Emergence of structures and forms in complex adaptive systems in nature

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Abstract:

Structure is an emergent property. The law of divergence (Second Law of Thermodynamics) proposes the increase of disorder with every spontaneous natural process. However, systems existing in nature constantly tend toward a state of increasing order. The self-organizing property present within these systems makes them robust and reliable to sustain changes in their surroundings. These systems inherently possess two intrinsic mechanisms. The first is a control mechanism that drives these systems away from a state of physical equilibrium or dead state, and the second is a feedback mechanism that enables these systems to adapt to the ever-changing environment through self-organization. The system elements organize themselves through various cycles of evolution by applying work on the system constraints in order to minimize them in the least possible time. Through this paper we investigate the cause of existence of an exergy gradient between the systems found in nature and their surrounding media. We relate the exergy flow current and constraint minimization to understand the existence of diverse forms and emergent complexity in nature, and we argue the existence of microscopic heat engines, which, by operating between two potentials, minimize physical constraints and optimize global exergy flow current in open adaptive complex dynamical systems, thereby giving rise to beautiful structural forms and varied species. Keywords: accelerate, action, asymmetry, complex adaptive systems, complexity, constraint, minimization, emergence, entropy, exergy flow, feedback loop, fractal, microscopic heat-engines, organization, Principle of Least Action, self-organization, symmetry in nature.

Introduction

Nature has inspired man time and time again to design structures, imagine processes, and propound theories. Natural systems are beautiful and complex in construction, exhibiting enormous varieties of shapes and structures. Geometric perfection is rarely observed in natural systems. It is almost impossible to find perfect symmetry in animate systems (systems which are continuously self-organizing and intrinsically adaptive with the externally surrounding environment) in nature. There is a gate in Japan, Neiko, which is sometimes called by the Japanese the most beautiful gate in all of Japan. The gate is very elaborate, with many gables, beautiful carvings, columns, and dragon headed princes carved into the pillars. But when one looks closely, they see that in the elaborate and complex design along one of the pillars one of the small design elements is carved upside down; otherwise, it is completely symmetrical. The error was purposely put so that the gods would not be jealous of man's perfection.¹A question arises, why is nature nearly symmetrical? What eludes that perfect geometry with sharp curves and faultless rounded circles that we spend time learning at schools?

Natural structures have living and evolving geometries that continuously optimize their struggle for better performance through progressive development.2 Near symmetry in nature is a topic of thorough discussion at various interdisciplinary levels. It would require physicists, chemists, mathematicians, biologists, complexity theorists, astrobiologists, and engineers, all focused to develop a common world view, to decipher nature's enduring mysteries. Architecture in nature has been extensively investigated by the Constructal Law, widely regarded as the Fourth Law of Thermodynamics.³ Natural structures evolve at various levels of hierarchy and become increasingly complex with time. Investigating such systems with continually optimizing geometries (architectures) becomes significantly difficult with passage of time. Also, with growth of complexity, systems become greatly organized at various hierarchical levels, namely physical, chemical, biological, societal, and technological.^{4, 5, 6, 7,} ^{8, 9, 10} However, in accordance to the Second Law of Thermodynamics, systems should come into a state of equilibrium with the surrounding by continuously dispersing energy. We need to develop certain fundamental laws and governing principles to

identify, measure, engineer, and re-engineer such systems for the development of society to ensure our sustainable existence.^{10, 11}

Prior to proposing any law or theory to investigate nature or natural processes, we must look into the most fundamental principles on which lies the foundation of all physical laws (i.e. the Principle of Least Action). The Principle of Least Action is an inherent law of nature which states that every spontaneous process tends to follow the path which will take least time to complete. Through this paper, we intend to present an idea on how the elements constituting a complex system obey the Principle of Least Action and hence, minimize the constraints by performing work on them. This is done through grouping and adding to the global exergy current flowing through the system, giving rise to various architectural structures and forms in nature. Exergy of a system is defined as the maximum possible work that it can perform. Mathematically, exergy is equal to Carnot efficiency times the heat contained by a system. In this paper, we present some analogies between the classical formulations of the motion of system elements, incorporating variational principles and thermo-dynamical aspects

of work, exergy and entropy. We believe every scientific theory must be broad in scope, present a constructive common worldview, be able to address a wide range of phenomena, and be able to sustain the tests of time like the laws of thermodynamics and quantum mechanics that have successfully explained countless natural phenomena. The long-range implications of our ideas have been presented in the discussion section.^{7, 11}, 12, 13, 14, 15, 16

Methodology

A complex system is a system composed of many interacting elements, often called agents, which display collective behaviour that does not follow the behaviours of the individual parts.¹⁷ The collective behaviour of the constituting elements is an emergent property. The word 'complex' has Latin origin, complexus, meaning consisting of many different and connected parts or not easy to understand. Nature inevitably consists of complex systems that are continuously evolving and optimizing themselves through various cycles of evolution to attain greater degrees of order; they recursively optimize their performance and get organized over time. Emergence is hard to quantify because, in a multi-element open system, there are infinite parameters controlling any emergent property of that system. Any change may cause that property to either totally disappear or appear altogether in a new form. Through this paper, we intend to present a new form of the Principle of Least Action to develop the idea as to how complexity evolves in multi-element systems, move into the domain of Constructal Theory, and eventually relate the two to describe the coherent interdependency between them.

Principle of Least Action: a multi-agent approach

The Principle of Least Action has emerged as the foundation for almost all physical laws, particularly those describing natural processes. There is not a broader and more fundamental principle in science than this. The Principle of Least Action has been refined time and again to describe a wide range of natural phenomena, be it the Gauss Principle of Least Constraint, Hertz's principle of Least Curvature, or the path integral formalism for quantum mechanics by Feynman. The conventional Least Action Principle is highly deterministic in nature and takes into account the variation in trajectory of a single system element between two pre-determined fixed points or states in space. 18 The cause of variation in trajectory of an element due to its interaction with other elements within the system needs to

be taken into consideration. The rational choices of the system elements or agents are to pursue the shortest possible path in order to organize in the least possible time span, which drives the system towards greater entropy generation and irreversibility.^{13, 19} In a networked complex system, each element will compute all possible paths from one node to another and thus will render the system towards a state of uncertainty.

In a one-dimensional state space, the equation of trajectory of a system element can be written as²⁰:

$$(x(t))_{actual} = (x(t))_{sortest} + \varepsilon(t)$$
 (1)

The above equation signifies that the actual path of a system element is greater than the shortest path by an amount ' ϵ (t)' termed as the 'variation parameter'. When a system element is free from mutual interactions, fields, and forces then, its actual path will always coincide with its shortest path, rendering the variation parameter to zero. For a two-element system, the elements in a system are labelled by α and β . So, the new trajectories of the two system elements due to mutual interaction are expressed as, for system element α :

$$x_{\alpha\beta}(t) = x_{\alpha\alpha}(t) + \varepsilon_{\alpha\beta}(t)$$
 (2)

In eqn. (2), the left side represents the actual trajectory of the system element α due to its interaction with β . The right-hand side of the equation consists of two parts, $x_{\alpha\alpha}(t)$, the trajectory of element α in absence of any other interacting element which must inherently be its shortest path and the variation parameter, $\epsilon_{\alpha\beta}(t)$, due to mutual interaction between the elements. Similarly, the trajectory of element β can be expressed similarly as:

$$x_{\alpha\beta}(t) = x_{\beta\beta}(t) + \varepsilon_{\alpha\beta}(t)$$
 (3)

The variation parameter employed here is different from the one generally used in the analysis of calculus of variation to evaluate the shortest path. In open systems there is an incessant in-flux and out-flux of mass, energy, and information so the final state of such a system is often indeterminate. The variation parameter employed in eqn. (1, 2 and 3) has to be, thus, weakly constrained. A natural question arises here as to how this parameter can establish the growth of complexity in a system with time.

Exponential growth of complexity: an empirical relationship

A system can also be defined as a

connected network. Systems found in nature are structurally complex. Complexity increases not with the amount of connections between the nodes that are present in a system but due to the numerous combinations of possible connections. In a system open to surrounding environment, the state of least action behaves as an attractor.6, 7, 8, 9 The system elements progressively optimize their trajectories to achieve the least action state but ultimately fail to achieve that stationary (least action) state. The pursuit of the system elements to reach the stationary state causes the action of the system as a whole to increase and gradually diverges the system away from equilibrium, but at the same time, mutual interactions between the system elements induce internal irreversibility within the system that make the process of selforganization irreversible, causing dissipation of free energy and information from the system and the entropy to rise.9, 21 The interaction or variation parameter thus plays a crucial role in complexation of a system.

According to the Principle of Least Action, the variation of the path is zero for any natural process occurring between two points of time, t₁ and t₂. Nature acts in the simplest way, in the shortest possible time.

Thus, the action integral is given by:

$$I = \int_{t_1}^{t_2} L dt = \int_{t_1}^{t_2} (T - V) dt$$
 (4)

Where L is the Lagrangian, T and V are the kinetic and the potential energies of the system (respectively) and L=T-V. For the motion of the system between time t_1 and t_2 , the Lagrangian, L, has a stationary value for the correct path of motion. This can be summarized as the Hamilton's Principle. Rewriting eqn. (4) in multi-agent notation:

$$I_{\alpha\alpha} = \int_{t_1}^{t_2} L_{\alpha\alpha} dt = \int_{t_1}^{t_2} (T_{\alpha\alpha} - V_{\alpha\alpha}) dt \qquad (5)$$

 $I_{\alpha\alpha}$ represents the action of element α in absence of any other element. Eqn. (5) on solving will give the shortest possible path between two points in state space at times t_1 and t_2 . In presence of a second element, β eqn. (5) gets modified into:

$$I_{\alpha\beta} = \int_{t_1}^{t_2} L_{\alpha\beta} dt = \int_{t_1}^{t_2} (T_{\alpha\beta} - V_{\alpha\beta}) dt \quad (6)$$

 $I_{\alpha\beta}$ is the action of element α in presence of other interacting system elements. From eqn. (2, 3) we can observe that the trajectory obtained by solving eqn. (6) is greater than that obtained by solving eqn. (5) by an amount $\epsilon_{\alpha\beta}$. Hence, $I_{\alpha\beta}$ is greater than $I_{\alpha\alpha}$. Neglecting the existence of any field, the potential energy term vanishes. So, we are left with:

$$(I_{\alpha\beta} - I_{\alpha\alpha}) = \int_{t_1}^{t_2} \frac{m_{\alpha}}{2} ((\dot{x}_{\alpha\beta}^2) - (\dot{x}_{\alpha\alpha}^2)) dt > 0$$

 $(I_{\alpha\beta} - I_{\alpha\alpha}) = \int_{t_1}^{t_2} \frac{m_{\alpha}}{2} ((\dot{x}_{\alpha\beta}^2) - (\dot{x}_{\alpha\alpha}^2)) dt =$
 $\int_{t_1}^{t_2} \frac{m_{\alpha}}{2} ((\dot{x}_{\alpha\alpha}^2 + 2\dot{x}_{\alpha\beta}\dot{x}_{\alpha\alpha} + \dot{x}_{\alpha\beta}^2) - (\dot{x}_{\alpha\alpha}^2)) dt > 0$ (7)

$$\int_{t_1}^{t_2} \dot{\varepsilon}_{\alpha\beta} dt > 0 \tag{8}$$

Eqn. (8) states that the integration of the time derivative of the variation parameter between times t_1 and t_2 is always positive. As it was discussed earlier, the interaction parameter induces irreversibility in a process due to mutual interactions and hence, generates entropy. Thus, the left side of the eqn. (8) represents the mechanical analogy of the entropy principle. For a system undergoing a process $1 \rightarrow 2$, with entropy transfer across the system boundary due to heat transaction, the entropy generation S_{gen} must be greater than zero.²²

$$S_2 - S_1 - \int_1^2 \frac{\delta Q}{T} = \dot{S}_{gen}$$
, and $\dot{S}_{gen} > 0$ (9)

From the above equation it can be seen that the rate of change in entropy is a monotonically increasing function. Further, if we compare eqn. (8) and (9) we can find an analogy between them. Both the rate of change of the variation parameter and entropy are related. With increase in interactions due to multiple elements in the system the loss in information about each element becomes increasingly significant and contributes to overall complexity growth. Thus, the growth of complexity with time is profound and has been found to follow an exponential distribution.^{7, 23, 24} The variation parameter controls the rational choices of the system elements to pursue specific trajectories to optimize their action.12 Optimizing this parameter through cycles of evolution is the process of self-organization and complexity growth. We therefore present an empirical (although incomplete) expression for this parameter here.

$$\varepsilon_{\alpha\beta}(t) = \{\phi(x_{\alpha\beta}, \dot{x}_{\alpha\beta})\}x_0e^{\lambda_{x_{\alpha\beta}}t}$$
 (10)

Eqn. (10) gives an expression for the growth of complexity in a system with time. Here, $\phi(x_{\alpha\beta}, \dot{x}_{\alpha\beta})$ is a function depending upon velocity and displacement, x_0 is the initial variation, i.e., at time t_1 and $\lambda x_{\alpha\beta}$ is the Lyapunov's exponent for the two element pair, α and β . In the metric formulation of complex systems action possessed and degree of orderliness are inversely related. ^{7,8,9}

$$\alpha = \frac{nm\hbar}{\sum_{i=0}^{n} \sum_{j=0}^{m} I_{ij}}$$
 (11)

In the above equation α is the measure of organization in a complex networked system.⁷ From the above equation it can be seen that action and organization are related inversely. Thereby, reduction in action with time and achieving a least action or maximum organized state is the elucidating motive of the system which it fails to achieve. This is because a maximum organized state for a system is also a state of maximum action.8 Such a state thus acts as an attractor.7, 8, 10, ^{11, 12} So, for natural systems the Lyapunov's exponent in eqn. (10) becomes negative. In a two-dimensional state space, the second Lyapunov's exponent should be positive in order to satisfy the Liouville'stheorem. 18For a two-dimensional state space:

$$\varepsilon_{\alpha\beta}^{x}(t) = \{\phi(x_{\alpha\beta}, \dot{x}_{\alpha\beta})\}x_{0}e^{-\lambda x_{\alpha\beta}t},$$

$$\varepsilon_{\alpha\beta}^{y}(t) = \{\phi(y_{\alpha\beta}, \dot{y}_{\alpha\beta})\}y_{0}e^{\lambda y_{\alpha\beta}t}$$
(12)

Constructal Theory: optimizing physical constraints in a system

The Constructal Law states that if a system has freedom to morph, it develops so that the flow architecture provides easier access to the currents that flow through it.2, ²⁵ The system's purpose is global existence. It is present along with fixed global constraints which may include the space allocated to the system, available material and components, allowable temperature, pressure or stress ranges, etc. The system designer brings together all components and optimizes the arrangement in order to reach maximum performance. In this way, he "constructs" the optimal flow architecture. Therefore the flow architecture shape and structure are deduced, not assumed in advance. A design engineer, thus, designs a system by optimizing the constraints and maximizing the performance. A flow system is also characterized by "performance" (function, objective) and "flow structure" (configuration, layout, geometry, architecture). Unlike the black box of classical thermodynamics, which represents a system at equilibrium, a flow system has performance and especially configuration. Each flow system has a drawing.26 Natural structures are flow structures through which heat, work, energy flow inwards and outwards with time. How do natural structures sustain and enhance their performances with time? How does nature create its structures?

The global exergy currents flowing through natural animate systems are fuel,

food, or perhaps information. In a complex networked system the global currents flowing in the network could be exergy and information. A very interesting property of complex adaptive systems is that they operate a feedback loop by which they interact with the surrounding and self-organize them with time (a continuous learning process). Also, at the same time, they possess an intrinsic control mechanism that diverges the system away from equilibrium.7, 11 It is due to the irreversibility generated because of mutual interaction (eqn. (8, 10)). The self-organizing process operating through a feedback loop is analogous to parallel computation processes by which information gets stored into the physical memory of the system.^{7,2}

In the earlier sections we analysed the cause of internal irreversibility and related it to entropy generation (eqn. (8, 9 and 10)), but we did not discuss the existence of any physical constraints present within the system. The physical constraints can be the physical boundary of the system, distance between nodes in a networked complex system, physical hindrance to the motion of the system elements within, or can be any energy barrier. In addition to the generated irreversibility due to motion and mutual interaction, the physical constraints provide another challenge for natural systems to organize, to sustain, and also enhance their performances with time. So, what mechanism does a natural system employ to organize itself with time? How does the system morph and transit from one level of complexity to the other?

Emergence of geometry in nature and enhancing performance through constraint minimization and parallel computation

It was discussed in earlier sections that geometries in nature are alive and imperfect. The existence of physical imperfections and asymmetry are signs that, they are continuously evolving and are living.2 Geometry and design are weakly connected in the sense that geometry is concerned with the shape and the form of a system, whereas design is concerned with its function and achievable performance. Thus, geometry represents the evolving system and design represents the sustaining system. In the empirical function to denote complexity we had multiplied a function $\phi(x_{\alpha\beta}, \dot{x}_{\alpha\beta})$ with the exponential term. The function depends upon the inter-nodal distance and instantaneous velocity of the elements crossing the nodes. Continuous self-organizing processes aim towards minimizing action and reducing the inter-nodal distance or displacement. With

self-organization the distances between the nodes shrink and the system's geometry varies continuously with time.7 shrinkage, the action of individual element is reduced and the elements evolve through time by approaching the shortest path. The continuous variation (geometrical) of a system or morphing would provide multiple pathways for the exergy currents to flow. Since a natural system is open, the search for shortest path for the exergy currents to flow through it will cause the system to continuously morph. On one hand, this would cause the system to change its shape by enlarging or shrinking its boundaries along its different axes of symmetry. On the other hand, the continuous search for shortest paths for currents to flow would cause internal differentiation of the paths at finer levels leading to alveoli structure of lungs, veins for blood circulation, a flowing river with numerous branches and tributaries, etc. Thus, the combined effect is seen in the form of asymmetrical structures showing self-similarity at finer levels. This "self-similarity" is signature in fractals.

Nature is full of fractals. Everything around us, animate or inanimate; for example, a leaf, a tree, the human body, a mountain range, snowflakes, etc. are all fractals. Fractals possess a great deal of self-similarity when examined successively at finer scales. The fractal dimension (d_p) of the least state attractor can be deduced from the Lyapunov's exponents; the negative exponent $(-\lambda x_{\alpha\beta})$ denoting shrinkage along the x-axis and the positive exponent $\lambda y_{\alpha\beta}$ denoting elongation along the y-axis. ¹⁸

$$d_F = 1 + \frac{\lambda_{y_{\alpha\beta}}}{\lambda_{x_{\alpha\beta}}}, \qquad 1 < d_F < 2 \quad (13)$$

Thus, the growth of complexity in natural systems along with progressive development or self-organization generates fractals in nature, the reason of enormous varieties of beautiful structural forms, asymmetry, and geometric imperfections. Sustenance and never-ending struggle for achieving better performance takes place by two proposed ways:

- 1. Allow better access to the flow of exergy currents within the system through constraint minimization.
- 2. Through numerous simultaneous (parallel) self-organizing processes operating between nodes within the networked system (Accelerated Natural Computation).

Interestingly, both the processes are intrinsically linked. The system elements apply work on the constraints and minimize them to make way for efficient exergy flow within the networked system. The flow of exergy currents through the system establishes a gradient between the system and the surrounding. Exergy of the surrounding is null, i.e. the surrounding environment can be considered as a dead state compared to the system. The existence of this exergy gradient between the system and the surrounding causes the system elements to act as microscopic heat engines. The work output of these heat engines (system elements) is used up in minimizing constraints. The energy rejected while performing work, by the Second Law, is dissipated into the surrounding media. According to the exergy principle, exergy of a system can never increase. The rate of decrease in exergy of a system is equal to difference between the internal irreversibility and cumulative exergy of the system elements.

$$\frac{dA}{dt} = \sum_{j} \left(1 - \frac{\tau_0}{\tau_j}\right) Q_j - I < 0 \qquad (14)$$

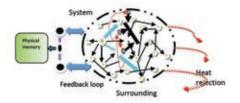
$$i > \sum_{J} \left(1 - \frac{\tau_0}{\tau_J} \right) Q_J$$
 (15)

So, eqn. (15) can be rewritten as:

$$T_0 \, \dot{S}_{gen} > \sum_j \left(1 - \frac{\tau_0}{\tau_j}\right) Q_j.$$

$$\dot{S}_{gen} > \frac{\sum_j q_j}{\tau_0} - \sum_j \frac{q_j}{\tau_j} \qquad (16)$$

From the above expression it can be deduced that internal irreversibility due to rise in complexity is greater than the difference between global entropy of the system and summation of local entropy for the all constituting system elements due to heat (energy, information) exchange with the surrounding media. Internal irreversibilities or inherent complexities restrain the system elements from performing maximum work to minimize constraint and limits the system from achieving least action state. The work lost due to irreversibility is the exergy destruction. Accelerating the self-organizing process causes the system elements to perform cumulative work on the constraints in a parallel arrangement. The parallel arrangement gives multiple simultaneous paths for exergy currents to flow by minimizing constraints along them. This also enhances the processing speed of the system by accelerating the feedback mechanism loop. This induces rapid decrease in action of the system as a whole and organizes the system at an accelerated rate. Thus, complexity present in the system generates irreversibility, preventing the elements from minimizing the constraint to a possible minimum, thereby preventing the system from reaching the least action state. Grouping causes multiple processing within the system thus accelerating the rate of self-organization. These two processes are constantly in operation, preventing any complex-adaptive system reaching the dead state and also allowing the system to grow and develop in time and progressively enhance its performance. The figure below shows an evolving complex networked system with various nodes (concentric circles, green), inter-nodal trajectory of system elements (thin arrows, black), the exergy flow current (thick arrows, cyan), and system elements (solid spheres, black). The system elements work between the exergy gradients of system and surrounding (dotted red arrows) and perform work (thick black arrows) to minimize physical constraint (thick black line). In the figure below, three elements group, and together they apply cumulative efforts to modify the constraint. This causes simultaneous work and accelerates the feedback loop. The history (in the form irreversibilties in shape or structural from) gets stored in the physical memory of the



Discussion

The aim of this section is to discuss the long-range implications of the ideas presented in this paper and the future work to be done using them.

- 1. In the future, we can work on defining the function $\phi(x_{\alpha\beta}, x_{\alpha\beta})$ and investigate complexity growth in greater detail. We can also work to formulate the multi-agent approach of the Principle of Least Action and quantify organization, parallel computation, and accelerated self-organization.^{7, 13}
- 2. According to the Space Time Energy Matter (STEM) compression, systems increasingly get localised in space and increase their performance efficiency. We have seen earlier how self-organization processes shrink inter-nodal distance in a system. Thus, STEM compression and approach to a least action state are analogous. Both appear to be unrealized attractors for the leading edge of complexity development (of emergent hierarchical intelligence) in the

universe.10

- 3. A debatable question has many a time erupted in our minds: are we alone in this universe? The Search for Extra Terrestrial Intelligence (SETI) project, Black hole intelligence, XRB's¹¹, the Cosmic Contact Censorship¹⁴ can be explained by the idea of exergy flow and constraint minimization.
- 4. Using geometry in nature to design more efficient structures that allow for better exergy flow can optimize our resources and allow our sustainable existence.
- 5. Are Cosmological Natural Selection (CNS) and natural selection of rational choices of system elements in game-theoretic formulation of complex systems related?

Conclusion

Through this paper, the key features of complex systems, their continuously evolving geometries, the process of self-organization (by operating a feedback loop), the growth of complexity with time, and accelerated rate of organization by minimization of physical constraints through parallel processes have been addressed by making use of the Constructal Law and Principle of Least Action for multi-agent systems. We believe these ideas will open up new gateways to engineer and redesign our future structures and technologies. Finally, in the words of Feynman, "God made the laws only nearly symmetrical so that we should not be jealous of His perfection!'

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