparing this year's economic projections with last year's is complicated by a change in CBO's approach. This year, the detailed economic projections account for the macroeconomic effects of fiscal policy; the detailed projections shown in Appendix A of last year's report, *The 2015 Long-Term Budget Outlook*, did not. Instead, the detailed 2015 economic projections were "benchmark" projections, consistent with a constant ratio of debt to gross domestic product (GDP) and constant marginal tax rates. Some of the macroeconomic effects of the fiscal policies embodied in the extended baseline, and their feedback effects on the budget, were presented separately last year.[2] The result is that the estimates of economic variables presented in this appendix are not strictly comparable to those CBO published last year. Where possible, this year's appendix highlights differences between this year's and last year's projections that incorporate the effects of fiscal policy. Nonetheless, most economic variables reported here are not strongly affected by fiscal policy, and the demographic projections are not affected at all. Where the effects did have a notable influence on CBO's projections, this appendix highlights those effects for this year's projections.

Economic Variables

The performance of the U.S. economy in coming decades will affect the federal government's tax revenues, spending, and debt accumulation. To estimate those effects, CBO projects trends in such key economic variables as the size and composition of the labor force, the number of hours worked, earnings per worker, capital accumulation, productivity, inflation, and interest rates. The agency also considers ways in which fiscal policy influences economic activity. (Chapter 6 of this volume discusses the economic effects of some alternative paths for deficits and debt accumulation.)

Gross Domestic Product

CBO projects that a recovery in aggregate demand will spur more rapid growth in real (inflation-adjusted) GDP over the next few years than the economy has experienced, on average, since the recession ended. Thereafter, real GDP is projected to grow at a pace that reflects increases in the supply of labor, capital services, and productivity that are consistent with the changes in marginal tax rates and increases in federal debt that CBO is projecting in its extended baseline.

CBO's projection of real GDP growth—an annual average of 2.1 percent over the 2016–2046 period—is similar to last year's projection. However, the growth rate is significantly slower than the 2.6 percent rate of the past three decades, primarily because of the anticipated slower growth of the labor force. Moreover, as the labor force

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[1] See Congressional Budget Office, *Updated Budget Projections: 2016 to 2026* (March 2016), www.cbo.gov/publication/51384, and *The Budget and Economic Outlook: 2016 to 2026* (January 2016), Chapter 2, www.cbo.gov/publication/51129.

[2] For a longer discussion of the projections that incorporated effects of fiscal policy under current law, see Congressional Budget Office, *The 2015 Long-Term Budget Outlook* (June 2015), Chapter 6, www.cbo.gov/publication/50250.

Table A-1.

Average Annual Values for Economic and Demographic Variables That Underlie CBO's Extended Baseline

	2016–2026	2027–2036	2037–2046	Overall, 2016–2046
		Economic Variables (Percent)		
Growth of GDP				
Real GDP	2.1	2.0	2.1	2.1
Nominal GDP	4.1	4.1	4.2	4.1
Growth of the Labor Force	0.6	0.3	0.4	0.4
Unemployment				
Unemployment rate	4.9	5.0	4.8	4.9
Natural rate of unemployment	4.8	4.7	4.6	4.7
Growth of Average Hours Worked	-0.1	-0.1	*	-0.1
Growth of Total Hours Worked	0.5	0.3	0.4	0.4
Earnings as a Share of Compensation	81	81	81	81
Growth of Real Earnings per Worker	1.2	1.3	1.3	1.3
Share of Earnings Below the Taxable Maximum	80	77	77	78
Growth of Capital Services	2.4	1.8	1.9	2.0
Growth of Productivity				
Total factor productivity	1.3	1.3	1.3	1.3
Labor productivity	1.6	1.7	1.8	1.7
Inflation				
Growth of the CPI-U	2.3	2.4	2.4	2.4
Growth of the GDP price index	2.0	2.0	2.0	2.0
Interest Rates				
Real rates				
On 10-year Treasury notes and the OASDI trust funds	1.6	1.9	2.2	1.9
On all federal debt held by the public	0.8	1.5	1.9	1.4
Nominal rates				
On 10-year Treasury notes and the OASDI trust funds	3.9	4.3	4.6	4.3
On all federal debt held by the public	3.1	4.0	4.3	3.7
		Demographic Variables		
Growth of the Population (Percent)	0.8	0.7	0.5	0.7
Fertility Rate (Children per woman)	1.9	1.9	1.9	1.9
Immigration Rate (Per 1,000 people in the U.S. population)	3.9	3.9	3.8	3.8
Life Expectancy at Birth, End of Period (Years)[a]	80.6	81.8	83.0	83.0
Life Expectancy at Age 65, End of Period (Years)[a]	20.2	20.9	21.6	21.6

Source: Congressional Budget Office.

The extended baseline generally reflects current law, following CBO's 10-year baseline budget projections through 2026 and then extending most of the concepts underlying those baseline projections for the rest of the long-term projection period.

CPI-U = consumer price index for all urban consumers; GDP = gross domestic product; OASDI = Old-Age, Survivors, and Disability Insurance (Social Security); * = between -0.05 percent and zero.

a. Life expectancy as used here is period life expectancy, which is the amount of time that a person in a given year would expect to survive beyond his or her current age on the basis of that year's mortality rates for various ages.

grows more slowly than the overall population, per capita real GDP is expected to increase more slowly than it has in the past—at an average annual rate of 1.4 percent over the 2016–2046 period, compared with 1.6 percent for the past 30 years.

Over the long term, total GDP is projected to be one-half of one percent below its potential (maximum sustainable) amount. That projection is based on CBO's estimate that actual GDP was roughly that much lower than potential GDP, on average, from 1961 to 2009 and lower than potential GDP, on average, in each of the past five business cycles. Those outcomes reflect CBO's assessment that actual output has fallen short of potential output during and after economic downturns to a larger extent and for longer periods than actual output has exceeded potential output during economic booms.[3]

Labor Market

Among the factors accounted for in CBO's labor market projections are the size of the labor force, the unemployment rate, the average number of hours that people work, and various measures of workers' earnings.

Growth of the Labor Force. The growth of the labor force has slowed progressively over the past few decades, but particularly since 2007. For the 2016–2046 period, CBO projects that the number of workers will increase by about 0.4 percent per year, on average. That rate is faster than the average since 2007 and similar to the rate CBO projected last year, but less than half the average for the past 30 years.

That slowdown in the pace relative to earlier decades is anticipated to result both from more workers' leaving the labor force (because of the burgeoning retirement of the baby-boom generation, despite the gradual increase in the average retirement age) and from fewer workers' entering it. The drop in new entrants will result from three trends. First, birth rates are declining: The nation's fertility rate has fallen by nearly 50 percent since 1960, to slightly below 2 today (discussed later under "Fertility" on page 103). As a result, the annual growth rate of the population between the ages of 20 and 64, which averaged about 1.0 percent over the past 30 years, is projected to slow to about 0.4 percent over the 2016–2046 period. Next, the participation of women in the labor force, which peaked in 1999, has declined slightly since then. (Participation rates among working-age men also have declined.) And finally, CBO estimates, some fiscal policies projected in the extended baseline would tend to reduce incentives to work. Notably, rising federal debt and increasing marginal tax rates (attributable to growth in real income) would limit the growth of after-tax wages, and continued growth in nongroup health insurance coverage under the Affordable Care Act over the next decade would reduce the need for employment-based coverage.

CBO expects that those forces will be modestly offset by a pair of trends working in the opposite direction. First, increasing longevity will lead people to work longer: In the coming decades, the average person is likely to work about three months longer for each additional year of life expectancy. Thus, if life expectancy was four years longer for one cohort of workers than for an earlier one, the longer-lived cohort would work about a year longer, all else being equal. Second, the population is becoming more educated, and workers with more education tend to stay in the labor force longer than do people with less education.

The Unemployment Rate. CBO projects that the unemployment rate will decline from 5.0 percent at the end of 2015 to 4.4 percent in 2017, rise again gradually to 5.0 percent by 2020, and then remain at that level through 2026. In the meantime, the natural rate of unemployment (which results from all sources other than fluctuations in overall demand related to the business cycle) will gradually decline from 4.9 percent to slightly less than 4.8 percent.[4] From 2021 onward, CBO projects, the unemployment rate will remain about one-quarter of a percentage point above the natural rate, which is consistent with the historical average relationship between the two measures and with the projected gap of one-half of one percent between actual and potential GDP.

After 2026, the actual and natural rates of unemployment are both projected to decline gradually as a result of changes in demographics and education: Older and more educated workers tend to have lower actual and natural rates of unemployment, so those rates will decline as the labor force ages and becomes increasingly more educated. By 2046, the natural rate is projected to be slightly less than 4.6 percent, and the actual rate is projected to be about 4.8 percent. The adoption of projections of age- and education-specific natural rates of unemployment results in lower rates than CBO published last year, when the agency projected that the natural rate of unemployment would gradually decline to about 5.3 percent by the end of 2017 and to 5.2 percent by the end of 2020 and remain at 5.0 percent from 2027 onward.

Average Hours Worked. Different subgroups of the labor force work different numbers of hours, on average. Men tend to work more hours than women do, and people

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[3] See Congressional Budget Office, *Why CBO Projects That Actual Output Will Be Below Potential Output on Average* (February 2015), www.cbo.gov/publication/49890.

[4] That decline reflects the decreasing share of younger workers and the rising share of older workers in the working-age population: Older workers have lower unemployment rates than younger ones, so the changing shares will reduce the overall rate.

between the ages of 30 and 40 tend to work more than people between the ages of 50 and 60. CBO's projections are based on the assumption that those differences among groups will remain stable. However, the agency also expects that over the long term, the composition of the labor force will shift toward groups that tend to work less (such as older workers). As a result, the average number of hours worked by the labor force as a whole will decline slightly. CBO estimates that by 2046, the average number of hours per worker will be about 2 percent less than it is today, about the same change in hours per worker that CBO projected last year.

Total Hours Worked. Total hours worked will increase at an average annual rate of 0.4 percent between 2016 and 2046, CBO estimates, on the basis of projections of the size of the labor force, average hours worked, and unemployment. That estimate matches last year's projection for the 2015–2040 period.

Earnings as a Share of Compensation. Workers' total compensation consists of taxable earnings and nontaxable benefits, such as paid leave and employers' contributions to health insurance and pensions. Over the years, the share of total compensation paid in the form of earnings has slipped—from about 90 percent in 1960 to about 81 percent in 2015—mainly because the cost of health insurance has risen more quickly than has total compensation.[5]

CBO expects that trend in health care costs to continue, and that by itself would further decrease the proportion of compensation that workers receive as earnings. However, starting in 2018, the Affordable Care Act will impose an excise tax on some employment-based health insurance plans that have premiums above specified amounts. Some employers and workers will respond by shifting to less expensive plans, thereby reducing the share of compensation consisting of health insurance premiums and increasing the share that consists of earnings. CBO projects that the effects of the tax on the mix of compensation will roughly offset the effects of rising costs for health care for a few decades; after that, the effects of rising health care costs will outweigh those of the excise tax.[6] As a result, the share of compensation that workers receive as earnings is projected to remain near 81 percent through 2046, which is about the same as CBO projected last year. (For more on the projected effects of the excise tax, see Chapter 5; for more on projected changes in health care costs, see Chapter 3.)

Growth of Real Earnings per Worker. Trends in prices, the growth of nonwage compensation (such as employment-based health insurance), and average hours worked imply that real earnings per worker will grow by an average of 1.2 percent annually over the 2016–2026 period and by 1.3 percent per year over the 2016–2046 period. Last year, CBO projected that growth in real earnings per worker would average 1.4 percent between 2015 and 2040. The current projection is lower because it accounts for changes in fiscal policy that would result in slower growth of output and earnings; the detailed economic projections published in *The 2015 Long-Term Budget Outlook* did not account for such effects.

Share of Earnings Below the Taxable Maximum. Social Security payroll taxes are levied only on earnings up to a maximum annual amount ($118,500 in 2016). Below that amount, earnings are taxed at a combined rate of 12.4 percent, split between the employer and employee (self-employed workers pay the full amount); no tax is paid on earnings above the cap. The taxable maximum has remained a nearly constant proportion of the average wage since the mid-1980s, but because earnings have grown more for higher earners than for others, the portion of covered earnings on which Social Security payroll taxes are paid has fallen from 90 percent in 1983 to 82 percent now.[7] CBO projects that the unequal growth in earnings will continue for the next decade and then stop: The portion of earnings subject to Social Security taxes is projected to fall below 78 percent by 2026 and to remain near that level thereafter. That share is about 1 percentage point lower than CBO projected last year.

The most recent projections, which reflect a reexamination of recent trends, show an increased rate of growth of wages and salaries for higher-income taxpayers relative to the growth of such income for other taxpayers and also

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[5] For more details, see Congressional Budget Office, *How CBO Projects Income* (July 2013), www.cbo.gov/publication/44433.

[6] CBO anticipates that the effects of the excise tax on the taxable share of compensation will diminish over time, both because it expects that most people will continue to want a significant amount of health insurance and because the Affordable Care Act set minimum amounts of coverage for health insurance plans. Therefore, the number of additional people moving to less expensive insurance plans will eventually dwindle.

[7] Covered earnings are those received by workers in jobs subject to Social Security payroll taxes. Most workers pay payroll taxes on their earnings, although a small number—mostly in state and local government jobs or in the clergy—are exempt.

relative to the growth rates that CBO had previously incorporated into its projections. That adjustment pushed more wages and salaries in CBO's projections above the taxable maximum.

Capital Services

Over the longer term, growth in the nation's stock of capital and in the flow of productive services from that stock will be driven by economic output, private saving, federal borrowing, marginal tax rates, and international flows of financial capital, CBO estimates. In particular, capital services will expand slightly more slowly than output after 2026 because of rising debt and increasing marginal tax rates.

CBO's projection of growth in the flow of real capital services is slightly below the rate it projected last year, largely because the agency improved its method for estimating the productive services that flow from different types of assets. That change led CBO to lower its estimates of historical and projected growth of capital services in the nonfarm business sector even though the historical data that the agency uses to estimate capital services are largely unchanged. In addition, in this year's projection, the greater accumulation of federal debt crowds out investment, further dampening the growth of capital services. As a result, CBO projects the flow of real capital services to grow at an average rate of 2.0 percent per year between 2016 and 2046.

Total Factor Productivity

The annual growth of total factor productivity (TFP, the average real output per unit of combined labor and capital services) is projected to increase from about 0.5 percent in 2015 to about 1.4 percent in 2022 and then to slow slightly through 2046, yielding an average annual growth rate of 1.3 percent from 2016 to 2046, or about 0.2 percentage points slower than the average annual rate of nearly 1.5 percent since 1950 and about the same as the average rate since 1990.

The projected path for TFP reflects several considerations that, in CBO's judgment, suggest growth in coming decades that is slower than the long-term historical average. For example, with the exception of a period of rapid growth in the late 1990s and early 2000s, productivity has tended to grow more slowly in recent decades than it has since the 1950s and 1960s. The long-term trend suggests that projections for the next few decades should place somewhat more weight on more recent, slower growth than on the more rapid growth of the more distant past. Thus, although CBO's projections include an acceleration in TFP from its particularly slow recent growth, the agency anticipates that TFP will return to a growth rate that is somewhat slower than its long-term average.

Some developments in particular support such projections for TFP, among them the recent slow growth in labor quality (a measure of workers' skills that accounts for educational attainment and work experience) following a relatively rapid rise over the past few decades. In CBO's judgment, that change results both from a gradual, persistent, long-term slowdown in the increase in average educational attainment and from the burgeoning retirement of a relatively large and skilled portion of the workforce—the baby-boom generation. The decline will be partly offset, however, by the aging of those remaining in the labor force over the next few decades, particularly as better health and longer life expectancy lead people to stay in the workforce longer than did members of previous generations. An older workforce generally has a larger proportion of more highly educated workers because those workers tend to remain in the labor force longer than do workers with less education.

Another factor that is projected to slow the growth of TFP is a reduction in the amount projected for federal investment. Under the assumptions used for CBO's baseline, the government's nondefense discretionary spending is projected to decline over the next decade to a much smaller percentage of GDP than it has averaged in the past. About half of nondefense discretionary spending from the 1980s onward consisted of federal investments in physical capital (such as roads), education and training, and research and development—all contributing to TFP growth. So lower nondefense discretionary spending as a percentage of GDP would mean less federal investment, causing growth in TFP to slow somewhat.

Although CBO's projection in 2015 was also for average TFP growth of 1.3 percent, that consistency is the product of offsetting changes. Because TFP reflects the portion of growth in real GDP that is not attributable to changes either in hours worked or in capital services, the downward revision to capital services in earlier years resulted in a corresponding increase in historical TFP. Higher historical growth in TFP in turn suggests higher growth in the future than CBO previously projected. That change, however, was offset in CBO's projections not only because CBO placed more weight on the considerations discussed above for trends in TFP but also because recent updates and revisions to historical output data led CBO, in developing its projections, to place more weight on the unexpected and persistent recent weakness in TFP growth.

Labor Productivity

The growth rates projected for the labor supply, the capital stock, and TFP result in CBO's projection of the average growth of labor productivity (real output per hour worked) of 1.7 percent annually over the 2016–2046 period. Last year, that growth was projected to average 1.8 percent between 2015 and 2040. The current projection is lower mainly because this year's estimate accounts for effects of fiscal policy in the extended baseline that would result in slower growth of investment.

Inflation

CBO projects the rate of inflation in the prices of various categories of goods and services as measured by the annual rate of change in the consumer price index for urban wage earners and clerical workers and in the consumer price index for all urban consumers (CPI-U). CBO projects that inflation will average 2.4 percent over the 2016–2046 period. (In the long term, both indexes are projected to increase at the same rate.) That long-term rate is slightly less than the average rate of inflation since 1990, when growth in the CPI-U averaged 2.5 percent per year, and slightly more than the 2.3 percent average rate that CBO projected last year for the 2015–2040 period. The change reflects the fact that CBO projected—accurately, as it turns out—that the rate of inflation would be particularly low in 2015, a year that is no longer encompassed by the long-term projections.

After 2018, the annual inflation rate for all final goods and services produced in the economy, as measured by the rate of increase in the GDP price index, is projected to average 0.4 percentage points less than the annual increase in the consumer price indexes.[8] The GDP price index grows more slowly than the consumer price indexes because it is based on the prices of a different set of goods and services and because it is based on a different method of calculation. The projected gap between the CPI-U and the GDP price index is unchanged from last year's estimate.

Interest Rates

CBO makes projections of the interest rates, both real and nominal, that apply to federal borrowing, including the rate on 10-year Treasury notes, the average rate on holdings of the Social Security trust funds, and the average rate on federal debt held by the public.

After considering several factors, including slower growth of the labor force, CBO expects real interest rates on federal borrowing to be lower in the future than they have been, on average, over the past few decades. The real interest rate on 10-year Treasury notes (calculated by subtracting the rate of increase in the consumer price index from the nominal yield on those notes) averaged roughly 3.1 percent between 1990 and 2007.[9] That rate has averaged 0.8 percent since 2009 and is projected to be 1.7 percent in 2026. In CBO's projections, the rate continues to rise thereafter, reaching 2.3 percent in 2046, 0.7 percentage points lower than its average over the past few decades.

Factors Affecting Interest Rates. Analysts who wish to use past trends as a starting point for long-term projections of interest rates must exercise judgment about which periods to examine. Real interest rates were low in the 1970s because of an unexpected surge in inflation; in the 1980s, when inflation declined at an unexpectedly rapid pace, real rates were high.[10] Interest rates fell sharply during the financial crisis and recession that began in 2007.

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[8] Final goods and services are those purchased directly by consumers, businesses (for investment), and governments, as well as net exports.

[9] Between 1970 and 2007, the real interest rate on 10-year Treasury notes averaged 3.2 percent; the average from 1953 to 2007 was 2.9 percent. Historical inflation rates are taken from the consumer price index, adjusted to account for changes over time in the way that the index measures inflation. See Bureau of Labor Statistics, "CPI Research Series Using Current Methods (CPI-U-RS)" (April 13, 2016), www.bls.gov/cpi/cpiurs.htm.

[10] CBO calculates real interest rates by subtracting expected rates of inflation from nominal interest rates. Borrowers and lenders agree to nominal interest rates after accounting for their expectations of what inflation will be. However, if rates are set under the expectation that inflation will be a certain percentage and it ends up being higher, real interest rates will turn out to be lower than anticipated. If inflation ends up lower than expected, the opposite will occur. CBO's approach is based on an assumption that the actual consumer price index, adjusted to account for changes over time in the way that the index measures inflation, is a useful proxy for expectations of inflation. One drawback is that if inflation trends are changing rapidly over time, changes in expectations may lag behind changes in actual inflation. Although CBO's approach could mismeasure expectations of inflation and real interest rates in some years, the way inflation has fluctuated over time suggests that CBO's approach yields useful measurements for 30-year averages.

To avoid using any of those possibly less representative periods, CBO considered average interest rates and their determinants for the 1990–2007 period and then judged how different those determinants might be over the long term.[11] Some factors reduce interest rates; others increase them. In CBO's assessment, over the 2016–2046 period, several factors will probably reduce interest rates on government securities relative to their 1990–2007 average:

- The labor force is projected to grow much more slowly than it has for the past few decades. If everything else remains equal (including the unemployment rate), that slower growth in the number of workers will tend to increase the amount of capital per worker in the long term, reducing the return on capital and therefore also reducing the return on government bonds and other investments.[12]

- The share of total income received by high-income households is expected to be larger in the future than it has been during the past few decades. Higher-income households tend to save a greater proportion of their income, so the difference in the distribution of income will increase the total amount of savings available for investment, other things being equal. As a consequence, the amount of capital per worker will rise and interest rates will fall.

- TFP will grow slightly more slowly in the future than it has in recent decades, CBO projects. For a given rate of investment, lower productivity growth reduces the return on capital and results in lower interest rates, all else being equal.

- The risk premium—the additional return that investors require to hold assets that are riskier than Treasury securities—will probably remain higher in the future than its average over the 1990–2007 period. Financial markets were already showing less appetite for risk in the early 2000s, resulting in higher risk premiums than in the 1990s. CBO expects the demand for low-risk assets to be further strengthened by the economic fallout from the financial crisis, the slow subsequent recovery, and financial institutions' response to increased regulatory oversight. Moreover, the greater riskiness perceived for investments in countries with emerging market economies is likely to increase demand for U.S. assets (particularly federal debt) that are considered to be relatively risk-free. That rise in demand will lead to lower returns on those assets (that is, to lower interest rates).

At the same time, in CBO's assessment, several factors will tend to boost interest rates on government securities relative to their average over the 1990–2007 period:

- Under the extended baseline, federal debt would be much larger as a percentage of GDP than it was before 2007—reaching 86 percent by 2026 and 141 percent by 2046. The latter figure is three and a half times the average of the 1990–2007 period. Higher federal debt tends to crowd out private investment in the long term, reducing the amount of capital per worker and increasing both the return on capital and interest rates.

- Net inflows of capital from other countries will be smaller as a percentage of GDP in the future than they have been, on average, in recent decades, CBO projects. In the 1990s and early-to-middle 2000s, rapid economic growth and high rates of saving in various nations with emerging market economies led to large flows of capital from those countries to the United States. Two types of developments are likely to affect those flows in the future. On one hand, as those nations' economies continue to grow, their consumption will probably increase relative to saving—because markets for their debt will develop and because average citizens will tend to receive more of the gains from economic growth—and their demand for domestic investment will rise. On the other hand, recent weakness in the outlook for global economic growth suggests that demand for investment abroad will be somewhat restrained. On net, that combination of changes is projected to reduce capital flows to the United States relative to those in the 1990s or early 2000s, decreasing domestic investment and the amount of capital per worker and boosting rates of return. (Those developments are consistent with CBO's projection that the U.S. trade deficit, the gap between its imports and its exports, will be narrower in the future as a percentage of GDP than it has been for the past few decades.)

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[11] A Bank of England study identified a similar set of determinants that account for the decline in real interest rates over the past 30 years. See Lukasz Rachel and Thomas D. Smith, *Secular Drivers of the Global Real Interest Rate*, Staff Working Paper 571 (Bank of England, December 2015), http://tinyurl.com/z6zqnb7 (PDF, 1.8 MB).

[12] For more information about the relationship between the growth of the labor force and interest rates, see Congressional Budget Office, *How Slower Growth in the Labor Force Could Affect the Return on Capital* (October 2009), www.cbo.gov/publication/41325.

- The capital share of income—the percentage of total income that is paid to owners of capital—has been on an upward trend for the past few decades, and CBO projects that it will remain higher than its average of recent decades. Although that share is expected to decline somewhat over the next decade from its current, historically high level, the factors that appear to have contributed to its rise (such as technological change and globalization) are likely to persist, keeping it above the historical average. In CBO's estimation, a larger share of income accruing to owners of capital will directly boost the return on capital and thus interest rates.

- The retirement of the baby-boom generation and slower growth of the labor force will reduce the number of workers in their prime saving years relative to the number of older people who are drawing down their savings, CBO projects. The result will be a decrease in the total amount of savings available for investment (all else being equal), which will tend to reduce the amount of capital per worker and thereby push interest rates up. (CBO estimates that this effect will only partially offset the positive effect on savings of increased income inequality, leaving a net increase in savings available for investment.)

CBO also has considered other influences on interest rates but has concluded that they will have relatively small effects.

In addition to its analysis of the factors listed above, CBO relies on information from financial markets in projecting interest rates for the long term. The current rate on 30-year Treasury bonds, for example, reflects market participants' judgments about the path of interest rates on short-term securities for 30 years into the future. That market forecast informs CBO's assessment of market expectations for the risk premium and for investment opportunities in the United States and abroad, and it points to considerably lower interest rates well into the future relative to those of recent decades.

Projections of Interest Rates. Some factors mentioned above are easier than others to quantify. For instance, the effect of labor force growth and rising federal debt can be estimated from available data, theoretical models, and estimates in the literature. But the extent to which other factors will affect interest rates is more difficult to compute. A shift in preferences for low- rather than high-risk assets is not directly observable, for instance. And although the distribution of income is observable, neither models nor empirical estimates offer much guidance for quantifying its effect on interest rates. Moreover, current interest rates are not a reliable indicator of investors' expectations about interest rates over the long term, in part because maturities of most of the government's outstanding debt securities are much shorter than the 30-year period that is the focus of CBO's long-term projections. In light of those sources of uncertainty, CBO relied on economic models, the research literature, and other information to guide its assessments of the effects of various factors on interest rates over the long term.

The estimates and assumptions that underlie CBO's extended baseline projections suggest a real interest rate on 10-year Treasury notes that averages about 1.9 percent over the 2016–2046 period. That rate is about 1.2 percentage points lower than the 3.1 percent average recorded for the 1990–2007 period, but it also implies that the real rate will gradually increase from its current unusually low level over the next three decades. In the final decade of the 30-year projection period, the rate is projected to average 2.2 percent.

The average interest rate on all federal debt held by the public tends to be somewhat below the rates on 10-year Treasury notes because interest rates are generally lower on shorter-term than on longer-term debt and because Treasury securities are expected to mature, on average, over periods of less than 10 years. The combination of CBO's projections of the interest rates for assets of different maturities and the average maturity of federal debt for the period beyond CBO's 10-year baseline leads to a 0.4 percentage-point difference between the rate on 10-year Treasury notes and the effective rate on federal debt. That difference is projected to average 0.8 percentage points over the next decade. The difference is larger over that period than is projected for later years because a significant portion of federal debt outstanding during that period was issued at the very low interest rates prevailing in the aftermath of the recession. (The average interest rate on all federal debt is projected to rise more slowly than the 10-year rate because only a portion of federal debt matures each year.) Thus, CBO projects, the average real interest rate on all federal debt held by the public (adjusted for the rate of increase in the CPI-U) will be about 1.4 percent for the 2016–2046 period.

The Social Security trust funds hold special-issue bonds that generally earn interest at rates that are higher than the average rate on federal debt. Therefore, in projecting the balances in the trust funds and calculating the present value of future streams of revenues and outlays for those funds, CBO used an interest rate that averages 1.9 percent for the 2016–2046 period.[13]

Combining CBO's projections of real interest rates with inflation, as measured by the growth of the CPI-U, yields projected nominal interest rates. CBO projects average nominal rates of 4.3 percent on 10-year Treasury notes and 3.7 percent on all federal debt held by the public for the 2016–2046 period.

Revisions to Projections of Interest Rates. The interest rate projections in this year's long-term budget outlook are substantially lower than last year's projections. The real rates on 10-year Treasury notes and the Social Security trust funds are projected to average 1.9 percent over the entire 30-year projection period and 2.2 percent in the final decade of the period. In particular, both rates are projected to be 2.2 percent in 2040 (the final year of the projection in *The 2015 Long-Term Budget Outlook*). Last year, after accounting for the effects of fiscal policy in the extended baseline, CBO projected both rates to be 2.6 percent in 2040.[14]

CBO's downward revisions to its interest rate projections are rooted in several factors. Since last year CBO has revised upward its estimates of the risk premium and of the net inflow of foreign capital relative to GDP. Both changes led to a downward revision in projected interest rates and both are consistent with signals from financial markets that participants expect interest rates to remain low well into the future. In addition, a release last July of revised historical data from the Bureau of Economic Analysis led CBO to revise downward its estimate of the share of income that is generated by capital; the new data showed that the share was lower than reported previously.[15] Finally, CBO expects TFP to grow more slowly relative to the growth experienced during the 1990–2007 period than it anticipated last year. The recent decline in the capital share and the slower expected growth in TFP both imply lower returns on capital and, in turn, lower interest rates.

Demographic Variables

In addition to influencing the overall performance of the economy, the size and composition of the U.S. population affects federal tax revenues and spending. Demographic projections incorporate estimated rates of fertility, immigration, and mortality, and the changes in those variables ultimately will affect the size of the labor force and the number of beneficiaries for such federal programs as Social Security and Medicare.

CBO anticipates that the annual growth rate of the U.S. population will decline gradually from about 0.8 percent in 2016 to about 0.5 percent in 2046 and that the total population will increase from 328 million at the beginning of 2016 to 400 million in 2046. Those values are somewhat below the estimates published in last year's report.

The population is projected not only to grow more slowly but also to become older, on average, than in the past. Because the elderly share of the population is growing and the working-age share is shrinking, the nation will face growing retirement and health care costs as a larger portion of the population receives Social Security and Medicare benefits while a smaller segment pays into the trust funds that support those federal programs.

Fertility

CBO estimates a total fertility rate of 1.9 children per woman for the 2016–2046 period.[16] (That rate is the average number of children that a woman would have in her lifetime if, at each age of her life, she experienced the birthrate observed or assumed for that year and if she

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[13] A present value is a single number that expresses a flow of future income or payments in terms of an equivalent lump sum received or paid at a specific point in time; the present value of a given set of cash flows depends on the rate of interest—known as the discount rate—that is used to translate them into current dollars.

[14] These comparisons address the economic projections that incorporate the effects of the fiscal policies embodied in the extended baseline. Last year's benchmark projections—that is, the projections consistent with the assumption of a constant ratio of debt to GDP and stable effective marginal tax rates beyond 2025—were different. In last year's benchmark, the real rate on 10-year Treasury notes averaged 2.2 percent over the entire projection period and 2.3 percent in the later years. Although this year's report does not use an economic benchmark, CBO estimated interest rates that are consistent with the assumption of a constant debt-to-GDP ratio and stable effective marginal tax rates beyond 2026. Those projections of 10-year Treasury note rates would be 1.7 percent over the 2016–2046 period, on average, and 1.8 percent in the later years. As a result, the comparable interest rates relative to last year's benchmark projections are revised downward by about 0.5 percentage points.

[15] See Stephanie H. McCulla and Shelly Smith, "The 2015 Annual Revision of the National Income and Product Accounts," *Survey of Current Business*, vol. 95, no. 8 (August 2015), pp. 1–31, http://go.usa.gov/x3Fe3 (PDF, 1.5 MB).

[16] Although CBO projects a total fertility rate, in its long-term model, the likelihood that a particular woman will have a child depends on such factors as that woman's education, marital status, immigration status, and childbearing history.

survived her entire childbearing period.) Fertility rates often decline during recessions and rebound during recoveries. However, after the 2007–2009 recession, the U.S. fertility rate (which in 2007 was 2.1) dropped and has remained below 1.9. CBO's projection is consistent with that recommended by the Social Security Advisory Board's 2015 Technical Panel on Assumptions and Methods and slightly below the average rate of 2.0 that CBO projected last year for the 2015–2040 period.[17] The change in projected fertility is the largest factor in this year's projection of slower population growth.

Immigration

CBO's immigration projections match those underlying its 10-year baseline through 2026. After 2026, net annual immigration (which accounts for all people who either enter or leave the United States in any year) is projected to decline slowly until 2036, when it is expected to equal the rate projected by the Census Bureau.[18] (CBO anticipates that net annual immigration will continue to match the Census Bureau's projections thereafter.) On that basis, the rate of net annual immigration to the United States is projected to be 4.0 per thousand people in the U.S. population in 2026 and 3.7 per thousand people in 2046. Net annual immigration is anticipated to rise from 1.4 million people in 2026 to 1.5 million people in 2046. The current projection is higher than the annual net immigration rate of 3.2 per thousand people after 2025 that CBO used in *The 2015 Long-Term Budget Outlook*. CBO increased its projection for the period after 2026 to be more consistent with the trend it anticipates for the next 10 years.

Mortality

The mortality rate, which is the number of deaths per thousand people, has generally declined in the United States for at least the past half century. During that period, the mortality rate has generally improved more quickly for younger people than for older people. In particular, a recent review of the data by CBO suggests that the differences in relative improvements in mortality exhibited by various age groups are significant and likely to continue. For example, mortality rates for people below 15 years old declined by about 80 percent between 1950 and 2012, an average drop of more than 2½ percent per year, whereas mortality rates for people over the age of 80 declined by an average of less than 1 percent per year over the same period. CBO projects that mortality rates for each five-year age group will continue to decline at the average pace experienced from 1950 through 2012. In contrast, in *The 2015 Long-Term Budget Outlook,* CBO projected that the rate of decline would be the same for all ages and both sexes. This year's projections show a slower rate of decline in mortality rates for people in older groups than for younger, but no difference by sex.

CBO's projections indicate an average life expectancy at birth of 82.3 years in 2040, compared with 79.2 years in 2016.[19] Similarly, CBO projects that life expectancy at age 65 in 2040 will be 21.2 years, or 1.8 years longer than life expectancy at age 65 in 2016.[20] The life expectancies projected for 2040 this year are a bit shorter than those reported last year: In last year's report, life expectancy at birth and at age 65 in 2040 was projected to be 82.6 years and 21.8 years, respectively.

After projecting average mortality rates for men and women in each age group, CBO incorporates differences in those rates on the basis of marital status, education, and lifetime household earnings. (For people under 30, the mortality projections account for age and sex only.) CBO projects a greater life expectancy for people who are married, have more education, and are in higher income groups.[21]

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[17] See 2015 Technical Panel on Assumptions and Methods, *Report to the Social Security Advisory Board* (September 2015), p. 9, http://go.usa.gov/cJYR5 (PDF, 3.4 MB); and Congressional Budget Office, *The 2015 Long-Term Budget Outlook* (June 2015), www.cbo.gov/publication/50250.

[18] See Census Bureau, "Population Projections, 2014 National Population Projections: Summary Tables," Table 1 (accessed July 8, 2016), http://go.usa.gov/x33DB.

[19] Life expectancy as used here is period life expectancy, which is the amount of time that a person in a given year would expect to survive beyond his or her current age on the basis of that year's mortality rates for various ages.

[20] CBO projects that life expectancy in 2090 will be 87.3 years at birth and 24.6 years at age 65. CBO's projections of life expectancies are longer than those of the Social Security trustees (85.9 and 23.6 years, respectively) but shorter than the projections (88.3 and 25.3 years, respectively) recommended in the report of the 2015 Technical Panel on Assumptions and Methods, *Report to the Social Security Advisory Board* (September 2015), pp. 13–20, http://go.usa.gov/cJYR5 (PDF, 3.4 MB).

[21] For more information about mortality differences among groups with different earnings, see Congressional Budget Office, *Growing Disparities in Life Expectancy* (April 2008), www.cbo.gov/publication/41681; and Julian P. Cristia, *The Empirical Relationship Between Lifetime Earnings and Mortality*, Working Paper 2007-11 (Congressional Budget Office, August 2007), www.cbo.gov/publication/19096.

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A SIMPLIFIED MODEL FOR PORTFOLIO ANALYSIS*

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This paper describes the advantages of using a particular model of the relationships among securities for practical applications of the Markowitz portfolio analysis technique. A computer program has been developed to take full advantage of the model: 2,000 securities can be analyzed at an extremely low cost—as little as 2% of that associated with standard quadratic programming codes. Moreover, preliminary evidence suggests that the relatively few parameters used by the model can lead to very nearly the same results obtained with much larger sets of relationships among securities. The possibility of low-cost analysis, coupled with a likelihood that a relatively small amount of information need be sacrificed make the model an attractive candidate for initial practical applications of the Markowitz technique.

1. Introduction

Markowitz has suggested that the process of portfolio selection be approached by (1) making probabilistic estimates of the future performances of securities, (2) analyzing those estimates to determine an *efficient set* of portfolios and (3) selecting from that set the portfolios best suited to the investor's preferences [1, 2, 3]. This paper extends Markowitz' work on the second of these three stages —*portfolio analysis.* The preliminary sections state the problem in its general form and describe Markowitz' solution technique. The remainder of the paper presents a simplified model of the relationships among securities, indicates the manner in which it allows the portfolio analysis problem to be simplified, and provides evidence on the costs as well as the desirability of using the model for practical applications of the Markowitz technique.

2. The Portfolio Analysis Problem

A security analyst has provided the following predictions concerning the future returns from each of N securities:

$E_i \equiv$ the expected value of R_i (the return from security i)

C_{i1} through C_{in} ; C_{ij} represents the covariance between R_i and R_j (as usual, when $i = j$ the figure is the variance of R_i)

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* Received December 1961.

† The author wishes to express his appreciation for the cooperation of the staffs of both the Western Data Processing Center at UCLA and the Pacific Northwest Research Computer Laboratory at the University of Washington where the program was tested. His greatest debt, however, is to Dr. Harry M. Markowitz of the RAND Corporation, with whom he was privileged to have a number of stimulating conversations during the past year. It is no longer possible to segregate the ideas in this paper into those which were his, those which were the author's, and those which were developed jointly. Suffice it to say that the only accomplishments which are unquestionably the property of the author are those of authorship—first of the computer program and then of this article.

The portfolio analysis problem is as follows. Given such a set of predictions, determine the set of *efficient portfolios*; a portfolio is efficient if none other gives either (a) a higher expected return and the same variance of return or (b) a lower variance of return and the same expected return.

Let X_i represent the proportion of a portfolio invested in security i. Then the expected return (E) and variance of return (V) of any portfolio can be expressed in terms of (a) the basic data (E_i-values and C_{ij}-values) and (b) the amounts invested in various securities:

$$E = \sum_i X_i E_i$$

$$V = \sum_i \sum_j X_i X_j C_{ij} .$$

Consider an objective function of the form:

$$\begin{aligned} \phi &= \lambda E - V \\ &= \lambda \sum_i X_i E_i - \sum_i \sum_j X_i X_j C_{ij} . \end{aligned}$$

Given a set of values for the parameters (λ, E_i's and C_{ij}'s), the value of ϕ can be changed by varying the X_i values as desired, as long as two basic restrictions are observed:

1. The entire portfolio must be invested:[1]

$$\sum_i X_i = 1$$

and 2. no security may be held in negative quantities:[2]

$$X_i \geqq 0 \quad \text{for all} \quad i.$$

A portfolio is described by the proportions invested in various securities—in our notation by the values of X_i. For each set of admissable values of the X_i variables there is a corresponding predicted combination of E and V and thus of ϕ. Figure 1 illustrates this relationship for a particular value of λ. The line ϕ_1 shows the combinations of E and V which give $\phi = \phi_1$, where $\phi = \lambda_k E - V$; the other lines refer to larger values of ϕ ($\phi_3 > \phi_2 > \phi_1$). Of all possible portfolios, one will maximize the value of ϕ;[3] in figure 1 it is portfolio C. The relationship between this solution and the portfolio analysis problem is obvious. The E, V combination obtained will be on the boundary of the set of attainable combinations; moreover, the objective function will be tangent to the set at that point. Since this function is of the form

$$\phi = \lambda E - V$$

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[1] Since cash can be included as one of the securities (explicitly or implicitly) this assumption need cause no lack of realism.

[2] This is the standard formulation. Cases in which short sales are allowed require a different approach.

[3] This fact is crucial to the critical line computing procedure described in the next section.

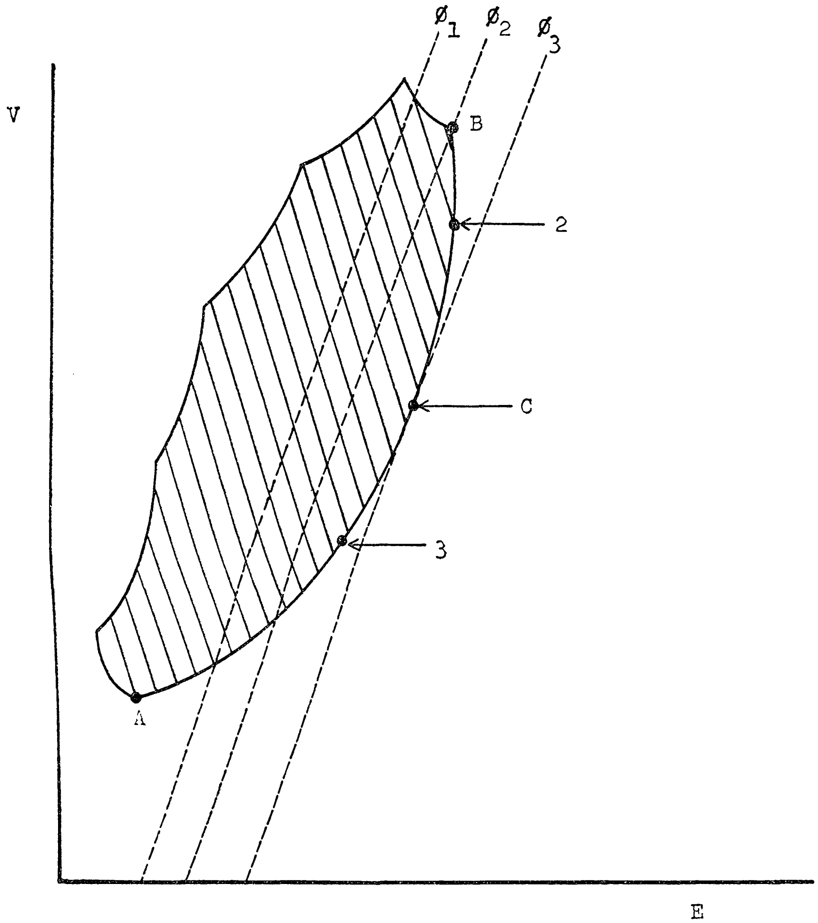

Figure 1

the slope of the boundary at the point must be λ; thus, by varying λ from + ∞ to 0, every solution of the portfolio analysis problem can be obtained.

For any given value of λ the problem described in this section requires the maximization of a quadratic function, ϕ (which is a function of X_i, X_i^2, and X_iX_j terms) subject to a linear constraint ($\sum_i X_i = 1$), with the variables restricted to non-negative values. A number of techniques have been developed to solve such *quadratic programming problems.* The critical line method, developed by Markowitz in conjunction with his work on portfolio analysis, is particularly suited to this problem and was used in the program described in this paper.

3. The Critical Line Method

Two important characteristics of the set of efficient portfolios make systematic solution of the portfolio analysis problem relatively straightforward. The first concerns the relationships among portfolios. Any set of efficient portfolios can be described in terms of a smaller set of *corner portfolios*. Any point on the E, V curve (other than the points associated with corner portfolios) can be obtained with a portfolio constructed by dividing the total investment between the two adjacent corner portfolios. For example, the portfolio which gives E, V combination C in Figure 1 might be some linear combination of the two corner portfolios with E, V combinations shown by points 2 and 3. This characteristic allows the analyst to restrict his attention to corner portfolios rather than the complete set of efficient portfolios; the latter can be readily derived from the former.

The second characteristic of the solution concerns the relationships among corner portfolios. Two corner portfolios which are adjacent on the E, V curve are related in the following manner: one portfolio will contain either (1) all the securities which appear in the other, plus one additional security or (2) all but one of the securities which appear in the other. Thus in moving down the E, V curve from one corner portfolio to the next, the quantities of the securities in efficient portfolios will vary until either one drops out of the portfolio or another enters. The point at which a change takes place marks a new corner portfolio.

The major steps in the critical line method for solving the portfolio analysis problem are:

1. The corner portfolio with $\lambda = \infty$ is determined. It is composed entirely of the one security with the highest expected return.[4]
2. Relationships between (a) the amounts of the various securities contained in efficient portfolios and (b) the value of λ are computed. It is possible to derive such relationships for any section of the E, V curve between adjacent corner portfolios. The relationships which apply to one section of the curve will not, however, apply to any other section.
3. Using the relationships computed in (2), each security is examined to determine the value of λ at which a change in the securities included in the portfolio would come about:
   a. securities presently in the portfolio are examined to determine the value of λ at which they would drop out, and
   b. securities not presently in the portfolio are examined to determine the value of λ at which they would enter the portfolio.
4. The next largest value of λ at which a security either enters or drops out of the portfolio is determined. This indicates the location of the next corner portfolio.
5. The composition of the new corner portfolio is computed, using the relationships derived in (2). However, since these relationships held only for the section of the curve between this corner portfolio and the preceding one, the solution process can only continue if new relationships are derived. The method thus returns to step (2) unless $\lambda = 0$, in which case the analysis is complete.

The amount of computation required to complete a portfolio analysis using

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[4] In the event that two or more of the securities have the same (highest) expected return, the first efficient portfolio is the combination of such securities with the lowest variance.

this method is related to the following factors:

1. The number of securities analyzed
   This will affect the extent of the computation in step (2) and the number of computations in step (3).
2. The number of corner portfolios
   Steps (2) through (5) must be repeated once to find each corner portfolio.
3. The complexity of the variance-covariance matrix
   Step (2) requires a matrix be inverted and must be repeated once for each corner portfolio.

The amount of computer memory space required to perform a portfolio analysis will depend primarily on the size of the variance-covariance matrix. In the standard case, if N securities are analyzed this matrix will have $\frac{1}{2}(N^2 + N)$ elements.

4. The Diagonal Model

Portfolio analysis requires a large number of comparisons; obviously the practical application of the technique can be greatly facilitated by a set of assumptions which reduces the computational task involved in such comparisons. One such set of assumptions (to be called the diagonal model) is described in this article. This model has two virtues: it is one of the simplest which can be constructed without assuming away the existence of interrelationships among securities and there is considerable evidence that it can capture a large part of such interrelationships.

The major characteristic of the diagonal model is the assumption that the returns of various securities are related only through common relationships with some basic underlying factor. The return from any security is determined solely by random factors and this single outside element; more explicitly:

$$R_i = A_i + B_i I + C_i$$

where A_i and B_i are parameters, C_i is a random variable with an expected value of zero and variance Q_i, and I is the level of some index. The index, I, may be the level of the stock market as a whole, the Gross National Product, some price index or any other factor thought to be the most important single influence on the returns from securities. The future level of I is determined in part by random factors:

$$I = A_{n+1} + C_{n+1}$$

where A_{n+1} is a parameter and C_{n+1} is a random variable with an expected value of zero and a variance of Q_{n+1}. It is assumed that the covariance between C_i and C_j is zero for all values of i and j $(i \neq j)$.

Figure 2 provides a graphical representation of the model. A_i and B_i serve to locate the line which relates the expected value of R_i to the level of I. Q_i indicates the variance of R_i around the expected relationship (this variance is assumed to be the same at each point along the line). Finally, A_{n+1} indicates the expected value of I and Q_{n+1} the variance around that expected value.

The diagonal model requires the following predictions from a security analyst:

1) values of A_i, B_i and Q_i for each of N securities
2) values of A_{n+1} and Q_{n+1} for the index I.

The number of estimates required from the analyst is thus greatly reduced: from 5,150 to 302 for an analysis of 100 securities and from 2,003,000 to 6,002 for an analysis of 2,000 securities.

Once the parameters of the diagonal model have been specified all the inputs required for the standard portfolio analysis problem can be derived. The relationships are:

$$E_i = A_i + B_i(A_{n+1})$$

$$V_i = (B_i)^2(Q_{n+1}) + Q_i$$

$$C = (B_i)(B_j)(Q_{n+1})$$

A portfolio analysis could be performed by obtaining the values required by the diagonal model, calculating from them the full set of data required for the standard portfolio analysis problem and then performing the analysis with the derived values. However, additional advantages can be obtained if the portfolio analysis problem is restated directly in terms of the parameters of the diagonal model. The following section describes the manner in which such a restatement can be performed.

5. The Analogue

The return from a portfolio is the weighted average of the returns from its component securities:

$$R_p = \sum_{i=1}^{N} X_i R_i$$

The contribution of each security to the total return of a portfolio is simply $X_i R_i$ or, under the assumptions of the diagonal model:

$$X_i(A_i + B_i I + C_i).$$

The total contribution of a security to the return of the portfolio can be broken into two components: (1) an investment in the "basic characteristics" of the security in question and (2) an "investment" in the index:

$$\begin{aligned} &(1) \quad X_i(A_i + B_i I + C_i) = X_i(A_i + C_i) \\ &(2) \quad\quad\quad\quad\quad\quad\quad\quad\quad\quad + X_i B_i I \end{aligned}$$

The return of a portfolio can be considered to be the result of (1) a series of investments in N "basic securities" and (2) an investment in the index:

$$R_p = \sum_{i=1}^{N} X_i(A_i + C_i) + \left[\sum_{i=1}^{N} X_i B_i\right] I$$

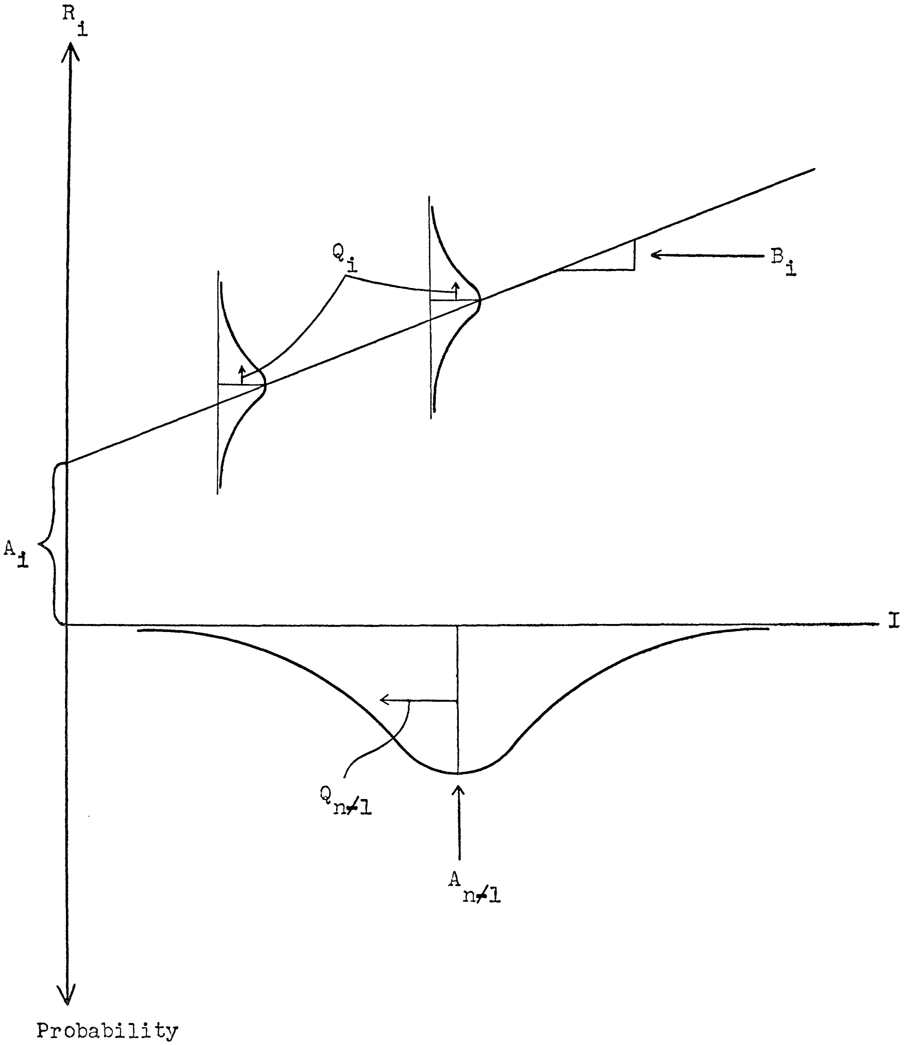

FIGURE 2

Defining X_{n+1} as the weighted average responsiveness of R_p to the level of I:

$$X_{n+1} \equiv \sum_{i=1}^{N} X_i B_i$$

and substituting this variable and the formula for the determinants of I, we obtain:

$$\begin{aligned} R_p &= \sum_{i=1}^{N} X_i(A_i + C_i) + X_{n+1}(A_{n+1} + C_{n+1}) \\ &= \sum_{i=1}^{N+1} X_i(A_i + C_i). \end{aligned}$$

The expected return of a portfolio is thus:

$$E = \sum_{i=1}^{N+1} X_i A_i$$

while the variance is:[5]

$$V = \sum_{i=1}^{N+1} X_i^2 Q_i$$

This formulation indicates the reason for the use of the parameters A_{n+1} and Q_{n+1} to describe the expected value and variance of the future value of I. It also indicates the reason for calling this the "diagonal model". The variance-covariance matrix, which is full when N securities are considered, can be expressed as a matrix with non-zero elements only along the diagonal by including an $(n + 1)$st security defined as indicated. This vastly reduces the number of computations required to solve the portfolio analysis problem (primarily in step 2 of the critical line method, when the variance-covariance matrix must be inverted) and allows the problem to be stated directly in terms of the basic parameters of the diagonal model:

$$\text{Maximize: } \lambda E - V$$

$$\text{Where: } E = \sum_{i=1}^{N+1} X_i A_i$$

$$V = \sum_{i=1}^{N+1} X_i^2 Q_i$$

$$\text{Subject to: } X_i \geqq 0 \text{ for all } i \text{ from 1 to } N$$

$$\sum_{i=1}^{N} X_i = 1$$

$$\sum_{i=1}^{N} X_i B_i = X_{n+1} .$$

6. The Diagonal Model Portfolio Analysis Code

As indicated in the previous section, if the portfolio analysis problem is expressed in terms of the basic parameters of the diagonal model, computing time and memory space required for solution can be greatly reduced. This section describes a machine code, written in the FØRTRAN language, which takes full advantage of the characteristics of the diagonal model. It uses the critical line method to solve the problem stated in the previous section.

The computing time required by the diagonal code is considerably smaller than that required by standard quadratic programming codes. The RAND QP

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[5] Recall that the diagonal model assumes $\text{cov}(C_i, C_j) = 0$ for all i and j $(i \neq j)$.

code[6] required 33 minutes to solve a 100-security example on an IBM 7090 computer; the same problem was solved in 30 seconds with the diagonal code. Moreover, the reduced storage requirements allow many more securities to be analyzed: with the IBM 709 or 7090 the RAND QP code can be used for no more than 249 securities, while the diagonal code can analyze up to 2,000 securities.

Although the diagonal code allows the total computing time to be greatly reduced, the cost of a large analysis is still far from insignificant. Thus there is every incentive to limit the computations to those essential for the final selection of a portfolio. By taking into account the possibilities of borrowing and lending money, the diagonal code restricts the computations to those absolutely necessary for determination of the final set of efficient portfolios. The importance of these alternatives, their effect on the portfolio analysis problem and the manner in which they are taken into account in the diagonal code are described in the remainder of this section.

A. The "lending portfolio"

There is some interest rate (r_l) at which money can be lent with virtual assurance that both principal and interest will be returned; at the least, money can be buried in the ground ($r_l = 0$). Such an alternative could be included as one possible security ($A_i = 1 + r_l$, $B_i = 0$, $Q_i = 0$) but this would necessitate some needless computation.[7] In order to minimize computing time, lending at some pure interest rate is taken into account explicitly in the diagonal code.

The relationship between lending and efficient portfolios can best be seen in terms of an E, σ curve showing the combinations of expected return and standard deviation of return ($= \sqrt{V}$) associated with efficient portfolios. Such a curve is shown in Figure 3 (FBCG); point A indicates the E, σ combination attained if all funds are lent. The relationship between lending money and purchasing portfolios can be illustrated with the portfolio which has the E, σ combination shown by point Z. Consider a portfolio with X_z invested in portfolio Z and the remainder $(1 - X_z)$ lent at the rate r_l. The expected return from such a portfolio would be:

$$E = X_z E_z + (1 - X_z)(1 + r_l)$$

and the variance of return would be:

$$V = X_z^2 V_z + (1 - X_z)^2 V_l + 2X_z(1 - X_z)(\text{cov}_{zl})$$

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[6] The program is described in [4]. Several alternative quadratic programming codes are available. A recent code, developed by IBM, which uses the critical line method is likely to prove considerably more efficient for the portfolio analysis problem. The RAND code is used for comparison since it is the only standard program with which the author has had experience.

[7] Actually, the diagonal code cannot accept non-positive values of Q_i; thus if the lending alternative is to be included as simply another security, it must be assigned a very small value of Q_i. This procedure will give virtually the correct solution but is inefficient.

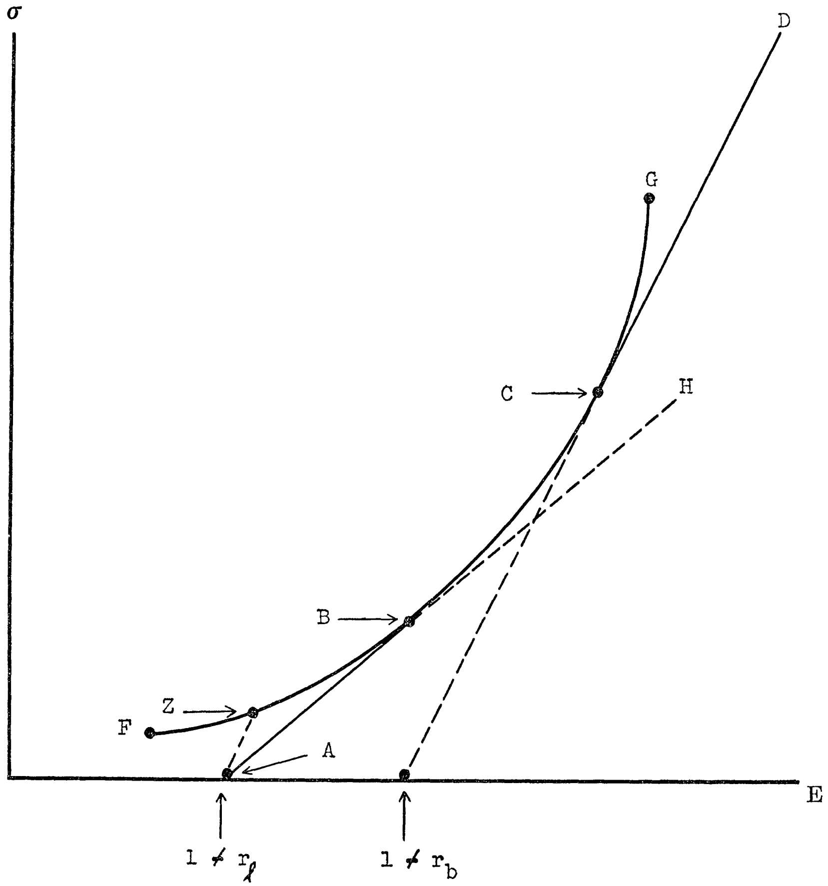

FIGURE 3

But, since V_l and cov_{zl} are both zero:

$$V = X_z^2 V_z$$

and the standard deviation of return is:

$$\sigma = X_z \sigma_z \, .$$

Since both E and σ are linear functions of X_z, the E, σ combinations of all portfolios made up of portfolio Z plus lending must lie on a straight line connecting points Z and A. In general, by splitting his investment between a portfolio and lending, an investor can attain any E, σ combination on the line connecting the E, σ combinations of the two components.

Many portfolios which are efficient in the absence of the lending alternative becomes inefficient when it is introduced. In Figure 3, for example, the possibility of attaining E, σ combinations along the line AB makes all portfolios along the original E, σ curve from point F to point B inefficient. For any desired level of

E below that associated with portfolio *B*, the most efficient portfolio will be some combination of portfolio *B* and lending. Portfolio *B* can be termed the "lending portfolio" since it is the appropriate portfolio whenever some of the investor's funds are to be lent at the rate r_l. This portfolio can be found readily once the E, σ curve is known. It lies at the point on the curve at which a ray from ($E = 1 + r_l$, $\sigma = 0$) is tangent to the curve. If the E, σ curve is not known in its entirety it is still possible to determine whether or not a particular portfolio is the lending portfolio by computing the rate of interest which *would* make the portfolio in question the lending portfolio. For example, the rate of interest associated in this manner with portfolio *C* is r_b, found by extending a tangent to the curve down to the *E*-axis. The diagonal code computes such a rate of interest for each corner portfolio as the analysis proceeds; when it falls below the previously stated lending rate the code computes the composition of the lending portfolio and terminates the analysis.

B. The "borrowing portfolio"

In some cases an investor may be able to borrow funds in order to purchase even greater amounts of a portfolio than his own funds will allow. If the appropriate rate for such borrowing were r_b, illustrated in figure 3, the E, σ combinations attainable by purchasing portfolio *C* with both the investor's funds and with borrowed funds would lie along the line *CD*, depending on the amount borrowed. Inclusion of the borrowing alternative makes certain portfolios inefficient which are efficient in the absence of the alternative; in this case the affected portfolios are those with E, σ combinations along the segment of the original E, σ curve from *C* to *G*. Just as there is a single appropriate portfolio if any lending is contemplated, there is a single appropriate portfolio if borrowing is contemplated. This "borrowing portfolio" is related to the rate of interest at which funds can be borrowed in exactly the same manner as the "lending portfolio" is related to the rate at which funds can be lent.

The diagonal code does not take account of the borrowing alternative in the manner used for the lending alternative since it is necessary to compute all previous corner portfolios in order to derive the portion of the E, σ curve below the borrowing portfolio. For this reason all computations required to derive the full E, σ curve above the lending portfolio must be made. However, the code does allow the user to specify the rate of interest at which funds can be borrowed. If this alternative is chosen, none of the corner portfolios which will be inefficient when borrowing is considered will be printed. Since as much as 65% of the total computer time can be spent recording (on tape) the results of the analysis this is not an insignificant saving.

7. The Cost of Portfolio Analysis with the Diagonal Code

The total time (and thus cost) required to perform a portfolio analysis with the diagonal code will depend upon the number of securities analyzed, the number of corner portfolios and, to some extent, the composition of the corner portfolios. A formula which gives quite an accurate estimate of the time required to perform an analysis on an IBM 709 computer was obtained by analyzing a series of runs during which the time required to complete each major segment of the program was recorded. The approximate time required for the analysis will be:[8]

$$\begin{aligned}\text{Number of seconds} = {} & .6 \\ & + .114 \times \text{number of securities analyzed} \\ & + .54 \times \text{number of corner portfolios} \\ & + .0024 \times \text{number of securities analyzed} \times \text{number of} \\ & \quad \text{corner portfolios.}\end{aligned}$$

Unfortunately only the number of securities analyzed is known before the analysis is begun. In order to estimate the cost of portfolio analysis before it is performed, some relationship between the number of corner portfolios and the number of securities analyzed must be assumed. Since no theoretical relationship can be derived and since the total number of corner portfolios could be several times the number of securities analysed, it seemed desirable to obtain some crude notion of the typical relationship when "reasonable" inputs are used. To accomplish this, a series of portfolio analyses was performed using inputs generated by a Monte Carlo model.

Data were gathered on the annual returns during the period 1940–1951 for 96 industrial common stocks chosen randomly from the New York Stock Exchange. The returns of each security were then related to the level of a stock market index and estimates of the parameters of the diagonal model obtained. These parameters were assumed to be samples from a population of A_i, B_i and Q_i triplets related as follows:

$$\begin{aligned} A_i &= \bar{A} + r_1 \\ B_i &= \bar{B} + \psi A_i + r_2 \\ Q_i &= \bar{Q} + \theta A_i + \gamma B_i + r_3 \end{aligned}$$

where r_1, r_2 and r_3 are random variables with zero means. Estimates for the parameters of these three equations were obtained by regression analysis and estimates of the variances of the random variables determined.[9] With this information the characteristics of any desired number of securities could be generated. A random number generator was used to select a value for A_i; this value, together with an additional random number determined the value of B_i; the value of Q_i was then determined with a third random number and the previously obtained values of A_i and B_i.

Figure 4 shows the relationship between the number of securities analyzed

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[8] The computations in this section are based on the assumption that no corner portfolios prior to the lending portfolio are printed. If the analyst chooses to print all preceding portfolios, the estimates given in this section should be multiplied by 2.9; intermediate cases can be estimated by interpolation.

[9] The random variables were considered normally distributed; in one case, to better approximate the data, two variances were used for the distribution—one for the portion above the mean and another for the portion below the mean.

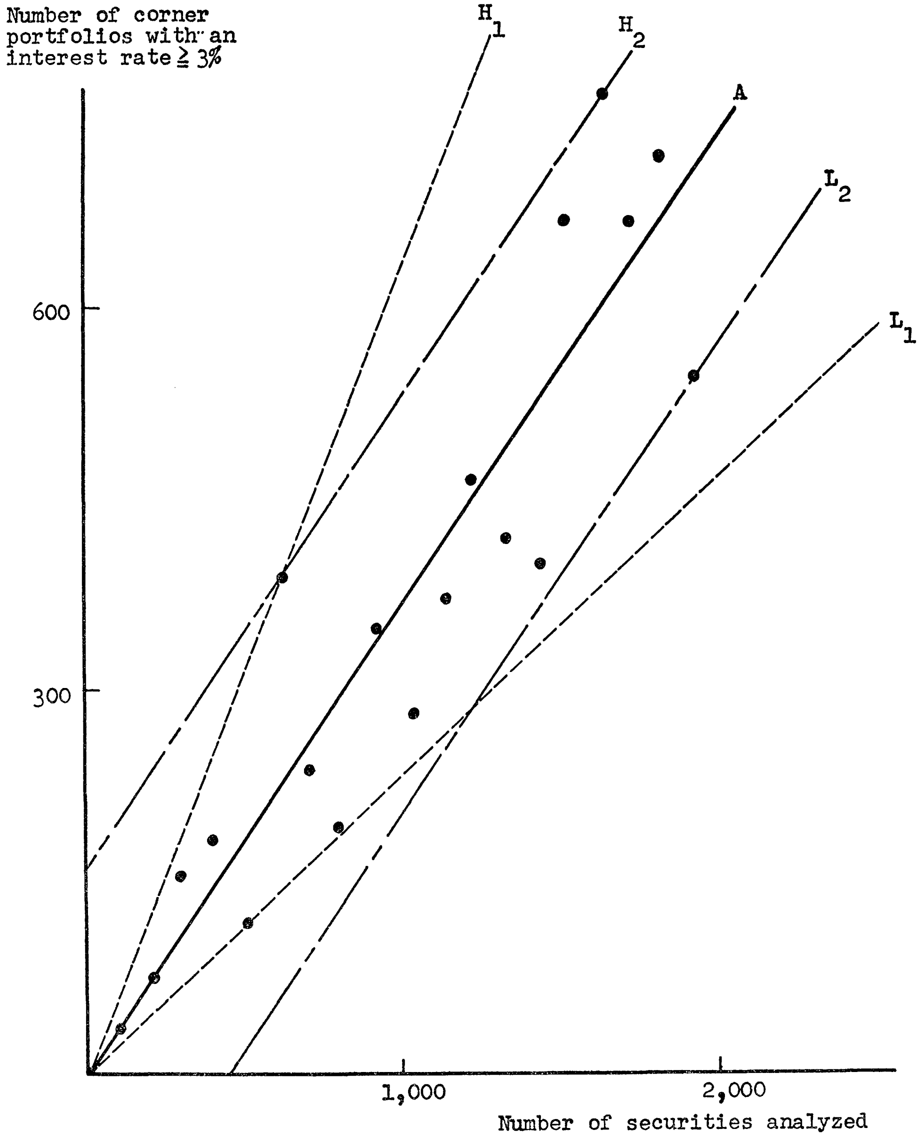

FIGURE 4

and the number of corner portfolios with interest rates greater than 3% (an approximation to the "lending rate"). Rather than perform a sophisticated analysis of these data, several lines have been used to bracket the results in various ways. These will be used subsequently as extreme cases, on the presumption that most practical cases will lie within these extremes (but with no presumption that these limits will never be exceeded). Curve *A* indicates the average relationship between the number of portfolios and the number of securities:

average $(N_p/N_s) = .37$. Curve H_1 indicates the highest such relationship: maximum $(N_p/N_s) = .63$; the line L_1 indicates the lowest: minimum $(N_p/N_s) = .24$. The other two curves, H_2 and L_2, indicate respectively the maximum deviation above (155) and below (173) the number of corner portfolios indicated by the average relationship $N_p = .37\ N_s$.

In Figure 5 the total time required for a portfolio analysis is related to the number of securities analyzed under various assumptions about the relationship

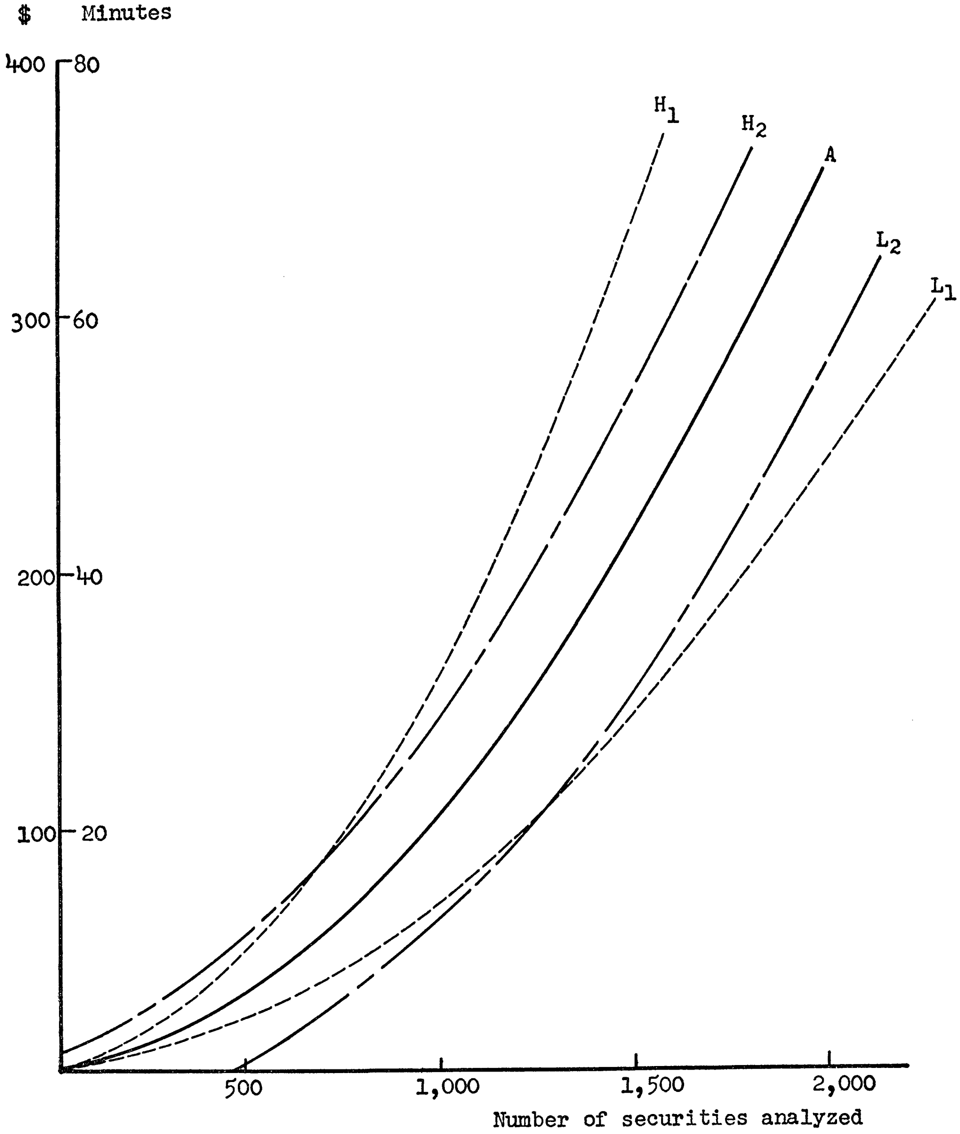

FIGURE 5

between the number of corner portfolios and the number of securities analyzed. Each of the curves shown in Figure 5 is based on the corresponding curve in Figure 4; for example, curve *A* in Figure 5 indicates the relationship between total time and number of securities analyzed on the assumption that the relationship between the number of corner portfolios and the number of securities is that shown by curve *A* in Figure 4. For convenience a second scale has been provided in Figure 5, showing the total cost of the analysis on the assumption that an IBM 709 computer can be obtained at a cost of $300 per hour.

8. The Value of Portfolio Analysis Based on the Diagonal Model

The assumptions of the diagonal model lie near one end of the spectrum of possible assumptions about the relationships among securities. The model's extreme simplicity enables the investigator to perform a portfolio analysis at a very small cost, as we have shown. However, it is entirely possible that this simplicity so restricts the security analyst in making his predictions that the value of the resulting portfolio analysis is also very small.

In order to estimate the ability of the diagonal model to summarize information concerning the performance of securities a simple test was performed. Twenty securities were chosen randomly from the New York Stock Exchange and their performance during the period 1940–1951 used to obtain two sets of

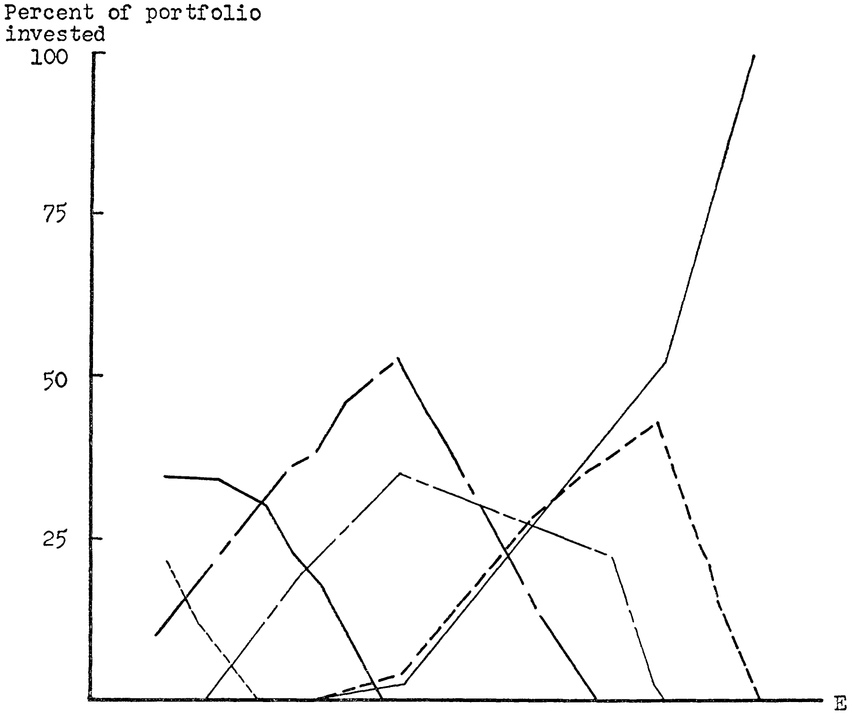

FIG. 6a. Composition of efficient portfolios derived from the analysis of the parameters of the diagonal model.

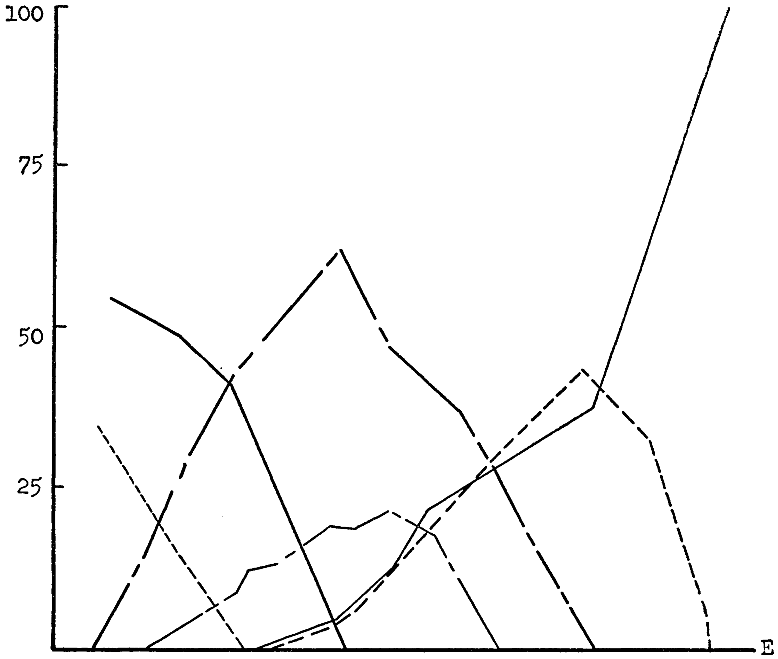

FIG. 6b. Composition of efficient portfolios derived from the analysis of historical data

data: (1) the actual mean returns, variances of returns and covariances of returns during the period and (2) the parameters of the diagonal model, estimated by regression techniques from the performance of the securities during the period. A portfolio analysis was then performed on each set of data. The results are summarized in Figures 6a and 6b. Each security which entered any of the efficient portfolios in significant amounts is represented by a particular type of line; the height of each line above any given value of E indicates the percentage of the efficient portfolio with that particular E composed of the security in question. The two figures thus indicate the compositions of all the efficient portfolios chosen from the analysis of the historical data (Figure 6b) and the compositions of all the portfolios chosen from the analysis of the parameters of the diagonal model (Figure 6a). The similarity of the two figures indicates that the 62 parameters of the diagonal model were able to capture a great deal of the information contained in the complete set of 230 historical relationships. An additional test, using a second set of 20 securities, gave similar results.

These results are, of course, far too fragmentary to be considered conclusive but they do suggest that the diagonal model may be able to represent the relationships among securities rather well and thus that the value of portfolio analyses based on the model will exceed their rather nominal cost. For these reasons it appears to be an excellent choice for the initial practical applications of the Markowitz technique.

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Triumph of the Optimists

101 YEARS OF GLOBAL INVESTMENT RETURNS

countries, over the whole of the 101-year period from 1900–2000. We also have century-long evidence on the small-firm and value/growth phenomena. We have put significant effort into compiling complete financial market histories, so that we can present consistent and comparable records for different countries. But *Triumph of the Optimists* is about much more than just data, since it has description and analysis at its core.

There is an obvious need for a reliable and truly international dataset for the investment industry as it continues relentlessly toward full market globalization. One of the many changes taking place in the investment business is the increasing demand for locally sourced research placed in a global context. Another innovation is the growing number of truly global mandates being given to fund managers. Globalization may be a cliché, but for portfolio managers it is fast becoming a reality. Access to a properly constituted and rigorously maintained international database is a sine qua non for the start of any investment process.

The period since spring 2000 has come as a shock to those who had become used to the bull market conditions of previous years. The bursting of the technology bubble, the rapid decline in economic growth rates, especially in the United States, and the advent of international terrorism raised questions about what we can expect for the future. We assert in this book that the single most important variable for making investment decisions is the equity risk premium, and we argue that high long-term returns on equities, relative to bonds, are unlikely to persist. Even after the setbacks of 2000–01, it is necessary to justify the relatively high rating of today's stock markets in terms of a historically low forward-looking equity risk premium. For the investment strategist this raises the most fundamental question of all: Do investors realize that returns are likely to revert to more normal levels, or do current valuations embody exaggerated expectations based on an imperfect understanding of history?

Good data is the key to understanding history. With this as our guiding principle, assembling the data for this book was a major task. For the United Kingdom, ABN AMRO supported us in compiling an authoritative record of UK equity market performance over the last 101 years. We did this because we were not satisfied with the data that previously existed, and there was anyway no comprehensive record of equity returns extending back to 1900. To construct our UK indexes, we devoted intensive efforts to financial archaeology. This involved transcribing original source data from dusty newspaper archives and ancient reference books into our database. A resulting benefit is that we have not simply assembled an index, but we also have the underlying stock-by-stock data, so we can now study the performance of segments of the market, such as industry sectors and market-capitalization bands. We also compiled a series of UK government bond indexes especially for this study.

For the other fifteen countries covered in this book, we have linked together the best quality indexes and returns data available from previous studies and other sources, a number of which are previously unpublished, and some of which are still work in progress. In addition to the United Kingdom, we cover two North American markets, the United States and Canada; ten other European markets, namely, Belgium, Denmark, France, Germany, Ireland, Italy, The Netherlands, Spain, Sweden and Switzerland; two Asia-Pacific markets, Australia

Chapter 3 Measuring long-term returns

Good measures of long-run returns should accurately reflect the outcome of an implementable investment strategy. The strategy should be one that could have been set up in advance, and followed in real life, and which is representative of the asset class and country in question. It is only too easy for researchers to fail to meet these criteria.

This chapter begins in section 3.1 by setting out the principles that need to be followed in constructing long-run return indexes. These provide a benchmark for assessing previous studies, and have been the guiding framework for this book. Given that our data goes back to the beginning of the last century and covers sixteen countries, we have not always been able to adhere to every principle, especially in the earliest years. Nevertheless, these standards have guided our choices, and we indicate where compromises have been necessary.

Next, in section 3.2 we take a closer look at equity index construction and at a bias that has afflicted some previous studies. When an index is compiled retrospectively, a crucial issue is how to avoid tilting its composition toward companies that, with hindsight, are known to have survived and/or to have been successful. In section 3.3, we review other issues that arise in index design, such as dividend reinvestment, index coverage, and index weighting.

In section 3.4 we consider how best to assemble a sample of international indexes. We show that reliance on data that is easy to acquire, such as indexes that start after the end of a war, tends to result in overstated performance. Both success bias and easy-data bias arise from a focus on assets that have survived or prospered over a particular period, and both can lead to overestimates of index returns and risk premia.

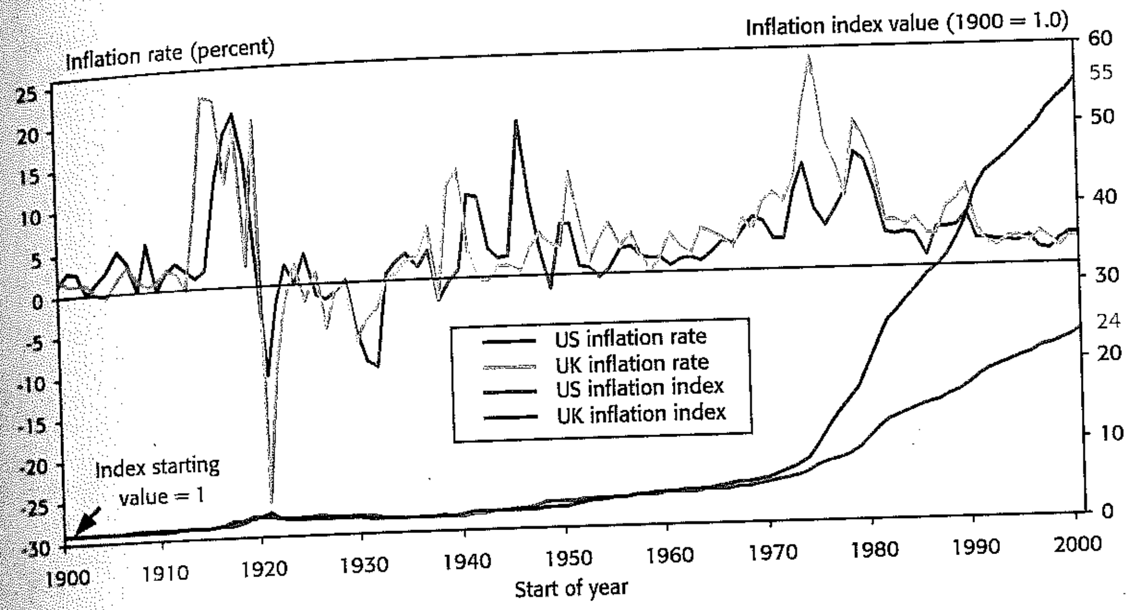
In section 3.5, we focus on the special problems that can arise when measuring inflation rates, as well as long-term returns on bonds, bills and currencies. We conclude in section 3.6 with a summary of the chapter.

3.1 Good indexes and bad

There are five guiding principles that underpin our measures of long-term performance. They are to avoid bias in index construction, to focus on total returns, to ensure the widest possible coverage within each market, to apply appropriate methods of weighting and averaging, and to maximize the extent to which comparisons can be made across national boundaries.

First, equity indexes should avoid bias. Good indexes follow an investment strategy that could be followed in real life. Apart from dealing costs, an investor should in principle have been able to replicate index performance. Indexes, especially when they are constructed retrospectively, must therefore be free of any look-ahead bias. They must be constructed solely from information that would have been available at the time of investment. Serious bias can arise if index constituents are tilted toward companies that subsequently survived or

Figure 5-2: US and UK annual inflation rates and cumulative inflation, 1900–2000



much higher than in the United States, peaking at 25 percent in 1975. The cumulative impact of these higher rates can be seen by comparing the two inflation indexes, which are plotted against the right-hand axis in Figure 5-2. The red line for the United Kingdom and the blue line for the United States are virtually coincident until the mid-1960s. From that point onward, the UK index rises to a value of fifty-five by end-2000, compared with twenty-four for the United States. From 1900–2000, UK consumer prices thus rose 55-fold, a factor of 2.3 times more than in the United States, with the difference almost entirely attributable to the last thirty-five years. Over the full 101-year period, the average annualized UK inflation rate was 4.1 percent per year, versus 3.2 percent for the United States.

5.2 Inflation around the world

While inflation was fairly similar in the United States and the United Kingdom, a number of other countries had quite different experiences. Table 5-1 provides international inflation rate comparisons across all sixteen countries covered in this book, showing the mean inflation rates from 1900–2000. Clearly, there were large differences between countries. At the same time, the standard deviations for each country show that there was also considerable variation in inflation rates over time. Taking the full 101-year period as a whole, there have been four high inflation rate countries, France, Germany, Italy, and Japan; two runner-ups, Belgium and Spain; and one low inflation country, Switzerland. The other countries fall in between, with inflation rates in the region of 3–4 percent per year. Note that the true 101-year means and standard deviations for inflation in Germany are much higher than shown in Table 5-1 since the statistics in the table omit the hyper-inflationary years of 1922–23.

and some of the studies span periods as brief as five years. These differences in research periods, methodologies, and definitions of "smallness" mean that the premia shown in Figure 9-5 are not directly comparable. In particular, it would be unwise to make inferences about the magnitudes or significance of any apparent size premium differences between countries.

In spite of this caveat, Figure 9-5 paints a very clear picture, namely, that the size premium was not restricted to the United States but was present in almost every country studied by the researchers. The sole exception was Korea, where a negative premium was reported, although this study used just five years of data. Furthermore, in most countries, researchers also looked at risk differences. They concluded, like Banz (1981), that the size premium could not be explained away by risk.

The pervasiveness and magnitude of the size effect, and the striking outperformance of smaller companies in most countries around the world, meant that the size effect rapidly became recognized as the premier stock market anomaly.

9.4 The reversal of the size premium

The "discovery" of the size effect in the United States by Banz (1981) and Reinganum (1981), and the publication and dissemination of their research, led to considerable interest in small-caps among investors in the United States. This spurred the launch of significant new small-cap investment vehicles led by Dimensional Fund Advisors, who raised several billion dollars within a couple of years of their 1981 launch. This honeymoon period lasted for approximately two years, until the end of 1983, and during this period, US small-caps continued to outperform. But subsequently, and over much of the period since, US small-caps have underperformed.

The UK experience was remarkably similar. When the HGSC was launched in 1987, its back-history showed that smaller companies had outperformed the UK market by 5.2 percent per year. This dramatic outperformance attracted substantial media attention, and there were over two hundred follow-up articles in the UK press. By the end of 1988, at least thirty open- and closed-end funds had been launched to exploit the perceived outperformance of small-caps, and numerous investment institutions developed a strategy of investing in smaller companies as a distinct asset class. Again, the honeymoon lasted just two years. In the decade that followed, smaller companies were to underperform by a large margin.

This reversal in the fortunes of US and UK small-cap stocks led us to write an article in 1999 entitled "Murphy's Law and Market Anomalies." Murphy's Law is often summarized as "bread always falls with the buttered side down." Figure 9-6 shows the performance record of US and UK small-caps at the time of our article, and shows why this appeared like a classic case of Murphy's Law. The left-hand side of Figure 9-6 shows the historical small- and micro-cap premia for the United States and the United Kingdom from the start date of the original research studies until the end of the post publication honeymoon period (i.e., 1926–

Our subsequent research has shown that the small-cap reversal extended beyond the United Kingdom and United States, and was a worldwide phenomenon. The line of investigation we followed here was to revisit all of the research studies that have been conducted into the size effect in different countries, and to estimate the size premium over the years since the research was published. These studies were discussed earlier in section 9.3 and their findings were summarized in Figure 9-5. We found that they showed evidence of a significant size premium in every country examined, with the sole exception of Korea, where the research covered just a five-year period. Most of these research studies were published in the 1980s.

To update these studies, we estimated the size premium in each country over the period since each study was published. For consistency, we again measured the size premium as the difference between the average monthly returns on the smallest and the largest stocks. For the United States, we use the CRSP NYSE Decile 10 and Decile 1 returns as our respective measures of small and large stock returns, as this most closely approximates Banz's (1981) earlier research, and gives results close to his over his earlier period. Similarly, for the United Kingdom we adopt the same definition as was used in Figure 9-5, namely, the difference between HGSC returns and overall UK equity returns.

For all other countries, we use the size-based indexes published by either Independence International Associates (IIA) or by FTSE International. IIA publish large- and small-cap indexes for a number of countries starting in 1975. They define small as the bottom 30 percent by capitalization of their universe, and large as the top 70 percent. FTSE publish a similar set of large and medium-small-cap indexes for a larger population of countries, but only from 1987, with some countries starting even later. FTSE define medium/small-cap as the bottom 25 percent by capitalization, and large-cap as the balancing 75 percent. For countries where we had a choice between both IIA and FTSE Indexes, we have used the IIA series since they provide a longer time series and generally have somewhat wider coverage.

The results of our research are shown in Figure 9-7. Countries are listed in alphabetical order, and for each country, the size premium reported by the original research studies and plotted earlier in Figure 9-5 in shown in green. Alongside this, the yellow bar shows the size premium calculated over the period since the original research was published, that is, over the period starting at the beginning of the year immediately following publication and ending at New Year 2001. No size-based indexes were available for Korea or Taiwan, so we omitted these countries. We have, however, included the four countries covered in this book, but which did not feature in Figure 9-5 due to the absence of any research study on the size premium. For these countries, we have omitted the "initial research" bars in Figure 9-7, while the "subsequent period" bars show the size premium over the period from 1990–2000.

It is clear from Figure 9-7 that there was a global reversal of the size effect in virtually every country, with the size premium not just disappearing but going into reverse. Researchers around the world universally fell victim to Murphy's Law, with the very effect they were documenting—and inventing explanations for—promptly reversing itself shortly after their studies were published. The only country experiencing a size premium, as opposed to a size discount, in the period subsequent to the original research was Switzerland. However, the Swiss size premium was statistically insignificant, and its magnitude was just 0.05 percent.

Table 12-2: Worldwide equity risk premia relative to long bond returns, 1900–2000

	Annual equity risk premium relative to long-term bonds						Ten year risk premium		
Country	Geometric mean	Arithmetic mean	Standard error	Standard deviation	Minimum premium	Maximum premium	Geometric mean	Arithmetic mean	Standard deviation
Australia	6.3	8.0	1.9	18.9	-30.6	66.3	6.3	6.4	4.6
Belgium	2.9	4.8	2.1	20.7	-35.1	76.6	3.0	3.2	5.1
Canada	4.5	6.0	1.8	17.8	-36.8	54.7	4.6	4.7	5.4
Denmark	2.0	3.3	1.7	16.9	-35.9	74.9	1.8	1.9	4.1
France	4.9	7.0	2.1	21.6	-32.7	83.7	4.9	5.1	6.8
Germany*	6.7	9.9	2.9	28.4	-38.6	117.6	8.2	8.5	9.1
Ireland	3.2	4.6	1.7	17.4	-37.0	73.3	3.0	3.2	4.8
Italy	5.0	8.4	3.0	30.0	-39.6	152.2	5.0	5.4	9.2
Japan	6.2	10.3	3.3	33.2	-43.3	193.0	6.7	7.2	11.5
The Netherlands	4.7	6.7	2.1	21.4	-43.9	107.6	4.3	4.5	6.5
South Africa	5.4	7.1	2.0	19.7	-29.2	70.9	6.2	6.3	5.0
Spain	2.3	4.2	2.0	20.3	-34.0	69.1	2.2	2.3	5.5
Sweden	5.2	7.4	2.2	22.1	-38.3	87.8	4.8	5.0	7.7
Switzerland†	2.7	4.2	1.9	17.9	-34.4	52.2	2.0	2.1	5.1
United Kingdom	4.4	5.6	1.7	16.7	-38.0	80.8	4.8	4.9	4.5
United States	5.0	7.0	2.0	20.0	-40.8	57.7	4.9	5.0	5.2
World	4.6	5.6	1.4	14.5	-31.2	37.4	4.6	4.7	4.8

* All statistics for Germany exclude 1922–23. † Premia for Switzerland are from 1911.

In this table, the first six columns give summary statistics for the annual premia, while the last three columns relate to rolling ten-year premia. The first column shows the geometric means that were plotted as bars in Figure 12-6. The fourth column shows the standard deviations. The 20.0 percent figure for the United States is close to the 19.6 percent standard deviation for the premia relative to bills shown earlier in Table 12-1. For some countries, however, the distribution of premia relative to bonds is narrower than relative to bills. For the United Kingdom, for example, the standard deviation is 16.7 percent, compared with 19.9 percent relative to bills. This is because, in the United Kingdom, there was a fairly high correlation between annual equity returns and long bond returns (0.56), while the correlation between equities and bills was lower (0.29). This propensity for good bond years to coincide with good equity years, and vice versa, has tended to lower the annual difference between equity and bond returns in the United Kingdom. This was particularly marked in the best and worst years on record for UK equities, namely, 1975 and 1974 respectively.

12.5 Summary

In this chapter, we have used 101 years of stock market history for sixteen different countries and for the world index to take a fresh look at the equity risk premium. In the past, the historical evidence for the US market, and to a lesser extent for the United Kingdom, has heavily influenced views about the magnitude of the risk premium. For the United States, the most widely cited source is Ibbotson Associates (2000), who estimate a geometric risk premium of

The chapter addresses four questions: Which historical equity risk premium should one use as the starting point? Why has it typically been so high? What is a good forward-looking predictor for the future? How can one use variables such as the dividend yield to improve forecasts of the risk premium?

We stress the central role in finance of the equity premium. The historical premium is often summarized in the form of an annualized rate of return. This is a geometric mean. It provides information on past performance. For the future, what is required is the arithmetic mean of the distribution of equity premia, which is larger than the geometric mean. For markets that have been particularly volatile, the arithmetic mean of past equity premia may exceed the geometric mean premium by several percentage points. We adjust the arithmetic mean for (i) the differences between the variability of the stock market over the last 101 years, and the variability that we might anticipate today, and (ii) the impact of unanticipated cash flows and of declines in the required risk premium. The result is a forward-looking, *geometric* mean risk premium for the United States, United Kingdom and world of around 2½ to 4 percent and an *arithmetic* mean risk premium for US, UK, and world equities that falls within a range from a little below 4 to a little above 5 percent.

These equity risk premia are lower than those cited in surveys of finance academics. They are also lower than frequently quoted historical averages, such as those from Ibbotson Associates, which cover a somewhat briefer interval. We show that the historical risk premium, even if it embraces countries that have been less successful than the United States, is supported by two factors. Over the second half of the last century, equity cash flows almost certainly exceeded expectations, and the required rate of return doubtless fell as investment risk declined and the scope for diversification increased. Stock markets rose, in both the United States and other countries, for reasons that are unlikely to be repeated. Even after the setbacks of 2001, the prospective risk premium is markedly lower than the historical risk premium.

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Chapter 7

Company Size and Return

One of the most remarkable discoveries of modern finance is the finding of a relationship between company size and return.[1] Historically on average, small companies have higher returns than those of large ones. Earlier chapters of this book document this phenomenon for the smallest stocks on the New York Stock Exchange, or NYSE. The relationship between company size and return cuts across the entire size spectrum; it is not restricted to the smallest stocks. This chapter examines returns across the entire range of company size.

Construction of the Size Decile Portfolios

The portfolios used in this chapter are those created by the Center for Research in Security Prices, or CRSP, at the University of Chicago's Booth School of Business. CRSP has refined the methodology of creating size-based portfolios and has applied this methodology to the entire universe of NYSE/AMEX/NASDAQ-listed securities going back to 1926.

The NYSE universe excludes closed-end mutual funds, preferred stocks, real estate investment trusts, foreign stocks, American Depository Receipts, unit investment trusts, and Americus Trusts. All companies on the NYSE are ranked by the combined market capitalization of all their eligible equity securities. The companies are then split into 10 equally populated groups or deciles. Eligible companies traded on the NYSE, the NYSE MKT LLC (formerly known as the American Stock Exchange, or AMEX), and the NASDAQ Stock Market (formerly the NASDAQ National Market) are then assigned to the appropriate deciles according to their capitalization in relation to the NYSE breakpoints. The portfolios are rebalanced using closing prices for the last trading day of March, June, September, and December. Securities added during the quarter are assigned to the appropriate portfolio when two consecutive month-end prices are available. If the final NYSE price of a security that becomes delisted is a month-end price, then that month's return is included in the quarterly return of the portfolio. When a month-end NYSE price is missing, the month-end value is derived from merger terms, quotations on regional exchanges, and other sources. If a month-end value is not available, the last available daily price is used.

In October 2008, NYSE Euronext acquired the American Stock Exchange and rebranded the index as NYSE Amex. Later, in May 2012, it was renamed NYSE MKT LLC. For the sake of continuity, we refer to this index as AMEX, its historical name.

Base security returns are monthly holding period returns. All distributions are added to the month-end prices. Appropriate adjustments are made to prices to account for stock splits and dividends. The return on a portfolio for one month is calculated as the value weighted average of the returns for the individual stocks in the portfolio. Annual portfolio returns are calculated by compounding the monthly portfolio returns.

Aspects of the Company Size Effect

The company size phenomenon is remarkable in several ways. First, the greater risk of small-cap does not, in the context of the capital asset pricing model, fully account for their higher returns over the long term. In the CAPM only systematic, or beta risk, is rewarded; small-cap stock returns have exceeded those implied by their betas.

Second, the calendar annual return differences between small- and large-cap companies are serially correlated. This suggests that past annual returns may be of some value in predicting future annual returns. Such serial correlation, or autocorrelation, is practically unknown in the market for large-cap stocks and in most other equity markets but is evident in the size premium series.

Table 7-8: Size-Decile Portfolios of the NYSE/AMEX/NASDAQ Returns in Excess of Decile 1 (%)

Decile	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec	[illegible]
2	0.86	0.52	-0.04	-0.17	0.09	-0.09	-0.11	0.18	0.02	-0.30	0.12	0.39	[illegible]
	68	60	42	34	45	44	38	47	46	42	51	49	
3	1.16	0.39	-0.04	-0.03	-0.12	-0.14	-0.13	0.36	-0.12	-0.33	0.45	0.34	[illegible]
	65	58	42	33	40	38	44	54	42	38	49	52	
4	1.38	0.60	0.06	-0.20	0.09	-0.07	-0.19	0.28	0.03	-0.77	0.31	0.55	[illegible]
	66	56	43	37	43	43	38	52	45	32	47	52	
5	2.11	0.62	-0.04	-0.15	-0.15	0.05	-0.14	0.28	0.02	-0.71	0.29	0.40	[illegible]
	66	55	43	38	39	43	44	51	46	37	51	50	
6	2.39	0.46	-0.05	-0.04	0.30	-0.05	-0.24	0.47	0.11	-1.11	0.18	0.33	[illegible]
	66	57	41	37	43	41	44	51	47	38	46	48	
7	2.99	0.61	-0.05	-0.09	0.16	-0.23	-0.17	0.21	0.23	-0.99	0.12	0.15	[illegible]
	67	56	47	37	38	37	39	44	49	32	47	44	
8	4.06	0.64	-0.05	-0.34	0.36	-0.32	-0.01	0.13	0.05	-0.98	0.12	-0.05	[illegible]
	66	51	46	34	37	41	41	41	47	37	41	41	
9	5.21	0.82	-0.24	-0.15	0.26	-0.30	-0.08	0.05	-0.05	-1.14	0.00	-0.64	[illegible]
	65	48	41	34	38	37	38	46	43	36	39	39	
10	8.53	0.84	-0.05	-0.03	0.57	-0.47	0.45	-0.19	0.53	-1.38	-0.59	-1.33	[illegible]
	81	45	46	36	38	38	41	34	44	32	33	35	

First row: Average excess return in percent Second row: Number of times excess return was positive (in 89 years)

Data from 1926–2014. Source: Morningstar and CRSP. Calculated (or Derived) based on data from CRSP US Stock Database and CRSP US Indices Database ©2015 Center for Research in Security Prices (CRSP®), The University of Chicago Booth School of Business. Used with permission.

Table 7-8 shows the returns of capitalization deciles 2-10 in excess of the return on decile 1; the excess returns are segregated into months. For each decile and for each month, the exhibit shows both the average excess return and the number of times the excess return was positive. These two statistics measure the seasonality of the excess return in different ways—the average excess return illustrates the size of the seasonality effect, while the number of positive excess returns shows its reliability.

Virtually all of the small-cap effect occurs in January, as the excess outcomes for small-company stocks are mostly negative in the other months of the year. Excess returns in January relate to size in a precisely rank-ordered fashion, and the January effect seems to pervade all size groups. Yet, simply demonstrating that the size premium is largely produced by the January effect does not refute the existence of such a premium.

Small-Cap Returns Are Unpredictable

Because investors cannot predict when small-cap returns will be higher than large-cap returns, it has been argued that they do not expect higher rates of return for small stocks. As was illustrated earlier in this chapter, even over periods of many years, investors in small stocks do not always earn returns that are higher than those of investors in large stocks. By simple definition, one cannot expect risky companies to always outperform less-risky companies; otherwise they would not be risky. Over the long-term, however, investors do expect small stocks to outperform large stocks.

The unpredictability of small-cap returns has given rise to another argument against the existence of a size premium: that markets have changed so that the size premium no longer exists. As evidence, one might observe the last 20 years of market data to see that the performance of large-cap stocks was basically equal to that of small-cap stocks. In fact, large-cap stocks have outperformed small-cap stocks in five of the last 10 years.

Equity Risk Premiums (ERP): Determinants, Estimation and Implications – The 2015 Edition

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coefficients to demand these premiums.[30] In the years since, there have been many attempts to provide explanations for this puzzle:

1. Statistical artifact: The historical risk premium obtained by looking at U.S. data is biased upwards because of a survivor bias (induced by picking one of the most successful equity markets of the twentieth century). The true premium, it is argued, is much lower. This view is backed up by a study of large equity markets over the twentieth century, which concluded that the historical risk premium is closer to 4% than the 6% cited by Mehra and Prescott.[31] However, even the lower risk premium would still be too high, if we assumed reasonable risk aversion coefficients.
2. Disaster Insurance: A variation on the statistical artifact theme, albeit with a theoretical twist, is that the observed volatility in an equity market does not fully capture the potential volatility, which could include rare but disastrous events that reduce consumption and wealth substantially. Reitz, referenced earlier, argues that investments that have dividends that are proportional to consumption (as stocks do) should earn much higher returns than riskless investments to compensate for the possibility of a disastrous drop in consumption. Prescott and Mehra (1988) counter than the required drops in consumption would have to be of such a large magnitude to explain observed premiums that this solution is not viable. [32] Berkman, Jacobsen and Lee (2011) use data from 447 international political crises between 1918 and 2006 to create a crisis index and note that increases in the index increase equity risk premiums, with disproportionately large impacts on the industries most exposed to the crisis.[33]
3. Taxes: One possible explanation for the high equity returns in the period after the Second World War is the declining marginal tax rate during that period. McGrattan and Prescott (2001), for instance, provide a hypothetical illustration where a drop in the tax rate on dividends from 50% to 0% over 40 years would cause equity prices to rise about 1.8% more than the growth rate in GDP; adding the dividend yield to this expected price appreciation generates returns similar to

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[30] Mehra, Rajnish, and Edward C.Prescott, 1985, *The Equity Premium: A Puzzle*, Journal of Monetary Economics, v15, 145–61. Using a constant relative risk aversion utility function and plausible risk aversion coefficients, they demonstrate the equity risk premiums should be much lower (less than 1%).

[31] Dimson, E., P. March and M. Staunton, 2002, *Triumph of the Optimists*, Princeton University Press.

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The Equity Risk Premium in 2016

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ABSTRACT

We analyze the history of the equity risk premium from surveys of U.S. Chief Financial Officers (CFOs) conducted every quarter from June 2000 to June 2016. The risk premium is the expected 10-year S&P 500 return relative to a 10-year U.S. Treasury bond yield. The average risk premium in 2016, 4.02%, is slightly higher than the average observed over the past 16 years. We also provide results on the risk premium disagreement among respondents as well as asymmetry or skewness of risk premium estimates. We also link our risk premium results to survey-based measures of the weighted average cost of capital and investment hurdle rates. The hurdle rates are significantly higher than the cost of capital implied by the market risk premium estimates.

JEL Classification: *G11, G31, G12, G14*

Keywords: *Cost of capital, financial crisis, equity premium, WACC, hurdle rate, long-term market returns, stock return forecasts, long-term equity returns, expected excess returns, disagreement, individual uncertainty, skewness, asymmetry, survey methods, TIPs, VIX, credit spreads*

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Introduction

We analyze the results of the most recent survey of Chief Financial Officers (CFOs) conducted by Duke University and *CFO* Magazine. The survey closed on June 2, 2016 and measures expectations beginning in the second quarter of 2016. In particular, we poll CFOs about their long-term expected return on the S&P 500. Given the current U.S. 10-year Treasury bond yield, we provide estimates of the equity risk premium and show how the premium changes through time. We also provide information on the disagreement over the risk premium as well as average confidence intervals. Finally, we link the equity risk premium to measures used to evaluate firm's investments: the weighted average cost of capital (WACC) and the investment hurdle rate.

1. Method

2.1 Design

The quarterly survey of CFOs was initiated in the third quarter of 1996.[1] Every quarter, Duke University polls financial officers with a short survey on important topical issues (Graham and Harvey, 2009). The usual response rate for the quarterly survey is 5%-8%. Starting in June of 2000, a question on expected stock market returns was added to the survey. Fig. 1 summarizes the results from the risk premium question. While the survey asks for both the one-year and ten-year expected returns, we focus on the ten-year expected returns herein, as a proxy for the market risk premium.

The executives have the job title of CFO, Chief Accounting Officer, Treasurer, Assistant Treasurer, Controller, Assistant Controller, or Vice President (VP), Senior VP or Executive VP of Finance. Given that the majority of survey respondents hold the CFO title, for simplicity we refer to the entire group as CFOs.

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[1] The surveys from 1996Q3-2004Q2 were partnered with a national organization of financial executives. The 2004Q3 and 2004Q4 surveys were solely Duke University surveys, which used Duke mailing lists (previous survey respondents who volunteered their email addresses) and purchased email lists. The surveys from 2005Q1 to present are partnered with *CFO magazine*. The sample includes both the Duke mailing lists and the *CFO* subscribers that meet the criteria for policy-making positions.

2.2 Delivery and response

In the early years of the survey, the surveys were faxed to executives. The delivery mechanism was changed to the Internet starting with the December 4, 2001 survey. Respondents are given four business days to fill out the survey, and then a reminder is sent allowing another four days. Usually, two-thirds of the surveys are returned within two business days.

The response rate of 5-8% could potentially lead to a non-response bias. There are six reasons why we are not overly concerned with the response rate. First, we do not manage our email list. If we deleted the email addresses that had not responded to the survey in the past 12 quarters, our response rate would be in the 15-20% range – which is a good response rate. Second, Graham and Harvey (2001) conduct a standard test for non-response biases (which involves comparing the results of those that fill out the survey early to the ones that fill it out late) and find no evidence of bias. Third, Brav, Graham, Harvey and Michaely (2005) conduct a captured sample survey at a national conference in addition to an Internet survey. The captured survey responses (to which over two-thirds participated) are qualitatively identical to those for the Internet survey (to which 8% responded), indicating that non-response bias does not significantly affect their results. Fourth, Brav et al. contrast survey responses to archival data from Compustat and find archival evidence for the universe of Compustat firms that is consistent with the responses from the survey sample. Fifth, Campello, Graham, and Harvey (2011) show that the December 2008 response sample is fairly representative of the firms included in the commonly used Compustat database. Sixth, Graham, Harvey, Popadak and Rajgopal (2016) update the non-response bias test in a survey of 1,900 CFOs and find no evidence of non-response bias.

2.3 Data integrity

In each quarter, implement a series of rules to ensure the integrity of the data. We have, on average, 355 responses each quarter. However, in recent years the average number of responses has exceeded 400. There are a total of 23,086 survey observations. There are six key pieces of data: 1) the 10-year forecast (LT); 2) lower 10% of 10-year forecast (LLT); and 3) upper 10% of the

10-year forecast (ULT). We collect the analogous information for the one-year S&P 500 forecasts too (ST). This paper focuses on the 10-year forecasts but the short-term forecasts factor into our data filters.

Our exclusion rules are the following:

1. Delete all missing forecasts, LT, ST
2. Delete all negative LT forecasts (not ST forecasts)
3. Delete all observations that failed to use percentages (forecasts<1.0 for both ST and LT)
4. Delete observations where they failed to annualize, i.e. delete if LT>30% (does not apply to ST)
5. Delete is ST>100%.
6. Delete if lower intervals inconsistent, i.e. LST>=ST or LLT>=LT.
7. Delete if upper intervals inconsistent, i.e. UST<=ST or ULT<=LT.
8. Delete if ST-LST and UST-ST both equal 1 (we call this a lazy answer)
9. Delete if LT-LLT and ULT-LT both equal 1 (again, a lazy answer)

2.4 The 2016 results

The expected market return questions are a subset of a larger set of questions in the quarterly survey of CFOs. The survey usually contains between eight and ten questions. Some of the questions are repeated every quarter and some change through time depending on economic conditions. The historical surveys can be accessed at http://www.cfosurvey.org. Appendix 1 shows the risk premium question in the most recent survey.

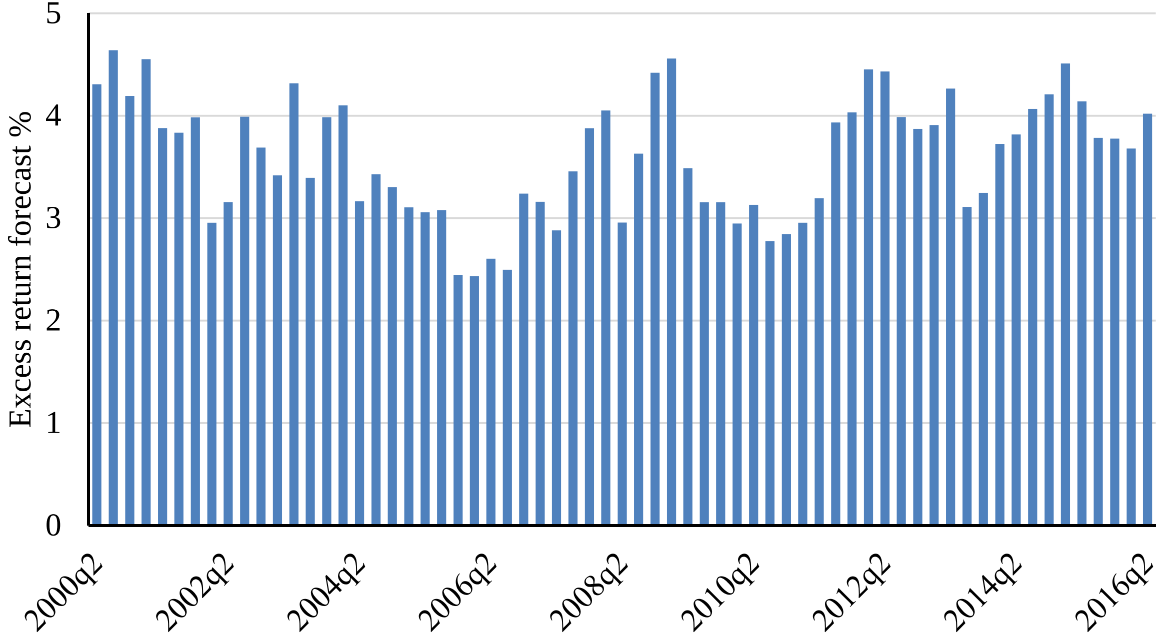
While the survey is anonymous, we collect demographic information on seven firm characteristics, including industry, sales revenue, number of employees, headquarters location, ownership (public or private), and proportion of foreign sales.

During the past 16 years, we have collected over 23,000 responses to the survey. Panel A of Table 1 presents the date that the survey window opened, the number of responses for each survey, the 10-year Treasury bond rate, as well as the average and median expected excess returns. There is relatively little time variation in the risk premium. This is confirmed in Fig. 1a, which displays the historical risk premiums contained in Table 1. The current premium, 4.02%, is close to the historical average. The June 2016 survey shows that the expected annual S&P 500 return is 5.83% (=4.02%+1.81%) which is below the overall average of 7.19%. The total return forecasts are presented in Fig. 1b.[2]

Panel B of Table 1 presents some summary statistics that pool all responses through the 16 year history of the survey. The overall average ten-year risk premium return is 3.58%.[3] The standard deviation of the individual responses is 2.91% (see Panel B). The standard deviation of the quarterly risk premium estimates is 0.58% (not reported in the Table).

Figure 1a

10-year forecasted S&P 500 (mean) annual returns over and above the 10-year Treasury bond yield



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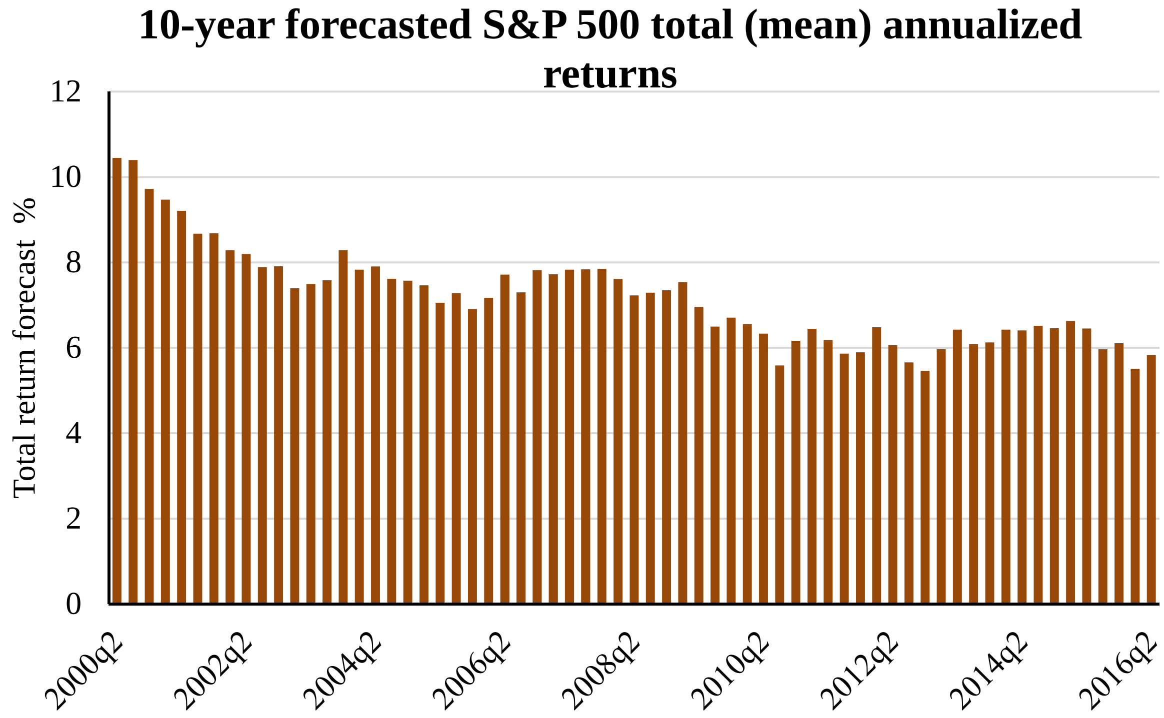
[2] See, for example, Ghysels (1998), Welch (2000, 2001, 2009), Ghysels (1998), Fraser (2001), Harris and Marston (2001), Pástor and Stambaugh (2001), Fama and French (2002), Goyal and Welch (2003), Graham and Harvey (2003), Ang and Bekaert (2005), Fernandez (2004, 2006, 2009) for studies of the risk premium.

[3] Using the Ibbotson Associates data from January 1926 through July 2010, the arithmetic (geometric) average return on the S&P 500 over and above the 30-day U.S. Treasury bill is 7.75% (5.80%). Using data from April 1953-July 2010, the arithmetic (geometric) risk premium is 6.27% (5.12%). The risk premium over the 10 year bond should be reduced by 212 basis points for the arithmetic premium and 174 basis points for the geometric premium. Fama and French (2002) study the risk premium on the S&P 500 from 1872-2000 using fundamental data. They argue that the ex ante risk premia is between 2.55% and 4.32% for 1951-2000 period. Ibbotson and Chen (2001) estimate a long-term risk premium between 4 and 6%. Also see Siegel (1999), Asness (2000), Heaton and Lucas (2000) and Jagannathan, McGratten and Scherbina (2001). A recent treatment is Sharpe and Suarez (2013).

Figure 1b



10-year forecasted S&P 500 total (mean) annualized returns

The cross-sectional standard deviation across the individual CFO forecasts in a quarter is a measure of the disagreement or dispersion of the participants in each survey. Dispersion sharply increased during the global financial crisis. The average disagreement in 2005 was 2.39%. Disagreement increased in 2006 to 2.64%. As the crisis began in 2007, disagreement increased to 2.98 by March 2008. The peak disagreement was recorded in February 2009 (4.13%). The most recent observation is 3.24%.

We also report information on the average of the CFOs' assessments of the one in ten chance that the market will exceed or fall below a certain level. In the most recent survey, the worst case total return is +0.39% which is lower than the historic average of 1.52%. The best-case return is 9.71% which is also slightly lower than the average of 10.97%.

With information on the 10% tails, we construct a probability distribution for each respondent. We use Davidson and Cooper's (1976) method to recover each respondent's probability distribution:

$$\text{Variance} = ([x(0.90)-x(0.10)]/2.65)^2$$

where $x(0.90)$ and $x(0.10)$ represent the 90th and 10th percentiles of the respondent's distribution, ULT and LLT. Keefer and Bodily (1983) show that this simple approximation is the preferred method of estimating the variance of a probability distribution of random variables, given information about the 10th and 90th percentiles. Like disagreement, the average of individual volatilities peaked in February 2009 at 4.29%. The current level, 3.52%, is very close to the overall average.

There is also a natural measure of asymmetry in each respondent's response. We look at the difference between each individual's 90% tail and the mean forecast and the mean minus the 10% tail. Hence, if the respondent's forecast of the excess return is 6% and the tails are -8% and +11%, then the distribution is negatively skewed with a value of -9% (=5%-14%). As with the usual measure of skewness, we cube this quantity and standardize by dividing by the cube of the individual standard deviation. In every quarter's survey, there is on average negative skewness in the individual forecasts. The average asymmetry -0.63 which is slightly lower than the average of -0.47.

Table 1

Summary statistics based on the responses from the
65 CFO Outlook Surveys from June 2000 to June 2016 (Maximums in red, minimums in green)

A. By quarter

#	Survey date	Survey quarter	Number of survey responses	10-year bond yield	Total market return forecast	Average risk premium	Median risk premium	Disagreement (standard deviation of risk premium estimates)	Average of individual standard deviations	Average of individuals' worst 10% market return scenario	Average of individuals' best 10% market return scenario	Skewness of risk premium estimates	Average of individuals' asymmetry	% who forecast negative excess return
1	6/6/2000	2000Q2	209	**6.14**	**10.45**	4.31	3.86	3.22				0.95		9.09
2	9/7/2000	2000Q3	188	5.76	10.40	**4.64**	4.24	3.03				0.83		4.79
3	12/4/2000	2000Q4	243	5.53	9.72	4.19	4.47	2.52				0.53		4.12
4	3/12/2001	2001Q1	140	4.92	9.47	4.55	4.58	2.91				0.78		3.57
5	6/7/2001	2001Q2	208	5.33	9.21	3.88	3.67	2.64				0.58		5.77
6	9/10/2001	2001Q3	199	4.84	8.67	3.83	3.16	2.53				**0.13**		3.52
7	12/4/2001	2001Q4	279	4.70	8.68	3.98	3.30	2.43				0.61		2.15
8	3/11/2002	2002Q1	233	5.33	8.29	2.96	2.67	2.43	3.28	**3.68**	12.42	1.06	-0.28	11.16
9	6/4/2002	2002Q2	316	5.04	8.20	3.16	2.96	2.61	3.50	3.00	12.28	1.86	-0.39	10.44
10	9/16/2002	2002Q3	361	3.90	7.89	3.99	4.10	2.31	3.39	3.05	12.03	0.86	-0.25	2.77
11	12/2/2002	2002Q4	285	4.22	7.91	3.69	3.78	2.56	3.23	3.32	11.87	1.24	-0.28	4.91
12	3/19/2003	2003Q1	184	3.98	7.40	3.42	3.02	2.37	3.59	1.95	11.47	0.83	-0.62	4.35
13	6/16/2003	2003Q2	366	3.18	7.50	4.32	**4.82**	2.34	3.74	2.16	12.07	0.90	-0.33	3.28
14	9/18/2003	2003Q3	167	4.19	7.58	3.39	3.81	**2.07**	**2.83**	3.31	10.83	0.35	-0.43	6.59
15	12/10/2003	2003Q4	220	4.30	8.29	3.98	3.70	2.66	3.29	3.40	12.10	1.74	-0.45	2.27
16	3/24/2004	2004Q1	206	3.73	7.83	4.10	4.27	2.37	3.46	2.85	12.02	0.50	-0.29	3.88
17	6/16/2004	2004Q2	177	4.74	7.90	3.16	3.26	2.61	3.10	3.14	11.34	2.14	-0.40	6.21
18	9/10/2004	2004Q3	179	4.19	7.62	3.43	3.31	2.92	3.27	2.61	11.29	2.02	-0.52	8.94
19	12/3/2004	2004Q4	287	4.27	7.57	3.30	3.23	2.66	3.05	3.10	11.17	1.89	-0.37	5.92
20	2/28/2005	2005Q1	272	4.36	7.46	3.10	3.39	2.52	3.06	3.13	11.23	1.29	-0.33	6.62
21	5/31/2005	2005Q2	316	4.00	7.06	3.06	3.00	2.22	3.22	2.39	10.93	0.46	-0.26	6.65
22	8/29/2005	2005Q3	321	4.20	7.28	3.08	2.80	2.61	3.36	2.15	11.06	2.42	-0.52	7.48
23	11/21/2005	2005Q4	338	4.46	6.91	2.45	2.54	2.20	3.48	2.23	11.44	0.41	**-0.23**	9.76
24	3/6/2006	2006Q1	276	4.74	7.17	**2.43**	2.26	2.40	3.44	2.07	11.18	1.02	-0.37	8.70
25	6/1/2006	2006Q2	494	5.11	7.72	2.61	2.89	2.74	3.29	3.00	11.70	1.84	-0.24	**18.02**
26	9/11/2006	2006Q3	460	4.80	7.30	2.50	2.20	2.49	3.32	2.53	11.33	1.32	-0.33	7.83
27	11/21/2006	2006Q4	386	4.58	7.82	3.24	3.42	2.93	3.36	2.94	11.82	1.91	-0.30	6.99
28	3/1/2007	2007Q1	380	4.56	7.72	3.16	3.44	2.39	3.38	2.73	11.67	1.80	-0.39	5.53
29	6/1/2007	2007Q2	419	4.95	7.83	2.88	3.05	2.14	3.21	3.08	11.58	0.56	-0.37	3.58
30	9/7/2007	2007Q3	479	4.38	7.84	3.46	3.62	2.82	3.12	3.33	11.59	1.80	-0.34	5.22
31	11/30/2007	2007Q4	458	3.97	7.85	3.88	4.03	2.75	3.31	2.93	11.70	1.38	-0.32	3.28
32	3/7/2008	2008Q1	381	3.56	7.61	4.05	4.44	2.99	3.21	3.08	11.58	2.23	-0.30	3.94
33	6/13/2008	2008Q2	384	4.27	7.23	2.96	2.73	2.60	3.32	2.44	11.24	1.50	-0.41	9.38
34	9/5/2008	2008Q3	432	3.66	7.29	3.63	3.34	2.79	3.31	2.30	11.06	1.71	-0.42	4.63
35	11/28/2008	2008Q4	534	2.93	7.35	4.42	4.07	3.19	3.73	1.77	11.64	1.94	-0.37	2.81
36	2/26/2009	2009Q1	443	2.98	7.54	4.56	4.02	**4.13**	**4.29**	1.18	**12.54**	1.80	-0.47	5.87
37	5/29/2009	2009Q2	427	3.47	6.96	3.49	3.53	3.12	3.73	1.37	11.26	1.79	-0.42	6.56
38	9/11/2009	2009Q3	536	3.34	6.50	3.16	2.66	2.88	3.87	0.62	10.86	1.82	-0.46	10.82
39	12/11/2009	2009Q4	457	3.55	6.71	3.16	2.45	3.56	3.86	0.64	10.88	2.38	-0.52	9.85
40	2/26/2010	2010Q1	478	3.61	6.56	2.95	2.39	3.28	3.96	0.39	10.86	2.31	-0.68	9.41
41	6/4/2010	2010Q2	444	3.20	6.33	3.13	2.80	3.08	3.90	0.33	10.64	**2.61**	-0.64	9.91
42	9/10/2010	2010Q3	451	2.81	5.59	2.78	**2.19**	2.53	4.21	**-1.16**	9.99	0.77	-0.67	8.65
43	12/10/2010	2010Q4	402	3.32	6.17	2.85	2.68	2.62	3.91	0.26	10.63	1.89	-0.55	10.70
44	3/4/2011	2011Q1	429	3.49	6.45	2.96	2.51	2.92	4.16	-0.27	10.76	2.44	**-0.70**	8.16
45	6/3/2011	2011Q2	406	2.99	6.18	3.19	3.01	2.90	3.90	0.12	10.45	2.09	-0.68	5.17
46	9/9/2011	2011Q3	397	1.93	5.86	3.93	3.07	3.11	3.79	0.04	10.09	2.41	-0.54	**2.02**
47	12/16/2011	2011Q4	439	1.86	5.89	4.03	3.14	2.98	4.07	-0.11	10.68	1.91	-0.36	3.42
48	3/1/2012	2012Q1	406	2.03	6.48	4.45	3.97	2.97	4.07	0.30	11.08	2.25	-0.59	2.71
49	5/30/2012	2012Q2	338	1.63	6.06	4.43	4.37	2.96	3.94	0.00	10.42	1.96	-0.59	2.37
50	9/7/2012	2012Q3	675	1.67	5.66	3.99	3.33	3.00	3.66	-0.01	9.67	2.04	-0.58	2.37
51	12/6/2012	2012Q4	325	**1.59**	**5.46**	3.87	3.41	2.59	3.69	-0.49	**9.25**	1.42	-0.62	3.08
52	3/8/2013	2013Q1	418	2.06	5.97	3.91	3.94	2.73	3.84	-0.14	10.02	2.01	-0.64	4.55
53	5/31/2013	2013Q2	300	2.16	6.43	4.27	3.84	2.91	4.02	0.10	10.76	1.63	-0.67	2.67
54	9/5/2013	2013Q3	404	2.98	6.09	3.11	3.02	2.73	3.41	0.75	9.77	1.71	-0.53	6.68
55	12/5/2013	2013Q4	320	2.88	6.13	3.25	3.12	2.95	3.81	0.18	10.26	1.69	-0.50	7.19
56	3/4/2014	2014Q1	291	2.70	6.43	3.73	3.30	2.63	3.32	1.35	10.13	0.64	-0.69	5.15
57	6/5/2014	2014Q2	325	2.59	6.41	3.82	3.41	3.23	3.76	0.50	10.46	1.89	-0.64	7.08
58	9/4/2014	2014Q3	316	2.45	6.52	4.07	3.55	3.33	3.69	0.90	10.68	2.56	-0.60	3.16
59	12/4/2014	2014Q4	398	2.25	6.46	4.21	4.50	2.51	3.79	0.46	10.51	1.22	-0.59	2.26
60	3/3/2015	2015Q1	414	2.12	6.63	4.51	3.88	3.50	3.72	0.81	10.68	1.92	-0.55	5.80
61	6/4/2015	2015Q2	399	2.31	6.45	4.14	3.69	3.03	3.96	0.20	10.68	1.93	-0.72	4.26
62	9/3/2015	2015Q3	376	2.18	5.96	3.78	2.82	3.17	3.48	0.28	9.49	2.72	-0.72	3.99
63	12/3/2015	2015Q4	347	2.33	6.11	3.78	2.67	3.58	3.55	0.54	9.94	1.92	-0.52	9.22
64	3/3/2016	2016Q1	476	1.83	5.51	3.68	3.17	2.55	3.12	1.04	9.29	0.99	-0.34	3.15
65	6/2/2016	2016Q2	472	1.81	5.83	4.02	3.19	3.24	3.52	0.39	9.71	2.14	-0.63	2.54
	Average of quarters		355	**3.58**	**7.19**	**3.61**	**3.37**	**2.80**	**3.57**	**1.52**	**10.97**	**1.54**	**-0.47**	**5.89**
	Standard deviation			1.18	1.13	0.58	0.63	0.38	0.34	1.33	0.80	0.66	0.15	3.05

B. By individual responses

Survey for											
All dates	23,086	**3.41**	**6.99**	**3.58**	**3.30**	**2.91**	**3.60**	**1.37**	**10.91**	**1.64**	**-0.48**	**5.95**

2.5 Risk premia, weighted average cost of capital and hurdle rates

The risk premia that we measure can be used in the calculation of the cost of capital. In a simple capital asset pricing model, the cost of equity capital would be the product of the company's beta times the risk premium along with the risk free rate. The average firm's cost of equity capital would be 6.63% (assuming a beta=1). Assuming the Baa bond yield is the borrowing rate and a 25% marginal tax rate, the weighted average cost of capital would be about 5.67%.

In previous surveys, we have asked CFOs about their weighted average cost of capital. For example, in March of 2011, companies told us that their internally calculated weighted average cost of capital was 10% (averaged across respondents). At the time, the cost of equity capital was similar to today, 6.45%. The bond yields were higher, with the Baa yielding 6.09%. The average firm (assuming average beta is 1.0) without any debt would have a WACC of 6.45%. When debt is introduced, the WACC would be less than 6.45% -- which is sharply lower than the reported 10%.

Why is there such a divergence? One possible reason is that companies consider other factors in calculating the WACC – perhaps a multifactor model.[4] However, there is no evidence supporting this hypothesis. For example, consultants often add a premium for smaller firms based on the results in many research papers of a size premium. However, in our survey the average WACC for firms with less than $25 million in revenue is 10.6% and the WACC for the largest firms with annual revenue greater than $10 billion is 10.5%.

This analysis was replicated in June of 2012 with similar results. Given the same assumptions, the WACC is 5.37%. However, the average self-reported WACC is 9.3%. Again, there is no evidence of a size premium. The smallest firms reported a WACC of 9.3% and the largest firms 9.7%.

The WACC should not be confused with the investment hurdle rate. The WACC is an analytical calculation that combines a model-based cost of equity (such as the CAPM) and the after-tax cost of debt (reflected in current borrowing rates). Given capital constraints, firms often impose a higher hurdle rate on their investments. For example, to allocate capital to an investment that

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[4] Graham and Harvey (2001) find that most companies use a 1-factor model for cost of capital calculations.

promises a projected return exactly at the firm's WACC is equivalent to accepting a zero net present value project.

The June 2012 survey also asked for the investment hurdle rates. They are much higher than the WACCs. The average rate was 13.5% (compared to the survey-reported WACC of 9.3% and the implied WACC from the survey based risk premium of 5.7%. Similar to the WACC results, there is no evidence that the hurdle rates are higher for small firms. Our evidence shows that the reported average hurdle rate for the smallest firms is 13.1% and for the largest firms the rate is 14.2%.

Even though we know from Graham and Harvey (2001) that three quarters of companies use the capital asset pricing model, there is a large gap between an imputed WACC and the WACC that people use. One way to reconcile this is that companies use very long term averages of equity and bond premia in their calculations. For example, suppose the cost of capital is being calculated with averages from 1926. Ibbotson (2013) reports an arithmetic average return of 11.8% over the 1926-2012 period. The average return on corporate bonds is 6.4%. Using the same parameters, we get an imputed WACC of 9.7%. This is very close to the average reported WACC and, indeed, identical to the WACC reported by the largest firms in our survey.

We learn the following: 1) the equity risk premium is much lower today than averages used over long-periods (e.g. from 1926) such as reported in Morningstar (2013) and Duff and Phelps (2015); 2) the survey questions asking directly about a company's WACC is consistent with companies routinely using long-horizon averages for inputs; and 3) WACCs should be thought as lower bounds – the Hurdle Rates used for actual investment decisions are 400bp higher than the stated WACCs.[5]

2.6 Recessions, the financial crisis and risk premia

Our survey spans two recessions: March 2001-September 2001 as well as the recession that begins in December 2007 and ends in June 2009. Financial theory would suggest that risk premia should vary with the business cycle. Premiums should be highest during recessions and lowest

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[5] Also see Sharpe and Suarez (2013) and Jagannathan et al. (2016) who analyze our CFO survey data.

during recoveries. Previous research has used a variety of methods including looking at ex post realized returns to investigate whether there is business-cycle like variation in risk premia.

While we only have 60 observations and this limits our statistical analysis, we do see important differences. During recessions, the risk premium is 3.92% and during non-recessions, the premium falls to 3.46%.

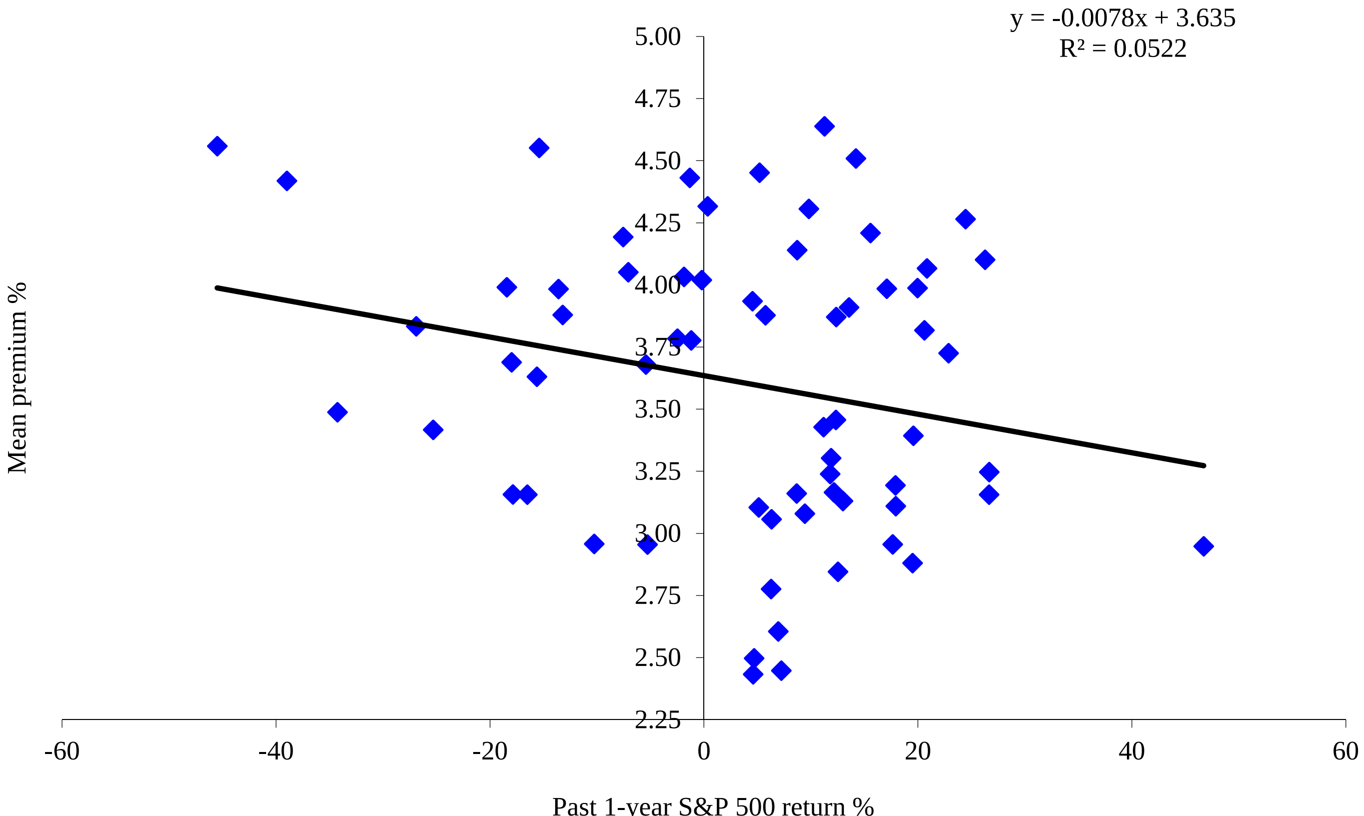
2.7 Explaining variation in the risk premium

While we document the level and a limited time-series of the long-run risk premium, statistical inference is complicated by the fact that the forecasting horizons are overlapping. First, we have no way of measuring the accuracy of the risk premiums as forecasts of equity returns. Second, any inference based on regression analysis is confounded by the fact that from one quarter to the next, there are 36 common quarters being forecasted. This naturally induces a moving-average process.

We do, however, try to characterize the time-variation in the risk premium without formal statistical tests. Figure 2 examines the relation between the mean premium and previous one-year returns on the S&P 500.

Figure 2

The ten-year equity risk premium and past 1-year returns on the S&P 500 index

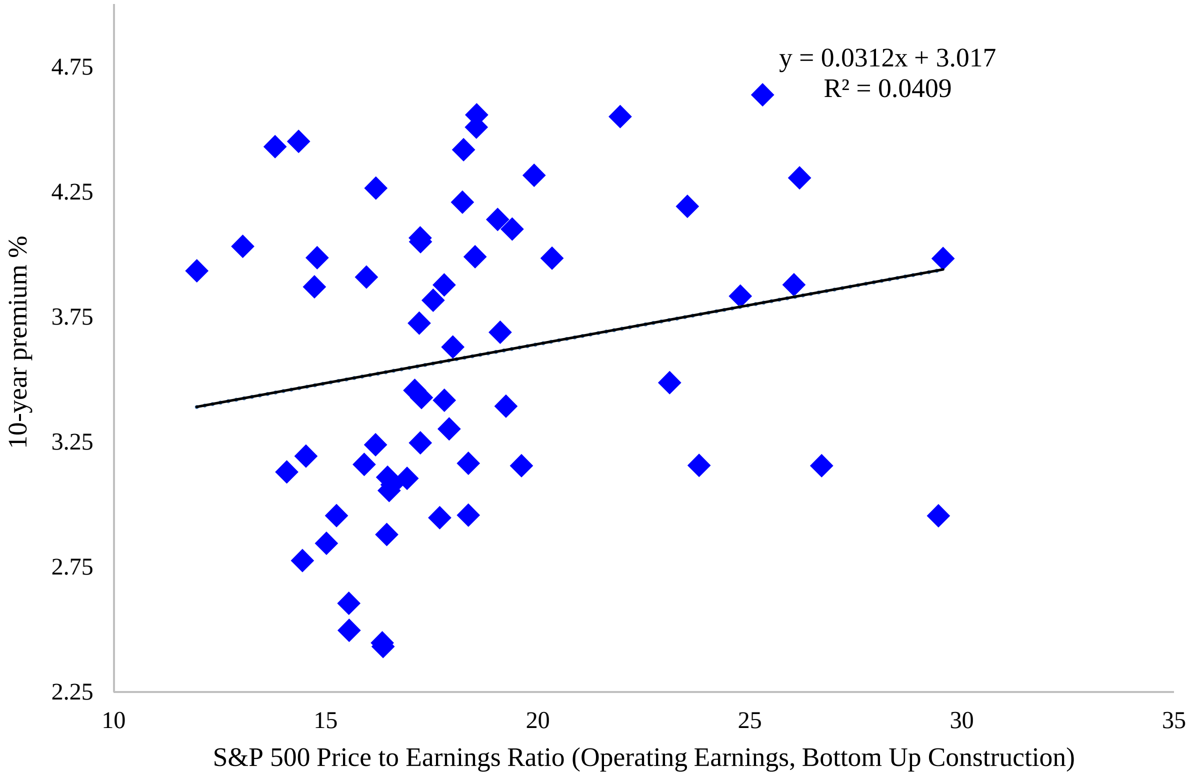


The evidence suggests that there is a weak negative correlation between past returns and the level of the long-run risk premium. This makes economic sense. When prices are low (after negative returns), expected return increase.

An alternative to using past-returns is to examine a measure of valuation. Figure 3 examines a scatter of the mean premium versus the forward price-to-earnings ratio of the S&P 500.

Figure 3

The equity risk premium and the S&P 500 forward price-to-earnings ratio

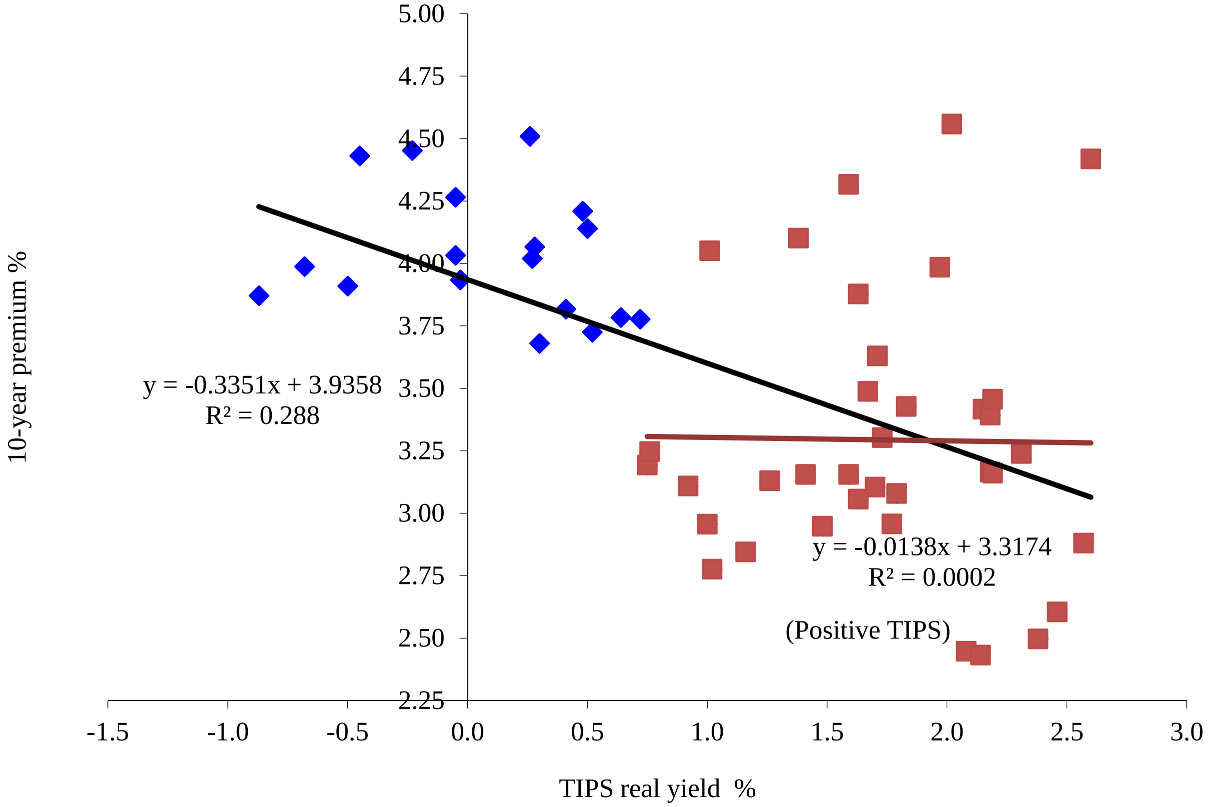


Looking at the data in Figure 3, it appears that the inference may be complicated by a non-linear relation. At very high levels of valuation, the expected return (the risk premium) was low.

We also examine the real yield on Treasury Inflation Indexed Notes. The risk premium is like an expected real return on the equity market. It seems reasonable that there could be a correlation between expected real rates of return stocks and bonds. Figure 4 examines the 10-year on the run yield on the Treasury Inflation Indexed Notes.

Figure 4

The equity risk premium and the real yield on Treasury Inflation Indexed Notes

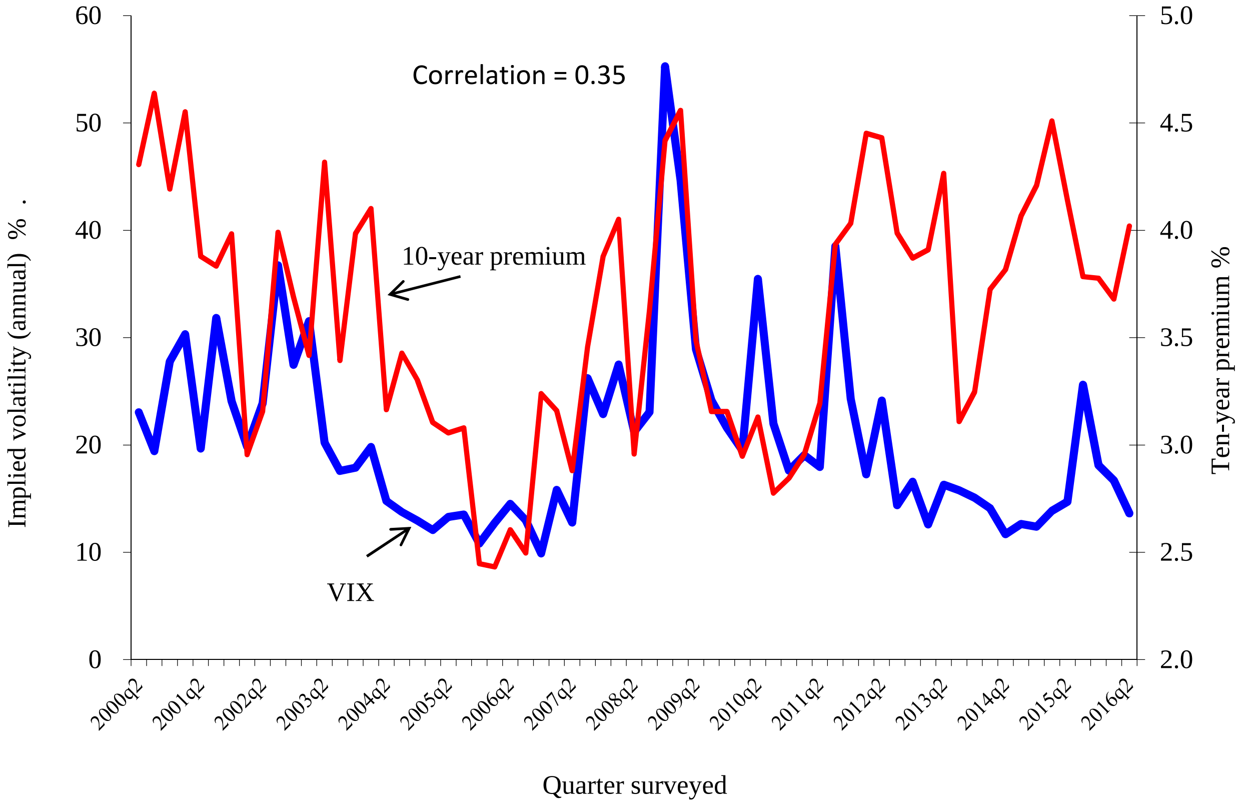


Overall, there is a negative correlation of -0.517. However, this correlation is driven by the negative TIPS yields. This is consistent with the idea that in periods of heightened uncertainty, investors engage in a flight to safety and accept low or negative TIPS yields – and at the same time demand a high risk premium for investing in the equity market.

Finally, we consider two alternative measures of risk and the risk premium. Figure 5 shows that over our sample there is evidence of a strong positive correlation between market volatility and the long-term risk premium. We use a five-day moving average of the implied volatility on the S&P index option (VIX) as our volatility proxy. The correlation between the risk premium and volatility is 0.35. If the closing day of the survey is used, the correlation is roughly the same. Asset pricing theory suggests that there is a positive relation between risk and expected return. While our volatility proxy doesn't match the horizon of the risk premium, the evidence, nevertheless, is suggestive of a positive relation. Figure 5 also highlights a strong recent divergence between the risk premium and the VIX.

Figure 5

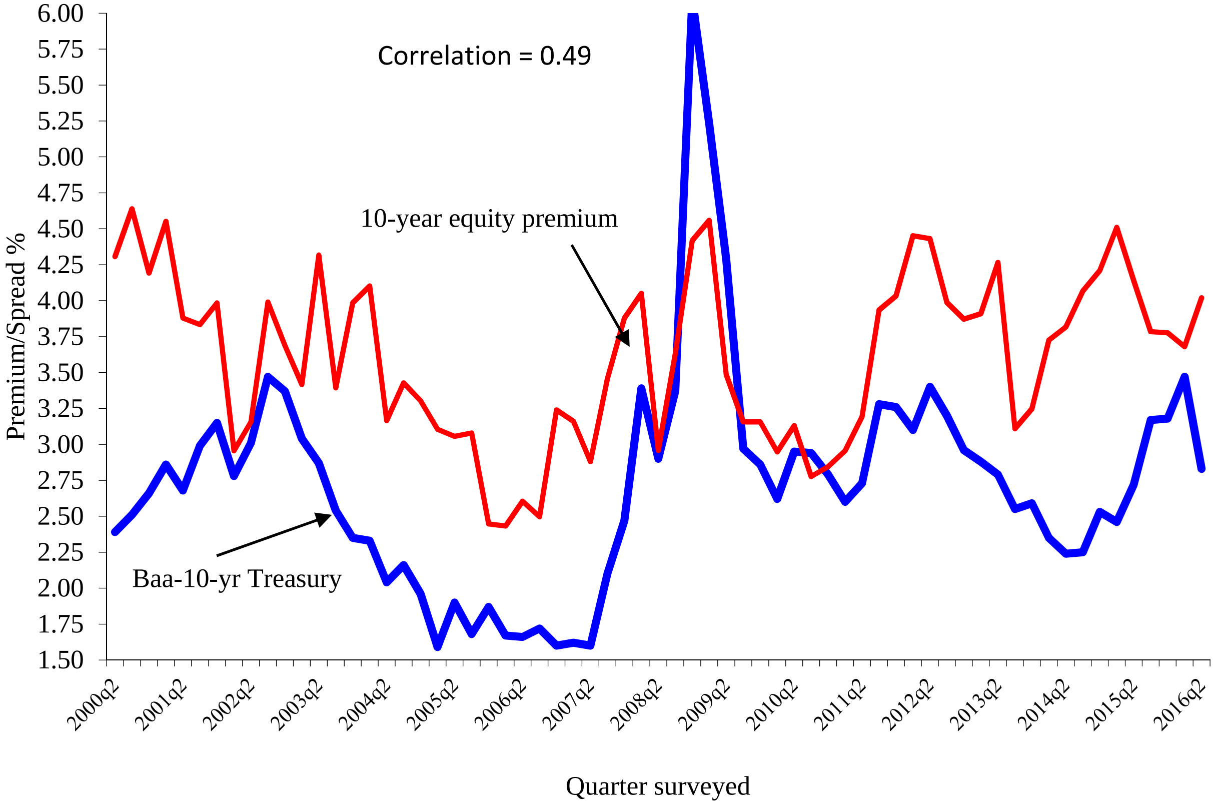
The equity risk premium and the implied volatility on the S&P 500 index option (VIX)



We also consider an alternative risk measure, the credit spread. We look at the correlation between Moody's Baa rated bond yields less the 10-year Treasury bond yield and the risk premium. Figure 6 shows a highly significant relation between the time-series with a correlation of 0.49. Similar to Figure 5, there is a strong recent divergence.

Figure 6

The equity risk premium and credit spreads



2.8 Other survey questions

The June 2016 survey contains a number of other questions. http://www.cfosurvey.org presents the full results of these questions. The site also presents results conditional on demographic firm characteristics. For example, one can examine the CFOs views of the risk premium conditional on the industry in which the CFO works.

2.9 Risk premium data and corporate policies

Research by Ben-David, Graham and Harvey (2013) uses the one-year risk premium forecasts as a measure of optimism and the 80% confidence intervals as a direct measure of overconfidence. By linking email addresses that respondents provide to archival corporate data, Ben-David et al. find that the tightness of the confidence intervals is correlated with corporate investment. Overconfident managers invest more.

Campello, Graham and Harvey (2010) use the survey during the financial crisis and the higher risk premiums to examine the implications of financial constraints on the real activities of the firm. They provide new evidence on the negative impact of financial constraints on firms' investment plans.

Campello, Giambona, Graham and Harvey (2011) use the survey during the financial crisis to study how firms managed liquidity during the financial crisis.

Graham, Harvey and Puri (2013) administer a psychometric test using the survey instrument and link CEO optimism and risk aversion to corporate financial policies.

Graham, Harvey and Puri (2015) use survey data to study how capital is allocated within the firm and the degree to which CEOs delegate decision making to CFOs.

Graham, Harvey and Rajgopal (2005) use survey data to study how managers manipulate earnings. Dichev, Graham, Harvey, and Rajgopal (2013) study earnings quality.

Graham, Harvey, Popadak and Rajgopal (2016) use a similar survey sample to study corporate culture.

2.10 CFO Survey compared to other surveys

Table 2 compares the predictive ability of the Duke-CFO survey with other popular surveys. The table reports the correlations between the current quarter Duke-CFO survey of either optimism about the economy or optimism about the firm's prospects with the subsequent quarter's realization for five surveys: UBS-Gallup, CEO Survey, Conference Board Consumer Confidence, University of Michigan Consumer Confidence and ISM Purchasing Manager's Index. Both of the Duke-CFO optimism measures significantly predict all five of these popular barometers of economic confidence. Related analysis shows that our CFO survey anticipates economic activity sooner (usually one quarter sooner) than do the other surveys.

Table 2
The ability of the Duke CFO survey to predict other surveys

	Predictive correlations	
Survey	Optimism about economy	Optimism about firm's prospects
UBS-Gallup	0.289	0.380
CEO Survey	0.814	0.824
Conference Board Consumer Confidence	0.513	0.767
University of Michigan Consumer Confidence	0.341	0.253
ISM Purchasing Managers Index	0.694	0.497

3. Conclusions

We provide a direct measure of ten-year market returns based on a multi-year survey of Chief Financial Officers. Importantly, we have a 'measure' of expectations. We do not claim it is the true market expectation. Nevertheless, the CFO measure has not been studied before.

While there is relatively little time-variation in the risk premium, premia are higher during recessions and higher during periods of uncertainty. We also link our analysis to the actual investment decisions of financial managers. We are able to impute the weighted average cost of capital given the CFO estimates of equity risk premia, current corporate bond yields and marginal tax rates. This imputed measure is significantly less than the WACCs that CFOs report using in project evaluation. One way to reconcile this is that CFOs use very long-term averages of equity premia and bond rates when calculating WACCs. We provide evidence on the actual hurdle rates used by companies. These hurdle rates are, on average, 400bp higher than the reported WACCs.

While we have over 23,000 survey responses in 16 years, much of our analysis uses summary statistics for each survey. As such, with only 65 unique quarters of predictions and a variable of interest that has a 10-year horizon, it is impossible to evaluate the accuracy of the market excess return forecasts. For example, the June 4, 2007 10-year annual forecast was 7.83% and the realized annual S&P 500 return through June 2, 2016 is 3.2%. Our analysis shows some weak correlation between past returns, real interest rates and the risk premium. In contrast, there is significant evidence on the relation between two common measures of economic risk and the risk premium. We find that both the implied volatility on the S&P index as well as a commonly used measure of credit spreads are correlated with our measured equity risk premium.

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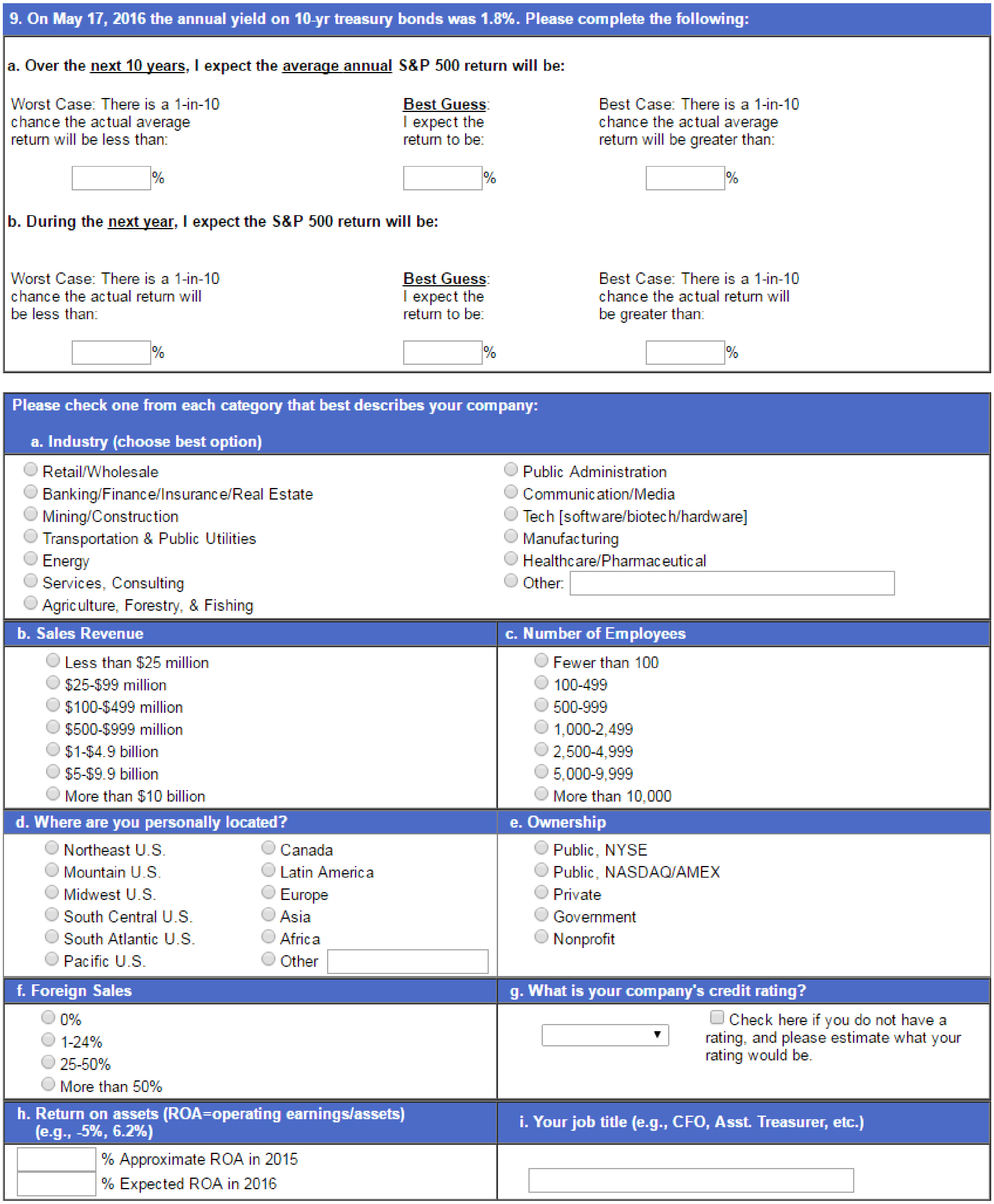
Appendix A

Excerpt from the Survey Instrument



9. On May 17, 2016 the annual yield on 10-yr treasury bonds was 1.8%. Please complete the following:

a. Over the next 10 years, I expect the average annual S&P 500 return will be:

Worst Case: There is a 1-in-10 chance the actual average return will be less than:	**Best Guess**: I expect the return to be:	Best Case: There is a 1-in-10 chance the actual average return will be greater than:
%	%	%

b. During the next year, I expect the S&P 500 return will be:

Worst Case: There is a 1-in-10 chance the actual return will be less than:	**Best Guess**: I expect the return to be:	Best Case: There is a 1-in-10 chance the actual return will be greater than:
%	%	%

Please check one from each category that best describes your company:

a. Industry (choose best option)

- ○ Retail/Wholesale
- ○ Banking/Finance/Insurance/Real Estate
- ○ Mining/Construction
- ○ Transportation & Public Utilities
- ○ Energy
- ○ Services, Consulting
- ○ Agriculture, Forestry, & Fishing
- ○ Public Administration
- ○ Communication/Media
- ○ Tech [software/biotech/hardware]
- ○ Manufacturing
- ○ Healthcare/Pharmaceutical
- ○ Other:

b. Sales Revenue

- ○ Less than $25 million
- ○ $25-$99 million
- ○ $100-$499 million
- ○ $500-$999 million
- ○ $1-$4.9 billion
- ○ $5-$9.9 billion
- ○ More than $10 billion

c. Number of Employees

- ○ Fewer than 100
- ○ 100-499
- ○ 500-999
- ○ 1,000-2,499
- ○ 2,500-4,999
- ○ 5,000-9,999
- ○ More than 10,000

d. Where are you personally located?

- ○ Northeast U.S.
- ○ Mountain U.S.
- ○ Midwest U.S.
- ○ South Central U.S.
- ○ South Atlantic U.S.
- ○ Pacific U.S.
- ○ Canada
- ○ Latin America
- ○ Europe
- ○ Asia
- ○ Africa
- ○ Other

e. Ownership

- ○ Public, NYSE
- ○ Public, NASDAQ/AMEX
- ○ Private
- ○ Government
- ○ Nonprofit

f. Foreign Sales

- ○ 0%
- ○ 1-24%
- ○ 25-50%
- ○ More than 50%

g. What is your company's credit rating?

☐ Check here if you do not have a rating, and please estimate what your rating would be.

h. Return on assets (ROA=operating earnings/assets) (e.g., -5%, 6.2%)

% Approximate ROA in 2015
% Expected ROA in 2016

i. Your job title (e.g., CFO, Asst. Treasurer, etc.)

Dockets UE-170485 & UG-170486
Exhibit DJG-17
Page 1 of 2

Market Risk Premium used in 71 countries in 2016:
a survey with 6,932 answers

Pablo Fernandez*, Alberto Ortiz** and Isabel Fernandez Acín***
IESE Business School. University of Navarra

ABSTRACT

This paper contains the statistics of the Equity Premium or Market Risk Premium (MRP) used in 2016 for **71 countries**. We got answers for more countries, but we only report the results for 71 countries with more than 8 answers.

54% of the MRP used in 2016 decreased (vs. 2015) and 38% increased.

Most previous surveys have been interested in the Expected MRP, but this survey asks about the Required MRP. The paper also contains the references used to justify the MRP, and comments from 46 persons.

JEL Classification: G12, G31, M21

Keywords: equity premium; required equity premium; expected equity premium; historical equity premium

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xLhMTPPP

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Table 2. Market Risk Premium (%) used for 71 countries in 2016

		Average	**Median**	**St Dev**	**Max**	**min**	**Q1**	**Q3**	**N**
1	USA	5,3%	5,0%	1,3%	20,0%	1,5%	4,5%	6,0%	2536
2	Spain	6,2%	6,0%	1,4%	12,0%	1,5%	5,0%	6,8%	817
3	Germany	5,3%	5,0%	1,7%	12,4%	1,2%	4,0%	6,0%	360
4	UK	5,3%	5,0%	1,4%	12,8%	1,5%	4,5%	6,0%	221
5	Italy	5,6%	5,5%	1,5%	10,1%	2,0%	4,8%	6,0%	152
6	Canada	5,4%	5,2%	1,3%	10,5%	3,0%	4,6%	6,0%	127
7	Brazil	8,2%	7,0%	4,9%	30,0%	1,8%	5,5%	8,7%	107
8	France	5,8%	5,5%	1,6%	11,4%	2,0%	5,0%	6,7%	105
9	Mexico	7,4%	7,0%	2,3%	15,0%	3,0%	6,0%	9,0%	103
10	South Africa	6,3%	6,0%	1,5%	11,8%	3,0%	5,5%	7,0%	99
11	China	8,3%	7,0%	4,4%	30,0%	3,8%	6,0%	10,0%	96
12	Netherlands	5,1%	5,0%	1,2%	11,6%	2,5%	4,5%	5,9%	93
13	Switzerland	5,1%	5,0%	1,1%	9,6%	3,0%	4,5%	5,6%	88
14	Australia	6,0%	6,0%	1,6%	15,0%	3,0%	5,0%	6,2%	87
15	India	8,1%	8,0%	2,4%	16,0%	2,3%	6,6%	9,0%	82
16	Russia	7,9%	7,0%	3,5%	25,0%	2,7%	6,0%	9,0%	81
17	Chile	6,1%	6,0%	1,6%	15,0%	3,0%	5,5%	7,0%	72
18	Sweden	5,2%	5,0%	1,0%	9,0%	3,0%	4,5%	5,9%	72
19	Austria	5,4%	5,3%	1,4%	14,3%	2,5%	5,0%	6,0%	71
20	Belgium	5,6%	5,5%	1,1%	8,1%	3,6%	5,0%	6,4%	71
21	Norway	5,5%	5,0%	1,8%	14,0%	3,0%	4,5%	6,0%	70
22	Denmark	5,3%	5,0%	1,7%	14,0%	2,0%	4,4%	6,0%	63
23	Japan	5,4%	5,0%	2,3%	16,7%	2,0%	4,0%	6,8%	58
24	Argentina	11,8%	11,0%	4,4%	28,7%	5,0%	9,0%	14,0%	57
25	Colombia	8,1%	7,8%	3,9%	20,5%	2,0%	6,5%	9,0%	56
26	Portugal	7,9%	8,0%	2,1%	14,0%	4,0%	6,6%	9,0%	55
27	Finland	5,5%	5,0%	1,6%	12,0%	3,0%	4,7%	6,0%	51
28	Poland	6,2%	5,8%	1,5%	10,0%	4,4%	5,0%	7,6%	50
29	Peru	7,8%	7,5%	2,6%	15,0%	3,5%	6,3%	8,3%	44
30	New Zealand	5,8%	6,0%	1,4%	8,0%	2,0%	5,0%	7,0%	42
31	Greece	13,0%	12,4%	5,2%	23,0%	6,5%	8,5%	17,9%	41
32	Luxembourg	4,7%	5,0%	1,1%	7,0%	2,0%	4,0%	5,4%	38
33	Israel	5,9%	6,0%	2,2%	15,0%	2,5%	5,0%	7,0%	37
34	Turkey	8,1%	8,0%	3,4%	18,0%	2,5%	5,5%	10,5%	37
35	Czech Republic	6,3%	6,5%	1,0%	8,0%	4,3%	5,5%	7,3%	32
36	Egypt	13,8%	13,0%	6,2%	30,3%	3,5%	9,0%	16,4%	32
37	Indonesia	8,0%	8,0%	2,1%	14,5%	4,5%	6,1%	9,3%	29
38	Ireland	6,6%	5,8%	2,2%	12,3%	4,0%	5,0%	8,2%	28
39	Pakistan	9,8%	6,5%	5,4%	18,0%	2,5%	6,0%	16,0%	26
40	Taiwan	7,9%	7,2%	2,1%	15,0%	4,3%	7,0%	8,4%	26
41	Korea	6,7%	7,0%	1,8%	11,1%	2,0%	6,0%	7,3%	25
42	Singapore	5,9%	6,0%	1,3%	9,6%	3,9%	5,5%	6,3%	25
43	Liechtenstein	4,8%	5,0%	1,0%	7,3%	3,0%	4,4%	5,0%	24
44	Hong Kong	7,6%	6,9%	2,6%	12,0%	3,5%	5,5%	10,0%	21

CAPITAL EQUIPMENT ANALYSIS: THE REQUIRED RATE OF PROFIT

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The interest in capital equipment analysis that has been evident in the business literature of the past five years is the product of numerous social, economic, and business developments of the postwar period. No conclusive listing of these developments can be attempted here. However, four should be mentioned which are of particular importance in this search for a more systematic method for discovering, evaluating, and selecting investment opportunities. These are: (1) the high level of capital outlays (in absolute terms); (2) the growth in the size of business firms; (3) the delegation of responsibility for initiating recommendations from top management to the profit center, which has been part of the general movement toward decentralization; and (4) the growing use of "scientific management" in the operations of the business firm.

These developments have motivated the current attempt to develop objective criteria whereby the executive committee in a decentralized firm can arrive at a capital budget. Since each of its profit centers submits capital proposals, the executive committee must screen these and establish an allocation and a level of capital outlays that is consistent with top management's criteria for rationing the firm's funds. Capital budgeting affords the promise that this screening process can be made amenable to some established criteria that are understandable to all the component parts of the firm. Consequently, capital budgeting appeals to top management, for, in the first place, each plant manager can see his proposal in the light of all competing proposals for the funds of the enterprise. This may not completely eliminate irritation among the various parts of the firm, but a rational capital budgeting program can go a long way toward maintaining initiative on the part of a plant manager, even though the executive committee may veto one or all of his proposals. In the second place, the use of a capital budgeting program serves to satisfy top management that each accepted proposal meets adequate predetermined standards and that the budget as a whole is part of a sound, long-run plan for the firm.

What specifically does a capital budgeting program entail? The focal points of capital budgeting are: (1) ascertaining the profit abilities of the array of capital outlay alternatives, and (2) determining the least profitability required to make an investment, i.e., a cut-off point. Capital budgeting also involves administrative procedures and organization designed to discover investment opportunities, process information, and carry out the budget; however, these latter aspects of the subject have been discussed in detail by means of case studies that have appeared in publications of the American Management Association and the

National Industrial Conference Board and in periodicals such as the *N.A.C.A. Bulletin*.[1] Hence, we will not concern ourselves with them here.

There are at least four methods for establishing an order-preference array of the capital expenditure suggestions. They are: (1) the still popular "payoff period"; (2) the average investment formula; (3) the present value formula with the rate of interest given; and (4) the present value formula used to find the rate of profit. It is not our intention in this paper to discuss these various methods specifically, since critical analyses of these alternatives are to be found in papers by Dean, by Lorie and Savage, and by Gordon in a recent issue of the *Journal of Business*,[2] which is devoted exclusively to the subject of capital budgeting.

However, it is of interest to note that in each of these methods the future revenue streams generated by the proposed outlays must be amenable to measurement if the method is to be operational. However, improvements in quality, more pleasant working conditions, strategic advantages of integration, and other types of benefits from a capital outlay are still recognized only in qualitative terms, and there is a considerable hiatus in the literature of capital budgeting with respect to the solution of this problem. Hence, in the absence of satisfactory methods for quantifying these types of benefits, the evaluation of alternative proposals is still characterized by intuitive judgments on the part of management, and a general quantitative solution to the capital budgeting problem is not now feasible. It appears to us that this problem affords one of the most promising opportunities for the application of the methods of management science. In fact, we anticipate that techniques for the quantification of the more important factors now treated qualitatively will soon be found.

Given the rate of profit on each capital outlay proposal, the size of the budget and its allocation are automatically determined with the establishment of the rate of profit required for the inclusion of a proposal in the budget. In the balance of this paper, a method for determining this quantity is proposed and its use in capital budgeting is analyzed.

II

We state that the objective of a firm is the maximization of the value of the stockholders' equity. While there may be legitimate differences of opinion as to whether this is the sole motivation of management, we certainly feel that there can be no quarrel with the statement that it is a dominant variable in manage-

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[1] American Management Association, *Tested Approaches to Capital Equipment Replacement*, Special Report No. 1, 1954; American Management Association, *Capital Equipment Replacement*; *AMA Special Conference*, May 3–4, 1954 (New York, 1954, American Management Association, 105 pp.); J. H. Watson, III, National Industrial Conference Board, *Controlling Capital Expenditures*, Studies in Business Policy, No. 62, April, 1953; C. I. Fellers, "Problems of Capital Expenditure Budgeting", *N.A.C.A. Bulletin*, 26 (May, 1955), 918–24; E. N. Martin, "Equipment Replacement Policy and Application", *N.A.C.A. Bulletin*, 35 (February, 1954), 715–30.

[2] *Journal of Business*, Vol. XXVIII, No. 3 (October, 1955).

ment's decisions. It has been shown by Lutz and Lutz in their *Theory of the Investment of the Firm*[3] and by others[4] that this objective is realized in capital budgeting when the budget is set so as to equate the marginal return on investment with the rate of return at which the corporation's stock is selling in the market. The logic and operation of this criterion will be discussed later. Now, we only wish to note the role assigned in capital budgeting to the rate of profit that is required by the market.

At the present time, the dividend yield (the current dividend divided by the price) and the earnings yield (the current income per share divided by the price) are used to measure the rate of profit at which a share is selling. However, both these yields fail to recognize that a share's payments can be expected to grow, and the earnings yield fails to recognize that the corporation's earnings per share are not the payments made to the stockholder.

The practical significance of these failures is evidenced by the qualifications with which these two rate-of-profit measures are used by investment analysts. In the comparative analysis of common stocks for the purpose of arriving at buy or sell recommendations, the conclusions indicated by the dividend and/or the earnings yield are invariably qualified by the presence or absence of the prospect of growth. If it is necessary to qualify a share's yield as a measure of the rate of profit one might expect to earn by buying the share, then it must follow that current yield, whether income or dividend, is inadequate for the purposes of capital budgeting, which is also concerned with the future. In short, it appears to us that the prospective growth in a share's revenue stream should be reflected in a measure of the rate of profit at which the share is selling. Otherwise, its usefulness as the required rate of profit in capital budgeting is questionable.

In his *Theory of Investment Value*[5], a classic on the subject, J. B. Williams tackled this problem of growth. However, the models he developed were arbitrary and complicated so that the problem of growth remained among the phenomena dealt with qualitatively. It is our belief that the following proposal for a definition of the rate of profit that takes cognizance of prospective growth has merit.

The accepted definition of the rate of profit on an asset is the rate of discount that equates the asset's expected future payments with its price. Let P_0 = a share's price at $t = 0$, let D_t = the dividend expected at time t, and let k = the rate of profit. Then, the rate of profit on a share of stock is the value of k that satisfies

$$(1) \qquad P_0 = \sum_{t=1}^{\infty} \frac{D_t}{(1 + k)^t}.$$

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[3] Friedrich and Vera Lutz, *The Theory of Investment of the Firm* (Princeton, N. J., 1951, Princeton University Press, 253 pp.), 41–43.

[4] Joel Dean, *Capital Budgeting: Top Management Policy on Plant, Equipment, and Product Development* (New York, 1951, Columbia University Press, 174 pp.); Roland P. Soule, "Trends in the Cost of Capital", *Harvard Business Review*, 31 (March, April, 1953), 33–47.

[5] J. B. Williams, *The Theory of Investment Value*, (Cambridge, Massachusetts, 1938, Harvard University Press), 87–96.

It is mathematically convenient to assume that the dividend is paid and discounted continuously at the annual rates D_t and k, in which case

$$P_0 = \int_0^\infty D_t e^{-kt}\,dt. \tag{2}$$

Since P_0 is known, estimating the rate of profit at which a share of stock is selling requires the determination of D_t, $t = 1, 2, \cdots, \infty$.

At the outset it should be made clear that our objective is not to find the rate of profit that *will actually be earned* by buying a share of stock. This requires knowledge of the dividends that will be paid in the future, the price at which the share will be sold, and when it will be sold. Unfortunately, such information is not available to us. The rate of profit of interest here is a relation between the present known price and the *expected future dividends*. The latter will vary among individuals with the information they have on a host of variables and with their personality. Therefore, by expected future dividends we mean an estimate that (1) is derivable from known data in an objective manner, (2) is derived by methods that appear reasonable, i.e., not in conflict with common sense knowledge of corporation financial behavior, and (3) can be used to arrive at a manageable measure of the rate of profit implicit in the expectation.

We arrive at D_t by means of two assumptions. One, a corporation is expected to retain a fraction b of its income after taxes; and two, a corporation is expected to earn a return of r on the book value of its common equity. Let Y_t equal a corporation's income per share of common after taxes at time t. Then the expected dividend at time t is

$$D_t = (1 - b)Y_t \tag{3}$$

The income per share at time t is the income at $(t - 1)$ plus r percent of the income at $(t - 1)$ retained, or

$$Y_t = Y_{t-1} + rbY_{t-1} \tag{4}$$

Equation (4) is simply a compound interest expression so that, if Y_t grows continuously at the rate $g = br$,

$$Y_t = Y_0 e^{gt}. \tag{5}$$

From Equations (3) and (5)

$$D_t = D_0 e^{gt}. \tag{6}$$

Substituting this expression for D_t in Equation (2) and integrating, yields

$$\begin{aligned} P_0 &= \int_0^\infty D_0 e^{gt}\, e^{-kt}\,dt \\ &= D_0 \int_0^\infty e^{-t(k-g)}\,dt \\ &= \frac{D_0}{k - g}. \end{aligned} \tag{7}$$

The condition for a solution is $k > g$, a condition that is easily satisfied, for otherwise, P_0 would be infinite or negative.

Solving Equation (7) for k we find that

$$k = \frac{D_0}{P_0} + g. \tag{8}$$

Translated, this means that the rate of profit at which a share of common stock is selling is equal to the current dividend, divided by the current price (the dividend yield), plus the rate at which the dividend is expected to grow. Since there are other possible empirical definitions of the market rate of profit on a share of stock, we will refer to k as the growth rate of profit.

III

Let us now review and evaluate the rationale of the model we have just established. Estimating the rate of profit on a share of stock involves estimating the future dividend stream that it provides, and the fundamental difference between this model and the dividend yield is the assumption of growth. The latter, as can be seen, assumes that the dividend will remain constant. Since growth is generally recognized as a factor in the value of a share and since it is used to explain differences in dividend yield among shares, its explicit recognition appears desirable. Future dividends are uncertain, but the problem cannot be avoided by ignoring it. To assume a constant rate of growth and estimate it to be equal to the current rate appears to be a better alternative.

Under this model the dividend will grow at the rate br, which is the product of the fraction of income retained and the rate of return earned on net worth. It is mathematically true that the dividend will grow at this rate if the corporation retains b and earns r. While we can be most certain that the dividend will not grow uniformly and continuously at some rate, unless we believe that an alternative method for estimating the future dividend stream is superior, the restriction of the model to the assumption that it will grow uniformly at some rate is no handicap. Furthermore, the future is discounted; hence, an error in the estimated dividend for a year in the distant future results in a considerably smaller error in k than an error in estimating the dividend in a near year.

It should be noted that this measure of the rate of profit is suspect, when *both* income and dividend are zero, and it may also be questioned when either falls to very low (or negative) values. In such cases, the model yields a lower rate of profit than one might believe that the market requires on a corporation in such difficulties. It is evident that the dividend and the income yields are even more suspect under these conditions and, hence, are subject to the same limitations.

There are other approaches to the estimation of future dividends than the extrapolation of the current dividend on the basis of the growth rate implicit in b and r. In particular, one can arrive at g directly by taking some average of the past rate of growth in a corporation's dividend. Whether or not this or some other measure of the expected future dividends is superior to the one presented earlier will depend on their relative usefulness in such purposes as the analysis of variation in prices among shares and the preferences of those who want an objective measure of a share's rate of profit.

So far, we have compared the growth rate of profit with the income and dividend yields on theoretical grounds. Let us now consider how they differ in practice, using the same measurement rules for the variables in each case. The numerical difference between the growth rate of profit and the dividend yield is simply the growth rate. However, the income yield, which is the measure of the rate of profit commonly recommended for capital budgeting, differs from the growth rate of profit in a more complex manner, and to establish this difference we first note that

$$b = \frac{Y - D}{Y} \text{ and } r = \frac{Y}{B} \tag{9}$$

where B = the net worth or book value per share. The growth rate of profit, therefore, may be written as

$$k = \frac{D}{P} + br = \frac{D}{P} + \frac{Y - D}{B}. \tag{10}$$

Next, the income yield can be decomposed as follows:

$$y = \frac{Y}{P} = \frac{D}{P} + \frac{Y - D}{P}. \tag{11}$$

We see then that y and k will be equal when book and market values are equal. It can be argued that the income yield overstates a share's payment stream by assuming that each payment is equal to the income per share and understates the payment stream by assuming that it will not grow. Hence, in this special case where book and market values are equal, the two errors exactly compensate each other.

Commonly market and book values differ, and y will be above k when market is below book, and it will be below k when market is above book. Hence, a share of IBM, for example, that is priced far above book had had an earnings yield of two to three percent in 1955. We know that the market requires a higher rate of profit on a common stock, even on IBM, and its growth rate of profit, k, is more in accord with the value suggested by common sense. Conversely, when U. S. Steel was selling at one-half of book value in 1950, the high income yield grossly overstated the rate of profit that the market was, in fact, requiring on the stock.

Furthermore, the growth rate of profit will fluctuate in a narrower range than the earnings yield. For instance, during the last few years, income, dividends, and book value have gone up more or less together, but market price has gone up at a considerably higher rate. Consequently, the growth rate of profit, dependent in part on book value, has fallen less than the earnings yield. Conversely, in a declining market k would rise less rapidly than y.

There is a widespread feeling that many accounting figures, particularly book value per share, are insensitive to the realities of the world, and some may feel that the comparative stability of k is merely a consequence of the limitations of accounting data. This is not true! The behavior of k is not a consequence of the supposed lack of realism in accounting data. Rather, book value appears in the model because it, and not market value, is used to measure the rate of return the corporation earns on investment, which, we have seen, is the rate of return that enters into the determination of the rate at which the dividend will grow. The comparative stability of k follows from the simple fact that, when a revenue stream is expected to grow, a change in the required rate of profit will give rise to a more than proportional change in the asset's price. Conversely, a change in the price reflects a less than proportional change in the rate of profit.

IV

Given the rate of profit expected on each item in the schedule of available investment opportunities and given the rate of profit at which the corporation's stock is selling, what should the capital budget be? As stated earlier, the accepted theory is that the budget should be set so as to equate the marginal return on investment with the rate of profit at which the stock is selling. The reasoning is, if the market requires, let us say, a 10 percent return on investment in the corporation's stock, and if the corporation can earn 15 percent on additional investment, obtaining the funds and making the investment will increase the earnings per share. As the earnings and the dividend per share increase or as the market becomes persuaded that they will increase, the price of the stock will rise. The objective, it will be recalled, is the maximization of the value of the stockholder's equity.

The conclusion drawn implicitly assumes that the corporation can sell additional shares at or above the prevailing market, or if a new issue depresses the market, the fall will be slight, and the price will soon rise above the previous level. However, some other consideration may argue against a new stock issue; for example, the management may be concerned with dilution of control, or the costs of floating a new issue may be very high, or a new issue may be expected to depress the price severely and indefinitely for reasons not recognized in the theory. Hence, it does not automatically follow that a new issue should be floated when a firm's demand for funds exceeds, according to the above criterion, those that are internally available.

In determining whether the required rate of profit is above or below r', the marginal return on investment, one can use y, the earnings yield, or k, the growth rate of profit as the required rate of profit. If y and k differ and if the reasoning in support of k presented earlier is valid, using y to estimate the direction in which a new issue will change the price of the stock may result in a wrong conclusion.

In arriving at the optimum size of a stock issue, the objective is to equate r' and y or k, depending on which is used. Internal data may be used to estimate the marginal efficiency of capital schedule. If the required rate of profit is considered a constant, its definition, $y = Y/P$ or $k = D/P + br$, provides its value. However, the required rate of profit may vary with the size of the stock issue or with the variables that may change as a consequence of the issue. In this event,

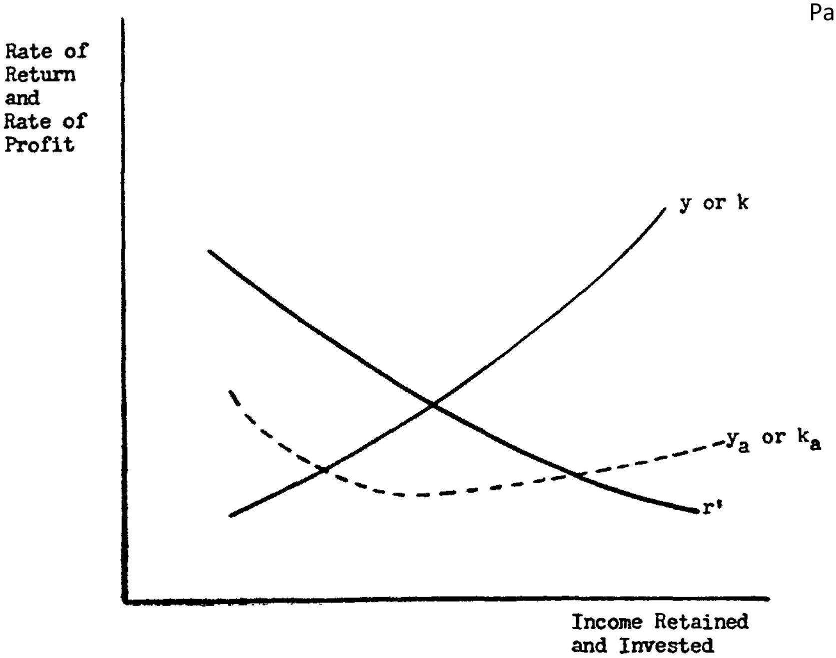

FIG. 1

finding the optimum size of a stock issue requires a model that predicts the variation in the required rate of profit with the relevant variables.

Borrowing is an alternative source of funds for investment. However, an analysis of this alternative requires the measurement of both (1) the variation in risk with debt, and (2) the difference between the rate of profit and the rate of interest needed to cover a given increase in risk. This has not been done as yet, which may explain the widespread practice of arbitrarily establishing a "satisfactory" financial structure and only borrowing to the extent allowed by it.

It has been stated by Dean[6] and Terborgh[7] that the long-term ceiling on a firm's capital outlays is the amount of its internally available funds. However, the share of its income a corporation retains is not beyond the control of its management; and, among the things we want from a capital budgeting model is guidance on whether the share of a corporation's income that is retained for investment should be raised or lowered.

Proceeding along traditional lines, the problem may be posed as follows. A firm estimates its earnings and depreciation allowances for the coming year and deducts the planned dividend to arrive at a preliminary figure for the capital budget. The marginal rate of return on investment in excess of this amount may be above or below the required rate of profit. We infer from theory that the two rates should be equated by (1) raising the budget and reducing the dividend

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[6] Dean, *op. cit.*, 53–55.

[7] George Willard Terborgh, *Dynamic Equipment Policy* (New York, 1949, McGraw-Hill, 290 pp.), 228–29.

when the marginal return on investment is above the required rate of return, and (2) raising the dividend and reducing the budget when the reverse holds. The conditions under which this process yields an equilibrium are illustrated in Figure 1. The marginal return on investment, r', should fall as the budget is increased, and the required rate of profit, y or k, should increase or it should fall at a lower rate than r'. The latter case is illustrated by the line y_a or k_a.

Changing the dividend so as to equate r' and say y should maximize the price of the stock. For instance, if r' is above y, the company can earn a higher return on investment than stockholders require, and a dollar used this way is worth more to the stockholders than the dollar distributed in dividends. In other words, the price should go up by more than the income retained.

There are, of course, a number of problems connected with the use of this model for arriving at the optimum dividend rate. First, there is the question whether y or k should be used to measure the required rate of profit. Second, there is no question that the required rate of profit varies with the dividend rate. Hence, the current rate of profit given by the definition does not tell what profit rate will be required with a different dividend rate. This requires a model which predicts the variation in y or k with the dividend rate and other variables. Third, there is a very nasty problem of the short and the long run. It is widely believed, though the evidence has limitations, that the price of a share of stock varies with the dividend rate, in which case a corporation should distribute all of its income. However, it is quite possible that a change in the dividend gives rise to the expectation that earnings and future dividends are changing in the same direction. Further, in the short run, the market is not likely to be informed on a firm's marginal efficiency of capital schedule. For these and other reasons, it is likely that the dividend rate should not be made to vary with short-run changes in the marginal efficiency of capital, and more sophisticated methods than those now in use are needed to establish the variation in price or required rate of profit with the dividend rate.

V

The major points developed in this paper may be summarized as follows. We presented a definition of the rate of profit required by the market on a share of common stock, and we noted some of its advantages. It is theoretically superior to the income and dividend yields because it recognizes that the revenue stream provided by a share can be expected to grow. Furthermore, its empirical characteristics are also superior to those of the income and dividend yields since its value is generally in closer agreement with common sense notions concerning the prevailing rate of profit on a share of stock and since its value fluctuates in a narrower range over time. We next examined some of the problems involved in using this definition of the rate of profit and the earnings yield in capital budgeting models. Finally, we saw that, before capital budgeting theory can be made a reliable guide to action, we must improve our techniques for estimating the future revenue on a capital outlay proposal, and we must learn a good deal more about how the rate of profit the market requires on a share of stock varies with the dividend, the growth rate, and other variables that may influence it.

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THE RELATIONSHIP BETWEEN RETURN AND MARKET VALUE OF COMMON STOCKS*

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This study examines the empirical relationship between the return and the total market value of NYSE common stocks. It is found that smaller firms have had higher risk adjusted returns, on average, than larger firms. This 'size effect' has been in existence for at least forty years and is evidence that the capital asset pricing model is misspecified. The size effect is not linear in the market value; the main effect occurs for very small firms while there is little difference in return between average sized and large firms. It is not known whether size *per se* is responsible for the effect or whether size is just a proxy for one or more true unknown factors correlated with size.

1. Introduction

The single-period capital asset pricing model (henceforth CAPM) postulates a simple linear relationship between the expected return and the market risk of a security. While the results of direct tests have been inconclusive, recent evidence suggests the existence of additional factors which are relevant for asset pricing. Litzenberger and Ramaswamy (1979) show a significant positive relationship between dividend yield and return of common stocks for the 1936-1977 period. Basu (1977) finds that price-earnings ratios and risk adjusted returns are related. He chooses to interpret his findings as evidence of market inefficiency but as Ball (1978) points out, market efficiency tests are often joint tests of the efficient market hypothesis and a particular equilibrium relationship. Thus, some of the anomalies that have been attributed to a lack of market efficiency might well be the result of a misspecification of the pricing model.

This study contributes another piece to the emerging puzzle. It examines the relationship between the total market value of the common stock of a firm and its return. The results show that, in the 1936-1975 period, the common stock of small firms had, on average, higher risk-adjusted returns

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*This study is based on part of my dissertation and was completed while I was at the University of Chicago. I am grateful to my committee, Myron Scholes (chairman), John Gould, Roger Ibbotson, Jonathan Ingersoll, and especially Eugene Fama and and Merton Miller, for their advice and comments I wish to acknowledge the valuable comments of Bill Schwert on earlier drafts of this paper

than the common stock of large firms. This result will henceforth be referred to as the 'size effect'. Since the results of the study are not based on a particular theoretical equilibrium model, it is not possible to determine conclusively whether market value *per se* matters or whether it is only a proxy for unknown true additional factors correlated with market value. The last section of this paper will address this question in greater detail.

The various methods currently available for the type of empirical research presented in this study are discussed in section 2. Since there is a considerable amount of confusion about their relative merit, more than one technique is used. Section 3 discusses the data. The empirical results are presented in section 4. A discussion of the relationship between the size effect and other factors, as well as some speculative comments on possible explanations of the results, constitute section 5.

2. Methodologies

The empirical tests are based on a generalized asset pricing model which allows the expected return of a common stock to be a function of risk β and an additional factor ϕ, the market value of the equity.[1] A simple linear relationship of the form

$$E(R_i)=\gamma_0+\gamma_1\beta_i+\gamma_2[(\phi_i-\phi_m)/\phi_m], \tag{1}$$

is assumed, where

$E(R_i)$ = expected return on security i,
γ_0 = expected return on a zero-beta portfolio,
γ_1 = expected market risk premium,
ϕ_i = market value of security i,
ϕ_m = average market value, and
γ_2 = constant measuring the contribution of ϕ_i to the expected return of a security.

If there is no relationship between ϕ_i and the expected return, i.e., $\gamma_2=0$, (1) reduces to the Black (1972) version of the CAPM.

Since expectations are not observable, the parameters in (1) must be estimated from historical data. Several methods are available for this purpose. They all involve the use of pooled cross-sectional and time series regressions to estimate γ_0, γ_1, and γ_2. They differ primarily in (a) the assumption concerning the residual variance of the stock returns (homoscedastic or heteroscedastic in the cross-sectional), and (b) the treatment of the

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[1] In the empirical tests, Φ_i and Φ_m are defined as the market proportion of security i and average market proportion, respectively. The two specifications are, of course, equivalent.

errors-in-variables problem introduced by the use of estimated betas in (1). All methods use a constrained optimization procedure, described in Fama (1976, ch. 9), to generate minimum variance (m.v.) portfolios with mean returns γ_i, $i=0,\ldots,2$. This imposes certain constraints on the portfolio weights, since from (1)

$$E(R_p)\equiv\gamma_i=\gamma_0\sum_j w_j+\gamma_1\sum_j w_j\beta_j + \gamma_2\left[\left(\sum_j w_j\phi_j-\phi_m\sum_j w_j\right)\Big/\phi_m\right], \qquad i=0,\ldots,2, \tag{2}$$

where the w_j are the portfolio proportions of each asset j, $j=1,\ldots,N$. An examination of (2) shows that $\hat{\gamma}_0$ is the mean return of a standard m.v. portfolio ($\sum_j w_j=1$) with zero beta and $\phi_p\equiv\sum_j w_j\phi_j=\phi_m$ [to make the second and third terms of the right-hand side of (2) vanish]. Similarly, $\hat{\gamma}_1$ is the mean return on a zero-investment m.v. portfolio with beta of one and $\phi_p=0$, and $\hat{\gamma}_2$ is the mean return on a m.v. zero-investment, zero-beta portfolio with $\phi_p=\phi_m$. As shown by Fama (1976, ch. 9), this constrained optimization can be performed by running a cross-sectional regression of the form

$$R_{it}=\gamma_{0t}+\gamma_{1t}\beta_{it}+\gamma_{2t}[(\phi_{it}-\phi_{mt})/\phi_{mt}]+\varepsilon_{it}, \qquad i=1,\ldots,N, \tag{3}$$

on a period-by-period basis, using estimated betas $\hat{\beta}_{it}$ and allowing for either homoscedastic or heteroscedastic error terms. Invoking the usual stationarity arguments the final estimates of the gammas are calculated as the averages of the T estimates.

One basic approach involves grouping individual securities into portfolios on the basis of market value and security beta, reestimating the relevant parameters (beta, residual variance) of the portfolios in a subsequent period, and finally performing either an ordinary least squares (OLS) regression [Fama and MacBeth (1973)] which assumes homoscedastic errors, or a generalized least squares (GLS) regression [Black and Scholes (1974)] which allows for heteroscedastic errors, on the portfolios in each time period.[2] Grouping reduces the errors-in-variables problem, but is not very efficient because it does not make use of all information. The errors-in-variables problem should not be a factor as long as the portfolios contain a reasonable number of securities.[3]

Litzenberger and Ramaswamy (1979) have suggested an alternative method which avoids grouping. They allow for heteroscedastic errors in the cross-section and use the estimates of the standard errors of the security

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[2] Black and Scholes (1974) do not take account of heteroscedasticity, even though their method was designed to do so.

[3] Black, Jensen and Scholes (1972, p. 116).

betas as estimates of the measurement errors. As Theil (1971, p. 610) has pointed out, this method leads to unbiased maximum likelihood estimators for the gammas as long as the error in the standard error of beta is small and the standard assumptions of the simple errors-in-variables model are met. Thus, it is very important that the diagonal model is the correct specification of the return-generating process, since the residual variance assumes a critical position in this procedure. The Litzenberger–Ramaswamy method is superior from a theoretical viewpoint; however, preliminary work has shown that it leads to serious problems when applied to the model of this study and is not pursued any further.[4]

Instead of estimating equation (3) with data for all securities, it is also possible to construct arbitrage portfolios containing stocks of very large and very small firms, by combining long positions in small firms with short positions in large firms. A simple time series regression is run to determine the difference in risk-adjusted returns between small and large firms. This approach, long familiar in the efficient markets and option pricing literature, has the advantage that no assumptions about the exact functional relationships between market value and expected return need to be made, and it will therefore be used in this study.

3. Data

The sample includes all common stocks quoted on the NYSE for at least five years between 1926 and 1975. Monthly price and return data and the number of shares outstanding at the end of each month are available in the monthly returns file of the Center for Research in Security Prices (CRSP) of the University of Chicago. Three different market indices are used; this is in response to Roll's (1977) critique of empirical tests of the CAPM. Two of the three are pure common stock indices — the CRSP equally- and value-weighted indices. The third is more comprehensive: a value-weighted combination of the CRSP value-weighted index and return data on corporate and government bonds from Ibbotson and Sinquefield (1977) (henceforth 'market index').[5] The weights of the components of this index are derived from information on the total market value of corporate and government bonds in various issues of the *Survey of Current Business* (updated annually) and from the market value of common stocks in the CRSP monthly index file. The stock indices, made up of riskier assets, have both higher returns

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[4] If the diagonal model (or market model) is an incomplete specification of the return generating process, the estimate of the standard error of beta is likely to have an upward bias, since the residual variance estimate is too large The error in the residual variance estimate appears to be related to the second factor. Therefore, the resulting gamma estimates are biased.

[5] No pretense is made that this index is complete, thus, the use of quotation marks It ignores real estate, foreign assets, etc ; it should be considered a first step toward a comprehensive index. See Ibbotson and Fall (1979)

and higher risk than the bond indices and the 'market index'.[6] A time series of commercial paper returns is used as the risk-free rate.[7] While not actually constant through time, its variation is very small when compared to that of the other series, and it is not significantly correlated with any of the three indices used as market proxies.

4. Empirical results

4.1. Results for methods based on grouped data

The portfolio selection procedure used in this study is identical to the one described at length in Black and Scholes (1974). The securities are assigned to one of twenty-five portfolios containing similar numbers of securities, first to one of five on the basis of the market value of the stock, then the securities in each of those five are in turn assigned to one of five portfolios on the basis of their beta. Five years of data are used for the estimation of the security beta; the next five years' data are used for the reestimation of the portfolio betas. Stock price and number of shares outstanding at the end of the five year periods are used for the calculation of the market proportions. The portfolios are updated every year. The cross-sectional regression (3) is then performed in each month and the means of the resulting time series of the gammas could be (and have been in the past) interpreted as the final estimators. However, having used estimated parameters, it is not certain that the series have the theoretical properties, in particular, the hypothesized beta. Black and Scholes (1974, p. 17) suggest that the time series of the gammas be regressed once more on the excess return of the market index. This correction involves running the time series regression (for $\hat{\gamma}_2$)

$$\hat{\gamma}_{2t} - R_{Ft} = \hat{\alpha}_2 + \hat{\beta}_2 (R_{mt} - R_{Ft}) + \hat{\varepsilon}_{2t}. \tag{4}$$

It has been shows earlier that the theoretical β_2 is zero. (4) removes the effects of a non-zero $\hat{\beta}_2$ on the return estimate $\hat{\gamma}_2$ and $\hat{\alpha}_2$ is used as the final estimator for $\hat{\gamma}_2 - R_F$. Similar corrections are performed for γ_0 and γ_1. The

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[6] Mean monthly returns and standard deviations for the 1926–1975 period are.

	Mean return	Standard deviation
'Market index'	0.0046	0.0178
CRSP value-weighted index	0.0085	0.0588
CRSP equally-weighted index	0.0120	0.0830
Government bond index	0.0027	0.0157
Corporate bond index	0.0032	0.0142

[7] I am grateful to Myron Scholes for making this series available The mean monthly return for the 1926–1975 period is 0.0026 and the standard deviation is 0.0021.

derivations of the $\hat{\beta}_i$, $i=0,\ldots,2$, in (4) from their theoretical values also allow us to check whether the grouping procedure is an effective means to eliminate the errors-in-beta problem.

The results are essentially identical for both OLS and GLS and for all three indices. Thus, only one set of results, those for the 'market index' with GLS, is presented in table 1. For each of the gammas, three numbers are reported: the mean of that time series of returns which is relevant for the test of the hypothesis of interest (i.e., whether or not $\hat{\gamma}_0$ and $\hat{\gamma}_1$ are different from the risk-free rate and the risk premium, respectively), the associated t-statistic, and finally, the estimated beta of the time series of the gamma from (4). Note that the means are corrected for the deviation from the theoretical beta as discussed above.

The table shows a significantly negative estimate for γ_2 for the overall time period. Thus, shares of firms with large market values have had smaller returns, on average, than similar small firms. The CAPM appears to be misspecified. The table also shows that γ_0 is different from the risk-free rate. As both Fama (1976, ch. 9) and Roll (1977) have pointed out, if a test does not use the true market portfolio, the Sharpe–Lintner model might be wrongly rejected. The estimates for γ_0 are of the same magnitude as those reported by Fama and MacBeth (1973) and others. The choice of a market index and the econometric method does not affect the results. Thus, at least within the context of this study, the choice of a proxy for the market portfolio does not seem to affect the results and allowing for heteroscedastic disturbances does not lead to significantly more efficient estimators.

Before looking at the results in more detail, some comments on econometric problems are in order. The results in table 1 are based on the 'market index' which is likely to be superior to pure stock indices from a theoretical viewpoint since it includes more assets [Roll (1977)]. This superiority has its price. The actual betas of the time series of the gammas are reported in table 1 in the columns labeled $\hat{\beta}_i$. Recall that the theoretical values of β_0 and β_1 are zero and one, respectively. The standard zero-beta portfolio with return $\hat{\gamma}_0$ contains high beta stocks in short positions and low beta stocks in long positions, while the opposite is the case for the zero-investment portfolio with return $\hat{\gamma}_1$. The actual betas are all significantly different from the theoretical values. This suggests a regression effect, i.e., the past betas of high beta securities are overestimated and the betas of low beta securities are underestimated.[8] Past beta is not completely uncorrelated with the error of the current beta and the instrumental variable approach to the error-in-variables problem is not entirely successful.[9]

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[8] There is no such effect for β_2 because that portfolio has both zero beta and zero investment, i e , net holdings of both high and low beta securities are, on average, zero

[9] This result is first documented in Brenner (1976) who examines the original Fama–McBeth (1973) time series of $\hat{\gamma}_{0t}$

Table 1

Portfolio estimators for γ_0, γ_1 and γ_2 based on the 'market index' with generalized least squares estimation[a]

$$R_{it}=\hat{\gamma}_{0t}+\hat{\gamma}_{1t}\hat{\beta}_{it}+\hat{\gamma}_{2t}[(\phi_{it}-\phi_{mt})/\phi_{mt}]$$

Period	$\hat{\gamma}_0-R_F$	$t(\hat{\gamma}_0-R_F)$	$\hat{\beta}_0$	$\hat{\gamma}_1-(R_M-R_F)$	$t(\hat{\gamma}_1-(R_M-R_F))$	$\hat{\beta}_1$	$\hat{\gamma}_2$	$t(\hat{\gamma}_2)$	$\hat{\beta}_2$
1936–1975	0.00450	2.76	0.45	−0.00092	−1.00	0.75	−0.00052	−2.92	0.01
1936–1955	0.00377	1.66	0.43	−0.00060	−0.80	0.80	−0.00043	−2.12	0.01
1956–1975	0.00531	2.22	0.46	−0.00138	−0.82	0.73	−0.00062	−2.09	0.01
1936–1945	0.00121	0.30	0.63	−0.00098	−0.77	0.82	−0.00075	−2.32	−0.01
1946–1955	0.00650	2.89	0.03	−0.00021	−0.26	0.75	−0.00015	−0.65	0.06
1956–1965	0.00494	2.02	0.34	−0.00098	−0.56	0.96	−0.00039	−1.27	−0.01
1966–1975	0.00596	1.43	0.49	−0.00232	−0.80	0.69	−0.00080	−1.55	0.01

[a] $\hat{\gamma}_0-R_F$ = mean difference between return on zero beta portfolio and risk-free rate, $\hat{\gamma}_1-(R_M-R_F)$ = mean difference between actual risk premium ($\hat{\gamma}_1$) and risk premium stipulated by Sharpe–Lintner model (R_M-R_F) $\hat{\gamma}_2$ = size premium. $\hat{\beta}_i$ = actual estimated market risk of $\hat{\gamma}_i$ (theoretical values. $\beta_0=0$, $\beta_1=1$, $\beta_2=0$), all β_0, β_1 are significantly different from the theoretical values. $t(\;)$ = t-statistic.

The deviations from the theoretical betas are largest for the 'market index', smaller for the CRSP value-weighted index, and smallest for the CRSP equally-weighted index. This is due to two factors: first, even if the true covariance structure is stationary, betas with respect to a value-weighted index change whenever the weights change, since the weighted average of the betas is constrained to be equal to one. Second, the betas and their standard errors with respect to the 'market index' are much larger than for the stock indices (a typical stock beta is between two and three), which leads to larger deviations -- a kind of 'leverage' effect. Thus, the results in table 1 show that the final correction for the deviation of $\hat{\beta}_0$ and $\hat{\beta}_1$ from their theoretical values is of crucial importance for maket proxies with changing weights.

Estimated portfolio betas and portfolio market proportions are (negatively) correlated. It is therefore possible that the errors in beta induce an error in the coefficient of the market proportion. According to Levi (1973), the probability limit of $\hat{\gamma}_1$ in the standard errors-in-the-variables model is

$$\operatorname{plim} \hat{\gamma}_1 = \gamma_1/(1+(\sigma_u^2 \cdot \sigma_2^2)/D) < \gamma_1,$$

with

$$D = (\sigma_1^2 + \sigma_u^2) \cdot \sigma_2^2 - \sigma_{12}^2 > 0,$$

where σ_1^2, σ_2^2 are the variances of the true factors β and ϕ, respectively, σ_u^2 is the variance of the error in beta and σ_{12} is the covariance of β and ϕ. Thus, the bias in $\hat{\gamma}_1$ is unambiguously towards zero for positive γ_1. The probability limit of $\hat{\gamma}_2 - \gamma_2$ is [Levi (1973)]

$$\operatorname{plim} (\hat{\gamma}_2 - \gamma_2) = (\sigma_u^2 \cdot \sigma_{12} \cdot \gamma_1)/D.$$

We find that the bias in $\hat{\gamma}_2$ depends on the covariance between β and ϕ and the sign of γ_1. If σ_{12} has the same sign as the covariance between $\hat{\beta}$ and ϕ, i.e., $\sigma_{12} < 0$, and if $\gamma_1 > 0$, then $\operatorname{plim}(\hat{\gamma}_2 - \gamma_2) < 0$, i.e., $\operatorname{plim} \hat{\gamma}_2 < \gamma_2$. If the grouping procedure is not successful in removing the error in beta, then it is likely that the reported $\hat{\gamma}_2$ overstates the true magnitude of the size effect. If this was a serious problem in this study, the results for the different market indices should reflect the problem. In particular, using the equally-weighted stock index should then lead to the smallest size effect since, as was pointed out earlier, the error in beta problem is apparently less serious for that kind of index. In fact, we find that there is little difference between the estimates.[10]

[10] For the overall time period, $\hat{\gamma}_2$ with the equally-weighted CRSP index is -0.00044, with the value weighted CRSP index -0.00044 as well as opposed to the -0.00052 for the 'market index' reported in table 1. The estimated betas of $\hat{\gamma}_0$ and $\hat{\gamma}_1$ which reflect the degree of the error in beta problems are 0.07 and 0.91, respectively, for the equally-weighted CRSP index and 0.13 and 0.87 for the value-weighted CRSP index.

Thus, it does not appear that the size effect is just a proxy for the unobservable true beta even though the market proportion and the beta of securities are negatively correlated.

The correlation coefficient between the mean market values of the twenty-five portfolios and their betas is significantly negative, which might have introduced a multicollinearity problem. One of its possible consequences is coefficients that are very sensitive to addition or deletion of data. This effect does not appear to occur in this case: the results do not change significantly when five portfolios are dropped from the sample. Revising the grouping procedure — ranking on the basis of beta first, then ranking on the basis of market proportion — also does not lead to substantially different results.

4.2. *A closer look at the results*

An additional factor relevant for asset pricing — the market value of the equity of a firm — has been found. The results are based on a linear model. Linearity was assumed only for convenience and there is no theoretical reason (since there is no model) why the relationship should be linear. If it is nonlinear, the particular form of the relationship might give us a starting point for the discussion of possible causes of the size effect in the next section. An analysis of the residuals of the twenty-five portfolios is the easiest way to look at the linearity question. For each month t, the estimated residual return

$$\hat{\varepsilon}_{it} = R_{it} - \hat{\gamma}_{0t} - \hat{\gamma}_{1t}\hat{\beta}_{it} - \hat{\gamma}_{2t}[(\phi_{it} - \phi_{mt})/\phi_{mt}], \qquad i=1,\ldots,25, \tag{5}$$

is calculated for all portfolios. The mean residuals over the forty-five year sample period are plotted as a function of the mean market proportion in fig. 1. Since the distribution of the market proportions is very skewed, a logarithmic scale is used. The solid line connects the mean residual returns of each size group. The numbers identify the individual portfolios within each group according to beta, '1' being the one with the largest beta, '5' being the one with the smallest beta.

The figure shows clearly that the linear model is misspecified.[11] The residuals are not randomly distributed around zero. The residuals of the portfolios containing the smallest firms are all positive; the remaining ones are close to zero. As a consequence, it is impossible to use $\hat{\gamma}_2$ as a simple size premium in the cross-section. The plot also shows, however, that the misspecification is not responsible for the significance of $\hat{\gamma}_2$ since the linear model underestimates the true size effect present for very small firms. To illustrate this point, the five portfolios containing the smaller firms are

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[11] The nonlinearity cannot be eliminated by defining ϕ_i as the log of the market proportion

deleted from the sample and the parameters reestimated. The results, summarized in table 2, show that the $\hat{\gamma}_2$ remain essentially the same. The relationship is still not linear; the new $\hat{\gamma}_2$ still cannot be used as a size premium.

Fig. 1 suggests that the main effect occurs for very small firms. Further support for this conclusion can be obtained from a simple test. We can regress the returns of the twenty-five portfolios in each result on beta alone and examine the residuals. The regression is misspecified and the residuals contain information about the size effect. Fig. 2 shows the plot of those residuals in the same format as fig. 1. The smallest firms have, on average, very large unexplained mean returns. There is no significant difference between the residuals of the remaining portfolios.

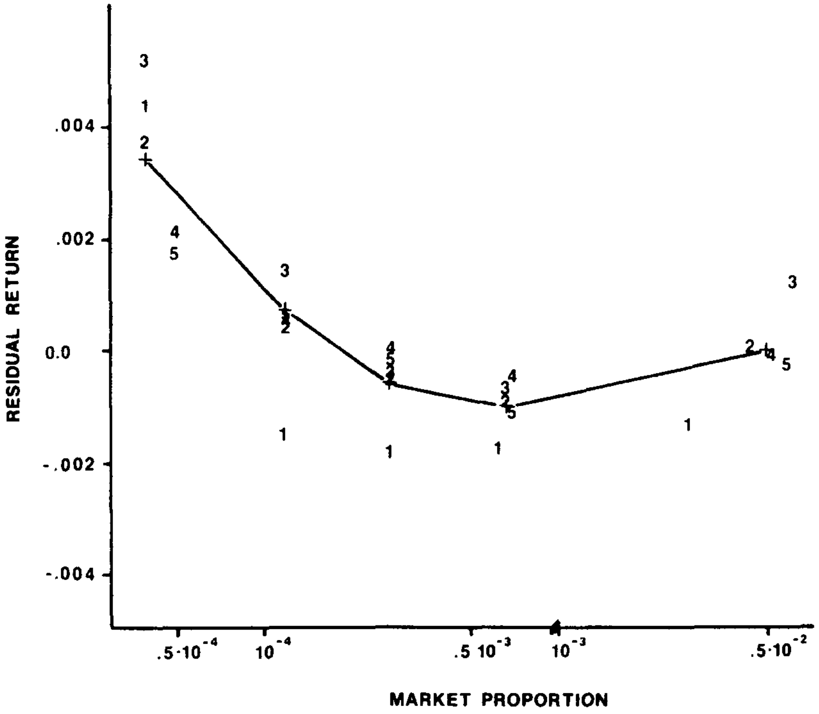

Fig. 1. Mean residual returns of portfolios (1936–1975) with equally-weighted CRSP index as market proxy. The residual is calculated with the three-factor model [eq. (3)]. The numbers 1,...,5 represent the mean residual return for the five portfolios within each size group (1: portfolio with largest beta,. .,5· portfolio with smallest beta) + represents the mean of the mean residuals of the five portfolios with similar market values.

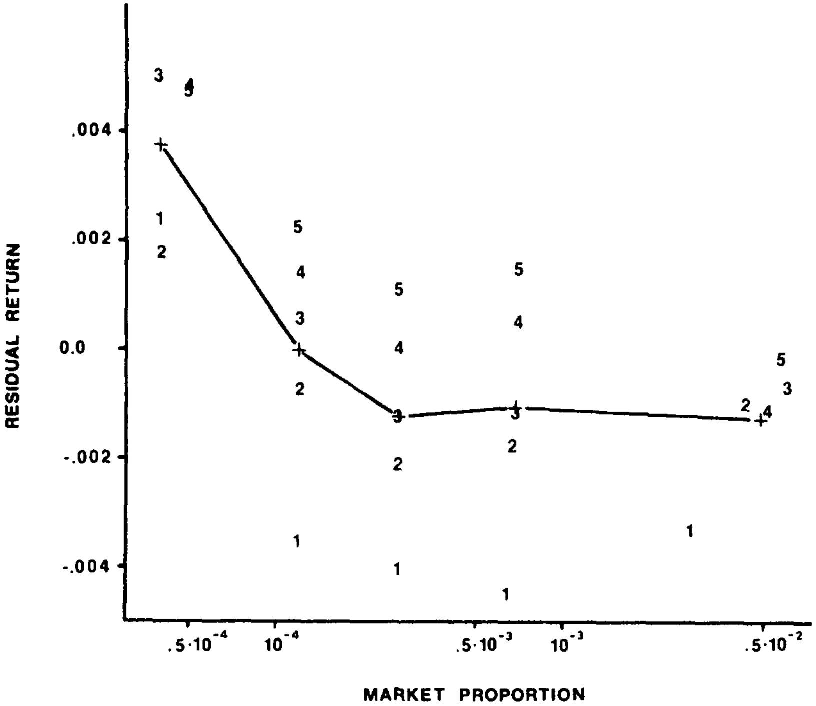

Fig. 2. Mean residual returns of portfolios (1936–1975) with equally-weighted CRSP index as market proxy. The residual is calculated with the two-factor model $(\hat{\varepsilon}_{it} = R_{it} - \hat{\gamma}_{0t} - \hat{\gamma}_{1t}\hat{\beta}_{it})$ The symbols are as defined for fig. 1.

4.3. *'Arbitrage' portfolio returns*

One important empirical question still remains: How important is the size effect from a practical point of view? Fig. 2 suggests that the difference in returns between the smallest firms and the remaining ones is, on average, about 0.4 percent per month. A more dramatic result can be obtained when the securities are chosen solely on the basis of their market value.

As an illustration, consider putting equal dollar amounts into portfolios containing the smallest, largest and median-sized firms at the beginning of a year. These portfolios are to be equally weighted and contain, say, ten, twenty or fifty securities. They are to be held for five years and are rebalanced every month. They are levered or unlevered to have the same beta. We are then interested in the differences in their returns,

$$R_{1t} = R_{st} - R_{lt}, \qquad R_{2t} = R_{st} - R_{at}, \qquad R_{3t} = R_{at} - R_{lt}, \tag{6}$$

Table 2
Portfolio estimators for γ_2 for all 25 portfolios and for 20 portfolios (portfolios containing smallest firms deleted) based on CRSP equally weighted index with generalized least-squares estimation.[a]

	Size premium $\hat{\gamma}_2$ with	
Period	25 portfolios	20 portfolios
1936–1975	−0.00044 (−2.42)	−0 00043 (−2.54)
1936–1955	−0.00037 (−1 72)	−0.00041 (−1.88)
1956–1975	−0.00056 (−1.91)	−0.00050 (−1.91)
1936–1945	−0.00085 (−2.81)	−0.00083 (−2.48)
1946–1955	0.00003 (0.12)	−0.00003 (−0 13)
1956–1965	−0.00023) (−0.81)	−0.00017 (−0.65)
1966–1975	−0.00091 (−1.78)	−0.00085 (−1.84)

[a] *t*-statistic in parentheses

where R_{st}, R_{at} and R_{lt} are the returns on the portfolios containing the smallest, median-sized and largest firms at portfolio formation time (and $R_{1t} = R_{2t} + R_{3t}$). The procedure involves (a) the calculation of the three differences in raw returns in each month and (b) running time series regressions of the differences on the excess returns of the market proxy. The intercept terms of these regressions are then interpreted as the $\bar{R}_i$, $i = 1,\dots,3$. Thus, the differences can be interpreted as 'arbitrage' returns, since, e.g., R_{1t} is the return obtained from holding the smallest firms long and the largest firms short, representing zero net investment in a zero-beta portfolio.[12] Simple equally weighted portfolios are used rather than more sophisticated minimum variance portfolios to demonstrate that the size effect is not due to some quirk in the covariance matrix.

Table 3 shows that the results of the earlier tests are fully confirmed. $\bar{R}_2$, the difference in returns between very small firms and median-size firms, is typically considerably larger than $\bar{R}_3$, the difference in returns between median-sized and very large firms. The average excess return from holding very small firms long and very large firms short is, on average, 1.52 percent

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[12] No *ex post* sample bias is introduced, since monthly rebalancing includes stocks delisted during the five years. Thus, the portfolio size is generally accurate only for the first month of each period

Table 3
Mean monthly returns on 'arbitrage' portfolios.[a]
$R_j - R_k = \hat{\alpha}_i + \hat{\beta}_i(R_m - R_F)$

	$\bar{\alpha}_1$[b]			$\bar{\alpha}_2$[c]			$\bar{\alpha}_3$[d]		
	$n=10$	$n=20$	$n=50$	$n=10$	$n=20$	$n=50$	$n=10$	$n=20$	$n=50$
Overall period									
1931–1975	0.0152 (2.99)	0.0148 (3.53)	0.0101 (3.07)	0.0130 (2.90)	0.0124 (3.56)	0.0089 (3.64)	0.0021 (1.06)	0.0024 (1.41)	0.0012 (0.85)
Five-year subperiods									
1931–1935	0.0589 (2.25)	0.0597 (2.81)	0.0427 (2.35)	0.0462 (1.92)	0.0462 (2.55)	0.0326 (2.46)	0.0127 (1.09)	0.0134 (1.49)	0.0101 (1.42)
1936–1940	0.0201 (0.82)	0.0182 (0.97)	0.0089 (0.67)	0.0118 (0.55)	0.0145 (0.90)	0.0064 (0.65)	0.0084 (1.20)	0.0037 (0.62)	0.0025 (0.49)
1941–1945	0.0430 (2.29)	0.0408 (2.46)	0.0269 (2.17)	0.0381 (2.29)	0.0367 (2.54)	0.0228 (2.02)	0.0049 (1.25)	0.0038 (1.09)	0.0041 (1.68)
1946–1950	−0.0060 (−1.17)	−0.0046 (−0.97)	−0.0036 (−0.97)	−0.0058 (−1.03)	−0.0059 (−1.29)	−0.0029 (−0.83)	−0.0002 (−0.07)	−0.0104 (−0.50)	−0.0007 (−0.38)
1951–1955	−0.0067 (−0.89)	−0.0011 (−0.21)	0.0013 (0.32)	−0.0004 (−0.07)	0.0026 (0.72)	0.0010 (0.39)	−0.0062 (−1.29)	−0.0037 (−0.99)	0.0003 (0.11)
1956–1960	0.0039 (0.67)	0.0008 (0.15)	0.0037 (0.89)	0.0007 (0.14)	−0.0027 (−0.64)	0.0011 (0.45)	0.0031 (0.88)	0.0035 (1.16)	0.0026 (0.97)
1961–1965	0.0131 (1.38)	0.0060 (0.67)	0.0024 (0.31)	0.0096 (1.11)	0.0046 (0.72)	0.0036 (0.77)	0.0035 (0.59)	0.0014 (0.24)	−0.0012 (−0.24)
1966–1970	0.0121 (1.64)	0.0117 (2.26)	0.0077 (1.91)	0.0129 (1.93)	0.0110 (2.71)	0.0071 (2.43)	0.0008 (0.23)	0.0007 (0.22)	0.0006 (0.27)
1971–1975	0.0063 (0.60)	0.0108 (1.23)	0.0098 (1.45)	0.0033 (0.39)	0.0077 (1.18)	0.0083 (1.79)	0.0030 (0.64)	0.0031 (0.72)	0.0015 (0.43)

[a] Equally-weighted portfolios with n securities, adjusted for differences in market risk with respect to CRSP value-weighted index, t-statistics in parentheses.

[b] Small firms held long, large firms held short.

[c] Small firms held long, median-size firms held short.

[d] Median-size firms held long, large firms held short.

per month or 19.8 percent on an annualized basis. This strategy, which suggests very large 'profit opportunities', leaves the investor with a poorly diversified portfolio. A portfolio of small firms has typically much larger residual risk with respect to a value-weighted index than a portfolio of very large firms with the same number of securities [Banz (1978, ch. 3)]. Since the fifty largest firms make up more than 25 percent of the total market value of NYSE stocks, it is not surprising that a larger part of the variation of the return of a portfolio of those large firms can be explained by its relation with the value-weighted market index. Table 3 also shows that the strategy would not have been successful in every five year subperiod. Nevertheless, the magnitude of the size effect during the past forty-five years is such that it is of more than just academic interest.

5. Conclusions

The evidence presented in this study suggests that the CAPM is misspecified. On average, small NYSE firms have had significantly larger risk adjusted returns than large NYSE firms over a forty year period. This size effect is not linear in the market proportion (or the log of the market proportion) but is most pronounced for the smallest firms in the sample. The effect is also not very stable through time. An analysis of the ten year subperiods show substantial differences in the magnitude of the coefficient of the size factor (table 1).

There is no theoretical foundation for such an effect. We do not even know whether the factor is size itself or whether size is just a proxy for one or more true but unknown factors correlated with size. It is possible, however, to offer some conjectures and even discuss some factors for which size is suspected to proxy. Recent work by Reinganum (1980) has eliminated one obvious candidate: the price–earnings (*P*/*E*) ratio.[13] He finds that the *P*/*E*-effect, as reported by Basu (1977), disappears for both NYSE and AMEX stocks when he controls for size but that there is a significant size effect even when he controls for the *P*/*E*-ratio, i.e., the *P*/*E*-ratio effect is a proxy for the size effect and not vice versa. Stattman (1980), who found a significant negative relationship between the ratio of book value and market value of equity and its return, also reports that this relationship is just a proxy for the size effect. Naturally, a large number of possible factors remain to be tested.[14] But the Reinganum results point out a potential problem with some of the existing negative evidence of the efficient market hypothesis. Basu believed to have identified a market inefficiency but his *P*/*E*-effect is

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[13] The average correlation coefficient between *P*/*E*-ratio and market value is only 0 16 for individual stocks for thirty-eight quarters ending in 1978. But for the portfolios formed on the basis of *P*/*E*-ratio, it rises to 0 82 Recall that Basu (1977) used ten portfolios in his study.

[14] E.g., debt–equity ratios, skewness of the return distribution [Kraus and Litzenberger (1976)].

just a proxy for the size effect. Given its longevity, it is not likely that it is due to a market inefficiency but it is rather evidence of a pricing model misspecification. To the extent that tests of market efficiency use data of firms of different sizes and are based on the CAPM, their results might be at least contaminated by the size effect.

One possible explanation involving the size of the firm directly is based on a model by Klein and Bawa (1977). They find that if insufficient information is available about a subset of securities, investors will not hold these securities because of estimation risk, i.e., uncertainty about the true parameters of the return distribution. If investors differ in the amount of information available, they will limit their diversification to different subsets of all securities in the market.[15] It is likely that the amount of information generated is related to the size of the firm. Therefore, many investors would not desire to hold the common stock of very small firms. I have shown elsewhere [Banz (1978, ch. 2)] that securities sought by only a subset of the investors have higher risk-adjusted returns than those considered by all investors. Thus, lack of information about small firms leads to limited diversification and therefore to higher returns for the 'undesirable' stocks of small firms.[16] While this informal model is consistent with the empirical results, it is, nevertheless, just conjecture.

To summarize, the size effect exists but it is not at all clear why it exists. Until we find an answer, it should be interpreted with caution. It might be tempting to use the size effect, e.g., as the basis for a theory of mergers — large firms are able to pay a premium for the stock of small firms since they will be able to discount the same cash flows at a smaller discount rate. Naturally, this might turn out to be complete nonsense if size were to be shown to be just a proxy.

The preceding discussion suggests that the results of this study leave many questions unanswered. Further research should consider the relationship between size and other factors such as the dividend yield effect, and the tests should be expanded to include OTC stocks as well.

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[15] Klein and Bawa (1977, p 102)

[16] A similar result can be obtained with the introduction of fixed holding costs which lead to limited diversification as well. See Brennan (1975), Banz (1978, ch. 2) and Mayshar (1979)

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Busting the Myth About Size

by Vitali Kalesnik, Ph.D., and Noah Beck

Many market participants (including investors, product providers, and analysts alike) assume that, just as value stocks on average outperform growth, small-cap stocks on average outperform large-caps. Unlike value, however, and contrary to popular opinion, there is little solid evidence that stock size affects performance.

A recent Research Affiliates article by Hsu and Kalesnik (2014) concluded that there are at best three factors from which investors can benefit through passive investing: market, value, and low beta. The size premium was conspicuously missing from that short list. In this article we explore empirical evidence behind the size premium in more detail. The summary below offers a preview of our findings. We let the reader examine the evidence and draw his or her own conclusion. In our opinion the preponderance of evidence does not support the existence of a size premium.

We are not arguing that investors should stop investing in small stocks. A portfolio of small stocks offers a certain level of diversification in an investment program dominated by large-stock strategies. Moreover, major anomalies are stronger in the universe of small stocks (likely because small stocks are more prone to mispricing). Thus, small stocks have the potential to serve as an alpha pool for skilled active managers and rules-based strategies that primarily target factors other than size. Nonetheless, we are skeptical that investors will earn a higher return simply by preferring small stocks over large.

Updating the Evidence

Banz (1981) reported that small-cap stocks outperformed large-cap stocks. For the subsequent decade the phenomenon Banz observed was considered a curious anomaly. The situation changed in 1993, when Eugene Fama and Kenneth French suggested that small stocks may expose investors to some undiversifiable risk that warrants a higher required rate of return. At that moment, the size factor took its place alongside the market and value factors in the original Fama–French three-factor model. Carhart (1997) then made the case for momentum as a fourth return factor. Today the most standard equity pricing model used in academia includes four factors: market, value, size, and momentum.

But consider this: What if a large company were split, on paper only, into two small companies? Suppose there is no change in operations, and imagine that one of the small companies booked all the cash flows on even-numbered days of the month, and the other one accounted for all the cash on odd days. In this scenario, it would be most surprising if the small companies both delivered higher returns than the original large company. Yet the size premium is precisely based on the expectation that small-cap stocks will outperform large-cap stocks!

Summary of Findings on the Size Premium

Arguments in Favor:

1. Over the period July 1926 to July 2014, there was a size premium of 3.4% per annum in the United States.
2. The U.S. size premium is statistically significant (with a p-value of 1.7%), *assuming the returns are normally distributed.*
3. In the 30+ years since the publication of Banz's (1981) article , there has been an average size premium of 1.0% per annum across 18 developed markets including the United States.

Arguments Against:

1. There is an upward bias in size premium estimates due to inaccurate returns on delisted stocks in major databases.
2. Indices and hypothetical portfolios ignore trading costs.
3. The statistical significance of the size premium estimates is likely overstated due to data-mining and reporting bias.
4. Even with the biases that favor small stocks, there is no unquestionably significant evidence in support of the size factor.
    - The estimate of the U.S. size premium is dominated by extreme outliers from the 1930s.
    - The assumption of normality used to obtain statistical significance in the U.S. sample is extremely dubious.
    - There is no statistical significance outside the United States.
5. Even with the biases that favor small stocks, there is no risk-adjusted performance advantage attributable to the size factor.

Source: Research Affiliates.

For any reasonable economic theory explaining why small-cap stocks are supposed to outperform large-cap stocks, there is an equally plausible theory explaining why the reverse should be true. The source of the specific risk postulated by Fama and French (1993) was unclear 21 years ago, and it is still murky today. Theoretical explanations for the size premium were provided after researchers observed the anomalous regularity in returns—not the other way around. Today investors believe in the size premium on the basis of empirical evidence, not on theoretical arguments. So let's turn to the evidence with updated data.

Following the methodology employed in Fama and French (2012), we grouped stocks in each country by size into two portfolios. The large stock portfolio consists of the top 90% of the market by market capitalization, and the small stock portfolio consists of the bottom 10% of the market. Stocks within the large and small portfolios are weighted by market capitalization. To measure the premium we looked at the arithmetic difference between the small and large stock portfolio returns. We report in **Table 1** the average annualized returns, volatilities, and *t*-statistics in 18 major developed countries from January 1982 to July 2014. Table 1 also displays data for the United States over the longer period from July 1926 to July 2014.

In the 88-year U.S. sample, the size premium is 3.4% per annum. Assuming a normal distribution of premium estimates (we will discuss later why this assumption may not be warranted), the size premium is statistically significant with a *t*-stat of 2.38, which corresponds to a *p*-value of 1.7%. After 1981, when Banz's paper appeared, the premium is positive in the United States and positive on average in the international sample, but it is not statistically significant anywhere. The substantial, statistically significant average return observed in the long-term U.S. dataset is the main reason why size is popularly believed to be one of the most important factors.

Examining the U.S. Data

Existence of the size premium in the United States is practically an article of faith in the practice of asset management as well as the academic literature. The empirical evidence, however, does not stand up very well to closer scrutiny. The data are doubtful for several reasons, including overestimated small-cap returns due to missing data on delisted stocks; the absence of transaction costs in the calculation of index returns; biases resulting from data-mining and the publishing process; and misestimated statistical measures based on the assumption of normality. In addition, there proves to be no return advantage on a risk-adjusted basis.

Table 1. Size Premium: U.S. and International Evidence

Nation	Average Return (Ann.)	Average Volatility (Ann.)	*t*-stat
Post Publication Period, 1982–2014			
Australia	−1.1%	10.2%	−0.64
Austria	2.0%	13.7%	0.85
Belgium	3.0%	10.7%	1.59
Canada	0.7%	9.2%	0.43
Denmark	−0.2%	13.0%	−0.09
France	2.9%	9.9%	1.67
Germany	−0.5%	10.5%	−0.27
Hong kong	−0.8%	16.5%	−0.26
Ireland	4.9%	18.3%	1.53
Italy	−0.8%	11.0%	−0.39
Japan	3.3%	13.9%	1.36
Netherlands	1.7%	10.8%	0.88
Norway	−0.2%	15.0%	−0.07
Singapore	2.3%	15.6%	0.83
Sweden	0.7%	12.6%	0.34
Switzerland	−2.2%	10.7%	−1.18
United Kingdom	0.8%	9.4%	0.48
United States	1.9%	9.4%	1.15
Equally Weighted Avg. of 18 Countries	1.0%	5.5%	1.05
Full Sample, United States, 1926–2014			
United States	3.4%	13.5%	2.38

Note: Within each country we split stocks into large and small portfolios. Following Fama and French (2012), the portfolio of large stocks comprises 90% of the national market and the small-stock portfolio comprises 10%. Portfolios are capitalization-weighted. The size premium is estimated as the arithmetic average of the differences in return between the small and the large portfolios

Source: Research Affiliates, using CRSP/Compustat and Worldscope/Datastream data.

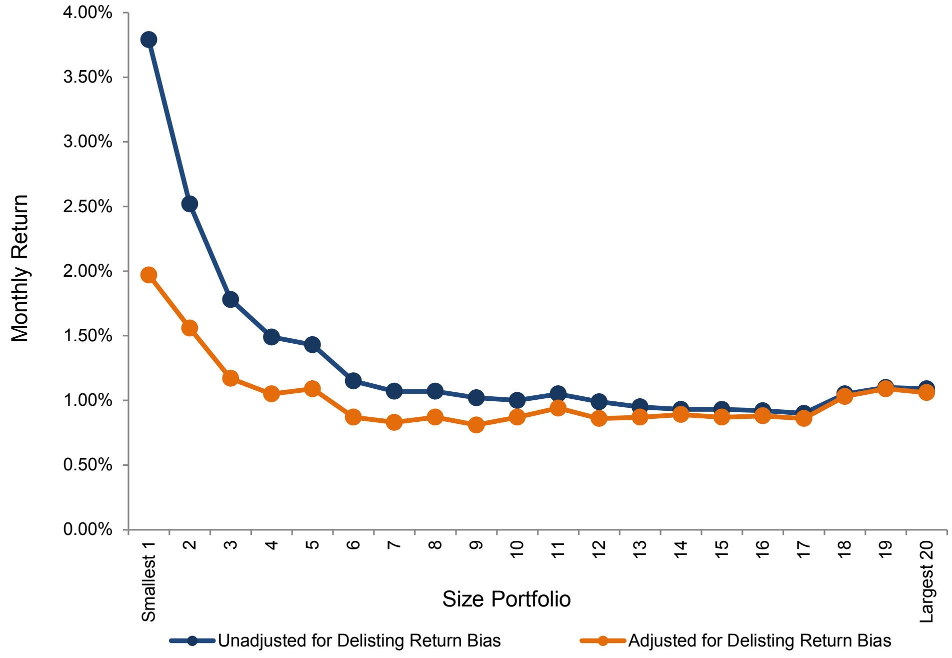
Delisting bias. Shareholders do not necessarily lose the full amount of their investment in a company when it is delisted from a major stock exchange. Often the stock can still be traded in the over-the-counter (OTC) market, and the investor may receive some residual value if the company is liquidated. Nonetheless, returns on stocks after they have been delisted are likely to be very negative. Moreover, all companies are subject to business and financial risks that might result in their stock's falling short of listing requirements, but small stocks by market capitalization are appreciably more likely to be removed from an exchange. Shumway (1997) pointed out that regular performance databases overestimated small-cap stock returns because they did not include returns on delisted stocks. If a database that is used in simulating portfolios omits the strongly negative returns of delisted stocks, the hypothetical results will be better than what actual portfolios can achieve in practice.

To estimate the impact of the delisting bias on the size premium, Shumway and Warther (1999) looked at the smallest and the most distressed stocks for which they could obtain reliable data, namely, stocks listed on the NASDAQ exchange. We represent their findings in **Figure 1**. The chart shows the average monthly returns for 20 groups of stocks sorted by size before and after correcting for the upward bias in the database. Clearly, the smallest stocks are significantly more affected by the delisting bias. After adjusting for the delisting bias, the statistical significance of the size premium completely disappears. It is unreasonable to suppose that the effect Shumway and Warther quantified for NASDAQ stocks is missing from other exchanges.

Transaction costs. Theoretical simulations ignore an important component of investment performance measurement: trading expenses—the actual costs of buying or selling investments. Small stocks by definition have much lower trading capacity and, correspondingly, much higher transaction costs. Soon after the first articles documenting the size effect appeared, researchers asked how much of the premium remains when trading costs are taken into account. Stoll and Whaley (1983) showed that transaction costs accounted for a significant part of the size premium for stocks listed on the New York Stock Exchange and the American Stock Exchange.

Data-mining and reporting bias. There are literally hundreds of known factors in the existing literature, and many papers documenting new factors are published every year. In our opinion the vast majority of these factors are spurious products of data-mining. We are not alone in taking a skeptical position. Lo and MacKinlay (1990), Black (1993), and MacKinlay (1995), among others, have argued that many factors, notably including size, are likely to be a result of data-mining.

Figure 1. Average Stock Returns by Size Group



Source: Research Affiliates, using data from Shumway and Warther (1999).

And, in finance no less than the physical and biological sciences, striking results—especially new discoveries—tend to win the competition for space in academic journals.

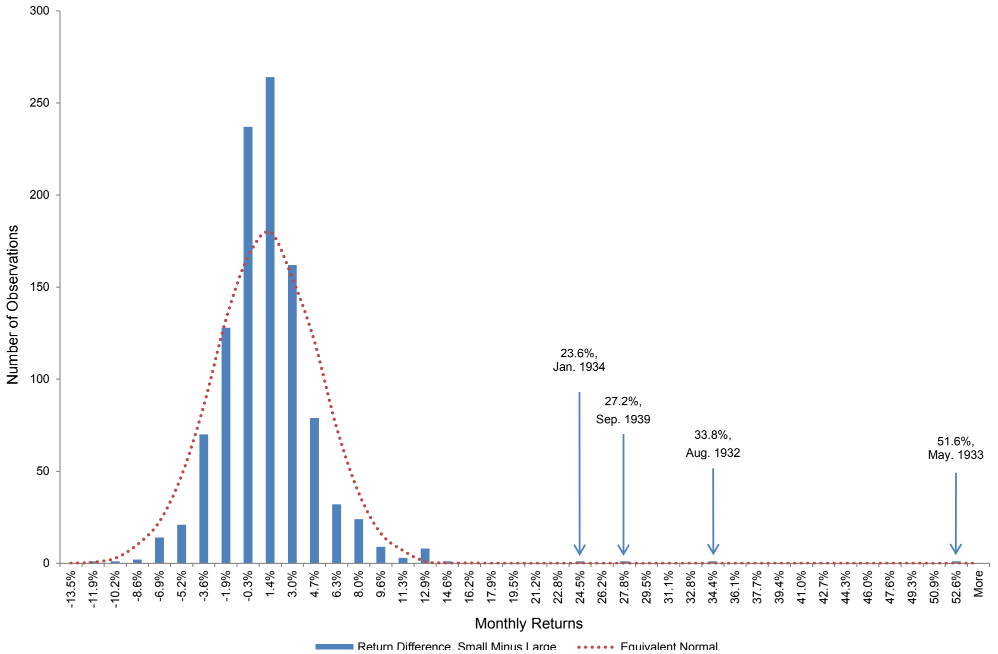
The standard procedure for determining whether a factor is statistically significant is to see if its *t*-stat crosses a certain threshold. Normally the threshold is set at 1.96 for a 5% confidence level. With a *t*-stat of 2.38, the U.S. size premium passes this test for the 1926–2014 sample. But Harvey, Liu, and Zhu (2014) rightly observed that if many researchers are looking for statistical irregularities, then the 1.96 criterion is too low; it allows many inherently random outliers to be misidentified as valid factors. They argue that the threshold for the size factor should have been closer to a *t*-stat of 2.50 in 1993.[1] Size does not pass this test.

Non-normality of returns. Standard statistical testing assumes that the estimate of a variable—in this case, the average of the size premium—quickly converges to a normal distribution.[2] If, however, the underlying data include large outliers, then the assumption of normality is unfounded. The differences between the small and large stock portfolio returns exhibit just such outliers. **Figure 2** is a histogram of the return differences. For comparison, we display on the same chart a normal distribution with the same mean and standard deviation.

We indicate on the chart four extreme outliers of 6 sigma or higher. "Sigma" may be an unfamiliar statistical term, so let us put these outlier returns in perspective. The 23.6% premium registered in January 1934 is a 6-sigma event. If it were drawn from normal distribution, this would be a one-in-67-million-year event, like the one that wiped out the dinosaurs. The 27.2% difference in returns in September 1939 is a 6.9-sigma event; in a normal distribution, it would have about a one-in-five chance of occurring in the 4.5 billion years since the planet earth came into existence. The 33.8% premium in August 1932 is an 8.6-sigma event, and the 51.6% premium in May 1933 is a 13.1-sigma event. If these last two outliers were drawn from a normal distribution, each would have much less than a one-in-a-hundred chance of occurring in the entire 13.8 billion years the universe has existed.

To add to the problem, all four outliers occurred in the 1930s. If they were removed, the estimated size premium in Table 1 would drop from 3.4% to 1.9% and lose statistical significance. (There is a similar outcome in the post-war period: The estimated size premium is about 1.9% premium with a *t*-stat of 1.52.) We do not argue, however, that truncating or otherwise transforming the sample will give us a better estimate. What happened in the 1930s is very valuable information about the economy and the stock market. The average return from the full sample, including the

Figure 2. Distribution of Return Differences



Source: Research Affiliates, using data from Shumway and Warther (1999).

unadjusted outliers, is the best estimate available as long as the statistical bounds around it are borne in mind. If the size premium is predicated on exceedingly rare events, then we'll have to wait many lifetimes to determine with confidence whether or not it exists.

No risk-adjusted benefit. Academics are interested in the arithmetic average returns in a simulated long/short portfolio, but practitioners are concerned with the actual risk-adjusted returns that they can generate from their investments—and the majority do not engage in short-selling. We display in **Table 2** the average geometrically chained cumulative returns of the long-only portfolios of small and large stocks. These results are produced using the same databases we used earlier in this article, so they contain the same biases that we noted above.

Small stocks outperform large stocks in this sample, but, because small stocks are generally more volatile, the Sharpe ratios reveal that small-cap investing provides a miniscule advantage in the risk-adjusted return. If investors are switching from large stocks to small in the hope of a premium, they should realize that they are increasing the volatility, too. The estimates of average returns are very noisy, and are likely overstated due to the biases we described earlier; the estimates of volatility on the other hand are real. (Estimates of the mean are always less certain than estimates of standard deviation.) We suggest that investors seeking higher returns consider boosting their overall equity allocation rather than chasing the illusory size premium in an attempt to add risk on the cheap within the existing allocation. A large-cap stock portfolio would have higher returns than a mix of small-cap stocks and risk-free assets designed to have the same volatility. In other words, the added risk of small-cap stocks is essentially uncompensated. Note that even in the only data set with a statistically significant size premium (i.e., the U.S. full sample from 1926–2014), the Sharpe ratio is actually lower for small stocks.

Concluding Remarks

We placed our inquiry in a historical context, starting with Banz's (1981) paper, because the widespread belief in a size premium is largely a result of its early discovery. Market capitalization data were readily available to early researchers writing doctoral dissertations and journal articles, and, as we have seen, the performance

Table 2. Average Returns on Long-Only Portfolios

	Small Stocks			Large Stocks			Difference		
Nation	Average Return	Average Volatility	Sharpe Ratio	Average Return	Average Volatility	Sharpe Ratio	Average Return	Average Volatility	Sharpe Ratio
				Post Publication Period, 1982–2014					
Australia	10.8%	24.9%	0.26	12.4%	23.4%	0.35	−1.6%	1.5%	−0.08
Austria	13.3%	21.5%	0.42	10.2%	24.4%	0.24	3.1%	−2.9%	0.18
Belgium	15.8%	18.7%	0.62	12.6%	20.3%	0.41	3.2%	−1.6%	0.21
Canada	11.2%	21.4%	0.33	11.1%	18.7%	0.37	0.1%	2.7%	−0.04
Denmark	12.1%	20.1%	0.39	12.6%	19.4%	0.43	−0.4%	0.7%	−0.04
France	15.7%	20.5%	0.56	12.5%	21.0%	0.39	3.2%	−0.5%	0.17
Germany	11.0%	18.4%	0.36	11.0%	21.4%	0.31	0.0%	−3.0%	0.05
Hong kong	10.6%	31.9%	0.20	12.5%	29.2%	0.28	−1.9%	2.7%	−0.08
Ireland	18.3%	23.6%	0.60	12.6%	23.8%	0.35	5.7%	−0.2%	0.24
Italy	8.1%	23.6%	0.16	8.7%	24.9%	0.18	−0.6%	−1.3%	−0.02
Japan	9.3%	23.8%	0.21	6.4%	21.8%	0.10	2.9%	2.0%	0.11
Netherlands	14.7%	20.0%	0.52	13.1%	19.0%	0.46	1.6%	1.0%	0.06
Norway	13.6%	24.9%	0.38	13.3%	25.9%	0.35	0.2%	−1.0%	0.02
Singapore	10.1%	31.7%	0.19	9.6%	24.3%	0.22	0.5%	7.3%	−0.03
Sweden	14.8%	24.7%	0.42	13.8%	24.9%	0.39	0.9%	−0.2%	0.04
Switzerland	11.0%	17.9%	0.38	13.5%	17.3%	0.53	−2.5%	0.6%	−0.16
United Kingdom	11.8%	19.8%	0.38	11.5%	17.7%	0.41	0.3%	2.1%	−0.03
United States	13.3%	19.1%	0.48	12.0%	15.2%	0.51	1.3%	3.9%	−0.04
Arithmetic average:	12.5%	22.6%	0.38	11.6%	21.8%	0.35	0.9%	0.8%	0.03
				Full Sample, United States, 1926–2014					
United States	11.8%	27.2%	0.31	9.8%	18.4%	0.34	2.1%	8.7%	−0.03

Note: Within each country we split stocks into capitalization-weighted large and small portfolios. Following Fama and French (2012), the large stock portfolio comprises 90% of the national market, and the small stock portfolio, 10%. The returns shown are the geometric average returns of the small and large stock portfolios. The difference columns represent the simple differences of the geometric average return, volatility, and Sharpe ratios.

Source: Research Affiliates, using CRSP/Compustat and Worldscope/Datastream data.

of small stocks was exceptional in the 1930s. Eugene Fama was one of Rolf Banz's professors at the University of Chicago; in fact, as a member of Banz's dissertation committee, he was intimately familiar with Banz's research on the small-cap anomaly.[3] Fama and Kenneth French included the size premium in their influential three-factor model, an analytical advance that opened the gate for empirical research into studying factors previously unexplained by then-existing theories. Riding on the popularity of the Fama–French theory, the size premium was soon entrenched in the pantheon of risk factors.

Berk (1997) argued that the size premium observed in the data is nothing more than a poor way of value investing. Value investing relies on buying cheaply priced companies as measured by a ratio of price to company fundamentals. Investing based on size, measured by company market capitalization, would use only the price side of the valuation measure. Because it would therefore use only a fraction of the relevant information, the strategy is significantly weaker than a value strategy that uses prices as they relate to company fundamentals. In our view, Berk's argument is, to date, the strongest explanation why the size premium is observed.

However, we go one step further. If Berk questioned the size premium as a separate factor, we question the size premium as a phenomenon. Today, more than 30 years after the initial publication of Banz's paper, the empirical evidence is extremely weak even before adjusting for possible biases. The return premium is not statistically significant in any of the international markets, whether taken alone or in combination. The U.S. long-term size premium is driven by the extreme outliers, which occurred three-quarters of a century ago. These extreme outliers confound the standard techniques of setting confidence bounds around the estimated premium. Finally, adjusting for biases, most notably the delisting bias, makes the size premium vanish. If the size premium were discovered today, rather than in the 1980s, it would be challenging to even publish a paper documenting that small stocks outperform large ones. All this evidence makes us question the existence of the size premium as such.

We are not arguing that investors should completely abandon small stocks. Small stocks are more volatile than large stocks, and they receive considerably less attention from sell-side analysts. Consequently, small stocks are more likely to be mispriced. The major anomalies are, in fact, stronger in the small-cap sector. Small stocks are more attractive as an alpha pool to be fished by skillful active managers and exploited by rules-based value and momentum strategies.

Endnotes

1. The authors argue further that "a newly discovered factor today should have a *t*-ratio that exceeds 3.0." Page 35.
2. This result relies on the central limit theorem, which says that, as the number of random observations increases, the arithmetic average converges to a normal distribution. If the observations include extreme outliers, the convergence can be either extremely slow or may not occur at all.
3. Fox (2009), page 204.

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				EPS Growth Rates			Market	Weighted		Weighted			Weighted			Weighted		
			Dividend	Value			Cap	Dividend Yield		Value Line			IBES			Zacks		
	Company	Ticker	Yield	Line	IBES	Zacks	($Millions)	Weight	Product	Mkt. Cap.	Weight	Product	Mkt. Cap.	Weight	Product	Mkt. Cap.	Weight	Product
	(a)		(b)	(c)	(d)	(b)	(b)											
1	3M Company	MMM	2.5%	8.5%	8.91%	9.67%	113,478	0.006375	0.000158	113,478	0.006670	0.000567	113,478	0.006557	0.000584	113,478	0.006375	0.000616
2	Abbott Laboratories	ABT	2.4%	7.5%	11.24%	10.77%	75,462	0.004239	0.000103	75,462	0.004436	0.000333	75,462	0.004361	0.000490	75,462	0.004239	0.000457
3	AbbVie Inc.	ABBV	3.9%	11.5%	14.61%	14.89%	103,710	0.005826	0.000229	103,710	0.006096	0.000701	103,710	0.005993	0.000876	103,710	0.005826	0.000868
4	Accenture PLC	ACN	2.1%	8.0%	9.42%	10.33%	72,432	0.004069	0.000084	72,432	0.004258	0.000341	72,432	0.004186	0.000394	72,432	0.004069	0.000420
5	Activision Blizzard, Inc	ATVI	0.6%	8.5%	16.13%	16.19%	37,111	0.002085	0.000013	37,111	0.002181	0.000185	37,111	0.002145	0.000346	37,111	0.002085	0.000338
6	Acuity Brands Inc	AYI	0.3%	19.0%	20.00%	19.67%	7,609	0.000427	0.000001	7,609	0.000447	0.000085	7,609	0.000440	0.000088	7,609	0.000427	0.000084
7	Advance Auto Parts Inc	AAP	0.2%	11.5%	12.52%	10.29%	10,524	0.000591	0.000001	10,524	0.000619	0.000071	10,524	0.000608	0.000076	10,524	0.000591	0.000061
8	Aetna Inc.	AET	0.8%	8.0%	13.28%	11.25%	45,313	0.002546	0.000020	45,313	0.002664	0.000213	45,313	0.002618	0.000348	45,313	0.002546	0.000286
9	Affiliated Managers Grou	AMG	0.5%	8.5%	14.01%	12.82%	9,092	0.000511	0.000003	9,092	0.000534	0.000045	9,092	0.000525	0.000074	9,092	0.000511	0.000065
10	Aflac Incorporated	AFL	2.4%	4.0%	6.87%	5.00%	29,266	0.001644	0.000039	29,266	0.001720	0.000069	29,266	0.001691	0.000116	29,266	0.001644	0.000082
11	Agilent Technologies, Inc	A	1.0%	6.5%	10.43%	9.65%	17,130	0.000962	0.000010	17,130	0.001007	0.000065	17,130	0.000990	0.000103	17,130	0.000962	0.000093
12	Air Products and Chemic	APD	2.8%	10.5%	-0.30%	8.83%	29,416	0.001652	0.000046	29,416	0.001729	0.000182	29,416	0.001700	(0.000005)	29,416	0.001652	0.000146
13	Alaska Air Group, Inc.	ALK	1.3%	13.0%	10.00%	11.37%	11,062	0.000621	0.000008	11,062	0.000650	0.000085	11,062	0.000639	0.000064	11,062	0.000621	0.000071
14	Albemarle Corporation	ALB	1.2%	11.0%	9.08%	13.80%	11,743	0.000660	0.000008	11,743	0.000690	0.000076	11,743	0.000679	0.000062	11,743	0.000660	0.000091
15	Alexandria Real Estate Ec	ARE	3.0%	NA	0.10%	6.09%	10,205	0.000573	0.000017	--	--	--	10,205	0.000590	0.000001	10,205	0.000573	0.000035
16	Allegion PLC	ALLE	0.9%	10.0%	13.00%	13.53%	7,132	0.000401	0.000003	7,132	0.000419	0.000042	7,132	0.000412	0.000054	7,132	0.000401	0.000054
17	Allergan PLC.	AGN	0.3%	10.0%	13.81%	13.42%	79,983	0.004493	0.000013	79,983	0.004702	0.000470	79,983	0.004622	0.000638	79,983	0.004493	0.000603
18	Alliance Data Systems Cc	ADS	0.8%	10.0%	12.29%	13.80%	14,169	0.000796	0.000007	14,169	0.000833	0.000083	14,169	0.000819	0.000101	14,169	0.000796	0.000110
19	Alliant Energy Corporati	LNT	3.2%	6.5%	6.35%	5.50%	9,078	0.000510	0.000016	9,078	0.000534	0.000035	9,078	0.000525	0.000033	9,078	0.000510	0.000028
20	Allstate Corporation (The	ALL	1.8%	5.5%	14.85%	8.00%	29,736	0.001670	0.000030	29,736	0.001748	0.000096	29,736	0.001718	0.000255	29,736	0.001670	0.000134
21	Altria Group	MO	3.4%	9.5%	7.87%	7.28%	138,513	0.007781	0.000266	138,513	0.008142	0.000773	138,513	0.008004	0.000630	138,513	0.007781	0.000566
22	Ameren Corporation	AEE	3.2%	6.0%	5.85%	6.50%	13,275	0.000746	0.000024	13,275	0.000780	0.000047	13,275	0.000767	0.000045	13,275	0.000746	0.000048
23	American Electric Power	AEP	3.5%	4.0%	2.40%	5.63%	33,250	0.001868	0.000065	33,250	0.001954	0.000078	33,250	0.001921	0.000046	33,250	0.001868	0.000105
24	American Express Compa	AXP	1.6%	4.5%	6.54%	8.67%	70,122	0.003939	0.000065	70,122	0.004122	0.000185	70,122	0.004052	0.000265	70,122	0.003939	0.000342
25	American International G	AIG	2.1%	10.0%	46.70%	10.67%	60,576	0.003403	0.000070	60,576	0.003561	0.000356	60,576	0.003500	0.001635	60,576	0.003403	0.000363
26	American Tower Corpora	AMT	1.9%	11.0%	20.09%	13.15%	52,096	0.002927	0.000056	52,096	0.003062	0.000337	52,096	0.003010	0.000605	52,096	0.002927	0.000385
27	American Water Works	AWK	1.9%	8.0%	7.40%	7.39%	13,852	0.000778	0.000015	13,852	0.000814	0.000065	13,852	0.000800	0.000059	13,852	0.000778	0.000058
28	AmerisourceBergen Corp	ABC	1.7%	10.0%	9.34%	10.09%	19,042	0.001070	0.000018	19,042	0.001119	0.000112	19,042	0.001100	0.000103	19,042	0.001070	0.000108
29	Amgen Inc.	AMGN	2.8%	7.5%	6.57%	6.49%	119,490	0.006713	0.000191	119,490	0.007024	0.000527	119,490	0.006905	0.000454	119,490	0.006713	0.000436
30	Amphenol Corporation	APH	0.9%	8.0%	7.04%	6.42%	21,515	0.001209	0.000011	21,515	0.001265	0.000101	21,515	0.001243	0.000088	21,515	0.001209	0.000078
31	AMTEK, Inc.	AME	0.7%	6.0%	10.67%	11.81%	12,469	0.000700	0.000005	12,469	0.000733	0.000044	12,469	0.000721	0.000077	12,469	0.000700	0.000083
32	Anadarko Petroleum Cor	APC	0.3%	NA	8.40%	9.33%	35,137	0.001974	0.000006	--	--	--	35,137	0.002030	0.000171	35,137	0.001974	0.000184
33	Analog Devices, Inc.	ADI	2.2%	16.0%	10.31%	10.40%	29,324	0.001647	0.000037	29,324	0.001724	0.000276	29,324	0.001695	0.000175	29,324	0.001647	0.000171
34	Anthem, Inc.	ANTM	1.6%	9.0%	11.06%	10.10%	43,956	0.002469	0.000039	43,956	0.002584	0.000233	43,956	0.002540	0.000281	43,956	0.002469	0.000249
35	Aon PLC	AON	1.1%	12.0%	8.97%	11.25%	31,352	0.001761	0.000020	31,352	0.001843	0.000221	31,352	0.001812	0.000163	31,352	0.001761	0.000198
36	Apache Corporation	APA	1.9%	7.0%	-10.60%	6.00%	20,144	0.001132	0.000021	20,144	0.001184	0.000083	20,144	0.001164	(0.000123)	20,144	0.001132	0.000068
37	Apartment Investment ar	AIV	3.2%	NA	7.10%	7.06%	7,044	0.000396	0.000013	--	--	--	7,044	0.000407	0.000029	7,044	0.000396	0.000028
38	Apple Inc.	AAPL	1.6%	10.0%	9.25%	9.86%	753,718	0.042342	0.000673	753,718	0.044304	0.004430	753,718	0.043555	0.004029	753,718	0.042342	0.004175
39	Applied Materials, Inc.	AMAT	1.0%	23.0%	21.62%	15.52%	41,660	0.002340	0.000024	41,660	0.002449	0.000563	41,660	0.002407	0.000520	41,660	0.002340	0.000363
40	Archer-Daniels-Midland	ADM	2.8%	5.0%	13.29%	20.29%	25,823	0.001451	0.000041	25,823	0.001518	0.000076	25,823	0.001492	0.000198	25,823	0.001451	0.000294
41	Arthur J. Gallagher & Co.	AJG	2.8%	13.5%	11.32%	9.30%	10,122	0.000569	0.000016	10,122	0.000595	0.000080	10,122	0.000585	0.000066	10,122	0.000569	0.000053
42	AT&T Inc.	T	4.8%	5.5%	7.40%	4.80%	249,348	0.014008	0.000677	249,348	0.014657	0.000806	249,348	0.014409	0.001066	249,348	0.014008	0.000672
43	Automatic Data Processir	ADP	2.2%	10.0%	11.52%	10.40%	45,676	0.002566	0.000057	45,676	0.002685	0.000268	45,676	0.002639	0.000304	45,676	0.002566	0.000267
44	AvalonBay Communities	AVB	3.1%	NA	4.73%	7.37%	25,548	0.001435	0.000044	--	--	--	25,548	0.001476	0.000070	25,548	0.001435	0.000106
45	Avery Dennison Corpora	AVY	2.1%	9.0%	10.48%	7.00%	7,090	0.000398	0.000008	7,090	0.000417	0.000038	7,090	0.000410	0.000043	7,090	0.000398	0.000028
46	Baker Hughes Incorporat	BHI	1.1%	NA	45.90%	7.00%	25,921	0.001456	0.000016	--	--	--	25,921	0.001498	0.000688	25,921	0.001456	0.000102
47	Ball Corporation	BLL	0.7%	14.5%	8.97%	12.00%	12,675	0.000712	0.000005	12,675	0.000745	0.000108	12,675	0.000732	0.000066	12,675	0.000712	0.000085
48	Bank of America Corpora	BAC	1.3%	17.0%	10.84%	8.50%	233,184	0.013100	0.000169	233,184	0.013707	0.002330	233,184	0.013475	0.001461	233,184	0.013100	0.001113
49	Bank Of New York Mello	BK	1.6%	10.5%	11.20%	8.00%	48,996	0.002752	0.000044	48,996	0.002880	0.000302	48,996	0.002831	0.000317	48,996	0.002752	0.000220
50	Baxter International Inc.	BAX	1.0%	-2.5%	14.43%	12.58%	28,469	0.001599	0.000016	28,469	0.001673	(0.000042)	28,469	0.001645	0.000237	28,469	0.001599	0.000201
51	BB&T Corporation	BBT	2.7%	7.5%	5.28%	6.14%	35,825	0.002013	0.000055	35,825	0.002106	0.000158	35,825	0.002070	0.000109	35,825	0.002013	0.000124
52	Becton, Dickinson and Cc	BDX	1.6%	9.0%	10.16%	9.94%	39,032	0.002193	0.000035	39,032	0.002294	0.000206	39,032	0.002256	0.000229	39,032	0.002193	0.000218
53	Bed Bath & Beyond Inc.	BBBY	1.3%	2.5%	6.33%	7.25%	5,874	0.000330	0.000004	5,874	0.000345	0.000009	5,874	0.000339	0.000021	5,874	0.000330	0.000024
54	Best Buy Co., Inc.	BBY	2.8%	9.5%	10.97%	10.54%	14,834	0.000833	0.000024	14,834	0.000872	0.000083	14,834	0.000857	0.000094	14,834	0.000833	0.000088
55	BlackRock, Inc.	BLK	2.6%	7.5%	11.60%	11.41%	62,320	0.003501	0.000091	62,320	0.003663	0.000275	62,320	0.003601	0.000418	62,320	0.003501	0.000399
56	Boeing Company (The)	BA	3.2%	7.0%	14.70%	13.00%	107,856	0.006059	0.000194	107,856	0.006340	0.000444	107,856	0.006233	0.000916	107,856	0.006059	0.000788
57	BorgWarner Inc.	BWA	1.5%	6.0%	7.63%	8.60%	8,086	0.000454	0.000007	8,086	0.000475	0.000029	8,086	0.000467	0.000036	8,086	0.000454	0.000039
58	Boston Properties, Inc.	BXP	2.2%	NA	2.69%	5.72%	20,700	0.001163	0.000026	--	--	--	20,700	0.001196	0.000032	20,700	0.001163	0.000067
59	Bristol-Myers Squibb Cor	BMY	2.9%	16.5%	13.62%	11.32%	87,795	0.004932	0.000145	87,795	0.005161	0.000852	87,795	0.005073	0.000691	87,795	0.004932	0.000558
60	Broadcom Limited	AVGO	1.9%	19.5%	15.00%	13.60%	87,296	0.004904	0.000092	87,296	0.005131	0.001001	87,296	0.005045	0.000757	87,296	0.004904	0.000667
61	C.H. Robinson Worldwid	CHRW	2.4%	6.5%	7.28%	9.40%	10,815	0.000608	0.000014	10,815	0.000636	0.000041	10,815	0.000625	0.000045	10,815	0.000608	0.000057
62	C.R. Bard, Inc.	BCR	0.4%	10.5%	11.35%	10.40%	18,008	0.001012	0.000004	18,008	0.001059	0.000111	18,008	0.001041	0.000118	18,008	0.001012	0.000105
63	CA Inc.	CA	3.2%	6.5%	0.60%	5.00%	13,292	0.000747	0.000024	13,292	0.000781	0.000051	13,292	0.000768	0.000005	13,292	0.000747	0.000037
64	Cabot Oil & Gas Corpora	COG	0.3%	32.5%	NA	30.00%	11,633	0.000654	0.000002	11,633	0.000684	0.000222	--	--	--	11,633	0.000654	0.000196
65	Campbell Soup Company	CPB	2.5%	5.5%	4.71%	5.34%	17,225	0.000968	0.000024	17,225	0.001012	0.000056	17,225	0.000995	0.000047	17,225	0.000968	0.000052
66	Capital One Financial Co	COF	1.9%	2.5%	7.08%	8.32%	40,658	0.002284	0.000043	40,658	0.002390	0.000060	40,658	0.002350	0.000166	40,658	0.002284	0.000190
67	Cardinal Health, Inc.	CAH	2.2%	13.0%	9.21%	9.36%	26,091	0.001466	0.000032	26,091	0.001534	0.000199	26,091	0.001508	0.000139	26,091	0.001466	0.000137
68	Carnival Corporation	CCL	2.4%	15.5%	13.46%	13.97%	31,646	0.001778	0.000042	31,646	0.001860	0.000288	31,646	0.001829	0.000246	31,646	0.001778	0.000248
69	Caterpillar, Inc.	CAT	3.2%	4.5%	26.17%	9.50%	56,197	0.003157	0.000101	56,197	0.003303	0.000149	56,197	0.003247	0.000850	56,197	0.003157	0.000300
70	CBOE Holdings, Inc.	CBOE	1.3%	9.5%	19.08%	16.05%	8,986	0.000505	0.000006	8,986	0.000528	0.000050	8,986	0.000519	0.000099	8,986	0.000505	0.000081
71	CBS Corporation	CBS	1.1%	13.0%	16.13%	13.37%	28,209	0.001585	0.000017	28,209	0.001658	0.000216	28,209	0.001630	0.000263	28,209	0.001585	0.000212
72	CenterPoint Energy, Inc.	CNP	3.8%	6.0%	6.07%	5.00%	12,024	0.000675	0.000026	12,024	0.000707	0.000042	12,024	0.000695	0.000042	12,024	0.000675	0.000034
73	CenturyLink, Inc.	CTL	8.8%	10.5%	0.03%	-2.65%	13,369	0.000751	0.000066	13,369	0.000786	0.000083	13,369	0.000773	0.000000	13,369	0.000751	(0.000020)
74	CF Industries Holdings, I	CF	4.1%	10.0%	NA	-1.23%	6,916	0.000389	0.000016	6,916	0.000407	0.000041	--	--	--	6,916	0.000389	(0.000005)
75	Chevron Corporation	CVX	4.0%	3.0%	NA	7.00%	206,897	0.011623	0.000459	206,897	0.012162	0.000365	--	--	--	206,897	0.011623	0.000814
76	Church & Dwight Compa	CHD	1.5%	7.0%	7.73%	8.92%	12,641	0.000710	0.000011	12,641	0.000743	0.000052	12,641	0.000730	0.000056	12,641	0.000710	0.000063
77	Cigna Corporation	CI	0.0%	11.5%	13.17%	11.94%	38,811	0.002180	0.000001	38,811	0.002281	0.000262	38,811	0.002243	0.000295	38,811	0.002180	0.000260
78	Cintas Corporation	CTAS	1.1%	8.0%	12.00%	10.50%	12,961	0.000728	0.000008	12,961	0.000762	0.000061	12,961	0.000749	0.000090	12,961	0.000728	0.000076
79	Cisco Systems, Inc.	CSCO	3.5%	7.0%	10.04%	7.59%	165,660	0.009306	0.000327	165,660	0.009738	0.000682	165,660	0.009573	0.000961	165,660	0.009306	0.000706
80	Citigroup Inc.	C	1.1%	11.5%	7.26%	9.15%	165,588	0.009302	0.000100	165,588	0.009733	0.001119	165,588	0.009569	0.000695	165,588	0.009302	0.000851
81	Citizens Financial Group,	CFG	1.6%	NA	16.41%	19.00%	17,786	0.000999	0.000016	--	--	--	17,786	0.001028	0.000169	17,786	0.000999	0.000190
82	Clorox Company (The)	CLX	2.4%	7.5%	6.97%	6.99%	17,053	0.000958	0.000023	17,053	0.001002	0.000075	17,053	0.000985	0.000069	17,053	0.000958	0.000067
83	CME Group Inc.	CME	2.2%	10.0%	8.92%	10.00%	40,014	0.002248	0.000050	40,014	0.002352	0.000235	40,014	0.002312	0.000206	40,014	0.002248	0.000225
84	CMS Energy Corporation	CMS	3.0%	6.5%	7.45%	6.00%	12,629	0.000709	0.000021	12,629	0.000742	0.000048	12,629	0.000730	0.000054	12,629	0.000709	0.000043
85	Coach, Inc.	COH	3.4%	9.5%	9.92%	10.64%	11,236	0.000631	0.000021	11,236	0.000660	0.000063	11,236	0.000649	0.000064	11,236	0.000631	0.000067
86	Coca-Cola Company (The	KO	3.5%	4.0%	3.01%	5.74%	183,140	0.010288	0.000357	183,140	0.010765	0.000431	183,140	0.010583	0.000319	183,140	0.010288	0.000591
87	Colgate-Palmolive Comp	CL	2.1%	12.0%	8.73%	9.30%	64,662	0.003632	0.000077	64,662	0.003801	0.000456	64,662	0.003737	0.000326	64,662	0.003632	0.000338
88	Comcast Corporation	CMCSA	1.7%	11.0%	11.74%	10.41%	181,179	0.010178	0.000168	181,179	0.010650	0.001171	181,179	0.010470	0.001229	181,179	0.010178	0.001060
89	Comerica Incorporated	CMA	1.4%	10.5%	22.27%	8.00%	11,991	0.000674	0.000009	11,991	0.000705	0.000074	11,991	0.000693	0.000154	11,991	0.000674	0.000054
90	ConAgra Foods Inc.	CAG	2.0%	1.0%	12.95%	8.00%	17,224	0.000968	0.000019	17,224	0.001012	0.000010	17,224	0.000995	0.000129	17,224	0.000968	0.000077
91	ConocoPhillips	COP	2.1%	9.5%	-39.00%	11.86%	61,618	0.003461	0.000074	61,618	0.003622	0.000344	61,618	0.003561	(0.001389)	61,618	0.003461	0.000411
92	Consolidated Edison Inc	ED	3.5%	3.0%	3.50%	3.07%	23,912	0.001343	0.000047	23,912	0.001406	0.000042	23,912	0.001382	0.000048	23,912	0.001343	0.000041
93	Constellation Brands Inc	STZ	0.9%	16.5%	15.96%	19.03%	33,677	0.001892	0.000018	33,677	0.001980	0.000327	33,677	0.001946	0.000311	33,677	0.001892	0.000360
94	Cooper Companies, Inc. (	COO	0.0%	16.5%	13.00%	11.60%	9,642	0.000542	0.000000	9,642	0.000567	0.000094	9,642	0.000557	0.000072	9,642	0.000542	0.000063

95	Corning Incorporated	GLW	2.3%	10.5%	8.90%	6.73%	24,826	0.001395	0.000032	24,826	0.001459	0.000153	24,826	0.001435	0.000128	24,826	0.001395	0.000094
96	Costco Wholesale Corpor	COST	1.1%	10.0%	10.09%	10.71%	74,629	0.004192	0.000044	74,629	0.004387	0.000439	74,629	0.004313	0.000435	74,629	0.004192	0.000449
97	Coty Inc.	COTY	2.9%	8.0%	-5.00%	9.52%	12,918	0.000726	0.000021	12,918	0.000759	0.000061	12,918	0.000746	(0.000037)	12,918	0.000726	0.000069
98	Crown Castle Internation	CCI	4.0%	8.5%	21.15%	2.45%	34,105	0.001916	0.000077	34,105	0.002005	0.000170	34,105	0.001971	0.000417	34,105	0.001916	0.000047
99	CSRA Inc.	CSRA	1.4%	NA	10.00%	10.00%	4,572	0.000257	0.000004	--	--	--	4,572	0.000264	0.000026	4,572	0.000257	0.000026
100	CSX Corporation	CSX	1.5%	6.5%	12.02%	13.75%	44,042	0.002474	0.000037	44,042	0.002589	0.000168	44,042	0.002545	0.000306	44,042	0.002474	0.000340
101	Cummins Inc.	CMI	2.7%	5.5%	NA	9.84%	25,192	0.001415	0.000039	25,192	0.001481	0.000081	--	--	--	25,192	0.001415	0.000139
102	CVS Health Corporation	CVS	2.6%	9.5%	11.66%	9.81%	79,766	0.004481	0.000117	79,766	0.004689	0.000445	79,766	0.004609	0.000537	79,766	0.004481	0.000440
103	D.R. Horton, Inc.	DHI	1.2%	11.0%	9.40%	10.97%	12,431	0.000698	0.000008	12,431	0.000731	0.000080	12,431	0.000718	0.000068	12,431	0.000698	0.000077
104	D/B/A Chubb Limited Ne	CB	2.1%	7.5%	5.03%	10.00%	64,297	0.003612	0.000074	64,297	0.003779	0.000283	64,297	0.003715	0.000187	64,297	0.003612	0.000361
105	Danaher Corporation	DHR	0.7%	8.0%	9.48%	11.84%	59,776	0.003358	0.000022	59,776	0.003514	0.000281	59,776	0.003454	0.000327	59,776	0.003358	0.000398
106	Darden Restaurants, Inc.	DRI	2.7%	14.5%	11.72%	10.41%	10,367	0.000582	0.000016	10,367	0.000609	0.000088	10,367	0.000599	0.000070	10,367	0.000582	0.000061
107	Deere & Company	DE	2.2%	5.0%	20.06%	7.58%	35,151	0.001975	0.000043	35,151	0.002066	0.000103	35,151	0.002031	0.000407	35,151	0.001975	0.000150
108	Delphi Automotive PLC	DLPH	1.5%	14.0%	12.17%	13.54%	20,229	0.001136	0.000018	20,229	0.001189	0.000166	20,229	0.001169	0.000142	20,229	0.001136	0.000154
109	Delta Air Lines, Inc.	DAL	1.8%	11.5%	NA	9.44%	33,075	0.001858	0.000033	33,075	0.001944	0.000224	--	--	--	33,075	0.001858	0.000175
110	DENTSPLY SIRONA Inc.	XRAY	0.6%	8.5%	8.39%	9.41%	14,479	0.000813	0.000005	14,479	0.000851	0.000072	14,479	0.000837	0.000070	14,479	0.000813	0.000077
111	Devon Energy Corporatio	DVN	0.6%	4.5%	2.28%	11.25%	22,495	0.001264	0.000007	22,495	0.001322	0.000060	22,495	0.001300	0.000030	22,495	0.001264	0.000142
112	Digital Realty Trust, Inc.	DLR	3.4%	NA	35.69%	5.93%	17,258	0.000970	0.000033	--	--	--	17,258	0.000997	0.000356	17,258	0.000970	0.000057
113	Discover Financial Servic	DFS	1.8%	5.0%	8.84%	7.53%	25,541	0.001435	0.000026	25,541	0.001501	0.000075	25,541	0.001476	0.000130	25,541	0.001435	0.000108
114	Dollar General Corporati	DG	1.4%	11.0%	7.31%	10.85%	19,133	0.001075	0.000015	19,133	0.001125	0.000124	19,133	0.001106	0.000081	19,133	0.001075	0.000117
115	Dominion Resources, Inc.	D	3.9%	5.5%	4.41%	6.00%	49,134	0.002760	0.000107	49,134	0.002888	0.000159	49,134	0.002839	0.000125	49,134	0.002760	0.000166
116	Dover Corporation	DOV	2.2%	2.0%	11.99%	11.25%	12,552	0.000705	0.000015	12,552	0.000738	0.000015	12,552	0.000725	0.000087	12,552	0.000705	0.000079
117	Dow Chemical Company	DOW	2.9%	8.0%	6.98%	6.88%	77,545	0.004356	0.000126	77,545	0.004558	0.000365	77,545	0.004481	0.000313	77,545	0.004356	0.000300
118	Dr Pepper Snapple Grou	DPS	2.4%	9.0%	8.73%	9.54%	17,688	0.000994	0.000024	17,688	0.001040	0.000094	17,688	0.001022	0.000089	17,688	0.000994	0.000095
119	DTE Energy Company	DTE	3.2%	5.0%	5.05%	5.95%	18,472	0.001038	0.000033	18,472	0.001086	0.000054	18,472	0.001067	0.000054	18,472	0.001038	0.000062
120	Duke Energy Corporatio	DUK	4.1%	4.5%	2.55%	5.03%	57,803	0.003247	0.000134	57,803	0.003398	0.000153	57,803	0.003340	0.000085	57,803	0.003247	0.000163
121	Dun & Bradstreet Corpor	DNB	1.9%	2.5%	4.95%	8.00%	3,942	0.000221	0.000004	3,942	0.000232	0.000006	3,942	0.000228	0.000011	3,942	0.000221	0.000018
122	E.I. du Pont de Nemours	DD	1.9%	8.0%	6.79%	7.88%	69,719	0.003917	0.000074	69,719	0.004098	0.000328	69,719	0.004029	0.000274	69,719	0.003917	0.000309
123	Eastman Chemical Comp	EMN	2.5%	8.0%	7.00%	7.70%	11,697	0.000657	0.000017	11,697	0.000688	0.000055	11,697	0.000676	0.000047	11,697	0.000657	0.000051
124	Eaton Corporation, PLC	ETN	3.2%	5.5%	9.48%	8.82%	33,521	0.001883	0.000060	33,521	0.001970	0.000108	33,521	0.001937	0.000184	33,521	0.001883	0.000166
125	Ecolab Inc.	ECL	1.2%	8.0%	11.98%	13.25%	36,318	0.002040	0.000024	36,318	0.002135	0.000171	36,318	0.002099	0.000251	36,318	0.002040	0.000270
126	Edison International	EIX	2.7%	3.5%	4.80%	6.33%	26,215	0.001473	0.000040	26,215	0.001541	0.000054	26,215	0.001515	0.000073	26,215	0.001473	0.000093
127	Eli Lilly and Company	LLY	2.4%	11.0%	12.32%	11.92%	94,182	0.005291	0.000129	94,182	0.005536	0.000609	94,182	0.005442	0.000671	94,182	0.005291	0.000631
128	Emerson Electric Compar	EMR	3.2%	5.0%	8.35%	7.46%	38,504	0.002163	0.000070	38,504	0.002263	0.000113	38,504	0.002225	0.000186	38,504	0.002163	0.000161
129	Entergy Corporation	ETR	4.6%	-2.5%	-5.96%	1.30%	13,670	0.000768	0.000035	13,670	0.000804	(0.000020)	13,670	0.000790	(0.000047)	13,670	0.000768	0.000010
130	EOG Resources, Inc.	EOG	0.7%	7.5%	NA	9.00%	56,590	0.003179	0.000022	56,590	0.003326	0.000249	--	--	--	56,590	0.003179	0.000286
131	EQT Corporation	EQT	0.2%	22.0%	23.60%	24.80%	11,005	0.000618	0.000001	11,005	0.000647	0.000142	11,005	0.000636	0.000150	11,005	0.000618	0.000153
132	Equifax, Inc.	EFX	1.2%	10.5%	11.00%	11.00%	16,428	0.000923	0.000011	16,428	0.000966	0.000101	16,428	0.000949	0.000104	16,428	0.000923	0.000102
133	Equinix, Inc.	EQIX	2.0%	19.5%	18.77%	16.30%	31,160	0.001750	0.000035	31,160	0.001832	0.000357	31,160	0.001801	0.000338	31,160	0.001750	0.000285
134	Equity Residential	EQR	3.2%	NA	-2.23%	6.09%	23,090	0.001297	0.000042	--	--	--	23,090	0.001334	(0.000030)	23,090	0.001297	0.000079
135	Essex Property Trust, Inc.	ESS	3.0%	NA	6.45%	5.82%	15,225	0.000855	0.000026	--	--	--	15,225	0.000880	0.000057	15,225	0.000855	0.000050
136	Estee Lauder Companies,	EL	1.6%	8.5%	9.18%	10.10%	30,914	0.001737	0.000028	30,914	0.001817	0.000154	30,914	0.001786	0.000164	30,914	0.001737	0.000175
137	Eversource Energy	ES	3.2%	7.0%	6.01%	6.33%	18,867	0.001060	0.000034	18,867	0.001109	0.000078	18,867	0.001090	0.000066	18,867	0.001060	0.000067
138	Exelon Corporation	EXC	3.6%	6.0%	1.53%	5.59%	33,607	0.001888	0.000068	33,607	0.001975	0.000119	33,607	0.001942	0.000030	33,607	0.001888	0.000106
139	Expedia, Inc.	EXPE	0.9%	21.5%	22.56%	21.80%	18,826	0.001058	0.000009	18,826	0.001107	0.000238	18,826	0.001088	0.000245	18,826	0.001058	0.000231
140	Expeditors International	EXPD	1.4%	8.5%	6.14%	9.33%	10,204	0.000573	0.000008	10,204	0.000600	0.000051	10,204	0.000590	0.000036	10,204	0.000573	0.000053
141	Extra Space Storage Inc	EXR	4.0%	NA	8.00%	7.67%	9,782	0.000550	0.000022	--	--	--	9,782	0.000565	0.000045	9,782	0.000550	0.000042
142	Exxon Mobil Corporation	XOM	3.6%	11.5%	32.80%	4.57%	344,202	0.019336	0.000698	344,202	0.020233	0.002327	344,202	0.019890	0.006524	344,202	0.019336	0.000884
143	Fastenal Company	FAST	2.5%	6.5%	11.72%	16.33%	14,738	0.000828	0.000021	14,738	0.000866	0.000056	14,738	0.000852	0.000100	14,738	0.000828	0.000135
144	Federal Realty Investmen	FRT	2.9%	NA	5.00%	4.87%	9,607	0.000540	0.000016	--	--	--	9,607	0.000555	0.000028	9,607	0.000540	0.000026
145	FedEx Corporation	FDX	0.8%	11.0%	11.72%	12.40%	52,403	0.002944	0.000024	52,403	0.003080	0.000339	52,403	0.003028	0.000355	52,403	0.002944	0.000365
146	Fidelity National Informa	FIS	1.5%	14.5%	13.21%	12.00%	26,171	0.001470	0.000021	26,171	0.001538	0.000223	26,171	0.001512	0.000200	26,171	0.001470	0.000176
147	Fifth Third Bancorp	FITB	2.2%	3.0%	4.77%	7.80%	18,901	0.001062	0.000024	18,901	0.001111	0.000033	18,901	0.001092	0.000052	18,901	0.001062	0.000083
148	FirstEnergy Corporation	FE	4.5%	5.0%	-7.32%	-0.40%	14,092	0.000792	0.000036	14,092	0.000828	0.000041	14,092	0.000814	(0.000060)	14,092	0.000792	(0.000003)
149	FLIR Systems, Inc.	FLIR	1.7%	7.5%	15.95%	15.00%	4,871	0.000274	0.000005	4,871	0.000286	0.000021	4,871	0.000282	0.000045	4,871	0.000274	0.000041
150	Flowserve Corporation	FLS	1.6%	2.5%	15.20%	12.56%	6,368	0.000358	0.000006	6,368	0.000374	0.000009	6,368	0.000368	0.000056	6,368	0.000358	0.000045
151	Fluor Corporation	FLR	1.6%	4.5%	14.05%	14.38%	7,311	0.000411	0.000007	7,311	0.000430	0.000019	7,311	0.000422	0.000059	7,311	0.000411	0.000059
152	FMC Corporation	FMC	0.9%	8.5%	-11.58%	10.88%	9,769	0.000549	0.000005	9,769	0.000574	0.000049	9,769	0.000565	(0.000065)	9,769	0.000549	0.000060
153	Foot Locker, Inc.	FL	1.5%	9.5%	9.29%	9.71%	9,487	0.000533	0.000008	9,487	0.000558	0.000053	9,487	0.000548	0.000051	9,487	0.000533	0.000052
154	Ford Motor Company	F	5.3%	3.0%	2.93%	8.22%	44,790	0.002516	0.000134	44,790	0.002633	0.000079	44,790	0.002588	0.000076	44,790	0.002516	0.000207
155	Fortive Corporation	FTV	0.5%	NA	5.68%	8.43%	20,958	0.001177	0.000005	--	--	--	20,958	0.001211	0.000069	20,958	0.001177	0.000099
156	Fortune Brands Home &	FBHS	1.2%	13.5%	9.55%	11.61%	9,494	0.000533	0.000006	9,494	0.000558	0.000075	9,494	0.000549	0.000052	9,494	0.000533	0.000062
157	Franklin Resources, Inc.	BEN	1.9%	6.0%	-4.58%	9.75%	23,721	0.001333	0.000025	23,721	0.001394	0.000084	23,721	0.001371	(0.000063)	23,721	0.001333	0.000130
158	Gap, Inc. (The)	GPS	3.8%	-0.5%	6.71%	8.75%	9,625	0.000541	0.000021	9,625	0.000566	(0.000003)	9,625	0.000556	0.000037	9,625	0.000541	0.000047
159	Garmin Ltd.	GRMN	4.1%	5.0%	1.01%	8.35%	9,920	0.000557	0.000023	9,920	0.000583	0.000029	9,920	0.000573	0.000006	9,920	0.000557	0.000047
160	General Dynamics Corpo	GD	1.8%	5.5%	9.22%	8.51%	56,518	0.003175	0.000057	56,518	0.003322	0.000183	56,518	0.003266	0.000301	56,518	0.003175	0.000270
161	General Electric Compan	GE	3.2%	12.0%	12.29%	11.08%	260,652	0.014643	0.000470	260,652	0.015321	0.001839	260,652	0.015062	0.001851	260,652	0.014643	0.001622
162	General Growth Properti	GGP	3.8%	NA	5.10%	6.05%	20,355	0.001143	0.000044	--	--	--	20,355	0.001176	0.000060	20,355	0.001143	0.000069
163	General Mills, Inc.	GIS	3.3%	6.0%	6.43%	7.44%	33,237	0.001867	0.000062	33,237	0.001954	0.000117	33,237	0.001921	0.000123	33,237	0.001867	0.000139
164	General Motors Compan	GM	4.5%	9.0%	-4.70%	9.42%	51,470	0.002891	0.000129	51,470	0.003025	0.000272	51,470	0.002974	(0.000140)	51,470	0.002891	0.000272
165	Genuine Parts Company	GPC	3.0%	6.5%	7.60%	4.80%	13,402	0.000753	0.000023	13,402	0.000788	0.000051	13,402	0.000774	0.000059	13,402	0.000753	0.000036
166	Gilead Sciences, Inc.	GILD	3.1%	1.5%	-4.81%	2.13%	86,905	0.004882	0.000153	86,905	0.005108	0.000077	86,905	0.005022	(0.000242)	86,905	0.004882	0.000104
167	Global Payments Inc.	GPN	0.1%	14.5%	18.20%	14.33%	11,908	0.000669	0.000000	11,908	0.000700	0.000101	11,908	0.000688	0.000125	11,908	0.000669	0.000096
168	Goldman Sachs Group, Ir	GS	1.1%	9.0%	11.45%	12.68%	90,951	0.005109	0.000058	90,951	0.005346	0.000481	90,951	0.005256	0.000602	90,951	0.005109	0.000648
169	H&R Block, Inc.	HRB	3.7%	10.0%	10.00%	11.67%	4,885	0.000274	0.000010	4,885	0.000287	0.000029	4,885	0.000282	0.000028	4,885	0.000274	0.000032
170	Halliburton Company	HAL	1.4%	8.0%	24.88%	17.50%	43,477	0.002442	0.000035	43,477	0.002556	0.000204	43,477	0.002512	0.000625	43,477	0.002442	0.000427
171	Hanesbrands Inc.	HBI	2.9%	12.5%	10.13%	17.33%	7,718	0.000434	0.000013	7,718	0.000454	0.000057	7,718	0.000446	0.000045	7,718	0.000434	0.000075
172	Harley-Davidson, Inc.	HOG	2.4%	9.0%	9.30%	10.67%	10,685	0.000600	0.000014	10,685	0.000628	0.000057	10,685	0.000617	0.000057	10,685	0.000600	0.000064
173	Harris Corporation	HRS	1.9%	6.5%	41.80%	4.40%	13,627	0.000766	0.000015	13,627	0.000801	0.000052	13,627	0.000787	0.000329	13,627	0.000766	0.000034
174	Hartford Financial Servic	HIG	1.9%	11.5%	9.38%	9.50%	17,799	0.001000	0.000019	17,799	0.001046	0.000120	17,799	0.001029	0.000096	17,799	0.001000	0.000095
175	Hasbro, Inc.	HAS	2.1%	12.0%	5.30%	10.00%	12,054	0.000677	0.000014	12,054	0.000709	0.000085	12,054	0.000697	0.000037	12,054	0.000677	0.000068
176	HCP, Inc.	HCP	4.7%	NA	2.50%	4.84%	14,708	0.000826	0.000039	--	--	--	14,708	0.000850	0.000021	14,708	0.000826	0.000040
177	Helmerich & Payne, Inc.	HP	4.1%	4.5%	12.00%	3.00%	7,383	0.000415	0.000017	7,383	0.000434	0.000020	7,383	0.000427	0.000051	7,383	0.000415	0.000012
178	Hershey Company (The)	HSY	2.3%	5.5%	8.61%	7.75%	23,128	0.001299	0.000029	23,128	0.001359	0.000075	23,128	0.001336	0.000115	23,128	0.001299	0.000101
179	Hess Corporation	HES	2.1%	-1.0%	-27.90%	5.00%	15,269	0.000858	0.000018	15,269	0.000898	(0.000009)	15,269	0.000882	(0.000246)	15,269	0.000858	0.000043
180	Hewlett Packard Enterpri	HPE	1.4%	1.5%	-9.68%	5.29%	29,872	0.001678	0.000024	29,872	0.001756	0.000026	29,872	0.001726	(0.000167)	29,872	0.001678	0.000089
181	Home Depot, Inc. (The)	HD	2.4%	11.5%	11.58%	12.72%	176,860	0.009935	0.000240	176,860	0.010396	0.001196	176,860	0.010220	0.001183	176,860	0.009935	0.001264
182	Honeywell International	HON	2.2%	8.5%	6.98%	9.28%	94,200	0.005292	0.000114	94,200	0.005537	0.000471	94,200	0.005443	0.000380	94,200	0.005292	0.000491
183	Hormel Foods Corporatic	HRL	2.0%	13.0%	9.88%	9.50%	18,184	0.001022	0.000020	18,184	0.001069	0.000139	18,184	0.001051	0.000104	18,184	0.001022	0.000097
184	Host Hotels & Resorts, In	HST	4.3%	NA	28.40%	5.00%	13,691	0.000769	0.000033	--	--	--	13,691	0.000791	0.000225	13,691	0.000769	0.000038
185	HP Inc.	HPQ	3.0%	-6.0%	1.78%	2.59%	29,736	0.001670	0.000050	29,736	0.001748	(0.000105)	29,736	0.001718	0.000031	29,736	0.001670	0.000043
186	Humana Inc.	HUM	0.8%	9.0%	13.27%	13.30%	30,675	0.001723	0.000013	30,675	0.001803	0.000162	30,675	0.001773	0.000235	30,675	0.001723	0.000229
187	Huntington Bancshares Ir	HBAN	2.4%	8.0%	10.16%	8.06%	14,390	0.000808	0.000020	14,390	0.000846	0.000068	14,390	0.000832	0.000084	14,390	0.000808	0.000065
188	Illinois Tool Works Inc.	ITW	2.0%	10.0%	9.27%	7.50%	45,497	0.002556	0.000051	45,497	0.002674	0.000267	45,497	0.002629	0.000244	45,497	0.002556	0.000192
189	Ingersoll-Rand PLC (Irela	IR	1.9%	9.5%	9.03%	8.10%	21,454	0.001205	0.000023	21,454	0.001261	0.000120	21,454	0.001240	0.000112	21,454	0.001205	0.000098
190	Intel Corporation	INTC	2.9%	8.0%	8.16%	8.42%	170,350	0.009570	0.000277	170,350	0.010013	0.000801	170,350	0.009844	0.000803	170,350	0.009570	0.000806
191	Intercontinental Exchang	ICE	1.3%	15.5%	12.55%	13.60%	35,750	0.002008	0.000027	35,750	0.002101	0.000326	35,750	0.002066	0.000259	35,750	0.002008	0.000273
192	Internationa Flavors & Fr	IFF	1.9%	7.5%	7.69%	8.80%	10,507	0.000590	0.000011	10,507	0.000618	0.000046	10,507	0.000607	0.000047	10,507	0.000590	0.000052

193	International Business Ma	IBM	3.3%	0.0%	2.56%	4.30%	162,657	0.009138	0.000297	162,657	0.009561	-	162,657	0.009399	0.000241	162,657	0.009138	0.000393
194	International Paper Comp	IP	3.6%	17.5%	9.19%	11.33%	20,953	0.001177	0.000043	20,953	0.001232	0.000216	20,953	0.001211	0.000111	20,953	0.001177	0.000133
195	Interpublic Group of Con	IPG	3.0%	13.0%	12.55%	11.62%	9,566	0.000537	0.000016	9,566	0.000562	0.000073	9,566	0.000553	0.000069	9,566	0.000537	0.000062
196	Intuit Inc.	INTU	1.2%	13.5%	12.80%	13.79%	30,134	0.001693	0.000019	30,134	0.001771	0.000239	30,134	0.001741	0.000223	30,134	0.001693	0.000233
197	Invesco PLC	IVZ	3.7%	4.5%	7.49%	10.14%	12,353	0.000694	0.000025	12,353	0.000726	0.000033	12,353	0.000714	0.000053	12,353	0.000694	0.000070
198	Iron Mountain Incorporat	IRM	6.2%	10.0%	3.43%	2.50%	9,309	0.000523	0.000033	9,309	0.000547	0.000055	9,309	0.000538	0.000018	9,309	0.000523	0.000013
199	J P Morgan Chase & Co	JPM	2.3%	6.0%	5.89%	5.00%	307,684	0.017285	0.000399	307,684	0.018086	0.001085	307,684	0.017780	0.001047	307,684	0.017285	0.000864
200	J.B. Hunt Transport Servi	JBHT	1.0%	9.0%	12.24%	13.50%	10,268	0.000577	0.000006	10,268	0.000604	0.000054	10,268	0.000593	0.000073	10,268	0.000577	0.000078
201	J.M. Smucker Company ('	SJM	2.3%	7.5%	5.22%	6.20%	15,072	0.000847	0.000020	15,072	0.000886	0.000066	15,072	0.000871	0.000045	15,072	0.000847	0.000052
202	Jacobs Engineering Grou	JEC	0.3%	8.0%	NA	9.81%	6,608	0.000371	0.000001	6,608	0.000388	0.000031	--	--	--	6,608	0.000371	0.000036
203	Johnson & Johnson	JNJ	2.6%	8.5%	5.89%	6.09%	338,997	0.019044	0.000488	338,997	0.019927	0.001694	338,997	0.019589	0.001154	338,997	0.019044	0.001160
204	Johnson Controls Interna	JCI	2.4%	2.0%	13.42%	12.42%	38,656	0.002172	0.000053	38,656	0.002272	0.000045	38,656	0.002234	0.000300	38,656	0.002172	0.000270
205	Juniper Networks, Inc.	JNPR	1.4%	8.0%	11.03%	10.48%	10,545	0.000592	0.000009	10,545	0.000620	0.000050	10,545	0.000609	0.000067	10,545	0.000592	0.000062
206	Kansas City Southern	KSU	1.5%	9.5%	11.15%	12.67%	9,267	0.000521	0.000008	9,267	0.000545	0.000052	9,267	0.000535	0.000060	9,267	0.000521	0.000066
207	Kellogg Company	K	2.9%	5.0%	6.88%	6.69%	25,347	0.001424	0.000041	25,347	0.001490	0.000074	25,347	0.001465	0.000101	25,347	0.001424	0.000095
208	KeyCorp	KEY	1.9%	9.0%	10.32%	7.50%	18,957	0.001065	0.000021	18,957	0.001114	0.000100	18,957	0.001095	0.000113	18,957	0.001065	0.000080
209	Kimberly-Clark Corporat	KMB	3.0%	10.0%	7.35%	7.23%	46,643	0.002620	0.000078	46,643	0.002742	0.000274	46,643	0.002695	0.000198	46,643	0.002620	0.000189
210	Kimco Realty Corporatio	KIM	5.0%	NA	5.91%	5.86%	9,228	0.000518	0.000026	--	--	--	9,228	0.000533	0.000032	9,228	0.000518	0.000030
211	Kinder Morgan, Inc.	KMI	2.3%	24.0%	10.00%	10.00%	48,556	0.002728	0.000063	48,556	0.002854	0.000685	48,556	0.002806	0.000281	48,556	0.002728	0.000273
212	KLA-Tencor Corporation	KLAC	2.3%	13.5%	11.97%	21.24%	15,073	0.000847	0.000019	15,073	0.000886	0.000120	15,073	0.000871	0.000104	15,073	0.000847	0.000180
213	Kohl's Corporation	KSS	5.6%	5.5%	8.60%	7.67%	6,824	0.000383	0.000021	6,824	0.000401	0.000022	6,824	0.000394	0.000034	6,824	0.000383	0.000029
214	Kroger Company (The)	KR	1.6%	10.0%	6.12%	9.17%	26,979	0.001516	0.000025	26,979	0.001586	0.000159	26,979	0.001559	0.000095	26,979	0.001516	0.000139
215	L Brands, Inc.	LB	5.0%	5.0%	8.96%	10.38%	13,640	0.000766	0.000038	13,640	0.000802	0.000040	13,640	0.000788	0.000071	13,640	0.000766	0.000080
216	L-3 Communications Hol	LLL	1.8%	10.0%	6.46%	5.77%	12,675	0.000712	0.000013	12,675	0.000745	0.000075	12,675	0.000732	0.000047	12,675	0.000712	0.000041
217	Lam Research Corporatic	LRCX	1.4%	19.5%	15.43%	12.34%	20,984	0.001179	0.000017	20,984	0.001233	0.000241	20,984	0.001213	0.000187	20,984	0.001179	0.000145
218	Lennar Corporation	LEN	0.3%	10.0%	NA	8.39%	11,938	0.000671	0.000002	11,938	0.000702	0.000070	--	--	--	11,938	0.000671	0.000056
219	Leucadia National Corpo	LUK	0.2%	19.0%	18.00%	18.00%	9,485	0.000533	0.000001	9,485	0.000558	0.000106	9,485	0.000548	0.000099	9,485	0.000533	0.000096
220	Lincoln National Corpora	LNC	1.8%	7.0%	8.62%	8.00%	14,817	0.000832	0.000015	14,817	0.000871	0.000061	14,817	0.000856	0.000074	14,817	0.000832	0.000067
221	Lockheed Martin Corpor	LMT	2.7%	8.5%	8.48%	5.82%	77,415	0.004349	0.000119	77,415	0.004551	0.000387	77,415	0.004474	0.000379	77,415	0.004349	0.000253
222	Lowe's Companies, Inc.	LOW	1.7%	14.5%	14.92%	14.59%	70,986	0.003988	0.000068	70,986	0.004173	0.000605	70,986	0.004102	0.000612	70,986	0.003988	0.000582
223	LyondellBasell Industries	LYB	3.8%	5.0%	-2.76%	8.00%	35,902	0.002017	0.000077	35,902	0.002110	0.000106	35,902	0.002075	(0.000057)	35,902	0.002017	0.000161
224	M&T Bank Corporation	MTB	2.0%	6.5%	7.18%	5.91%	23,582	0.001325	0.000026	23,582	0.001386	0.000090	23,582	0.001363	0.000098	23,582	0.001325	0.000078
225	Macerich Company (The)	MAC	4.4%	NA	-0.11%	5.57%	9,233	0.000519	0.000023	--	--	--	9,233	0.000534	(0.000001)	9,233	0.000519	0.000029
226	Macy's Inc	M	5.1%	5.0%	18.64%	8.50%	8,979	0.000504	0.000026	8,979	0.000528	0.000026	8,979	0.000519	0.000097	8,979	0.000504	0.000043
227	Marathon Oil Corporatio	MRO	1.2%	14.5%	-20.00%	9.00%	13,818	0.000776	0.000010	13,818	0.000812	0.000118	13,818	0.000798	(0.000160)	13,818	0.000776	0.000070
228	Marathon Petroleum Cor	MPC	2.9%	3.5%	-0.79%	9.00%	26,309	0.001478	0.000043	26,309	0.001546	0.000054	26,309	0.001520	(0.000012)	26,309	0.001478	0.000133
229	Marriott International	MAR	1.3%	13.0%	14.37%	8.50%	35,095	0.001972	0.000026	35,095	0.002063	0.000268	35,095	0.002028	0.000291	35,095	0.001972	0.000168
230	Marsh & McLennan Com	MMC	1.9%	9.0%	10.15%	11.98%	37,759	0.002121	0.000039	37,759	0.002219	0.000200	37,759	0.002182	0.000221	37,759	0.002121	0.000254
231	Martin Marietta Material:	MLM	0.8%	17.5%	NA	15.65%	13,762	0.000773	0.000006	13,762	0.000809	0.000142	--	--	--	13,762	0.000773	0.000121
232	Masco Corporation	MAS	1.2%	11.5%	20.80%	13.57%	10,824	0.000608	0.000007	10,824	0.000636	0.000073	10,824	0.000625	0.000130	10,824	0.000608	0.000083
233	Mastercard Incorporated	MA	0.8%	12.0%	14.92%	15.43%	121,190	0.006808	0.000053	121,190	0.007124	0.000855	121,190	0.007003	0.001045	121,190	0.006808	0.001050
234	Mattel, Inc.	MAT	6.0%	4.5%	15.65%	10.00%	8,674	0.000487	0.000029	8,674	0.000510	0.000023	8,674	0.000501	0.000078	8,674	0.000487	0.000049
235	McCormick & Company,	MKC	1.9%	7.5%	8.50%	8.81%	12,529	0.000704	0.000013	12,529	0.000736	0.000055	12,529	0.000724	0.000062	12,529	0.000704	0.000062
236	McDonald's Corporation	MCD	2.9%	6.5%	8.66%	8.87%	106,592	0.005988	0.000173	106,592	0.006266	0.000407	106,592	0.006160	0.000533	106,592	0.005988	0.000531
237	McKesson Corporation	MCK	0.8%	11.0%	7.85%	8.25%	30,714	0.001725	0.000013	30,714	0.001805	0.000199	30,714	0.001775	0.000139	30,714	0.001725	0.000142
238	Mead Johnson Nutrition (	MJN	1.9%	6.0%	2.25%	10.00%	16,396	0.000921	0.000017	16,396	0.000964	0.000058	16,396	0.000947	0.000021	16,396	0.000921	0.000092
239	Medtronic PLC	MDT	2.1%	6.0%	7.53%	7.83%	109,812	0.006169	0.000132	109,812	0.006455	0.000387	109,812	0.006346	0.000478	109,812	0.006169	0.000483
240	Merck & Company, Inc.	MRK	3.0%	5.5%	6.64%	6.12%	173,630	0.009754	0.000290	173,630	0.010206	0.000561	173,630	0.010033	0.000666	173,630	0.009754	0.000597
241	MetLife, Inc.	MET	3.0%	7.0%	11.14%	10.70%	57,266	0.003217	0.000098	57,266	0.003366	0.000236	57,266	0.003309	0.000369	57,266	0.003217	0.000344
242	Microchip Technology In	MCHP	2.0%	12.0%	22.31%	14.52%	15,912	0.000894	0.000018	15,912	0.000935	0.000112	15,912	0.000920	0.000205	15,912	0.000894	0.000130
243	Microsoft Corporation	MSFT	2.4%	8.0%	9.30%	9.19%	507,931	0.028534	0.000676	507,931	0.029857	0.002389	507,931	0.029351	0.002730	507,931	0.028534	0.002622
244	Mid-America Apartment	MAA	3.4%	NA	7.00%	7.05%	11,671	0.000656	0.000022	--	--	--	11,671	0.000674	0.000047	11,671	0.000656	0.000046
245	Molson Coors Brewing C	TAP	1.7%	17.0%	5.16%	8.73%	20,636	0.001159	0.000020	20,636	0.001213	0.000206	20,636	0.001192	0.000062	20,636	0.001159	0.000101
246	Mondelez International, I	MDLZ	1.7%	11.5%	10.62%	11.22%	67,339	0.003783	0.000065	67,339	0.003958	0.000455	67,339	0.003891	0.000413	67,339	0.003783	0.000424
247	Monsanto Company	MON	1.9%	9.5%	8.00%	12.13%	50,872	0.002858	0.000053	50,872	0.002990	0.000284	50,872	0.002940	0.000235	50,872	0.002858	0.000347
248	Moody's Corporation	MCO	1.3%	6.5%	9.92%	11.67%	21,650	0.001216	0.000016	21,650	0.001273	0.000083	21,650	0.001251	0.000124	21,650	0.001216	0.000142
249	Morgan Stanley	MS	1.9%	14.5%	13.72%	12.75%	78,547	0.004413	0.000084	78,547	0.004617	0.000669	78,547	0.004539	0.000623	78,547	0.004413	0.000563
250	Mosaic Company (The)	MOS	3.8%	3.5%	8.00%	9.50%	10,223	0.000574	0.000022	10,223	0.000601	0.000021	10,223	0.000591	0.000047	10,223	0.000574	0.000055
251	Motorola Solutions, Inc.	MSI	2.2%	10.5%	4.73%	5.20%	13,800	0.000775	0.000017	13,800	0.000811	0.000085	13,800	0.000797	0.000038	13,800	0.000775	0.000040
252	Nasdaq, Inc.	NDAQ	1.9%	10.0%	10.32%	9.48%	11,527	0.000648	0.000012	11,527	0.000678	0.000068	11,527	0.000666	0.000069	11,527	0.000648	0.000061
253	National Oilwell Varco, I	NOV	0.5%	-9.5%	48.27%	8.07%	14,922	0.000838	0.000004	14,922	0.000877	(0.000083)	14,922	0.000862	0.000416	14,922	0.000838	0.000068
254	Navient Corporation	NAVI	4.4%	NA	2.84%	8.00%	4,255	0.000239	0.000010	--	--	--	4,255	0.000246	0.000007	4,255	0.000239	0.000019
255	NetApp, Inc.	NTAP	1.9%	9.0%	15.11%	10.43%	11,123	0.000625	0.000012	11,123	0.000654	0.000059	11,123	0.000643	0.000097	11,123	0.000625	0.000065
256	Newell Brands Inc.	NWL	1.7%	18.0%	10.48%	11.96%	22,239	0.001249	0.000021	22,239	0.001307	0.000235	22,239	0.001285	0.000135	22,239	0.001249	0.000149
257	News Corporation	NWS	1.5%	NA	NA	11.80%	7,530	0.000423	0.000007	--	--	--	--	--	--	7,530	0.000423	0.000050
258	News Corporation	NWSA	1.6%	NA	6.00%	6.70%	4,800	0.000270	0.000004	--	--	--	4,800	0.000277	0.000017	4,800	0.000270	0.000018
259	NextEra Energy, Inc.	NEE	3.0%	6.0%	6.86%	7.23%	60,749	0.003413	0.000103	60,749	0.003571	0.000214	60,749	0.003510	0.000241	60,749	0.003413	0.000247
260	Nielsen N.V.	NLSN	3.0%	-2.5%	7.34%	10.27%	15,017	0.000844	0.000025	15,017	0.000883	(0.000022)	15,017	0.000868	0.000064	15,017	0.000844	0.000087
261	Nike, Inc.	NKE	1.3%	15.0%	11.75%	12.36%	91,228	0.005125	0.000067	91,228	0.005362	0.000804	91,228	0.005272	0.000619	91,228	0.005125	0.000633
262	NiSource, Inc	NI	2.9%	2.0%	8.00%	6.15%	7,711	0.000433	0.000013	7,711	0.000453	0.000009	7,711	0.000446	0.000036	7,711	0.000433	0.000027
263	Noble Energy Inc.	NBL	1.2%	NA	12.45%	10.00%	15,107	0.000849	0.000010	--	--	--	15,107	0.000873	0.000109	15,107	0.000849	0.000085
264	Nordstrom, Inc.	JWN	3.3%	0.5%	5.44%	6.00%	7,462	0.000419	0.000014	7,462	0.000439	0.000002	7,462	0.000431	0.000023	7,462	0.000419	0.000025
265	Norfolk Souther Corpora	NSC	2.1%	6.5%	11.78%	11.50%	36,006	0.002023	0.000043	36,006	0.002116	0.000138	36,006	0.002081	0.000245	36,006	0.002023	0.000233
266	Northern Trust Corporati	NTRS	1.8%	8.0%	11.85%	8.00%	19,828	0.001114	0.000020	19,828	0.001165	0.000093	19,828	0.001146	0.000136	19,828	0.001114	0.000089
267	Northrop Grumman Corp	NOC	1.5%	8.0%	7.53%	7.23%	41,373	0.002324	0.000035	41,373	0.002432	0.000195	41,373	0.002391	0.000180	41,373	0.002324	0.000168
268	Nucor Corporation	NUE	2.5%	24.5%	24.58%	12.00%	19,175	0.001077	0.000027	19,175	0.001127	0.000276	19,175	0.001108	0.000272	19,175	0.001077	0.000129
269	NVIDIA Corporation	NVDA	0.6%	14.5%	11.67%	10.30%	59,348	0.003334	0.000019	59,348	0.003489	0.000506	59,348	0.003429	0.000400	59,348	0.003334	0.000343
270	Occidental Petroleum Co	OXY	4.8%	3.5%	NA	6.50%	48,383	0.002718	0.000130	48,383	0.002844	0.000100	--	--	--	48,383	0.002718	0.000177
271	Omnicom Group Inc.	OMC	2.6%	9.0%	7.88%	7.51%	20,066	0.001127	0.000029	20,066	0.001180	0.000106	20,066	0.001160	0.000091	20,066	0.001127	0.000085
272	ONEOK, Inc.	OKE	4.4%	11.0%	18.34%	5.00%	11,872	0.000667	0.000029	11,872	0.000698	0.000077	11,872	0.000686	0.000126	11,872	0.000667	0.000033
273	Oracle Corporation	ORCL	1.4%	5.0%	8.75%	9.43%	181,787	0.010212	0.000139	181,787	0.010686	0.000534	181,787	0.010505	0.000919	181,787	0.010212	0.000963
274	PACCAR Inc.	PCAR	1.4%	5.0%	6.06%	10.00%	23,590	0.001325	0.000019	23,590	0.001387	0.000069	23,590	0.001363	0.000083	23,590	0.001325	0.000133
275	Pacific Gas & Electric Co.	PCG	2.9%	11.0%	3.74%	3.75%	34,204	0.001921	0.000056	34,204	0.002011	0.000221	34,204	0.001977	0.000074	34,204	0.001921	0.000072
276	Parker-Hannifin Corpora	PH	1.7%	6.5%	10.87%	9.39%	21,358	0.001200	0.000020	21,358	0.001255	0.000082	21,358	0.001234	0.000134	21,358	0.001200	0.000113
277	Patterson Companies, Inc	PDCO	2.2%	10.5%	7.35%	6.33%	4,340	0.000244	0.000005	4,340	0.000255	0.000027	4,340	0.000251	0.000018	4,340	0.000244	0.000015
278	Paychex, Inc.	PAYX	3.2%	9.0%	8.44%	8.88%	20,890	0.001174	0.000037	20,890	0.001228	0.000111	20,890	0.001207	0.000102	20,890	0.001174	0.000104
279	Pentair PLC.	PNR	2.2%	9.5%	9.29%	10.82%	11,435	0.000642	0.000014	11,435	0.000672	0.000064	11,435	0.000661	0.000061	11,435	0.000642	0.000070
280	People's United Financial	PBCT	3.8%	10.5%	2.00%	2.00%	6,159	0.000346	0.000013	6,159	0.000362	0.000038	6,159	0.000356	0.000007	6,159	0.000346	0.000007
281	Pepsico, Inc.	PEP	2.7%	7.0%	6.42%	7.16%	159,363	0.008953	0.000242	159,363	0.009368	0.000656	159,363	0.009209	0.000591	159,363	0.008953	0.000641
282	PerkinElmer, Inc.	PKI	0.5%	8.0%	8.01%	10.13%	6,351	0.000357	0.000002	6,351	0.000373	0.000030	6,351	0.000367	0.000029	6,351	0.000357	0.000036
283	Perrigo Company	PRGO	1.0%	NA	1.68%	7.58%	9,692	0.000544	0.000005	--	--	--	9,692	0.000560	0.000009	9,692	0.000544	0.000041
284	Pfizer, Inc.	PFE	3.8%	11.0%	6.35%	5.64%	203,129	0.011411	0.000428	203,129	0.011940	0.001313	203,129	0.011738	0.000745	203,129	0.011411	0.000644
285	Philip Morris Internation	PM	3.7%	5.0%	10.75%	10.80%	174,868	0.009824	0.000362	174,868	0.010279	0.000514	174,868	0.010105	0.001086	174,868	0.009824	0.001061
286	Phillips 66	PSX	3.3%	2.0%	-6.36%	6.65%	39,745	0.002233	0.000073	39,745	0.002336	0.000047	39,745	0.002297	(0.000146)	39,745	0.002233	0.000148
287	Pinnacle West Capital Co	PNW	3.1%	4.0%	5.93%	5.61%	9,424	0.000529	0.000016	9,424	0.000554	0.000022	9,424	0.000545	0.000032	9,424	0.000529	0.000030
288	Pioneer Natural Resource	PXD	0.0%	20.0%	18.00%	9.75%	32,336	0.001817	0.000001	32,336	0.001901	0.000380	32,336	0.001869	0.000336	32,336	0.001817	0.000177
289	PNC Financial Services G	PNC	1.8%	3.0%	8.40%	4.67%	57,979	0.003257	0.000060	57,979	0.003408	0.000102	57,979	0.003350	0.000281	57,979	0.003257	0.000152
290	PPG Industries, Inc.	PPG	1.5%	9.5%	9.06%	7.91%	27,018	0.001518	0.000023	27,018	0.001588	0.000151	27,018	0.001561	0.000141	27,018	0.001518	0.000120

291	PPL Corporation	PPL	4.2%	2.0%	2.44%	3.25%	25,468	0.001431	0.000060	25,468	0.001497	0.000030	25,468	0.001472	0.000036	25,468	0.001431	0.000046
292	Praxair, Inc.	PX	2.6%	6.0%	NA	12.00%	34,054	0.001913	0.000051	34,054	0.002002	0.000120	--	--	--	34,054	0.001913	0.000230
293	Principal Financial Grouŗ	PFG	2.9%	5.0%	7.89%	8.71%	18,086	0.001016	0.000029	18,086	0.001063	0.000053	18,086	0.001045	0.000082	18,086	0.001016	0.000088
294	Procter & Gamble Compa	PG	3.0%	7.5%	7.40%	7.86%	228,549	0.012839	0.000385	228,549	0.013434	0.001008	228,549	0.013207	0.000977	228,549	0.012839	0.001009
295	Progressive Corporation (	PGR	1.7%	7.5%	8.64%	9.00%	22,800	0.001281	0.000022	22,800	0.001340	0.000101	22,800	0.001318	0.000114	22,800	0.001281	0.000115
296	ProLogis, Inc.	PLD	3.3%	NA	10.60%	6.59%	28,098	0.001578	0.000052	--	--	--	28,098	0.001624	0.000172	28,098	0.001578	0.000104
297	Prudential Financial, Inc.	PRU	2.8%	5.5%	8.95%	8.50%	45,744	0.002570	0.000072	45,744	0.002689	0.000148	45,744	0.002643	0.000237	45,744	0.002570	0.000218
298	Public Service Enterprise	PEG	3.8%	2.5%	0.75%	2.35%	22,724	0.001277	0.000049	22,724	0.001336	0.000033	22,724	0.001313	0.000010	22,724	0.001277	0.000030
299	Public Storage	PSA	3.6%	NA	8.30%	5.15%	38,659	0.002172	0.000078	--	--	--	38,659	0.002234	0.000185	38,659	0.002172	0.000112
300	PulteGroup, Inc.	PHM	1.5%	16.5%	18.04%	14.01%	7,406	0.000416	0.000006	7,406	0.000435	0.000072	7,406	0.000428	0.000077	7,406	0.000416	0.000058
301	PVH Corp.	PVH	0.2%	6.0%	6.92%	10.69%	7,974	0.000448	0.000001	7,974	0.000469	0.000028	7,974	0.000461	0.000032	7,974	0.000448	0.000048
302	QUALCOMM Incorporat	QCOM	3.8%	6.0%	10.50%	8.83%	83,497	0.004691	0.000176	83,497	0.004908	0.000294	83,497	0.004825	0.000507	83,497	0.004691	0.000414
303	Quest Diagnostics Incorp	DGX	1.8%	9.0%	8.23%	8.61%	13,455	0.000756	0.000014	13,455	0.000791	0.000071	13,455	0.000778	0.000064	13,455	0.000756	0.000065
304	Ralph Lauren Corporatio	RL	2.5%	3.5%	2.80%	7.24%	6,635	0.000373	0.000009	6,635	0.000390	0.000014	6,635	0.000383	0.000011	6,635	0.000373	0.000027
305	Range Resources Corpora	RRC	0.3%	14.0%	4.35%	7.50%	7,401	0.000416	0.000001	7,401	0.000435	0.000061	7,401	0.000428	0.000019	7,401	0.000416	0.000031
306	Raymond James Financia	RJF	1.2%	9.5%	13.00%	17.00%	10,868	0.000611	0.000007	10,868	0.000639	0.000061	10,868	0.000628	0.000082	10,868	0.000611	0.000104
307	Raytheon Company	RTN	1.9%	8.0%	12.00%	7.54%	44,152	0.002480	0.000048	44,152	0.002595	0.000208	44,152	0.002551	0.000306	44,152	0.002480	0.000187
308	Realty Income Corporatio	O	4.2%	NA	7.30%	4.05%	16,313	0.000916	0.000039	--	--	--	16,313	0.000943	0.000069	16,313	0.000916	0.000037
309	Regency Centers Corpora	REG	3.1%	NA	11.00%	7.36%	6,914	0.000388	0.000012	--	--	--	6,914	0.000400	0.000044	6,914	0.000388	0.000029
310	Regions Financial Corpor	RF	1.8%	7.5%	8.67%	10.56%	17,512	0.000984	0.000018	17,512	0.001029	0.000077	17,512	0.001012	0.000088	17,512	0.000984	0.000104
311	Republic Services, Inc.	RSG	2.0%	8.0%	10.42%	9.84%	21,251	0.001194	0.000024	21,251	0.001249	0.000100	21,251	0.001228	0.000128	21,251	0.001194	0.000117
312	Reynolds American Inc	RAI	3.2%	14.0%	10.07%	9.21%	90,304	0.005073	0.000163	90,304	0.005308	0.000743	90,304	0.005218	0.000525	90,304	0.005073	0.000467
313	Robert Half International	RHI	2.1%	6.0%	8.20%	15.50%	5,844	0.000328	0.000007	5,844	0.000344	0.000021	5,844	0.000338	0.000028	5,844	0.000328	0.000051
314	Rockwell Automation, In	ROK	2.0%	4.0%	7.88%	7.57%	20,046	0.001126	0.000022	20,046	0.001178	0.000047	20,046	0.001158	0.000091	20,046	0.001126	0.000085
315	Rockwell Collins, Inc.	COL	1.3%	6.5%	8.33%	9.75%	13,066	0.000734	0.000010	13,066	0.000768	0.000050	13,066	0.000755	0.000063	13,066	0.000734	0.000072
316	Roper Technologies, Inc.	ROP	0.7%	7.0%	14.75%	12.33%	21,201	0.001191	0.000008	21,201	0.001246	0.000087	21,201	0.001225	0.000181	21,201	0.001191	0.000147
317	Ross Stores, Inc.	ROST	1.0%	9.0%	11.02%	10.50%	24,872	0.001397	0.000014	24,872	0.001462	0.000132	24,872	0.001437	0.000158	24,872	0.001397	0.000147
318	Royal Caribbean Cruises	RCL	1.9%	16.5%	14.50%	21.50%	21,218	0.001192	0.000023	21,218	0.001247	0.000206	21,218	0.001226	0.000178	21,218	0.001192	0.000256
319	Ryder System, Inc.	R	2.3%	7.5%	15.00%	15.00%	4,068	0.000229	0.000005	4,068	0.000239	0.000018	4,068	0.000235	0.000035	4,068	0.000229	0.000034
320	S&P Global Inc.	SPGI	1.3%	11.0%	12.02%	12.33%	33,603	0.001888	0.000024	33,603	0.001975	0.000217	33,603	0.001942	0.000233	33,603	0.001888	0.000233
321	Scana Corporation	SCG	3.7%	4.5%	5.87%	5.33%	9,475	0.000532	0.000020	9,475	0.000557	0.000025	9,475	0.000548	0.000032	9,475	0.000532	0.000028
322	Schlumberger N.V.	SLB	2.5%	10.0%	-0.81%	9.67%	109,606	0.006157	0.000156	109,606	0.006443	0.000644	109,606	0.006334	(0.000051)	109,606	0.006157	0.000595
323	Scripps Networks Interac	SNI	1.5%	9.0%	11.07%	9.53%	10,055	0.000565	0.000009	10,055	0.000591	0.000053	10,055	0.000581	0.000064	10,055	0.000565	0.000054
324	Seagate Technology PLC	STX	5.5%	4.5%	17.16%	8.17%	13,652	0.000767	0.000042	13,652	0.000802	0.000036	13,652	0.000789	0.000135	13,652	0.000767	0.000063
325	Sealed Air Corporation	SEE	1.5%	12.0%	8.08%	6.40%	8,331	0.000468	0.000007	8,331	0.000490	0.000059	8,331	0.000481	0.000039	8,331	0.000468	0.000030
326	Sempra Energy	SRE	3.0%	8.0%	9.30%	9.21%	27,504	0.001545	0.000046	27,504	0.001617	0.000129	27,504	0.001589	0.000148	27,504	0.001545	0.000142
327	Sherwin-Williams Compa	SHW	1.1%	8.5%	10.75%	10.71%	28,917	0.001624	0.000018	28,917	0.001700	0.000144	28,917	0.001671	0.000180	28,917	0.001624	0.000174
328	Signet Jewelers Limited	SIG	1.5%	11.5%	5.00%	6.50%	4,610	0.000259	0.000004	4,610	0.000271	0.000031	4,610	0.000266	0.000013	4,610	0.000259	0.000017
329	Simon Property Group, In	SPG	4.1%	NA	9.70%	7.74%	54,307	0.003051	0.000126	--	--	--	54,307	0.003138	0.000304	54,307	0.003051	0.000236
330	Skyworks Solutions, Inc.	SWKS	1.1%	12.5%	14.75%	14.44%	18,342	0.001030	0.000012	18,342	0.001078	0.000135	18,342	0.001060	0.000156	18,342	0.001030	0.000149
331	SL Green Realty Corpora	SLG	2.9%	NA	46.06%	4.53%	10,922	0.000614	0.000017	--	--	--	10,922	0.000631	0.000291	10,922	0.000614	0.000028
332	Snap-On Incorporated	SNA	1.7%	10.0%	10.00%	8.50%	9,531	0.000535	0.000009	9,531	0.000560	0.000056	9,531	0.000551	0.000055	9,531	0.000535	0.000046
333	Southern Company (The)	SO	4.5%	4.5%	3.68%	4.32%	49,723	0.002793	0.000125	49,723	0.002923	0.000132	49,723	0.002873	0.000106	49,723	0.002793	0.000121
334	Southwest Airlines Comp	LUV	0.8%	14.5%	10.66%	9.56%	32,842	0.001845	0.000014	32,842	0.001931	0.000280	32,842	0.001898	0.000202	32,842	0.001845	0.000176
335	Stanley Black & Decker, I	SWK	1.8%	9.0%	10.30%	10.07%	20,209	0.001135	0.000020	20,209	0.001188	0.000107	20,209	0.001168	0.000120	20,209	0.001135	0.000114
336	Staples, Inc.	SPLS	4.9%	1.5%	2.45%	2.47%	6,408	0.000360	0.000018	6,408	0.000377	0.000006	6,408	0.000370	0.000009	6,408	0.000360	0.000009
337	Starbucks Corporation	SBUX	1.7%	15.0%	15.53%	17.79%	84,413	0.004742	0.000082	84,413	0.004962	0.000744	84,413	0.004878	0.000758	84,413	0.004742	0.000844
338	State Street Corporation	STT	1.9%	5.5%	7.63%	6.61%	30,127	0.001692	0.000033	30,127	0.001771	0.000097	30,127	0.001741	0.000133	30,127	0.001692	0.000112
339	Stryker Corporation	SYK	1.3%	14.5%	9.74%	9.56%	48,787	0.002741	0.000036	48,787	0.002868	0.000416	48,787	0.002819	0.000275	48,787	0.002741	0.000262
340	SunTrust Banks, Inc.	STI	1.9%	7.0%	4.54%	7.45%	27,303	0.001534	0.000029	27,303	0.001605	0.000112	27,303	0.001578	0.000072	27,303	0.001534	0.000114
341	Symantec Corporation	SYMC	1.0%	5.0%	22.00%	11.98%	18,912	0.001062	0.000010	18,912	0.001112	0.000056	18,912	0.001093	0.000240	18,912	0.001062	0.000127
342	Synchrony Financial	SYF	1.6%	NA	10.23%	7.64%	26,943	0.001514	0.000024	--	--	--	26,943	0.001557	0.000159	26,943	0.001514	0.000116
343	Sysco Corporation	SYY	2.6%	12.0%	11.93%	8.69%	27,897	0.001567	0.000040	27,897	0.001640	0.000197	27,897	0.001612	0.000192	27,897	0.001567	0.000136
344	T. Rowe Price Group, Inc	TROW	3.3%	7.5%	8.16%	10.12%	16,762	0.000942	0.000031	16,762	0.000985	0.000074	16,762	0.000969	0.000079	16,762	0.000942	0.000095
345	Target Corporation	TGT	4.5%	9.5%	-0.65%	9.44%	29,607	0.001663	0.000075	29,607	0.001740	0.000165	29,607	0.001711	(0.000011)	29,607	0.001663	0.000157
346	TE Connectivity Ltd.	TEL	2.0%	7.5%	8.04%	9.00%	26,019	0.001462	0.000030	26,019	0.001529	0.000115	26,019	0.001504	0.000121	26,019	0.001462	0.000132
347	TEGNA Inc.	TGNA	2.2%	1.0%	10.00%	4.67%	5,498	0.000309	0.000007	5,498	0.000323	0.000003	5,498	0.000318	0.000032	5,498	0.000309	0.000014
348	Tesoro Corporation	TSO	2.8%	7.5%	-23.40%	10.00%	9,311	0.000523	0.000014	9,311	0.000547	0.000041	9,311	0.000538	(0.000126)	9,311	0.000523	0.000052
349	Texas Instruments Incorp	TXN	2.5%	8.0%	10.00%	9.60%	80,071	0.004498	0.000112	80,071	0.004707	0.000377	80,071	0.004627	0.000463	80,071	0.004498	0.000432
350	Textron Inc.	TXT	0.2%	15.5%	8.10%	8.92%	12,807	0.000719	0.000001	12,807	0.000753	0.000117	12,807	0.000740	0.000060	12,807	0.000719	0.000064
351	The AES Corporation	AES	4.3%	7.0%	11.30%	7.58%	7,358	0.000413	0.000018	7,358	0.000433	0.000030	7,358	0.000425	0.000048	7,358	0.000413	0.000031
352	The Charles Schwab Corp	SCHW	0.8%	14.0%	20.24%	17.02%	53,552	0.003008	0.000024	53,552	0.003148	0.000441	53,552	0.003095	0.000626	53,552	0.003008	0.000512
353	The Goodyear Tire & Rub	GT	1.1%	10.0%	6.49%	14.30%	8,870	0.000498	0.000006	8,870	0.000521	0.000052	8,870	0.000513	0.000033	8,870	0.000498	0.000071
354	The Kraft Heinz Compan	KHC	2.6%	NA	3.38%	14.77%	110,699	0.006219	0.000164	--	--	--	110,699	0.006397	0.000216	110,699	0.006219	0.000919
355	The Travelers Companies	TRV	2.2%	1.5%	2.97%	7.67%	33,689	0.001893	0.000042	33,689	0.001980	0.000030	33,689	0.001947	0.000058	33,689	0.001893	0.000145
356	Thermo Fisher Scientific I	TMO	0.4%	9.5%	10.56%	12.25%	59,836	0.003361	0.000013	59,836	0.003517	0.000334	59,836	0.003458	0.000365	59,836	0.003361	0.000412
357	Tiffany & Co.	TIF	1.9%	7.5%	7.62%	10.80%	11,671	0.000656	0.000013	11,671	0.000686	0.000051	11,671	0.000674	0.000051	11,671	0.000656	0.000071
358	Time Warner Inc.	TWX	1.6%	11.0%	10.95%	9.18%	76,117	0.004276	0.000070	76,117	0.004474	0.000492	76,117	0.004399	0.000482	76,117	0.004276	0.000393
359	TJX Companies, Inc. (The	TJX	1.4%	10.5%	10.34%	10.70%	49,581	0.002785	0.000038	49,581	0.002914	0.000306	49,581	0.002865	0.000296	49,581	0.002785	0.000298
360	Torchmark Corporation	TMK	0.8%	7.5%	7.05%	6.76%	8,980	0.000504	0.000004	8,980	0.000528	0.000040	8,980	0.000519	0.000037	8,980	0.000504	0.000034
361	Total System Services, Inc	TSS	0.8%	10.5%	11.58%	12.50%	9,692	0.000544	0.000004	9,692	0.000570	0.000060	9,692	0.000560	0.000065	9,692	0.000544	0.000068
362	Tractor Supply Company	TSCO	1.4%	10.5%	13.34%	16.40%	9,033	0.000507	0.000007	9,033	0.000531	0.000056	9,033	0.000522	0.000070	9,033	0.000507	0.000083
363	Twenty-First Century Fox	FOX	1.2%	NA	11.30%	10.91%	56,540	0.003176	0.000037	--	--	--	56,540	0.003267	0.000369	56,540	0.003176	0.000347
364	Twenty-First Century Fox	FOXA	1.2%	12.0%	13.26%	8.95%	57,595	0.003235	0.000038	57,595	0.003385	0.000406	57,595	0.003328	0.000441	57,595	0.003235	0.000290
365	Tyson Foods, Inc.	TSN	1.4%	9.5%	2.27%	11.00%	22,353	0.001256	0.000018	22,353	0.001314	0.000125	22,353	0.001292	0.000029	22,353	0.001256	0.000138
366	U.S. Bancorp	USB	2.2%	4.0%	5.60%	6.50%	86,947	0.004884	0.000106	86,947	0.005111	0.000204	86,947	0.005024	0.000281	86,947	0.004884	0.000317
367	Union Pacific Corporatio	UNP	2.3%	8.0%	11.27%	10.50%	86,844	0.004879	0.000110	86,844	0.005105	0.000408	86,844	0.005018	0.000566	86,844	0.004879	0.000512
368	United Dominion Realty '	UDR	3.4%	NA	-34.21%	5.67%	9,637	0.000541	0.000019	--	--	--	9,637	0.000557	(0.000191)	9,637	0.000541	0.000031
369	United Parcel Service, Inc	UPS	3.1%	9.5%	8.38%	9.40%	92,806	0.005214	0.000162	92,806	0.005455	0.000518	92,806	0.005363	0.000449	92,806	0.005214	0.000490
370	United Technologies Cor	UTX	2.4%	6.5%	7.39%	8.51%	90,118	0.005063	0.000119	90,118	0.005297	0.000344	90,118	0.005208	0.000385	90,118	0.005063	0.000431
371	UnitedHealth Group Inco	UNH	1.5%	12.5%	15.41%	13.29%	159,529	0.008962	0.000135	159,529	0.009377	0.001172	159,529	0.009219	0.001421	159,529	0.008962	0.001191
372	Universal Health Services	UHS	0.3%	10.5%	8.58%	9.43%	11,976	0.000673	0.000002	11,976	0.000704	0.000074	11,976	0.000692	0.000059	11,976	0.000673	0.000063
373	Unum Group	UNM	1.7%	10.5%	7.05%	7.00%	10,664	0.000599	0.000010	10,664	0.000627	0.000066	10,664	0.000616	0.000043	10,664	0.000599	0.000042
374	V.F. Corporation	VFC	3.1%	10.0%	8.00%	8.86%	22,463	0.001262	0.000039	22,463	0.001320	0.000132	22,463	0.001298	0.000104	22,463	0.001262	0.000112
375	Valero Energy Corporatic	VLO	4.3%	4.5%	20.32%	11.80%	29,350	0.001649	0.000071	29,350	0.001725	0.000078	29,350	0.001696	0.000345	29,350	0.001649	0.000195
376	Ventas, Inc.	VTR	4.8%	NA	6.90%	5.51%	22,963	0.001290	0.000062	--	--	--	22,963	0.001327	0.000092	22,963	0.001290	0.000071
377	Verizon Communications	VZ	4.8%	3.0%	1.81%	4.29%	197,436	0.011091	0.000529	197,436	0.011606	0.000348	197,436	0.011409	0.000207	197,436	0.011091	0.000476
378	Viacom Inc.	VIAB	1.8%	1.0%	6.21%	3.95%	18,090	0.001016	0.000018	18,090	0.001063	0.000011	18,090	0.001045	0.000065	18,090	0.001016	0.000040
379	Visa Inc.	V	0.7%	13.5%	16.10%	16.63%	188,696	0.010600	0.000078	188,696	0.011092	0.001497	188,696	0.010904	0.001756	188,696	0.010600	0.001763
380	Vornado Realty Trust	VNO	2.8%	19.5%	42.04%	4.60%	19,264	0.001082	0.000030	19,264	0.001132	0.000221	19,264	0.001113	0.000468	19,264	0.001082	0.000050
381	Vulcan Materials Compa	VMC	0.8%	20.5%	21.70%	21.72%	15,830	0.000889	0.000007	15,830	0.000931	0.000191	15,830	0.000915	0.000199	15,830	0.000889	0.000193
382	W.W. Grainger, Inc.	GWW	2.1%	6.0%	7.46%	10.50%	13,494	0.000758	0.000016	13,494	0.000793	0.000048	13,494	0.000780	0.000058	13,494	0.000758	0.000080
383	Walgreens Boots Alliance	WBA	1.8%	11.0%	13.00%	10.63%	88,145	0.004952	0.000091	88,145	0.005181	0.000570	88,145	0.005094	0.000662	88,145	0.004952	0.000526
384	Wal-Mart Stores, Inc.	WMT	2.9%	2.0%	4.41%	6.86%	216,648	0.012171	0.000348	216,648	0.012735	0.000255	216,648	0.012519	0.000552	216,648	0.012171	0.000835
385	Walt Disney Company (T	DIS	1.4%	8.5%	9.50%	8.06%	178,760	0.010042	0.000139	178,760	0.010508	0.000893	178,760	0.010330	0.000981	178,760	0.010042	0.000809
386	Waste Management, Inc.	WM	2.3%	7.5%	10.38%	9.49%	32,291	0.001814	0.000042	32,291	0.001898	0.000142	32,291	0.001866	0.000194	32,291	0.001814	0.000172
387	WEC Energy Group, Inc.	WEC	3.4%	6.0%	6.53%	6.00%	19,194	0.001078	0.000037	19,194	0.001128	0.000068	19,194	0.001109	0.000072	19,194	0.001078	0.000065
388	Wells Fargo & Company	WFC	2.8%	4.5%	7.46%	10.80%	277,037	0.015563	0.000428	277,037	0.016285	0.000733	277,037	0.016009	0.001194	277,037	0.015563	0.001681

389	Welltower Inc.	HCN	4.9%	NA	2.90%	3.95%	25,998	0.001461	0.000071	--	--	--	25,998	0.001502	0.000044	25,998	0.001461	0.000058
390	Western Digital Corporat	WDC	2.4%	7.5%	2.00%	12.11%	24,224	0.001361	0.000032	24,224	0.001424	0.000107	24,224	0.001400	0.000028	24,224	0.001361	0.000165
391	Western Union Company	WU	3.6%	8.0%	6.96%	7.15%	9,329	0.000524	0.000019	9,329	0.000548	0.000044	9,329	0.000539	0.000038	9,329	0.000524	0.000037
392	Westrock Company	WRK	3.1%	12.5%	9.18%	9.50%	12,988	0.000730	0.000022	12,988	0.000763	0.000095	12,988	0.000751	0.000069	12,988	0.000730	0.000069
393	Weyerhaeuser Company	WY	3.6%	12.5%	5.00%	5.00%	25,773	0.001448	0.000052	25,773	0.001515	0.000189	25,773	0.001489	0.000074	25,773	0.001448	0.000072
394	Whirlpool Corporation	WHR	2.4%	10.5%	13.35%	16.36%	12,452	0.000700	0.000017	12,452	0.000732	0.000077	12,452	0.000720	0.000096	12,452	0.000700	0.000114
395	Whole Foods Market, Inc	WFM	1.8%	5.0%	-2.63%	7.50%	9,815	0.000551	0.000010	9,815	0.000577	0.000029	9,815	0.000567	(0.000015)	9,815	0.000551	0.000041
396	Williams Companies, Inc	WMB	4.0%	14.0%	10.00%	10.00%	24,981	0.001403	0.000056	24,981	0.001468	0.000206	24,981	0.001444	0.000144	24,981	0.001403	0.000140
397	Willis Towers Watson Pu	WLTW	1.6%	NA	11.09%	11.03%	17,574	0.000987	0.000016	--	--	--	17,574	0.001016	0.000113	17,574	0.000987	0.000109
398	Wynn Resorts, Limited	WYNN	1.7%	5.0%	8.00%	8.00%	12,011	0.000675	0.000011	12,011	0.000706	0.000035	12,011	0.000694	0.000056	12,011	0.000675	0.000054
399	Xcel Energy Inc.	XEL	3.2%	5.5%	NA	5.43%	22,641	0.001272	0.000041	22,641	0.001331	0.000073	--	--	--	22,641	0.001272	0.000069
400	Xerox Corporation	XRX	3.5%	0.5%	-0.11%	10.00%	7,248	0.000407	0.000014	7,248	0.000426	0.000002	7,248	0.000419	(0.000000)	7,248	0.000407	0.000041
401	Xilinx, Inc.	XLNX	2.3%	6.5%	8.66%	8.25%	14,025	0.000788	0.000018	14,025	0.000824	0.000054	14,025	0.000810	0.000070	14,025	0.000788	0.000065
402	XL Group Ltd.	XL	2.2%	8.0%	NA	9.00%	10,591	0.000595	0.000013	10,591	0.000623	0.000050	--	--	--	10,591	0.000595	0.000054
403	Xylem Inc.	XYL	1.4%	9.5%	14.03%	15.00%	9,150	0.000514	0.000007	9,150	0.000538	0.000051	9,150	0.000529	0.000074	9,150	0.000514	0.000077
404	Yum! Brands, Inc.	YUM	1.9%	4.0%	13.00%	11.00%	22,841	0.001283	0.000024	22,841	0.001343	0.000054	22,841	0.001320	0.000172	22,841	0.001283	0.000141
405	Zimmer Biomet Holdings	ZBH	0.8%	12.5%	9.73%	9.93%	24,187	0.001359	0.000011	24,187	0.001422	0.000178	24,187	0.001398	0.000136	24,187	0.001359	0.000135
406	Zions Bancorporation	ZION	0.8%	11.0%	12.12%	9.33%	8,401	0.000472	0.000004	8,401	0.000494	0.000054	8,401	0.000485	0.000059	8,401	0.000472	0.000044
407	Zoetis Inc.	ZTS	0.8%	11.5%	13.36%	12.63%	25,897	0.001455	0.000012	25,897	0.001522	0.000175	25,897	0.001497	0.000200	25,897	0.001455	0.000184
							17,800,912	**1.000000**		**17,012,244**	**1.000000**		**17,305,101**	**1.000000**		**17,800,912**	**1.000000**	
	Weighted Average								**2.4%**			**8.7%**			**9.6%**			**9.2%**
	Average												**9.2%**					

NA -- Not Available

(a) Dividend paying components of S&P 500 index from zacks.com (retrieved Apr. 7, 2017)
(b) www.zacks.com (retrieved Apr. 7, 2017)
(c) www.valueline.com (Apr. 7, 2017)
(d) http://finance.yahoo.com (retrieved Apr. 9, 2017)