The Development of Finite-Element Models and the Horizontal Thrust of Guastavino Domes

SEZER ATAMTURKTUR AND THOMAS E. BOOTHBY

Modern engineering tools applied to structural analysis of two existing Guastavino tile domes prove the existence of horizontal thrust, in spite of the elder Rafael Guastavino’s claims to the contrary.

Introduction

The Guastavino vaulting system, initially promoted by Rafael Guastavino Moreno and continued by his son, Rafael Guastavino Esposito, has inspired numerous studies in architectural history and historic preservation. It is the intention of this article to further the understanding and appreciation of Guastavino domes and to present means of investigating their structural behavior with modern engineering tools. For these purposes the finite-element (FE) method is extensively employed through the computerized package ANSYS. The findings of these studies are used to assess the characteristics of Guastavino vaulting, specifically focusing on the question of horizontal thrust in domes. The results presented are supported by analytical models and field and laboratory tests conducted on the City-County Building entrance-vestibule domes in Pittsburgh and the New York State Education Building Reading Room domes in Albany, both of which were designed by Henry Hornbostel.

Because the FE modeling of a historic masonry structure poses two major challenges — the identification of the boundary conditions and the effective material properties — two separate model-refinement efforts were undertaken to achieve an acceptably accurate representation of a structural system. In the early stages of this research, a form of dynamic-vibration testing was used for the verification and updating of the boundary conditions. To obtain effective mechanical characteristics of the assembly, static compressive tests on tile and mortar specimens obtained from the State Education Building were conducted, and the combined elastic constraints of tile and mortar were homogenized. These results, although more reliable than nondestructive dynamic testing, cannot be obtained for every building under investigation; consequently the mechanical properties that were calibrated based on the nondestructive dynamic tests were compared to the results of direct measurement on specimens in laboratories and found to be in good agreement. Once the appropriate boundary conditions and material properties were obtained through the ANSYS model, the domes were then modeled in the frame-analysis program SAP 9.0 nonlinear, a program widely used by engineers in practice.

Overview: Rafael Guastavino and Cohesive Construction

Rafael Guastavino transferred a thousand-year-old system of Catalan vaulting from Spain to the United States, with the additional refinement of replacing the traditional mortar with rapidly hardening portland cement. Using this version of Catalan vaulting, he achieved spans of three to five times the typical span of a traditional Catalan vault. The capabilities of Guastavino vaulting, or, in the elder Guastavino’s words, “cohesive construction,” contributed to the structure of many important buildings between the 1890s and 1940s and resulted in more than 1,000 examples in North America.

During his career the elder Guastavino, a successful marketer, developed a number of rationales to promote his novel technique. One argument promoted by Guastavino, which appears to be widely accepted by historians, is that cohesive domes exert no thrust on their supports. A dome without a lateral thrust does not require thick walls; hence, lower building costs give it a great advantage over other systems. This argument surely contributed to Guastavino’s reputation for inexpensive and quickly...
The cracks are confined to a small zone around the intersection between the tile buttresses and the back of the dome.

The dome thickness varies from three to seven layers. The interior surface is finished with decorative tiles that create a highly ornamented and colorful finish. The intradoses of the vestibule domes are exposed to the weather, whereas the extradoses are less susceptible to climatic changes. The upper balcony floor, mainly carried by concrete beams, is in contact with the apex of each dome. Although the structural effect of this connection and the extent of the support of the domes to the balcony floor are unclear, this study assumed the domes to be nonbearing and independent from the balcony.

**New York State Education Building (1911).** In the State Education Building, twelve identical rib-and-dome units are arranged in a three-by-four grid: a repetitive circular system. The spherical domes are truncated from a 21-foot radius (6.3 m) over 29-foot-by-29-foot (8.9 m) bays with 6-foot (1.8 m) rise. The transformation from a circular plan to a square bay is accomplished by means of pendentives and slender ribbed arches (Fig. 4). The domes are supported by slender iron columns that continue past the vaults and join the trusses that carry the upper floor loads. The truss members contact the dome shell at every 45 degrees in plan, where small tile blocks 16 inches (40 cm) by 40 inches (150 cm) connect the truss chords to the dome shell (Fig. 5). The domes are nonbearing, as they are fully detached from the upper floor. The tile vaults have a thickness of three courses. No apparent cracks on the back of the tile domes were observed during the visual site survey.

**Analytical Methodology: Finite-Element Method**

In general, analytical modeling of masonry structures is a particularly challenging task. The true behavior of masonry, both in tension and in compression, is nonlinear; however, the needed stress-strain law is seldom available. Due to the orientation of the mortar joints, the material is anisotropic and inhomogeneous. On the other hand, significant progress in the assessment of a masonry structure can be made assuming masonry as a linearly elastic isotropic and homogenous material, thus avoiding the complications in the constitutive law, as well as the convergence issues associated with nonlinear analysis. In the present paper, these uncertainties are mitigated by obtaining material properties experimentally from specimens of the material used in construction and by experimentally verifying the assumed boundary conditions used in modeling the structure.
**Mechanical Properties of the Masonry Assembly**

The elder Guastavino's writings about the tests conducted on tile and mortar specimens reveal that maintaining consistency in tile and mortar manufacturing was difficult in the late-nineteenth and early-twentieth centuries. In addition to the inconsistent materials, it is apparent that the R. Guastavino Company underwent a continuous process of experimentation. As a result, it is desirable today to obtain properties of the specific materials that were used in construction.

The fundamental material-stiffness property used in this analysis is an effective modulus of elasticity representing the combined effect of tile units, mortar joints, and voids. If the properties of both mortar and tile are known, a simple formula can be used to average the two materials, based on the ratios of tile-unit and mortar-joint dimensions. Saliklis studied the material characteristics of Guastavino tile by conducting a survey on numerous small tile samples and found that the terra-cotta tile has significant orthotropic properties over the transverse and longitudinal directions of the unit. Noting that the tiles are oriented in both directions to break the joints, the study suggested an average elastic-modulus value of about 16.5 GPa (2390 ksi). Saliklis's study provides a general guideline for the mechanics of the tile unit; however, the mechanics of mortar used in Guastavino construction have been studied far less in the literature. Lane, in studying the chemical constitution of mortar specimens extracted from numerous Guastavino vaults, has shown that there is not one particular composition used by the Guastavino Company; instead, each building has a case-specific recipe for mortar.

Specimens of loose tile and mortar were collected from the debris above the State Education Building vaults. The elastic properties of these specimens were examined by static compressive tests in laboratory conditions, and the material characteristics of mortar and tile were identified both as individuals and as a homogenized assembly. The compressive tests obtained the Young's modulus (E) and Poisson's ratio (ν). Subsequently, these values are homogenized based on the methods of mechanics of materials to obtain the effective values to be entered into the FE model.

**Boundary conditions for Guastavino domes.** In addition to the difficulty in assessing the material properties, the assessment of the physical definition of the connections between adjacent members is particularly challenging for complex masonry systems. Because none of the theoretical fixed or free boundary conditions available on analysis programs exist in real structures, their use introduces approximation; the choice of one or the other depends on an accurate understanding of both the behavior of the structure and the technology and capacity of the FE software. Additionally, the elastic joint restraints in a masonry structure are dependent on the physical characteristics and configuration of members, as well as on the physical properties of masonry, rather than on intentionally designed pins or points of fixity, as in a steel structure.

The verification of the support conditions of a structural model is particularly challenging when dealing with large-scale masonry vaults, since the nondestructive means of obtaining empirical data from such systems is limited. Investigating the full-scale behavior of a masonry vault under artificial static loading is infeasible, due to the risk and the difficulty of placing a large static load and in loading the structure sufficiently to obtain a measurable response. Visual methods, such as comparing the crack locations on the existing structure to the tension zones in the FE model, are susceptible to error, especially in structures with support movements. Laboratory tests conducted on scaled vault samples, which may be successful in determining the stress distribution within the system caused by various loading conditions, often overlook the actual elastic boundary conditions, which are influenced by the adjacent elements within the system.

In the early stages of this study a form of nondestructive vibration testing called experimental modal analysis (EMA) was used to obtain natural frequencies and modes of vibration of the
vaults. Using EMA the natural frequencies and associated mode shapes of the system can be determined and compared to those calculated by FE analysis. Since the dynamic parameters are intrinsic to each structure and are directly related to the spatial distribution of mass and stiffness in the system, modal testing provides an opportunity to verify the uncertain model variables.

The procedure used in this study is the manual refinement of the uncertain boundary conditions and is based on the comparisons of dynamic parameters estimated by the FE model to those determined by modal testing. In this iterative procedure, the FE model is accepted as valid when overall agreement between the model and the dynamic parameters (mode shapes and natural frequencies) is achieved by means of numeric comparisons. The adjustments in material properties and boundary conditions of the model are made based on observable conditions, such as measured material properties or the contribution of visible elements to the boundary conditions. The results obtained through the analytical model, along with the established literature on Guastavino’s construction technique, support the discussion presented here on Guastavino vaults.12

Experimental Methodology: Static (Compression) Test

During the site visits to the State Education Building, a loose tile sample measuring 6 inches (15.8 cm) by 8 inches (20 cm) and an irregular mortar specimen measuring 4 inches (10 cm) by 5.5 inches (14 cm), believed to be left from the construction, were obtained from the rubble behind the domes. The specimens were subjected to longitudinal and transverse static compressive testing closely following the compressive-strength-testing procedure of ASTM C67 (2000). The tiles were cut into 6-inch-by-6-inch (15 cm) squares, ground, and canted with plaster of paris. The tests yielded a linearly elastic stress-strain curve whose slope determined the Young’s modulus of the tile: 1910 ksi (13.2 GPa) in the longitudinal direction and 2230 ksi (15.4 GPa) in the transverse direction. When the mortar specimen was tested, an approximate Young’s modulus of 430 ksi (2.97 GPa) was obtained.

While Saliklis’s results display a 1:1.8 ratio in orthotropic directions, the findings of this study show an almost isotropic behavior. Such a difference between Saliklis’s findings and those of this study is possibly due to the inconsistency in tile manufacturing or to the difference in measurement techniques, as Saliklis’s results are based on wave-propagation velocity.

Given the individual properties of tile and mortar (Table 1), the effective values that incorporate the combined effect of the two materials can be obtained by a homogenization procedure, based on a rule of mixtures

\[
(E_{\text{effective}} \times L_{\text{total}}) = (E_{\text{mortar}} \times L_{\text{mortar}}) + (E_{\text{layer}} \times L_{\text{layer}})
\]

corresponding to the z-axis. Generally speaking, the tile units are 0.8 inches by 6 inches by 12 inches (2 cm by 15 cm by 30 cm) in size, and the typical mortar joint is 0.8 inches (2 cm) in all directions. Consequently, an effective Young’s modulus value of 1,100 ksi (7.6 GPa) is obtained. When repeated for Poisson’s ratio, the same procedure yields a value of 0.26. The effective density of the assembly is obtained from a simple computation of the weight and volume of the specimens. Assuming an equal contribution of tile and mortar to the total volume, one can linearly average the densities of both materials and obtain the effective density of the assembly as 112 lbs/ft³ (1,800 kg/m³). Table 2 presents the findings of these tests, which are directly entered into the initial FE models.

Due to the lack of original tile or mortar specimens, laboratory test data for the material properties was not obtained for the City-County Building. Noting that both the State Education Building and the City-County Building were constructed in the same decade, the material-property values of the State Education Building specimens were entered into the preliminary FE model of the City-County Building.

Table 1. Individual Mechanical Values for Tile and Mortar Specimens

<table>
<thead>
<tr>
<th>Material</th>
<th>Modulus (E)</th>
<th>Poisson’s Ratio (v)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tile</td>
<td>1.91 x 10^6 lbs/in²</td>
<td>13.2 x 10^3 N/m²</td>
</tr>
<tr>
<td>Mortar</td>
<td>4.30 x 10^6 lbs/in²</td>
<td>2.97 x 10^3 N/m²</td>
</tr>
</tbody>
</table>

Table 2. Effective Material Properties Entered into the FE Model

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Young’s Modulus (E)</td>
<td>1.10 x 10^6 lbs/in²</td>
</tr>
<tr>
<td>Poisson’s Ratio (v)</td>
<td>0.26</td>
</tr>
<tr>
<td>Density (d)</td>
<td>110 lbs/ft³</td>
</tr>
</tbody>
</table>

Application of Finite-Element Method and Updating by Experimental Modal Analysis

The domes of both buildings were modeled in the commercially available finite-element software ANSYS v. 8.0. The physical dimensions of the systems were determined based on construction drawings of the City-County Building and the architectural drawings of the State Education Building; on-site survey data was used to assure the geometric accuracy of the drawings. The domes were modeled as symmetric spherical segments with square plans, meshed with thin, elastic shell elements (SHELL93 in ANSYS) of an approximate mesh size of 8 inches to 12 inches (20 to 30 cm). Using the impact-echo technique on the dome of the State Education Building, this study determined an approximate thickness of 5.4 inches (13.5 cm). The dome of the State Education Building is modeled as a constant thickness of three tile layers, and the dome of City-County Building...
is modeled as a constant thickness of five tile layers. In the initial phases of the analysis, the modal-analysis feature of the finite-element program was used. The natural frequencies, as well as the mode shapes of the vibrations of the dome, were determined subject to the material properties outlined above and boundary conditions based on visual inspection of the structure.

The techniques of experimental modal analysis were used to investigate the structural interaction of an interior tile dome at the State Education Building with the adjacent domes, as well as with the steel trusses and tile buttresses that restrain and partially support the domes. In this procedure, an array of accelerometers was placed on the back of the dome, and the dome is excited using a measured impact force at various points within the array (Fig. 6). The typical output from a single accelerometer consists of a frequency response function (Fig. 7). This graph shows the ratio of output acceleration to input acceleration as a function of frequency; peaks in the graph represent natural frequencies of the system. By investigating the output of multiple accelerometers, the natural frequencies of vibration of the structure and associated mode shapes can be determined. An example of the determination of a mode shape is quadrature response analysis (Fig. 8).

In this figure the frequency-response functions for accelerometers placed in four separate quadrants of the vault are either in phase or out of phase with each other, and the symmetries of the vibration mode can be determined and compared to the analytically determined vibration modes.

The results of the procedure led to conclusions about the elastic support restraints (Fig. 9). The peripheral edges of the dome are restrained from a horizontal translation perpendicular to the plane of symmetry but are generally allowed to translate vertically and to rotate. At the base line, where the dome webbing meets the supports, translation in all three axes is restrained; the tile buttresses between the adjacent domes provide vertical restraint while the diagonal buttresses allow...
vertical movements, the truss members, which contact the dome surface along the pendentive edges at four corners, restrict rotation in all directions. Although the boundary conditions determined by the initial inspection had to be modified in the FE model on the basis of the dynamic results, the changes all reflect observable conditions: the arches between vaults are sufficiently slender to permit rotation, and the tile buttresses are directly connected to a very solid system of steel trusses. A final detailed comparison of the experimental and analytical frequencies and mode shapes leads to the comparison between results shown in Table 3.

In City-County Building, three tile domes rest on massive brick arches: preliminary dynamic testing determined that these arches prevent dynamic interaction between adjacent domes. This observation leads to the modeling of a single dome on fixed supports. Although not measured, the surcharge volume is observed to have a low rise, and its influence on the structural behavior is therefore excluded from the analysis. The cracks at the intersection of the tile buttresses and the dome, previously described, are not modeled because they are local in character, appearing to result from restrained expansion of the dome at the buttresses. They have no measurable influence on the global behavior of the dome. Moreover, the need is shown later to include the buttresses in the boundary conditions to replicate the experimental mode shapes. The significance of the restraint provided by the tile buttress shown in Figure 2 implies that the shear across the crack is sufficient to transfer the restraint of the buttress to the dome.

The final boundary conditions obtained through the comparisons of FE solutions to experimental results differ from the preliminary boundary conditions only in the fixity of the support of the dome and in the addition of the effect of the tile buttresses (Fig. 10). The stone arches provide translational and rotational fixity along the periphery; the tile buttresses, extending between the dome webbing and the steel columns, restrain lateral translation.

**Table 3. Comparison of the Sequence of Analytically and Experimentally Determined Natural Frequencies for the New York State Education Building Domes**

<table>
<thead>
<tr>
<th></th>
<th>Experimental Modal Analysis</th>
<th>Finite-Element Method</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>41.0 Hz</td>
<td>40.91 Hz</td>
</tr>
<tr>
<td>2</td>
<td>x</td>
<td>41.47 Hz</td>
</tr>
<tr>
<td>3</td>
<td>48.33 Hz</td>
<td>50.19 Hz</td>
</tr>
<tr>
<td>4</td>
<td>52.04 Hz</td>
<td>51.47 Hz</td>
</tr>
<tr>
<td>5</td>
<td>58.50 Hz</td>
<td>53.53 Hz</td>
</tr>
<tr>
<td>6</td>
<td>x</td>
<td>57.35 Hz</td>
</tr>
<tr>
<td>7</td>
<td>64.50 Hz</td>
<td>60.19 Hz</td>
</tr>
<tr>
<td>8</td>
<td>73.00 Hz</td>
<td>64.6 Hz</td>
</tr>
</tbody>
</table>
Table 4. Horizontal and Vertical Reaction Results for the New York State Education Building and the City-County Building

<table>
<thead>
<tr>
<th>Building</th>
<th>Feature</th>
<th>Reaction (Vertical)</th>
<th>Reaction (Horizontal)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>single</td>
<td>all</td>
</tr>
<tr>
<td>State Education Building</td>
<td>boundary arches</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>top of pier</td>
<td>6.14 k</td>
<td>27.5 k</td>
</tr>
<tr>
<td></td>
<td>steel truss framework</td>
<td>3.51 k</td>
<td>15.7 k</td>
</tr>
<tr>
<td></td>
<td>surcharge</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>City-County Building</td>
<td>boundary arches</td>
<td>16.2 k</td>
<td>72.6 k</td>
</tr>
<tr>
<td></td>
<td>top of pier</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>tile buttresses</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Discussion: Presence of Horizontal Thrust and Elastic Thin-Shell Behavior

Guastavino and later scholars researching his work often argued that a cohesive dome does not exert any horizontal thrust on its supports due to its rigid nature. Guastavino (the father) illustrated his claim on the absence of horizontal thrust in domes with the following statement: 

"Suppose we take a big block of stone, say ten feet long and ten feet wide, and one foot or one foot 6 inches thick; if we support that on the four sides, just as a lintel, we have practically no thrust, and if we make a cavity on the under side, making a curve like dome, we will have a dome arch but will have no thrust." 

Interestingly, in the same essay, he corrects the misunderstanding on the absence of horizontal thrust for tile arches:

"It is frequently seen that the greatest friends of the system sometimes go too far in their enthusiasm and favor of the new idea... For instance it is said that arches under system have no thrust... Thus the barrel arch has some thrust, and requires some material to counterbalance this, that is rods. That is one of the causes, which makes the barrel construction more expensive than the dome."

Contrary to Guastavino’s assertion, a thin-shell dome, including a Guastavino dome, is not perfectly rigid, nor is it free from bending moment. Therefore, the tendency of the dome to bend downwards forces the peripheral abutments to spread; hence, there is a need for a horizontal-force component at the supports to balance this internal moment. The monolithic block argument could be made similarly for an arch carved from a stone lintel and would be instantly recognizable as fallacious. The gravity-loading behavior of the domes, assessed by static analysis from the FE model, also leads to the same conclusion.

Engineering perspective on thrust in Guastavino domes. The investigation of reaction forces for the City-County Building under gravity loading reveals that a horizontal thrust of 54.7 kN is transferred to the massive arches, while the buttresses contribute little in horizontal support (Table 4). It is also observed that the tile arches carry the entire vertical load, 72.6 kN, and ultimately transfer it to the piers. Almost no vertical load is resisted at the base, where the vault web meets the surcharge volume.

Similarly, reaction forces are obtained under gravity loading for the State Education Building. The results show that the steel girders contribute significantly to the vertical load transfer of the total dome weight. Approximately 15.7 kN is carried by each steel truss at the buttress locations between the domes, while approximately 27.5 kN is transferred through each surcharge volume. The horizontal reaction force counterbalanced by the adjacent domes is calculated to be 20.7 kN in total along one side of the pendentive dome, and the horizontal reaction forces where the dome web meets the surcharge is observed to be limited to 3.4 kN.

The FE solutions reveal the significance of the lateral thrust in this construction system. For both the City-County Building and the State Education Building, the magnitude of the horizontal load supported by the stone arches is equal to 75% of the vertical load transferred to one pier. These findings point out that the horizontal thrust is unavoidable for this type of structure.

Historical perspective on thrust in Guastavino domes. A review of Guastavino’s writing also reveals another remarkable point on the issue. Despite his passionate claims, horizontal thrust is in fact taken into account in Guastavino’s calculations and erections of domes. His formula for cohesive arches is based on the simple statics of the voussoir arch, which is widely known to exert a lateral thrust. Guastavino’s equations for domes are
Fig. 11. Guastavino's analysis of the forces in an arch. Horizontal forces in a dome are halved. C represents the compressive strength of the tile, which is multiplied by the area (12 inches/ft and the thickness) to obtain the strength as a function of the crown thrust.

extensions of the arches, assuming that one-half of the loads are carried in each direction: longitudinal and transverse. Hence, he calculates the center thickness of a dome with the formula for the arch and divides by two. In all cases, the existence of the lateral thrust at the support is considered (Fig. 11). The formula used by Guastavino implies a horizontal thrust equal to LS/16r, where L is the total load, S is the span of the dome, and r is the rise of the dome. Noting that the vertical support reaction is LS/2, the ratio of horizontal/vertical reaction reduces to 1/8 the span/raise ratio. For the State Education Building, with a span of 29.2 feet (8.90 m) and a rise above the pendentives of 5.9 feet (1.8 m), there is an estimated horizontal thrust of 62% of the weight of the dome. For the City-County Building, with a span of 32.8 feet (10 m) and a rise of 7.2 feet (2.2 m), there is an estimated horizontal thrust of 56% of the weight of the dome. These values are comparable to the ratios of total horizontal thrust to weight determined in this study to be 37% for the State Education Building and 83% for the City-County Building.

Although the elder Guastavino confidently argued that his domes do not have thrust due to their monolithic nature, the younger Guastavino apparently did not trust the tensile capacity of the cohesive system. On July 31, 1908, four months after his father's death, the younger Guastavino filed an application for a patent on reinforcement of the tile arches and domes with steel bars. The strengthening of domes is indicated clearly to account for the tension zones occurring in the webbing. The patent specification of the reinforcement permits an interesting inference from the following sentence: "As the greatest outward thrust of a semicircular dome is near the base, I place the metal rod extending around the dome closer near the bottom."20

This statement reveals that in 1908 the younger Guastavino was aware of the existence of a lateral thrust in his domes. On January 18, 1910, he was granted the patent on reinforced masonry domes. A carefully researched historical perspective on this discussion is provided by Santiago Huerta. He considers the elder Guastavino to be of two minds on the question of horizontal thrust in domes: on the one side arguing that the horizontal thrust does not exist, and on the other side, inserting metal reinforcement strategically in the domes to counteract this horizontal thrust. The younger Guastavino is much more single-minded in his understanding of this issue, using very refined methods of graphical analysis to determine the magnitude of the horizontal thrust and to design reinforcement to resist it.21

Conclusions

To place the question of lateral thrust in Guastavino domes in perspective, a comprehensive investigation of the structural behavior of the vestibule domes of the City-County Building and of the Reading Room domes of the State Education Building was completed. For these purposes the computerized tools of FE method were used extensively. Although it has long been known that the computerized FE tools have the ability to provide accurate results for given inputs, care must be exercised to enter physically sound and appropriate input values and conditions into the program.

In this study, the vault webbing is modeled with thin-shell elements of approximately 7.9-to-11.8-inch (20-30 cm) mesh size. Although the site survey revealed slight deviations from symmetry, the simplified symmetrical geometric model revealed satisfactory correlation with the experimental data. Typically in a Guastavino dome the tile layers decrease from spring point up to the apex. Approximation of the dome thickness to an average value was found to yield adequate results.

One of the difficulties associated with analyzing such structures with the tools of the FE method is the uncertain elastic boundary conditions caused by the complex functioning of the elements of a monumental vaulted structure (ribs, web, piers, buttresses, etc.). The boundary conditions of the FE models described herein were refined in the earlier stages of the study through a process of comparing the modal-parameter (natural frequencies and mode shapes) estimates of the FE model to those delivered by experiments. The updated boundary conditions for the domes in consideration yielded the following inferences:

- While massive arches supporting the domes provide complete fixity (translational and rotational) to the dome webbing, slender supporting arches do not restrain the periphery of the dome.
- The symmetric configuration of the system results in horizontal restraint perpendicular to the plane of symmetry.
- Tile buttresses, when present, restrain horizontal movement in the direction of their main axis.
- The base, where the dome shell rests on the piers, can be simplified as fixed in translation.

When masonry structures are analyzed with the FE method, another challenge is the identification of the material properties. To determine the mechanical behavior of Guastavino tile and mortar, static compressive tests were conducted separately on samples obtained from the State Education Building. The test results were homogenized according to the special constitution of cohesive construction. Through the homogenization routine, an effective Young's modulus of 7.6 GPa (1100 ksi), effective Poisson's ratio of 0.26, and effective density 1,800 kg/m³ were obtained. Although the tile and mortar manufactured by the Guastavino Company is known to have undergone continuous experimentation and have inconsistent nature, these values can be adapted to the initial FE modeling of Guastavino domes and vaults in future studies.

The FE models are employed to analyze the structures under the forces of gravity. This static analysis enables one to identify the support reactions...
exerted by the dome to the unmodeled adjacent members. The findings of this study illustrate the existence and, more importantly, the significance of the horizontal thrust in a Guastavino dome.

SEZER ATAMTURKTUR is a doctoral candidate in the Department of Architectural Engineering at Pennsylvania State University. She recently completed a master’s degree on the assessment of Guastavino domes.

THOMAS E. BOOTHBY, PhD, is a professor of architectural engineering at Pennsylvania State University. He was appointed to the faculty in 1992. He has more than 15 years of experience in the assessment of historic unreinforced-masonry structures.

Acknowledgements
This research was funded by National Center for Preservation Technology and Training. The authors are also grateful to Bruce Padolf of the City of Pittsburgh and George Webb of the New York State Education Department for the opportunity to test the City-County Building vestibule vaults and the State Education Building Reading Room domes. Also, the editorial help of Corinna Fisher and Lori Smith and the assistance of Sally Gimbert in the material tests are greatly appreciated. The authors also wish to express their thanks to the peer reviewers of this manuscript for their thoughtful and thought-provoking comments, and the editors and staff of Mount Ida Press for their indispensable editorial assistance.

Notes
1. Salvador Tarragó, ed., Guastavino Co. (1885–1962) Catalogue of Works in Catalonia and America (Barcelona: Col·legi d’Arquitectes de Catalunya, 2002). Tarragó’s introductory essay, “Considerations on Guastavino’s Work in Catalonia” (pp. 7–15), evaluates work of Guastavino in Spain and provides a general overview of timber vaulting. In the essay Tarragó states that the use of portland cement enabled Guastavino to widen the distance between pillars up to 26 feet (8 m), significantly greater than the standards of the time.

2. George Collins, “The Transfer of Thin Masonry Vaulting from Spain to America,” Journal of the Society of Architectural Historians 27 (Oct. 1968): 176–201. Collins’s article provides an overview of the background of Guastavino and history of his construction technique. The prestige that the Guastavino Company gained is evident in statistics of the work the company has performed. Collins emphasizes that the high visibility of their projects is far more impressive than the number of the projects with which the company was involved.

3. Rafael Guastavino, Essay on the Theory and History of Cohesive Construction, 2nd ed. (Boston: Ticknor and Company, 1892). In this article Rafael Guastavino the elder claims that with the cohesive-construction technique, one can build domes that do not exert horizontal thrust to their supports (pp. 78–79). In “The Transfer of Thin Masonry Vaulting from Spain to America,” George Collins supports Guastavino’s rationale and states that due to the monolithic nature of cohesive construction the domes exert very little thrust to their supports (p. 180). The language that Guastavino used to support his stone-block argument — rigid membrane, monolithic nature of domes — is also unchallenged and repeated extensively in the literature. In her master’s thesis, “Guastavino Tile Construction” (University of Pennsylvania, 1992), K. Ann Milkovich, in common with other architectural historians, perpetuates the claims of Guastavino uncritically (p. 29).


5. Section drawing, Reading Room, Albany State Education Building, Drawing A156A.1 #01959, Guastavino Collection, Avery Library, Columbia University.


8. Guastavino, 58. In this 1892 article the elder Guastavino indicates a series of tests conducted with engineer A. V. Abbott at the Fairbanks Scale Company in May 1887. Although no detailed information is provided regarding the test setup or methodology, obtained values for four different sets of compression tests are presented as 15.60 MPa (2260 psi), 11.19 MPa (1620 psi), 9.86 MPa (1430 psi) and 20.07 MPa (2910 psi). For future calculations Guastavino used the mean of these significantly varying results. The inconsistency in the strength capacity of the tile samples is evident even in laboratory environments.

9. Guastavino, 144–145. Guastavino the elder calls for the assistance of architects and manufacturers to perfect the building materials and in particular notes the necessity to manufacture lighter tiles.


11. Daniel Lane, “Putting Guastavino in Context: A Scientific and Historic Analysis of His Materials, Method, and Technology” (master’s thesis, Columbia University, 2001). Lane investigates the chemical composition of mortar specimens collected from several structures built by two Guastavinos. The findings of his study reveal differences in the mortar constitution that are not justified by technical advances; instead, Lane considers that Guastavino was experimenting on cement or simply using any available material that he could come up with (p. 104). Lane also notes the association of Guastavino with Jose Francisco Navarro, an established cement producer, and stresses Guastavino’s dedication to make his products better.


14. Atamturktur, 35–38 and 60–61. It is noteworthy that the impact-energy test that determined the thickness of this vault was done before the authors were made aware that the thickness of the vault is three tiles, or approximately 12–13 cm (5 in) (see note 5).


17. Huerta traces the origins of this argument to the 1754 writings of the Comte d’Espie, 93–95.

18. Guastavino, 136, 140.

