Bayesian Updating of Soil Parameters for Braced Excavations Using Field Observations

C. Hsein Juang, F.ASCE1; Zhe Luo, A.M.ASCE2; Sez Atamturkturk, M.ASCE3; and Hongwei Huang4

Abstract: A Bayesian framework using field observations for back-analysis and updating of soil parameters in a multistage braced excavation is presented. Because of the uncertainties originating from the poorly known soil parameters, the imperfect analysis model, and other factors such as construction variability, the soil parameters can only be inferred as probability distributions. In this paper, these posterior distributions are derived using the Markov chain Monte Carlo (MCMC) sampling method implemented with the Metropolis-Hastings algorithm. In the proposed framework, Bayesian updating is first realized with one type of response observation (maximum wall deflection or maximum settlement), and then this Bayesian framework is extended to allow for simultaneous use of two types of response observations in the updating. The proposed framework is illustrated with a quality excavation case and shown effective regardless of the prior knowledge of soil parameters and type of response observations adopted. DOI: 10.1061/(ASCE)GT.1943-5606.0000782. © 2013 American Society of Civil Engineers.

CE Database subject headings: Bayesian analysis; Markov process; Monte Carlo method; Simulation; Algorithms; Excavation; Walls; Deflection; Bracing.

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Introduction

Back-analysis or inverse analysis based on field observations (or measurements) in a braced (or supported) excavation process has been widely reported (Whittle et al. 1993; Calvello and Finno 2004; Hashash et al. 2004, 2010; Chua and Goh 2005; Hsiao et al. 2008; Tang and Kung 2009). Back-analysis with field observations is usually performed for important and/or difficult excavation projects only. Because braced excavations are generally carried out in stages, back-analysis to update key soil parameters (such as the normalized undrained shear strength and the normalized initial tangent modulus in an excavation in clays) is generally realized in multiple stages; before the first excavation stage, the wall and ground responses are predicted with limited field tests and/or laboratory data. Because these data often involve significant uncertainty, the predictions of the wall and ground responses, even those made with the more advanced FEM, often fail to match the observed responses at the end of this excavation stage. After the first-stage excavation is completed and wall and/or ground responses are measured, the key soil parameters can be updated with the observed responses to refine the knowledge of the soil parameters. With the updated soil parameters, the wall and/or ground responses in the subsequent excavation stages may be predicted with improved fidelity. This process can be repeated stage by stage until the final excavation depth is reached.

The soil parameters updated with field observations through back-analysis are not necessarily the true values of these parameters, because the wall and ground responses in a supported excavation may also be affected by construction quality (workmanship) and other environmental factors (such as temperature variation), in addition to the soil-wall interaction mechanism. Nevertheless, updated soil parameters allow for more accurate predictions of the wall and ground responses in the subsequent stages of excavation, which can be critical in some projects for developing remedial measures for preventing damage to adjacent buildings and infrastructures.

Many approaches are available for back-analysis of soil parameters, e.g., least squares method (Xu and Zheng 2001), maximum likelihood method (Ledesma et al. 1996), Bayesian method (Zhang et al. 2010a), and so on. The Bayesian probabilistic framework is demonstrated to be a robust approach for updating input parameters and response predictions (Tang 1971; Beck and Au 2002; Zhang et al. 2010a; Ching et al. 2010; Wang et al. 2010), and it offers a procedure to update the probability distribution function of input parameters provided that the observations are available (Miranda et al. 2009). Many successful applications of the Bayesian updating approach in geotechnical engineering have been reported, e.g., pile capacity analysis (Kay 1976), study on embankment on soft clay (Honjo et al. 1994), serviceability assessment of braced excavations in clay (Hsiao et al. 2008), and slope stability study (Zhang et al. 2010b). In the work reported by Hsiao et al. (2008), the Bayesian observational method is implemented with a semiempirical model known as the Kung-Juang-Hsiao-Hashash (KJHH) model (Kung et al. 2007c) for predicting wall and ground settlement, and the refinement of settlement predictions is realized in a stage-by-stage manner through updating of the bias factor of the prediction model.
In the early stages (the first or second stage) of excavation using a diaphragm wall, the deformation of the wall generally exhibits a cantilever shape and then changes into a concave shape at later stages (e.g., after the first or second stage). Thus, back-analysis for the purpose of updating soil parameters based on field observations in the early stages of excavation may not be meaningful because of the inevitable change of deformation pattern. Furthermore, the effectiveness of back-analysis in the early stages is also compromised by the fact that the excavation responses (maximum wall deflection and ground settlement) in these early stages are generally very small and are often prone to measurement errors.

Another practical issue arises when the excavation is carried out in a heterogeneous or layered soil deposit. Back-analysis (or updating) of soil parameters can become quite complex, because soil parameters of each layer, such as the undrained shear strength and initial tangent modulus, have to be treated as separate variables. In theory, this is not a problem if the analysis model, which is a required component in any updating scheme based on field observations, allows for consideration of soil parameters separately in each of multiple layers, as in the case of finite-element modeling of excavation in a layered soil deposit. However, back-analysis using FEM that involves multiple sets of soil parameters is computationally intensive, especially under the proposed Bayesian updating framework. Furthermore, increasing the number of parameters to be updated would demand greater experimental resources or field observations. In this paper, the focus is on the proposed updating methodology, and thus, its application to staged excavation is limited to soil deposits that can be modeled with a set of soil parameters. For the same reason, a semiempirical model is adopted herein as the analysis model, in lieu of the FEM model, so that the computational effort can be greatly reduced.

With the limitations described previously, the goal of this paper is to develop a framework that combines the Bayesian analysis and the observational method for updating soil parameters in a braced excavation in clay. The updated soil parameters are represented by their posterior distributions and sample statistics. In this framework, the updating process starts with an assumption for the prior distributions for soil parameters. After the initial excavation stage is conducted, the maximum wall deflection and maximum ground settlement are observed (or measured). Those observations are used to update the soil parameters through comparison with the predictions, and the updated soil parameters are then used to predict the responses in the subsequent stages. This procedure is repeated stage by stage as the excavation proceeds, and the soil parameters are updated accordingly.

In the traditional back-analysis, the focus is on finding a set of fixed values for the input parameters of concern. Because of the high degree of uncertainty involved in a braced excavation, the fixed parameter values may not be feasible or physically meaningful. In the current study, parameters of concern are treated as random variables, and the updated parameters are expressed in terms of posterior distributions. To update these parameters, a Markov chain Monte Carlo (MCMC) simulation is carried out using the Metropolis-Hastings algorithm. In this solution process, a prior distribution of each of the parameters of concern is needed. The prior distribution may be assumed based on prior knowledge (i.e., published literature and/or local engineering experience). As is shown later, converged results can be obtained even if the prior knowledge is imperfect. Finally, the proposed framework for updating soil parameters can be implemented with one type of field response observation (in this paper, either the maximum wall deflection or maximum settlement observation) or multiple types of observations. The proposed framework, which deals with updating of multiple soil parameters using multiple types of response observations from multiple stages of a braced excavation, is considered significant. The comprehensive updating analyses of braced excavations through MCMC simulations in this paper produce many critical insights.

**Review of the Kung-Juang-Hsiao-Hashash Model**

The KJHH model is a semiempirical model proposed by Kung et al. (2007c) for predicting the maximum wall deflection and maximum settlement in excavations in clay. It is based on an extensive series of regression analyses of 33 well-documented excavation cases and hundreds of FEM simulations using a proven small-strain constitutive relationship called the modified pseudoplasticity model. The KJHH model consists of equations that can be used to predict wall deflection and ground settlement using the following parameters: excavation depth ($H_e$); excavation width ($B$); the system stiffness [$S = EI/\gamma_w h_d^4$, as defined in Clough and O’Rourke (1990), where $E$ is the Young’s modulus of wall material, $I$ is the moment of inertia of the wall section, $\gamma_w$ is the unit weight of water, and $h_d$ is the average spacing of the struts]; the normalized clay layer thickness $\Sigma H_{clay}/H_{wall}$ (where $\Sigma H_{clay}$ is the total thickness of all clay layers and $H_{wall}$ is the wall length; for a pure clay site, this ratio is 1.0); the normalized undrained shear strength ($s_u/\sigma'_c$); and $\sigma'_c$ (denotes the vertical effective stress); and the normalized initial tangent modulus ($E_u/\sigma'_c$). In the KJHH model, the maximum wall deflection ($\delta_{wmax}$) is determined as $R^2 = 0.83$, per Kung et al. (2007c)

$$\delta_{wmax} (mm) = a_0 + a_1 X_1 + a_2 X_2 + a_3 X_3 + a_4 X_4 + a_5 X_5 + a_6 X_1 X_2 + a_7 X_1 X_3 + a_8 X_1 X_5$$

where $X_1 = t(H_e)$; $X_2 = t[ln(EI/\gamma_w h_d^4)]$; $X_3 = t(B/2)$; $X_4 = t(s_u/\sigma'_c)$; and $X_5 = t(E_u/\sigma'_c)$. The coefficients for Eq. (1) determined through the least-square regression are given as $a_0 = -13.41973$; $a_1 = -0.49351$; $a_2 = -0.09872$; $a_3 = 0.06025$; $a_4 = 0.23766$; $a_5 = -0.15406$; $a_6 = 0.00093$; $a_7 = 0.00285$; and $a_8 = 0.00198$.

Variables $X_i (i = 1, \ldots, 5)$ are the transformed variables of the five input parameters (Kung et al. 2007c):

$$X_i = t(x_i) = b_1 x_i^2 + b_2 x_i + b_3 \quad (2)$$

where $x_i (i = 1, 5)$ is corresponding input parameters. The transformation coefficients $b_1$, $b_2$, and $b_3$ for each of the five input parameters obtained by Kung et al. (2007c) are summarized in Table 1.

The next step is to calculate the deformation ratio ($R$), which is equal to the ratio of the maximum settlement over the maximum wall deflection ($R^2 = 0.92$, per Kung et al. 2007c):

$$R = c_0 + c_1 Y_1 + c_2 Y_2 + c_3 Y_3 + c_4 Y_1 Y_2 + c_5 Y_1 Y_3 + c_6 Y_2 Y_3 + c_7 Y_3^3 + c_8 Y_1 Y_2 Y_3$$

Table 1. Coefficients for Transformation of Input Variables

<table>
<thead>
<tr>
<th>Variables $x$</th>
<th>Coefficients of Eq. (2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H_e$ (m)</td>
<td>$b_1$</td>
</tr>
<tr>
<td>$\ln(EIF/\gamma_w h_d^4)$</td>
<td>11.5</td>
</tr>
<tr>
<td>$B/2$ (m)</td>
<td>-0.04</td>
</tr>
<tr>
<td>$s_u/\sigma'_c$</td>
<td>3.225</td>
</tr>
<tr>
<td>$E_u/\sigma'_c$</td>
<td>0.00041</td>
</tr>
</tbody>
</table>

Note: Data from Kung et al. (2007c).
where \( Y_1 = \sum H_{day}/H_{wall}; \ Y_2 = s_u/\sigma_u; \ Y_3 = E_u/1.000\sigma_u; \) and the coefficients for Eq. (3) determined through the least-square regression are as follows: \( c_0 = 4.55622; \ c_1 = -3.40151; \ c_2 = -7.37697; \ c_3 = -4.99407; \ c_4 = 7.14106; \ c_5 = 4.60055; \ c_6 = 8.74863; \ c_7 = 0.38092, \) and \( c_8 = -10.58958. \) Finally, the maximum settlement \((\delta_{sm})\) is obtained

\[
\delta_{sm} = R \cdot \delta_{hm}
\]  

(4)

Hsiao et al. (2008) found that the wall and ground responses in a given braced excavation in soft to medium clays are most sensitive to the uncertainty in \(s_u/\sigma_u\), \(E_u/\sigma_u\). Accordingly, for a given design of a braced excavation in clay, key soil parameters to be calibrated or updated are \(s_u/\sigma_u\) and \(E_u/\sigma_u\).

**Framework of Bayesian Updating with Markov Chain Monte Carlo Simulation**

### Updation Soil Parameters Using One Type of Response Observation

In this paper, the Bayesian updating framework is adapted for the KJHH model. The implementation starts with expressing the KJHH model for predicting the maximum wall deflection or maximum settlement as follows:

\[
y = c \cdot \delta(\theta)
\]  

(5)

where \(y = \) predicted \(\delta_{hm}\) or \(\delta_{sm}\) in a braced excavation; \(\delta(\theta) = \) prediction model for either \(\delta_{hm}\) or \(\delta_{sm}\) (Eq. (1) or Eq. (4)); \(\theta = \) vector of the soil parameters \((s_u/\sigma_u\) and \(E_u/\sigma_u\) in this paper); \(c = \) model bias factor \((c_b\) for \(\delta_{hm}\) model, \(c_p\) for \(\delta_{sm}\) model), which represents the model uncertainty. For both \(\delta_{hm}\) and \(\delta_{sm}\), the previous study (Kung et al. 2007c) showed that \(c\) can be approximately modeled as a normally distributed random variable with a mean value of 1.0; for the adopted \(\delta_{hm}\) prediction model, the SD of \(c\) (denoted as \(\sigma_{c_b}\)) is found to be 0.25; for the adopted \(\delta_{sm}\) model, the SD of \(c\) (denoted as \(\sigma_{c_p}\)) is found to be 0.34.

Based on Eq. (5), the likelihood that the prediction (\(y\)) is equal to the observation \((Y_{obs})\) can be expressed as a conditional probability density function (PDF) of \(\theta\):

\[
L(\theta)|y = Y_{obs}) = N[Y_{obs}/\delta(\theta)]
\]  

(6)

where \(L(\theta)|y = Y_{obs})\) is likelihood; and the notation \(N = \) normal PDF that is a function of \([Y_{obs}/\delta(\theta)]\). It is noted that at a given \(Y_{obs}\), the term \(N[Y_{obs}/\delta(\theta)]\) is a function of \(\theta\) only. Recalling that \(c = y/\delta(\theta)\), this normal PDF can be characterized with a mean of \(\mu_c\) and a SD of \(\sigma_{c}\). In a Bayesian framework, given a prior PDF, \(f(\theta)\), the posterior PDF of \(\theta\) can be obtained as follows (Wang et al. 2010; Zhang et al. 2010b):

\[
f(\theta|y = Y_{obs}) = m_1 \cdot N[Y_{obs}/\delta(\theta)] \cdot f(\theta)
\]  

(7)

where \(m_1\) is a normalization factor that guarantees a unit for the cumulative probability over the entire range of \(\theta\).

**Updating Soil Parameters Using Two Types of Response Observations**

To update the soil parameters with the observations of both \(\delta_{hm}\) and \(\delta_{sm}\) (denoted herein as \(Y_{obs1}\) and \(Y_{obs2}\), respectively), the likelihood that the predicted wall deflection \(y_1\) and the predicted settlement \(y_2\) are equal to the corresponding observations is a conditional probability density function (PDF) of \(\theta\):

\[
L(\theta|y_1 = Y_{obs1}, y_2 = Y_{obs2}) = N_2[Y_{obs1}/\delta_1(\theta), Y_{obs2}/\delta_2(\theta)]
\]  

(8)

where \(\delta_1(\theta)\) and \(\delta_2(\theta)\) is Eqs. (1) and (4), respectively; \(N_2 = \) PDF of a bivariate normal distribution with a mean vector \([\mu] = [\mu_{c_b}, \mu_{c_p}]\) and a covariance matrix of

\[
[\sigma^2] = \begin{bmatrix} \sigma_{c_b}^2 & \sigma_{c_b} \sigma_{c_p} \\ \sigma_{c_b} \sigma_{c_p} & \sigma_{c_p}^2 \end{bmatrix}
\]  

(9)

where \(\sigma_{c_b}^2 = \sigma_{c_p}^2 = \rho \sigma_{c_b} \sigma_{c_p}\), and \(\rho = \) correlation coefficient between the two model bias factors \(c_b\) and \(c_p\). The preceding formulation is simply an extension of the formulation presented in Eq. (6) from using one type of observation to using two types of observations. Similarly, the posterior PDF of \(\theta\) updated with two types of observations can be obtained as follows:

\[
f(\theta|y_1 = Y_{obs1}, y_2 = Y_{obs2}) = \frac{m_2 \cdot N_2[Y_{obs1}/\delta_1(\theta), Y_{obs2}/\delta_2(\theta)] \cdot f(\theta)}{\text{normalization factor}}
\]  

(10)

where \(m_2\) is normalization factor that guarantees a unity for the cumulative probability over the entire range of \(\theta\).

The posterior distribution may be obtained through optimization or sampling techniques. In this study, the MCMC simulation method, an efficient sampling technique that can yield samples of a posterior distribution (Beck and Au 2002), is adopted. MCMC performs a random walk within the domain defined by the uncertain soil parameters according to their prior distributions. At each random walk, if the likelihood of model predictions matching the observations is increased, then the candidate point is accepted. Otherwise, the candidate is rejected. One advantage of MCMC is that the computation of the normalization factor may be avoided, which is generally difficult for multiple-dimensional problems (Gamerman 1997). Furthermore, the Metropolis-Hastings algorithm (Metropolis et al. 1953; Hastings 1970) is adopted in this study for its efficiency to implement MCMC sampling of the key parameter \(\theta\) for its posterior PDF [see Eq. (10)].

**Procedure of Markov Chain Monte Carlo Simulation Using the Metropolis-Hastings Algorithm**

The procedure for MCMC simulation (or sampling) of \(\theta\) for its posterior PDF using the Metropolis-Hastings algorithm (Metropolis et al. 1953; Hastings 1970) can be summarized as follows:

1. At Stage \(k = 1\), determine the first point \(\theta_1\) in the Markov chain. This first instance \(\theta_1\) may be obtained by random selection from the prior distribution or may simply be assigned the mean value.
2. At next Stage \(k\) (\(k\) starts from 2), randomly generate a new \(\theta_p\) from a proposal distribution \(T(\theta_p|\theta_{k-1})\), which is assumed to be a multivariate normal distribution where the mean is set to be the current point \(\theta_{k-1}\) in the Markov chain and the covariance matrix is equal to \(s \cdot C_{p}\), where \(s\) is a scaling factor and \(C_{p}\) is the covariance matrix of the prior distribution of \(\theta\). The multivariate normal distribution is chosen for its good convergence properties in the Bayesian inference (Draper 2006).
3. Generate a random number \(u\) from a uniform distribution \(U(0, 1)\).
4. Compute the ratio of densities \(r\):
indicating that this is a clay-dominant site. Thus, the maximum wall and modulus ($E_i$) of this excavation [see Eq. (3)] is equal to 0.87, indicating that this is a clay-dominant site. Thus, the maximum wall and ground responses in this excavation are mainly influenced by the normalized shear strength ($s_u/\sigma_0'$) and normalized initial tangent modulus ($E_i/\sigma_0'$) of the clay (Hsiao et al. 2008). In this paper, the focus is to update these two soil parameters with field observations in the staged excavation using the Bayesian framework.

\[ r = \frac{q(\theta_p|y = y_{obs})}{q(\hat{\theta}_k|y = y_{obs})} \leq 1 \]  

(11)

where $q(\theta|y = y_{obs})$ is the unnormalized posterior PDF. In this study, $q(\theta|y = y_{obs})$ is essentially Eq. (7) or Eq. (10) without the normalization factor. Note that $q(\theta|y = y_{obs}) = q(\theta)$ evaluated at $y = y_{obs}$.  

5. Determine whether $\theta_p$ is acceptable (and thus yields a new point in the Markov chain) with the following acceptance rule: if $u \leq r$, then $\theta_p$ is acceptable and set $\theta_k = \theta_p$; otherwise, set $\theta_k = \theta_{k-1}$. Then go back to Step 2.

6. Repeat Steps 2–5 until the target number of samples (i.e., Markov chain length) is reached.

The Metropolis-Hastings algorithm randomly samples from the posterior distribution. Typically, initial samples are not completely valid because the Markov chain has not stabilized. These initial samples may be discarded as burn-in samples. Several factors influence the efficiency of sampling posterior distribution with the MCMC approach, such as the proposal distribution, Markov chain length, and number of burn-in samples. Therefore, the construction of a Markov chain is problem specific and needs to be examined case by case. It should be noted that other MCMC algorithms may be used in this procedure as long as efficient Markov chains can be achieved.

Example Application: Taipei National Enterprise Center Excavation Case

A well-documented excavation case, known as the Taipei National Enterprise Center (TNEC) case in Taiwan (Ou et al. 1998), is used herein as an example to illustrate the Bayesian framework for updating soil parameters using observed wall and/or ground responses in the staged excavation. At TNEC, the excavation width is 41.2 m, and the length of the 0.9-m-thick diaphragm wall is 35 m. The excavation was performed using the top-down construction method in seven stages (with a final excavation depth of 19.7 m) with the support of steel struts and floor slabs. The soil profile and the excavation depth in each of the seven stages are shown in Fig. 1. It is noted that the normalized undrained shear strength ($s_u/\sigma_0'$) and normalized initial tangent modulus ($E_i/\sigma_0'$) of the two clay layers are approximately the same (Kung et al. 2007a). The normalized clay layer thickness ($\Sigma H_{clay}/H_{wall}$) of this excavation [see Eq. (3)] is equal to 0.87, indicating that this is a clay-dominant site. Thus, the maximum wall and ground responses in this excavation are mainly influenced by the normalized shear strength ($s_u/\sigma_0'$) and normalized initial tangent modulus ($E_i/\sigma_0'$) of the clay (Hsiao et al. 2008). In this paper, the focus is to update these two soil parameters with field observations in the staged excavation using the Bayesian framework.

Parametric Study on the Construction of Markov Chains

The effectiveness and efficiency of the MCMC approach depend on the parametric setting. First, the prior distribution of soil parameter $\theta$ (i.e., vector of soil parameters) must be assumed. As an example, Prior Distribution 1, listed in Table 2, is assumed in the parametric analysis presented herein. The effect of choosing other prior distributions on the MCMC solutions is examined later.

Next, proper selection of the scaling factor $s$ is deemed critical to an efficient MCMC simulation. To test this notion, three Markov chains are simulated using the following scaling factors: $s = 0.01$; $s = 3$; and $s = 20$. For this analysis, the updating of soil parameters is based on two types of response observations (both maximum settlement and maximum wall deflection observations). The first 2,000 samples of $s_u/\sigma_0'$ in the Markov chains are shown in Fig. 2. It is apparent that $s$ has a significant effect on the efficiency of the MCMC simulation. The low efficiency of the MCMC simulation is observed when $s$ is either too small or too large as shown in Figs. 2(a) and (c), respectively: in Fig. 2(a), the overall trend of the samples fluctuates drastically, which means a longer time is needed to reach a steady state of the generated samples; in Fig. 2(c), a large number of the simulated samples are of the same value because many horizontal segments are observed, which also indicates low efficiency in the sampling process (as many rejections occurred). As a comparison, when $s = 3$, the Markov chain samples show an active simulation behavior as in Fig. 2(b).

To further examine the effect of scaling factor $s$ on the Markov chain construction, Fig. 3 shows the relationship between the acceptance rate and the scaling factor. Gelman et al. (2004) suggested that the Markov chain simulation is most efficient when the acceptance rate is between 20 and 40%. In this case, the acceptance rate falls within 20–40% if $s = 3$ for each of the excavation stages. Therefore, in this study, the scaling factor is set as 3 when updating with both observed maximum settlement and observed maximum wall deflection. It is found that when the soil parameters are updated with just one type of field observation (either observed maximum settlement or observed maximum wall deflection), a scaling factor

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Mean</th>
<th>COV</th>
<th>Mean</th>
<th>COV</th>
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</thead>
<tbody>
<tr>
<td>Prior Distribution 1</td>
<td>0.25</td>
<td>0.16</td>
<td>500</td>
<td>0.16</td>
</tr>
<tr>
<td>Prior Distribution 2</td>
<td>0.31</td>
<td>0.16</td>
<td>650</td>
<td>0.16</td>
</tr>
<tr>
<td>Prior Distribution 3</td>
<td>0.27</td>
<td>0.16</td>
<td>550</td>
<td>0.16</td>
</tr>
<tr>
<td>Prior Distribution 4</td>
<td>0.35</td>
<td>0.16</td>
<td>750</td>
<td>0.16</td>
</tr>
</tbody>
</table>

Note: COV suggested by Hsiao et al. (2008) for Taipei clays. Effect of assuming other COVs is examined separately.

Fig. 1. Soil profile [with basic data, liquid limit (LL); standard penetration test (SPT) blow count (N); plasticity index (PI); moisture content (w)] and excavation depths at various stages of TNEC (adapted from Kung et al. 2007c, © ASCE)

Table 2. Statistics of Four Prior Distributions Used in the Bayesian Updating Scheme

of 2 yields the most efficient Markov chain construction. However, determination of \( s \) should be carefully evaluated for other excavation cases.

With a properly selected scaling factor (\( s = 3 \)), the soil parameters can be updated stage by stage with the observations. In this study, the first 100 samples in the Markov chains are discarded as the burn-in samples. The updated probability density and sample statistics are obtained after the burn-in samples are discarded. The MCMC samples should be continuously generated until the sample statistics of the posterior distribution have converged within a preset tolerance (Zhang et al. 2010b). Fig. 4 shows an example of the variation of the sample statistics with the chain length obtained with 30 different Markov chains with \( s = 3 \). As can be seen from the results, at a Markov chain length of 50,000, the coefficient of variation (COV) for all statistics of the posterior distribution of \( \theta \) from the 30 Markov chains is less than 1%, the preset tolerance in this study. Therefore, it is concluded that robust posterior statistics can be achieved with the aforementioned parametric settings (e.g., scaling factor, burn-in sample, chain length) in the MCMC simulation.

On the basis of the selected parametric settings, the updating of soil parameters is carried out using the MCMC simulation with field observations in the TNEC excavation. As an example, Fig. 5 shows the MCMC sampling process with a Markov chain length of 50,000 based on the observations from Stage 6 excavation at TNEC (when the excavation reached the depth of 17.3 m, which is prior to the start of Stage 7 excavation). Fig. 6 shows the histograms of the Markov chain samples in Fig. 5. It is observed that the posterior distributions of both \( s_u/\sigma_s \) and \( E_i/\sigma_{s_i} \) are very close to a normal distribution. The updated distributions of these soil parameters from other stages of excavation all follow approximately a normal distribution.

Previous reliability analysis of braced excavations using the KJHH model as a performance function shows that the discrepancy between normal and lognormal assumptions has a negligible influence on the predicted excavation-induced serviceability failure probability (Hsiao et al. 2008). Furthermore, the MCMC sampling...
using prior distributions as listed in Table 2 seldom generates negative numbers. The very rarely generated negative numbers may be disregarded, and a truncated normal distribution may be used (Most and Knabe 2010). Thus, normal distribution is assumed for both prior and posterior distributions of soil parameters in this study. For simplicity, the predictions of the wall and ground responses in the subsequent stages can be realized with the mean values of the posterior distributions, and the reliability assessment of the wall and ground responses can be evaluated with the obtained posterior distribution and its statistics.

**Updating Soil Parameters Using One Type of Response Observation**

As noted previously, back-analysis in the early stages (the first or second stage) of excavation may not be effective because of the change of the deformed shape of the wall from the cantilever to concave-type shape (Kung et al. 2007a) and because the responses in these early stages under normal workmanship are generally small (Kung et al. 2007b; Hsiao et al. 2008).

In this section, the updating is carried out with only one type of response observation. First, the soil parameters are updated using only the observed settlements. Here, the maximum settlements at various target depths (z = 8.6, 11.8, 15.2, 17.3, and 19.7 m at the end of the corresponding excavation Stages 3–7) are computed using the KJHH model with the assumed Prior Distribution 1 (Table 2). The computed settlements are random variables because soil parameters are treated as random variables. Fig. 7(a) shows the mean values of these predictions versus the field observations. As an example, the predictions made prior to excavation, referred to herein as the as-design predictions, are labeled with the symbol □. These are the predictions made without the benefit of any field observations and involving no updating. As can be seen in Fig. 7(a), the as-design predictions using the assumed prior information deviated much from the 1:1 line (predicted settlement versus observed settlement).

As noted previously, back-analysis for the purpose of updating soil parameters began from Stage 3 excavation. After Stage 3 excavation is completed, the maximum settlement at the depth of 8.6 m is observed at 18 mm, which is much smaller than the as-design prediction of 47 mm [Fig. 7(a)]. Using this observation and the Bayesian updating framework presented earlier, the MCMC samples for posterior distributions of parameters $s_u / \sigma'_v$ and $E_i / \sigma'_v$ are generated. For simplicity, sample statistics (mean and COV) of these posterior distributions are obtained. The mean values of the updated parameters $s_u / \sigma'_v$ and $E_i / \sigma'_v$ are then used to predict the maximum settlement at various target depths prior to Stage 4 excavation. These maximum settlement predictions are labeled with the symbol X in Fig. 7(a). After Stage 4 is completed, the maximum settlement at the depth of 11.8 m is observed to be 34 mm, and this new observation is used to further update $s_u / \sigma'_v$ and $E_i / \sigma'_v$. This process is repeated stage by stage until the end.

The maximum settlement predictions at the final excavation depth (19.7 m in this case), which are generally of primary concern,
made prior to the start of Stage 3, 4, 5, 6, and 7 excavations are shown in Fig. 8. Note that the depths at which the maximum settlement predictions were made prior to the start of Stages 3, 4, 5, 6, and 7 are 4.9, 8.6, 11.8, 15.2, and 17.3 m, respectively. Prior to Stage 7 (when the excavation reached the depth of 17.3 m), the mean of the maximum settlement prediction for the final excavation depth of 19.7 m is 79 mm, which compares very well with the observed maximum settlement of 78 mm at the final excavation depth. For comparison, it is noted that the maximum settlement prediction for the final excavation depth of 19.7 m is 114 mm prior to Stage 3 when the excavation reached the depth of 4.9 m. The accuracy of the predictions made with the updated soil parameters increases in the Bayesian updating process, as shown in Fig. 8. In fact, for this TNEC excavation, the maximum settlement predictions for the final excavation depth prior to Stage 5 (starting at 11.8 m) and Stage 6 (starting at 15.2 m) are already very close to the observed maximum settlement of 78 mm. The predicted maximum wall deflection and ground settlement are random variables because the updated soil parameters are random variables. Thus, Fig. 8 also shows the 1 SD bounds of the predicted maximum settlement at the final excavation depth of 19.7 m in this TNEC case.

Next, the soil parameters are updated using the observed wall deflection observations only. Using the same procedure, the soil parameters are updated with the observed maximum wall deflections, and then the predictions for the subsequent stages are obtained accordingly, as shown in Fig. 7(b). The updated wall deflection predictions for the final excavation depth (19.7 m in this TNEC case) made prior to Stages 3, 4, 5, 6, and 7 (with corresponding starting depths of 4.9, 8.6, 11.8, 15.2, and 17.3 m, respectively) are shown in Fig. 9. The predicted maximum wall deflection for the final excavation depth of 19.7 m made prior to the Stage 7 excavation is 116 mm, and the 1 SD bounds are 102–130 mm. The observation at the final excavation depth is 106 mm, which falls within the predicted 1 SD bounds. Meanwhile, the predicted mean (116 mm) at the final excavation
depth is significantly improved over the mean prediction (143 mm) prior to Stage 3.

The Bayesian framework is shown to be effective and efficient when the soil parameters are updated with only one type of response observation (either maximum settlement or maximum wall deflection) in a braced excavation. Updating with two types of response observations is presented in the next section.

**Updating Soil Parameters Using Two Types of Response Observations**

In this section, updating of soil parameters using observations of both maximum settlements ($\delta_{um}$) and maximum wall deflections ($\delta_{vm}$) is demonstrated with the TNEC case. As in the previous section, Prior Distribution 1 (listed in Table 2) is assumed for the two key soil parameters $s_u/\sigma'_v$ and $E_u/\sigma'_v$. The procedure for MCMC sampling of the posterior distribution of $\theta$ using the Metropolis-Hastings algorithm is basically the same as described previously for updating using one type of response observation except that Eq. (10) is used in lieu of Eq. (7). In the analysis presented herein, the correlation coefficient between the two model bias factors ($c_h$ and $c_v$), which is defined in the covariance matrix [Eq. (9)], is assumed to be $\rho = 0$. The effect of this correlation is examined later in a separate section.

Fig. 10(a) shows the updated maximum settlement predictions using the posterior distribution of $\theta$ that is updated based on two types of observations (both maximum settlement and maximum wall deflection). For comparison, the updated maximum settlement prediction using the posterior distribution of $\theta$ that is updated based on one type of observation (either maximum settlement or wall deflection) is also included in Fig. 10(a). Similarly, Fig. 10(b) shows the updated maximum wall deflection predictions based on the three updating schemes. In Fig. 10, all predictions are for the final excavation depth of 19.7 m in this TNEC case, and these predictions are made prior to Stages 3, 4, 5, 6, and 7 (with corresponding starting depths of 4.9, 8.6, 11.8, 15.2, and 17.3 m, respectively). The following observation is made from Fig. 10: all three updating schemes (based on observations of maximum wall deflection only, maximum settlement only, and both maximum wall deflection and maximum settlement) are effective, although the updating based only on the wall deflection is slightly less effective than the other two schemes.

The results presented in Fig. 10 are the mean values of the updated predictions of the wall and ground responses. It should be of interest to examine the variation of the updated predictions. As an example, Fig. 11 shows the distribution of the final predictions (i.e., prior to the final stage of excavation) of the maximum settlement and maximum wall deflection with each of the three updating schemes. The observed responses at the end of excavation are also shown in Fig. 11. Two observations can be made from Fig. 11. First, among the three updating schemes, the one using two types of response observations (maximum wall deflection and maximum settlement observations) yielded the smallest variation. Second, because of this inevitable variation, the claim of excellent agreement between the prediction and the observation in the traditional back-analysis of a case history, often reported in the literature, may not be all that meaningful if the variation in the input parameters and/or the variation in the computed responses are not fully characterized.

**Effect of Prior Distribution on the Outcome of Bayesian Updating**

In this section, the effect of the assumed prior distribution of soil parameters on the outcome of the Bayesian updating is examined.

The focus is to determine whether approximately the same final outcome can be obtained after various stages of Bayesian updating regardless of what the initial assumption regarding the prior distribution is. Of course, the posterior distribution as a result of Bayesian updating relies heavily on the prior information. Thus, the premise for this analysis and discussion of the effect of prior distribution is that a reasonable prior (albeit uncertain) distribution can be estimated based on experience and available test data. To this end, three other prior distributions are assumed (Table 2) in addition to Prior Distribution 1 that has been used in all previous analyses. In all four prior distributions, the COVs are assumed to be 0.16 (Hsiao et al. 2008), and the influence of various assumed COV values are examined later. The mean values for these prior distributions are assumed based on the results of 17 small-strain triaxial tests on Taipei clay reported by Kung (2003). These four prior distributions are selected to cover the range of situations from where excavation responses would be underestimated to where the responses would be overestimated. The purpose of this series of analysis is to demonstrate that with a reasonable prior assumption (albeit uncertain), the soil parameters updated through Bayesian updating can converge.

![Updated settlement prediction](image_url)

![Updated wall deflection prediction](image_url)

Fig. 10. Comparisons of updated predictions with three updating schemes (using Prior distribution 1): (a) updated settlement prediction; (b) updated wall deflection prediction
The same Bayesian updating procedure using two types of observations is performed with the assumption of Prior Distributions 2, 3, and 4, and the updated mean values and COVs of $s_u/s_r'$ and $E_i/s_r'$ are shown in Figs. 12 and 13, respectively. As shown in Fig. 12, the updated mean values are converging as the excavation progresses in this case. Fig. 13 shows that the COVs of the updated parameters decrease as the excavation progresses in this case. It should be cautioned that the results presented in Figs. 12 and 13 are predicated on the reasonableness of the assumed prior distribution. Thus, a reasonable estimate (but not necessarily the best estimate) of the prior distribution may still be required to ensure the accuracy of the updated posterior distribution of soil parameters and the updated predictions of excavation responses.

The effect of the assumed COVs of soil parameters are further examined within the Bayesian framework. For demonstration purposes, the mean values of Prior Distribution 2 (Table 2) are adopted, and a series of prior COVs for $s_u/s_r'$ and $E_i/s_r'$ are selected: 0.10, 0.16, and 0.30. Then the same updating procedure is performed with the previously selected COVs, and the updated soil parameters are obtained. It is found that the updated mean values of $s_u/s_r'$ and $E_i/s_r'$ are almost the same regardless of which levels of prior COVs are used. Fig. 14 shows the COVs of the updated soil parameters prior to Stages 3, 4, 5, 6, and 7 of excavations using various prior distributions (at corresponding starting depths of 4.9, 8.6, 11.8, 15.2, and 17.3 m, respectively). As shown in Fig. 14, regardless of the level of the assumed COV in the prior distribution, the COVs of the updated soil parameters decreased from earlier stages to latter stages (i.e., with increasing number of observations and updating from multiple stages). Fig. 15 further compares the variation of soil parameters before and after updating, using the case of prior COV $0.3$ from Fig. 14 as an example. The COVs for both $s_u/s_r'$ and $E_i/s_r'$ after multistage updating with field response observations are reduced to less than 0.10. Thus, the uncertainty of soil parameters is shown to reduce significantly with the application of the Bayesian updating framework.

The preceding results show that, although prior knowledge is important, the Bayesian updating with observations through stages
of excavation can reduce the influence of this prior knowledge, and converged results can be obtained even if the prior knowledge is imperfect. The results demonstrate that the proposed Bayesian updating framework is effective regardless of the assumption of the prior distribution.

**Effect of the Correlation between Bias Factors of the Two Response Models**

When both observations of maximum wall deflection and maximum settlement are used within the proposed Bayesian updating framework, the effect of the correlation between bias factors $c_h$ and $c_v$ [Eq. (9)] on the updating outcome needs to be assessed. Previous studies indicate that the maximum settlement generally increases with maximum wall deflection (Clough and O’Rourke 1990; Hsieh and Ou 1998; Kung et al. 2007a, b, c). However, the bias factors $c_h$ and $c_v$ may be positively or negatively correlated. Therefore, the effect of the coefficient of correlation $\rho$ is examined by repeating the previous analysis with assumptions of $\rho = 0.8, 0.5, 0, -0.5$. Fig. 16 shows the updated predictions of the maximum settlement (Part a) and maximum wall deflection (Part b) made prior to Stages 3, 4, 5, 6, and 7 of the TNEC excavations (at corresponding starting depths of 4.9, 8.6, 11.8, 15.2, and 17.3 m, respectively) with four correlation scenarios. A couple of observations are worth mentioning: (1) the effect of this correlation on the outcome of Bayesian updating appears to be quite limited, and (2) the assumption of no correlation ($\rho = 0$) appears to yield no inferior outcome in the Bayesian updating. This finding is based on analysis of a single excavation case, and further study to confirm the finding is warranted. Nonetheless, the results of these sensitivity analyses clearly demonstrate the effectiveness and high flexibility of the proposed Bayesian updating framework.
Summary and Conclusions

The proposed Bayesian framework for updating soil parameters in a braced excavation is presented in this paper. This framework uses the KJHH model as an example for predicting maximum wall deflection and maximum settlement before a given excavation stage and Bayes’ theorem to update key soil parameters so that model predictions prior to next stage of excavation match well with field observations afterward. Unlike the traditional back-analysis, in which fixed parameter values are backcalculated based on observations, the proposed framework allows for consideration of the variation in these parameters and yields their posterior distributions. Thus, not only is the mean prediction improved, but also the variation of the prediction is reduced. The proposed framework uses the Metropolis-Hastings algorithm-based MCMC method to construct the posterior distributions of soil parameters. The effectiveness and flexibility of the proposed framework is examined through a case study of TNEC excavation.

The following conclusions are drawn based on the results presented in this paper:

1. The proposed Bayesian framework is shown effective for updating key soil parameters in the staged excavation based on either maximum settlement or maximum wall deflection observation or both types of observations. Although all three updating schemes (based on maximum settlement or maximum wall deflection or both) are effective, the one that uses both types of observations yields the least variation in the updated predictions of the maximum wall and ground responses.

2. Bayesian updating is shown effective in reducing the uncertainty (in terms of coefficient of variation) of the updated soil parameters and model predictions through multistage observations and updating. In the case study of the staged excavation, the mean values of the response predictions are getting increasingly closer to the field observations as the excavation proceeds. However, both the updated soil parameters and the updated response predictions still have to be expressed with a probability distribution to capture and reflect the uncertainty that cannot be removed completely in the updating process.
3. The uncertain nature of the updated soil parameters and response predictions, even after multistage Bayesian updating, has an important implication. The claim of excellent agreement between the prediction and the observation in the traditional back-analysis of an excavation case history, often reported in the literature, may not be all that meaningful if the variation in the input parameters and/or in the computed responses is not fully characterized and reported.

4. The final outcome of the Bayesian updating is not much affected by the assumed prior distributions and the levels of the coefficient of variation of the soil parameters. Thus, while prior knowledge is important, the Bayesian updating with observations through stages of excavation can reduce the influence of this prior knowledge, and converged results can be obtained even if the prior knowledge is imperfect.

5. The effect of the correlation between the maximum wall deflection and the maximum settlement, through the model factors, on the outcome of Bayesian updating appears to be quite limited. The assumption of no correlation appears to yield no inferior outcome in the Bayesian updating in the case study of TNEC excavation. Although further study to confirm this finding is warranted, it is postulated that the proposed framework through multistage observations and updating is able to compensate for the deficiency, if any, of this assumption.

6. MCMC simulation implemented with the Metropolis-Hastings algorithm is shown effective in this study. The construction of an effective Markov chain is, however, problem specific and should be examined case by case through a proper parametric analysis. The MCMC parameter settings are likely to be valid for similar excavation problems.

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