OPTIMIZATION OF DESIGN OF SUPPORTED EXCAVATIONS IN MULTI-LAYER STRATA

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ABSTRACT

In this paper, the authors present their robust geotechnical design (RGD) methodology for the design of braced excavations in multi-layer strata with a mix of sand and clay layers. The essence of RGD is to derive an optimal design through a careful adjustment of the design parameters so that the response of the braced excavation system is insensitive to the variation of uncertain soil parameters (called noise factors). Within the RGD framework, the effect of the uncertainties of soil parameters on the variation of the system response is evaluated using first order second moment (FOSM) method in conjunction with the finite element method (FEM). Furthermore, the design robustness is sought along with the cost efficiency and safety. Thus, the RGD methodology involves a multi-objective optimization, in which robustness and cost are treated as the objectives and the safety requirement is treated as a constraint. As cost and robustness are conflicting objectives in a braced excavation design, such optimization often leads to a Pareto front. Finally, through the use of the knee point concept, the most preferred design that meets the safety requirement and strikes a balance between the two objectives (cost and robustness) is identified on the Pareto front. The significance of the RGD methodology is illustrated with a braced excavation design example in multi-layer strata.

Key words: Excavation, optimization, pareto front, robustness, uncertainty.

1. INTRODUCTION

Urban area construction involving braced excavations inevitably induces deformation of the ground at the site, which detrimentally affects any adjacent structures. Supported or braced excavations in urban, congested areas are considered "risky" geotechnical operations that have significant adverse social and economic effects, as evidenced by the recently completed Central Artery/Tunnel Project (CA/T) in the heart of Boston (publicly known as the 'Big Dig'), in which the project completion suffered from significant delays and budget overruns. Because of the difficulty in accurately predicting excavation-induced wall deflections, engineers often face the conflicting goals of either over-designing for liability control or under-designing for cost-savings. The two conflicting goals of braced excavation design are satisfying the code requirements to ensure the safety of the public and workers, and simultaneously minimizing the cost of the excavation project.

The design must satisfy the minimum factors of safety for the stability requirements of the applicable codes. The design that satisfies the stability requirements must then be analyzed for wall deformation to prevent damage to adjacent structures, which often controls the braced excavation design in urban areas (Schuster *et al.* 2009). The maximum wall deflection must not exceed the allowable deflection specified by the codes or the owner. Of course, the design must also satisfy the budgetary constraints of the project.

The traditional design of a braced excavation system is mainly a trial-and-error process. Multiple candidate designs are checked until the computed responses such as the factor of safety against failure and the maximum wall deflection satisfy the requirements set by the owner or specified by the codes. The least cost design is then identified from the acceptable design pool. Because of the uncertainty in soil parameters, the least cost design may experience unsatisfactory performance when the variability of soil parameters is underestimated. Thus, the design requirements are prone to violation because of the possible high variability of the system response caused by high variability in soil parameters. When confronted with the high variability of the system response, the designer may choose an unduly over-conservative design, which is economically inefficient. One possible solution to this problem is to consider robustness in the design to reduce the variation of the system response. Robust design concept, originated in the field of Industrial Engineering (Taguchi 1986) to make the product of a process insensitive to uncertain parameters, has been applied to many other design fields such as mechanical and structural design (*e.g.*, Phadke 1989; Chen *et al.* 1996; Lee and Park 2001; Zhang *et al.* 2005; Doltsinis and Kang 2006; Lagaros *et al.* 2010). This concept has also been recently used in various geotechnical applications (*e.g.*, Juang *et al.* 2012; Juang and Wang 2013; Wang *et al.* 2013; Juang *et al.* 2013a & b).

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In this paper, we focus on the robustness against uncertainties in the design of braced excavations. A design is deemed robust if the system response of concern (such as the maximum wall deflection in the case of braced excavation) is insensitive to the variation of uncertain soil parameters. In the context of robust design, these uncertain soil parameters are known as "noise factors." In a robust design, "easy-to-control" design parameters are optimized so as to minimize the variability of system response that is caused by the variation in the "hard-to-control" noise factors. As shown in Fig. 1, unlike the initial design that yields a large variation in the system response, the design robustness can be achieved by adjusting the design parameters to make the system response less sensitive (or more robust) to the noise factors.

As is presented later, three main concerns in the robust design of braced excavation are safety, robustness, and cost. The safety of the excavation is ensured by the constraints based on stability and serviceability requirements. The safety constraints may be based on a deterministic (factor of safety) assessment or reliability assessment. The design robustness is achieved by minimizing the variation of the system response of concern (*e.g.*, maximum wall deflection in the case of braced excavation). The construction cost is simultaneously optimized, along with robustness, to enhance the cost-efficiency of the design.

To consider robustness, cost, and safety simultaneously, multi-objective optimization is adopted, in which both the robustness and cost are set as the design objectives and safety is guaranteed through constraints. Because robustness and cost are conflicting objectives in the case of braced excavation design, no single best design can be obtained. Rather, a set of non-dominated designs, collectively forming a Pareto front, is obtained. The Pareto front enables the designer to choose a preferred design based upon a desired level of robustness and/or budget. A knee point concept may be further used for identifying the most preferable design.

2. DETERMINISTIC MODEL FOR ASSESSING EXCAVATION-INDUCED WALL DEFLECTION

In the design of a braced excavation system, the maximum wall deflection is usually used as an index to assess the stability and serviceability of the system. Thus, the design with excessive
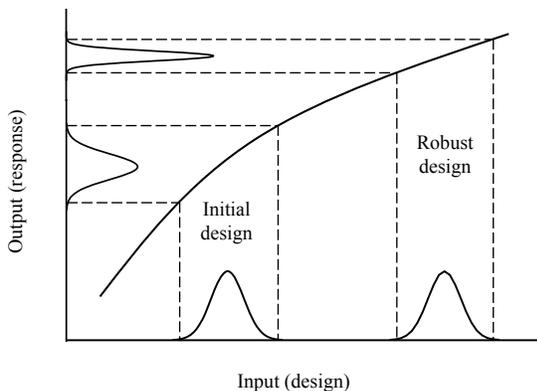

Fig. 1   Illustration of the robustness concept (modified after Phadke 1989)

wall deflection (*i.e.*, the actual maximum wall deflection exceeds the allowable maximum wall deflection) is not permissible. The wall deflection is accompanied with ground surface settlement, thus, the latter may also be used to assess the serviceability. However, the wall deflection is easier than the ground settlement to measure in the field during the construction, and is easier to analyze during the design phase. Thus, in this paper the maximum wall deflection is adopted as the system response of concern in the context of robust design. Although factors of safety against various stability problems are also system responses, they are treated as the constraints in the robust design described in this paper.

Though many empirical methods (*e.g.*, Mana and Clough 1981; Clough and O'Rourke 1990; Kung *et al.* 2007a) are available to predict the maximum wall deflection induced by excavation, these methods are more applicable for the design of excavations in homogeneous soils. They are not as accurate in the more complicated site conditions with mixed layers of sands and clays. Here, a commercially available computer program known as TORSA (Taiwan Originated Retaining Structure Analysis) is adopted for excavation analysis. This computer program, which is based upon the beam on elastic foundation theory, has been validated by hundreds of real-world design cases for braced excavations in Taiwan (Sino-Geotechnics 2010).

TORSA is a special-purpose FEM code based on the beam on elastic foundation theory (Biot 1937) developed by Trinity Foundation Engineering Consultants (TFEC), Taipei, Taiwan (Sino-Geotechnics 2010). In the beam on elastic foundation model, the diaphragm wall is assumed to be a beam on an elastic foundation. The pressure acting on the back of the diaphragm wall is assumed to be the active earth pressure and the resistance of soil inside the excavation is modeled as a series of soil springs (Sino-Geotechnics 2010). At each excavation stage, the active earth pressure on the back of the diaphragm wall is balanced by the forces of struts and soil springs inside the excavation. The magnitude the soil spring force is the modulus of horizontal subgrade reaction (k_h) multiplied by the deformation of soil spring. If the force on a given soil spring is smaller than the corresponding passive earth pressure, the soil spring is in the elastic state. When the deformation of the soil spring is large enough and the soil spring force reaches the threshold of the passive earth pressure, the soil spring is in the plastic state and the spring force becomes a constant and remains at the same level as the threshold passive earth pressure (Ou 2006).

When considering all the applied forces on the wall after the excavation at a given stage, including the active earth pressure, water pressure, surcharge effect, internal strut stiffness and preload, and the passive earth pressure and soil spring force, the beam on elastic foundation model can be formulated based on the limit equilibrium principle. This model is then solved via the finite element method (FEM) implemented in the TORSA code. The interested readers are referred to Ou (2006) and Sino-Geotechnics (2010) for further details about the finite element formulation of the beam on elastic foundation model.

In a braced excavation in sand, the main soil parameters affecting the system responses are friction angle (ϕ) and modulus of horizontal subgrade reaction (k_h). In a braced excavation in clay, the main soil parameters affecting the system responses are undrained shear strength (s_u) and modulus of horizontal subgrade reaction (k_h). These soil parameters, together with the surcharge

behind the wall (q_s) are the controlling parameters that affect the wall deflection in a given design of braced excavation system (Ou 2006). The possible high variations of these parameters necessitated their treatment as "noise factors" in the robust design of braced excavations.

3. ROBUST GEOTECHNICAL DESIGN METHODOLOGY FOR BRACED EXCAVATIONS

3.1   Concept of Robust Design for Braced Excavations

In a diaphragm wall-supported excavation system, the geometric dimensions of the excavation (length of excavation L_E, width of excavation B_E, and final excavation depth H_f) are predetermined according to the structural or architectural requirements. In the robust design of braced excavation, the wall thickness (t), the wall length (L), the vertical spacing of the struts (S), the stiffness of strut (EA) are considered as the design parameters. On the other hand, soil parameters in each layer of a multi-layer strata, specifically ϕ and k_h for the sand layer and s_u and k_h for the clay layer, as well as the surcharge behind the wall q_s, are treated as noise factors. Figure 2 shows the elements of a robust braced excavation design.

The goal of any robust excavation design is to identify a suite of optimal design parameter settings (t, L, S, EA) that maximizes the design robustness (by minimizing the variation of system response caused by noise factors) and minimizes the construction cost of braced excavation, while simultaneously satisfying the safety constraints. For ease of reference in this analysis, the robust design concept that is implemented specifically in geotechnical engineering problems is recognized as the robust geotechnical design (RGD) methodology.

3.2   Analytical Procedure for Deriving Mean and Standard Deviation

Uncertainty in the input parameters causes the uncertainty in the computed wall deflections. Thus, evaluating the variability of the maximum wall deflection caused by uncertain noise factors is a main task in the robust design of braced excavations. Here, the first-order second-moment (FOSM) method based upon the Taylor Series Expansion (Dang *et al.* 2012) is used for the uncertainty propagation.

The response of concern (such as the maximum wall deflection) of an excavation system, denoted as y, may be expressed as follows:

$$y = f_{FEM}(X, D) = f(x_1, x_2, ..., x_n) \tag{1}$$

where

$\quad f_{FEM}$ = a model such as an FEM code that turns a set of input data into an output y,

$\quad\quad X$ = a vector of noise factors (uncertain variables) = $\{x_1, x_2, x_3, ..., x_n\}$,

$\quad\quad D$ = a vector of design parameters (fixed-value parameters).

Recall that a smaller variation of the system response indicates a greater robustness. Thus, to assess the robustness of a design here, the mean and standard deviation of the response y must be evaluated. The mean and standard deviation of the response y based on FOSM may be computed as follows (Dang *et al.* 2012):
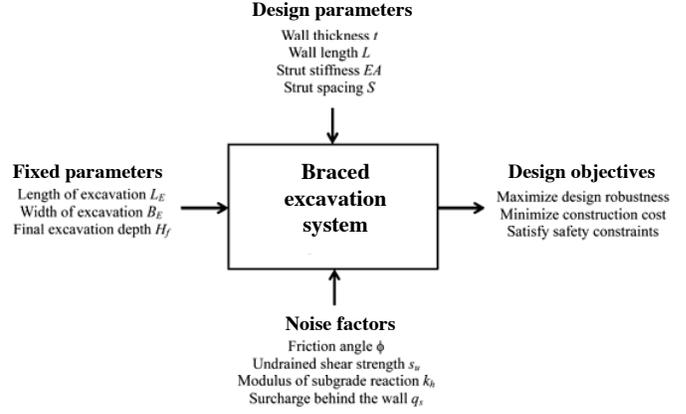

Fig. 2   Illustration of the robust geotechnical design of braced excavations

$$\mu_y = f(\mu_{x_1}, \mu_{x_2}, \mu_{x_3}, ..., \mu_{x_n}) \tag{2}$$

$$\sigma_y^2 = \sum_{i=1}^{n}\sum_{j=1}^{n} \frac{\partial y}{\partial x_i}\frac{\partial y}{\partial x_j}(\rho_{ij}\,\sigma_{x_i}\,\sigma_{x_j})$$

$$= \sum_{i=1}^{n}\left(\frac{\partial y}{\partial x_i}\right)^2\sigma_{x_i}^2 + \sum_{i=1}^{n}\sum_{j=1, j\neq i}^{n}\left(\frac{\partial y}{\partial x_i}\frac{\partial y}{\partial x_j}\right)(\rho_{ij}\sigma_{x_i}\sigma_{x_j}) \tag{3}$$

where μ_y = mean of maximum wall deflection; σ_y = standard deviation of maximum wall deflection; μ_{x_i} = mean of noise factors; σ_{x_i}, σ_{x_j} = standard deviation of noise factor x_i and x_j; $\partial y/\partial x_i$ = derivative of FEM solution function evaluated at μ_{x_i}; $\partial y/\partial x_j$ = derivative of FEM solution function evaluated at μ_{x_j}; ρ_{ij} is the correlation coefficient between x_i and x_j.

For our purposes, all noise factors set at their mean values are regarded as the baseline model. According to Dang *et al.* (2013), the first-order derivatives of the FEM solution in Eq. (3) can be calculated using a central finite difference method. The effect of a specific uncertain noise factor (*e.g.*, x_i) on the system response is investigated by varying the value of this specific noise factor while keeping other noise factors fixed at their mean values. The step-by-step implementation of this FOSM approach in conjunction with FEM solution can be summarized accordingly (after Dang *et al.* 2013):

1. Compute the response of excavation with FEM solution by adopting all the mean values of these noise factors, with a resulting mean response of μ_y.

2. Compute the standard deviation of response Δy_1 caused by variation in the first noise factor x_1 by completing the following three steps:

   a. Compute the resulting response of FEM solution y^+ with a mean plus one standard deviation for the first noise factor x_1, while all other noise factors are set at their mean values.

   b. Compute the resulting response of FEM solution y^- with mean minus one standard deviation for the first noise factor x_1, while all other noise factors are set at their mean values.

   c. Compute the standard deviation of response caused by the first noise factor $\Delta y_1 = |y^+ - y^-|/2$.

3. Follow a similar three-step procedure in step 2, compute the standard deviation of response Δy_i caused by variation in the ith noise factor x_i.

4. Compute the standard deviation of response σ_y caused by variation in the all noise factors as follows:

$$\sigma_y^2 = \sum_{i=1}^{n} (\Delta y_i)^2 + \sum_{i=1}^{n} \sum_{j=1, j \neq i}^{n} (\rho_{ij} \, \Delta y_i \, \Delta y_j) \tag{4}$$

As illustrated above, Step 1 describes the evaluation of Eq. (2) and Step 4 describes the evaluation of Eq. (3), which is further simplified into Eq. (4). Furthermore, according to Dang *et al.* (2013), the above procedure that combines the FOSM approach with FEM solution is effective in the calculation of the mean and variation of the response of the braced excavation. The computed variation of the system response of concern in a braced excavation (*e.g.*, maximum wall deflection) is then used as a basis for evaluating the robustness of a given design within the RGD framework.

3.3 Framework for Robust Geotechnical Design (RGD) of Braced Excavations

The robust geotechnical design (RGD) framework by Juang *et al.* (2012) is demonstrated with a design example of braced excavation in multiple strata, and summarized below (in reference to Fig. 3):

Step 1: To define the problem of concern and classify the design parameters and the noise factors for the geotechnical system (*e.g.*, braced excavation system), both of which have been defined in the previous section for the braced excavation problem.

Step 2: To quantify the uncertainty of the noise factors and specify the design domain. Here, the mean and coefficient of variation (COV) of the noise factors should be estimated. The design domain should consist of discrete design parameters and be specified based upon the design and construction experiences. Thus, there will be a finite number of designs (say, *M* designs) in the discrete design domain.

Step 3: To derive the mean and standard deviation of the system response for each design in the design space. Here, for a given set of design parameters, the FOSM procedure described previously is combined with the FEM analysis (using TORSA) for evaluating the variation in system response caused by uncertain noise factors. The FOSM procedure involves the execution of the adopted FEM analysis for evaluating the system response at each of the *N* sets of the sampling points of the noise factors ($N = 2n + 1$, where *n* is the number of noise factors). For a given design in the design space, a total of *N* numerical outputs are obtained for the *N* sets of the sampling points of noise factors. These *N* numerical outputs are then used to compute the mean and standard deviation of system response based on the FOSM formulation, as reflected by the inner loop of Figure 3. Next, the FOSM procedure combined with FEM analysis is repeated for each of the *M* designs in the design space. Through the *M* number of repetitions, the mean and standard devia-

tion of system response for each of the *M* designs in the design space are obtained, as represented by the outer loop of Fig. 3.

Step 4: To locate the satisfactory solutions that are optimal to both robustness and cost using the multi-objective optimization. The optimality in robustness is achieved by minimizing the standard deviation of the system response while the optimality in cost is achieved by minimizing the construction cost. The safety requirements are verified to ensure a "satisfactory design" through constraints of both stability and deformation requirements, which may be verified using a deterministic model or a probabilistic model.

In a multi-objective optimization, no single best design can be obtained if the objectives to be optimized are conflicting. Though a set of designs may be obtained that are superior to all others with all objectives considered, within that set, none is superior or inferior to each other with all objectives. These designs constitute an optimum set called the Pareto front as shown in Fig. 4(a). In this paper, a fast and elitist Non-dominated Sorting Genetic Algorithm (NSGA-II) developed by Deb *et al.* (2002) was used to establish the Pareto front, in which, the optimal designs are searched in the discrete design domain (Lin and Hajela 1992).

Step 5: To identify the most preferable design based on sacrifice-gain relationship on the Pareto front. Although Pareto front itself is a useful design guide, additional step may be taken so that the designer may select the "best" design out of a set of alternatives on the Pareto front. The authors used the knee point concept (Deb *et al.* 2011) to select the most preferable design. The knee point is the point on the Pareto front in which a small improvement in one objective will cause a large deterioration in the other objective, a scenario that makes moving in either direction not advantageous (Branke *et al.* 2004; Deb *et al.* 2011).

A normal boundary intersection method (Bechikh *et al.* 2010; Deb *et al.* 2011) was employed to find the knee point on the Pareto front. As illustrated in Fig. 4(b), a boundary line *L* is constructed by connecting two extreme points P_1 and P_2 in the normalized space of the Pareto front. Then for each point on the Pareto front, except for the two extreme points, the distance from the boundary line *L* is computed. The point with the maximum distance from the boundary line *L* is then identified, which is called the knee point, as denoted in Fig. 4(b).

4. COST ESTIMATES FOR BRACED EXCAVATIONS

For a robust geotechnical design, the cost-efficiency should be explicitly considered, which requires the estimation of cost for braced excavations (Zhang *et al.* 2011a). For a specific project, the site dimensions and excavation depth are fixed according to either the structural or the architectural requirements. The major cost in optimization is the cost of supporting system, which depends upon the design parameters.
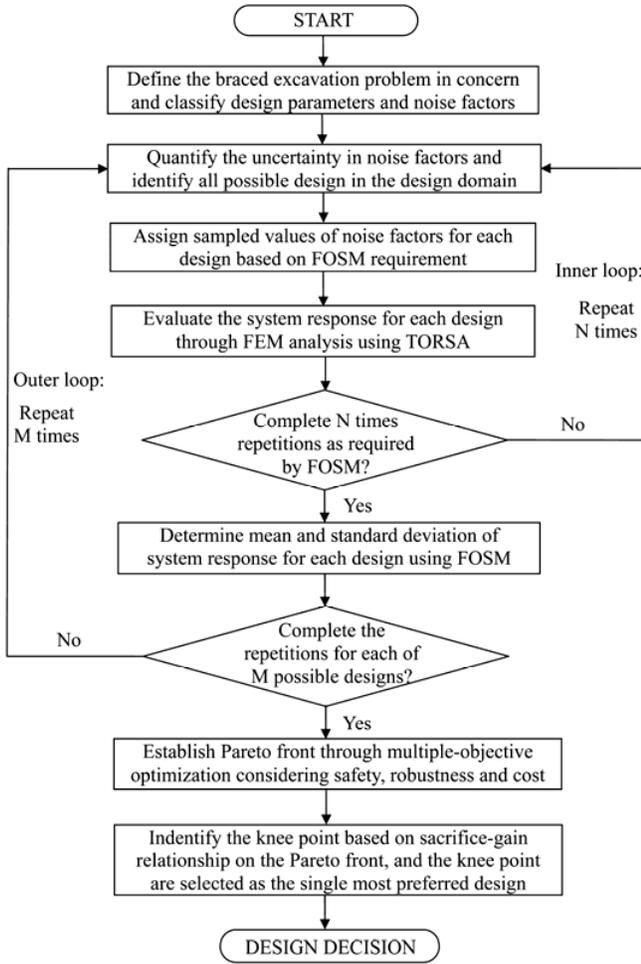
Fig. 3   Flowchart illustrating the robust geotechnical design of braced excavations (modified after Juang et al. 2013b)
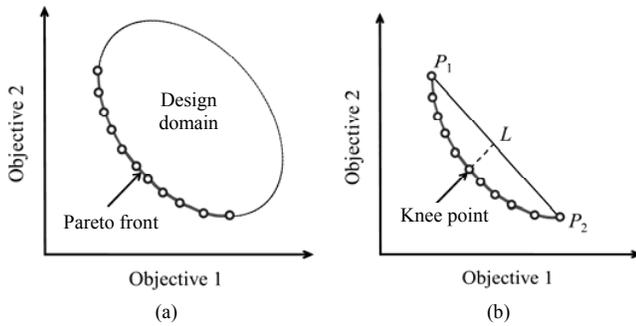

Fig. 4   Illustration of the Pareto front and knee point identification (modified after Bechikh et al. 2010)

The total cost for the supporting system Z includes the cost of both the diaphragm wall and bracing system. For a typical diaphragm wall constructed in Taiwan, the cost is proportional to the volume of the wall with a unit cost of approximately NT $10, 000/m^3 ($\approx$ 330 USD/m^3 according to Juang et al. 2013b). The volume of the wall is the multiplication of the wall length, the wall thickness and the perimeter of the excavation. For a bracing

system with H-section steels, the cost is proportional to the number of vertical levels of struts, the number of struts per level and the area of the excavation with a unit cost of a bracing system of approximately NT $1, 000/m^3 ($\approx$ 33 USD/m^3 as per Juang et al. 2013b). Thus, the total cost for the supporting system Z can be determined by a set of design parameters. It should be pointed out that for the design parameter "strut stiffness", there are five choices of struts per level in local practice in Taipei using TORSA: H300, H350, H400, 2@H350 and 2@H400 (note: 2@H350 means two H350 struts used per level; 2@H400 means two H400 struts used per level; see Table 1 for stiffness values of these strut designations). The main cost for struts is the cost in their installation, and the difference in the material costs among H300, H350 and H400 is generally negligible. Thus, the cost of the struts is related to the number of struts used per level, which corresponds to the design parameters of the strut stiffness within the RGD framework.

5. ROBUST GEOTECHNICAL DESIGN OF BRACED EXCAVATION — AN ILLUSTRATED EXAMPLE

5.1   Brief Summary of the Example Case

A hypothetical excavation case in the TORSA user manual (Sino-Geotechnics 2010) was adopted as an example to demonstrate our robust geotechnical design (RGD). The excavation was conducted in layered soils with mixed sand and clay layers. The soil profiles and soil properties for each layer are shown in Fig. 5 and Table 2, respectively. The groundwater table is found at 1 m below the ground surface, and the dimensions of the excavation site are pre-specified according to architectural and structural requirements. The shape of the excavation site is rectangular with a length of 33 m and a width of 13 m, and the final excavation depth is 10 m. A diaphragm wall is used as the retaining structure and the H-section steels are used as the bracing structures to support the diaphragm wall. For soil parameters, the values listed in Table 2 under the columns, s_u, ϕ and k_h are the mean values for each soil layer. For the clay layer, the undrained strength s_u is assumed to have a COV of 0.2 and the modulus of horizontal subgrade reaction k_h is assumed to have a COV of 0.5. The two key soil parameters s_u and k_h in the clay layer are assumed to be positively correlated with a correlation coefficient of 0.7. For the sand layer, the internal friction angle ϕ is assumed to have a COV of 0.1 and the modulus of horizontal subgrade reaction k_h is assumed to have a COV of 0.5. In the sand layer, ϕ and k_h are assumed to be positively correlated with a correlation coefficient of 0.3. The surcharge behind the wall q_s has a mean of 1 ton/m and a COV of 0.2. These statistics are estimated based upon local experience and published literatures (Phoon et al. 1995; Ou 2006; Hie 2007; Zhang et al. 2011b; Luo et al. 2012; Juang et al. 2013b). Succinctly, there are totally eleven uncertain noise factors in the RGD design of the braced excavation when the uncertainties in soil properties in each layer and the surcharge are all accounted for.

For each of the design parameters, the wall length (L), the wall thickness (t), the strut stiffness (EA), and the vertical spacing of the struts (S), the range and the search increment have to be specified. For example, the wall length L typically ranges from

Table 1   Strut stiffness (*EA*) for the five strut designations

Strut designation	*E* (GPa)	*A* (cm^2)	*EA* (GPa·m^2)
H300	200	119.78	2.40
H350	200	173.87	3.48
H400	200	218.69	4.37
2@H350	200	347.74	6.95
2@H400	200	437.38	8.75

Table 2   Basic soil properties in the example case of braced excavation (data from Sino-Geotechnics 2010)

Layer No.	Soil type	Depth (m)	γ_t (t/m^3)	s_u (t/m^2)	ϕ (°)	k_h (t/m^3)
1	ML	7.7	2	0	30	1000
2	SM	13.5	1.94	0	31	1400
3	SM	20.6	1.95	0	31.5	1700
4	CL	25.7	1.84	4	0	800
5	SM	35	1.91	0	31	1600

Note: The values in columns of s_u, ϕ and k_h represent the mean values of these soil properties
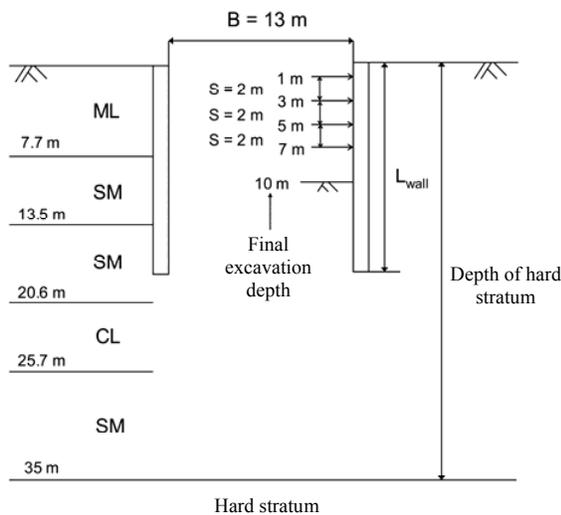
Fig. 5   Illustration of the soil profile for the example case of braced excavation

15 m to 25 m and the search for the final design can be performed with an increment of 0.5 m. Likewise, the wall thickness *t* ranges from 0.5 m to 1.3 m with an increment of 0.1 m. The strut stiffness *EA* in local practice has five choices: H300, H350, H400, 2@H350 and 2@H400. In any routine braced excavation design, the preload of strut is a fixed number depending upon the type of strut (*e.g.*, H300 using a preload of 50 tons, H350 using a preload of 75 tons, and H400 using a preload of 100 tons) as illustrated in Sino-Geotechnics (2010). For a typical braced excavation, the first-level strut is placed 1 m below the ground surface and the final-level strut is placed 3 m above the final excavation depth. Except for the final stage (Kung *et al*. 2007a; Sino-Geotechnics 2010), the location of a strut is typically 1 m above the excavation depth at that stage. Thus, for this excavation case with a final

excavation depth of 10 m, the vertical spacing of struts *S* has four alternations: 1.5 m, 2 m, 3 m and 6 m. If the spacing *S* = 1.5 m in this hypothetical excavation case, then the total number of struts will be 5 based on the strut arrangement described previously. Similarly, the total number of struts will be 4, 3 and 2, respectively, if the spacing is *S* = 2 m, 3 m, and 6 m correspondingly. In summary, in the design domain, the numbers of choices for the design parameters *L*, *t*, *EA*, and *S* are 21, 9, 5, and 4, respectively. Therefore, the total number of possible designs in the design domain is 21 × 9 × 5 × 4 = 3780.
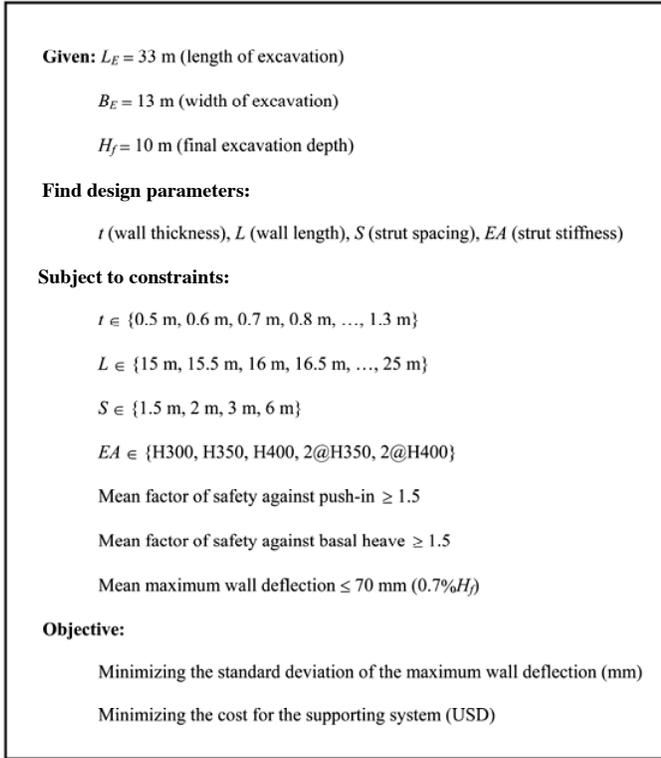
5.2   Optimization of Braced Excavation to Establish Pareto Front

Following the RGD procedure outlined previously, for each of the possible designs in the design domain, TORSA is used to analyze the system response, including factor of safety against basal heave failure (TGS 2001), factor of safety against wall "push-in" failure (TGS 2001), and the maximum wall deflection. Because of the uncertainties in the noise factors (uncertain soil parameters), TORSA analysis is integrated into the FOSM formulation, and the mean and standard deviation of each of these system responses are computed. These system responses are used in the multi-objective optimization to screen for the optimal design. Figure 6 shows the optimization settings. It is noted that for each candidate design, the mean values of the three system responses (basal heave, push-in, and maximum wall deflection) are used to check whether the safety requirements are satisfied. This validation of the safety requirements is implemented as constraints in the optimization. The standard deviation of the maximum wall deflection, representing the variation of the system response of concern, is used to gauge the design robustness, and the objective is to minimize this standard deviation (or to maximize the robustness). The other objective is to minimize the cost of the supporting system. Thus, all three aspects, safety, robustness and cost are explicitly considered in the RGD of this braced excavation system.
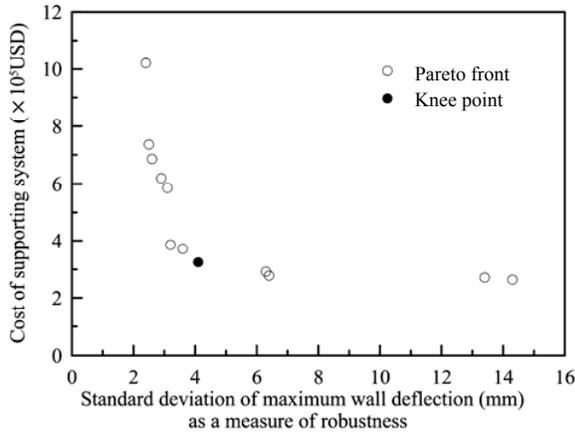
Through the NSGA-II algorithm (Deb *et al*. 2002), 12 designs out of the total 3780 possible designs in the design domain were selected onto the converged Pareto front, as shown in Fig. 7. The corresponding design parameters of all 12 designs on the Pareto front are listed in Table 3. As shown in Fig. 7, a tradeoff relationship between robustness (in terms of standard deviation of wall deflection) and cost is implied and the improvement in robustness (*i.e.*, decrease in standard deviation of the maximum wall deflection) requires an increase in the cost of the designed braced excavation system. This conflict between two objectives reveals the essential characteristic of the Pareto front.

Through this obtained Pareto front, designers can express their preference between different objectives. It is noted that engineering judgment can play a significant role in the selection of the final design. For example, based on experience of similar projects and with the engineering judgment, an acceptable budget for the given case may be estimated at, say, 3×10^5 USD. Then, the design with design parameters *t* = 0.5 m, *L* = 15.5 m, *S* = 2 m and *EA* = H400 (No. 4 design in Table 3), which is the most robust design within that cost level, is selected as the final design.

Finally, it should be of interest to frame the conventional design in the context of Pareto front. In the conventional practice, the least cost design among all the designs that satisfy the safety

Given: $L_E = 33$ m (length of excavation)

   $B_E = 13$ m (width of excavation)

   $H_f = 10$ m (final excavation depth)

Find design parameters:

   t (wall thickness), L (wall length), S (strut spacing), EA (strut stiffness)

Subject to constraints:

   $t \in \{0.5$ m, 0.6 m, 0.7 m, 0.8 m, $…, 1.3$ m$\}$

   $L \in \{15$ m, 15.5 m, 16 m, 16.5 m, $…, 25$ m$\}$

   $S \in \{1.5$ m, 2 m, 3 m, 6 m$\}$

   $EA \in \{$H300, H350, H400, 2@H350, 2@H400$\}$

   Mean factor of safety against push-in ≥ 1.5

   Mean factor of safety against basal heave ≥ 1.5

   Mean maximum wall deflection ≤ 70 mm $(0.7\%H_f)$

Objective:

   Minimizing the standard deviation of the maximum wall deflection (mm)

   Minimizing the cost for the supporting system (USD)

Fig. 6   Optimization settings for robust geotechnical design of braced excavation

Fig. 7   Pareto front optimized to both cost and robustness based upon the prescribed protection requirement, the maximum wall deflection not to exceed 0.7% of the final depth of excavation

Table 3   List of designs on the Pareto front based upon the prescribed protection requirement*

No.	T (m)	L (m)	S (m)	EA	Robustness** (mm)	Cost ($\times 10^5$USD)
1	0.5	15.5	6	H400	14.3	2.64
2	0.5	16	6	H400	13.4	2.71
3	0.5	15.5	3	H400	6.4	2.78
4	0.5	15.5	2	H400	6.3	2.92
5	0.6	15.5	3	H400	4.1	3.25
6	0.7	15.5	3	H400	3.6	3.72
7	0.7	15.5	2	H400	3.2	3.86
8	1.1	15	3	2@H350	3.1	5.86
9	1.1	16	3	2@H350	2.9	6.19
10	1.2	16.5	3	2@H350	2.6	6.86
11	1.3	16.5	3	2@H350	2.5	7.36
12	1.3	23	2	2@H350	2.4	10.21

*   The maximum wall deflection (δ_{hm}) is not permissible to exceed 0.7% of the final depth of excavation (H_f).

**   Robustness is evaluated in terms of standard deviation of the maximum wall deflection. Higher standard deviation such as the one for Design 1 or Design 2 indicates lower design robustness.

tradeoff relationship between safety and cost, it is inferred that a design with higher factor of safety tends to result in higher design robustness.

5.3 Knee Point Identification for Most Preferred Design

If the target cost/robustness level is not specified, it would be desirable to locate the most preferred design on the Pareto front. In such situation, the knee point concept can be adopted to identify the single most preferred design on the Pareto front. Using the normal boundary intersection method described previously, the knee point in Fig. 7 is identified as the design with parameters $t = 0.6$ m, $L = 15.5$ m, $S = 3$ m and $EA =$ H400 (No. 5 design in Table 3). This most preferred design (denoted as the knee point in Fig. 7) has a design cost of 3.25×10^5USD. As shown in Fig. 7, at the knee point, moving in either direction beyond this point is not attractive because a significant sacrifice is required in one objective to gain a marginal improvement in the other objective.

To further validate the global optimum of the normal boundary intersection method in the knee point search, a marginal utility-based method is employed to check the identified knee point. The marginal utility function $U'(X, \lambda)$ for an individual design on the Pareto front is defined as follows (Branke *et al.* 2004):

$$U'(X_i, \lambda) = \min_{j \neq i}\left(U(X_j, \lambda) - U(X_i, \lambda)\right) \tag{5}$$

where $U(x, \lambda)$ follows a linear utility form as follows (Branke *et al.* 2004):

$$U(X, \lambda) = \lambda f_1(X) + (1 - \lambda)f_2(X) \tag{6}$$

where X denotes one set of design parameters, $X = (t, L, S, EA)$ in this case; $f_1(X)$ denotes the normalized objective value for ro-

and performance requirements is usually selected as the final design. Because the conventional design is generally optimized with respect to only cost, it will yield a design that is the same as the least cost design among all the designs on the Pareto Front. Thus, the final design obtained in the conventional practice is a special case (with least cost) on the Pareto Front using the RGD method. Furthermore, based on the tradeoff relationship between robustness and cost presented in this paper, and the well-recognized

bustness, which is represented by the standard deviation of maximum wall deflection that is normalized within [0,1] in this case; $f_2(X)$ denotes the normalized objective value for cost, which is represented by the cost of supporting system that is normalized within [0, 1] in this case; λ is a random variable with a uniform distribution and $\lambda \in [0,1]$.

By sampling a large amount of random values for λ using Monte Carlo simulations, the marginal utility function is computed for each design on the Pareto front. The knee point is located by picking the design with the maximum expected value of marginal utility. Using the marginal utility-based method (Branke *et al*. 2004), the same knee point is identified ($t = 0.6$ m, $L = 15.5$ m, $S = 3$ m and $EA = $ H400), as shown in Fig. 7.

6.   FURTHER DISCUSSIONS

In the above analysis, the limiting wall deflection was based on the requirement of an adjacent structures protection level adopted in a design code of China (PSCG 2000), in which the maximum wall deflection (δ_{hm}) is not permissible to exceed 0.7% of the final depth of excavation (H_f). In an actual design scenario in Shanghai, China or elsewhere, however, the client may prescribe a different level of performance requirement (in terms of limiting wall deflection) to meet the project need. The RGD framework can be easily and readily adapted to meet this need. For illustration purposes, the excavation design for the example case is re-done considering a more restricted protection level in which the constraint of $\delta_{hm} \le 0.3\% \; H_f$ is implemented.

Following the RGD procedures outlined previously, a new Pareto front is obtained for this protection level ($\delta_{hm} \le 0.3\% \; H_f$ but all other requirements remain the same), as shown in Fig. 8. The Pareto front is now constituted by 10 designs, the design parameters of which are listed in Table 4. The most preferred design (represented by the knee point on the Pareto front) in Fig. 8 has the following design parameters: $t = 0.7$ m, $L = 15.5$ m, $S = 2$ m and $EA = $ H400 with a cost of 3.86×10^5 USD (No. 5 design in Table 4). As expected, the cost of the most preferred design under this stricter protection requirement ($\delta_{hm} \le 0.3\% \; H_f$) is greater than the cost of the most preferred design obtained previously for the protection level of $\delta_{hm} \le 0.7\% \; H_f$.

The flexibility of the RGD framework may further summarized. First, in the previous analyses, the constraint of $\delta_{hm} \le 0.3\% \; H_f$ is implemented in a deterministic manner (*i.e.*, yes-or-no type of assessment). This constraint may be replaced with a probabilistic assessment, in which the probability of exceeding a specified limiting wall deflection is compared with an allowable probability of exceedance. This is an extension of the deterministic assessment, which may be view as equivalent to a probabilistic assessment with an allowable probability of exceedance of 50%. Since the data for the probabilistic assessment, including the mean and standard deviation of the maximum wall deflection, is readily available, the constraint with a probabilistic assessment is readily implementable in the RGD framework. Similarly, the constraints regarding the factors of safety may be replaced by the probability of failure against different failure modes.

Second, the robustness in the previous optimization analysis is measured with the standard deviation of the maximum wall deflection caused by the uncertainty in the noise factors. This robustness measure may be replaced with other measures such as
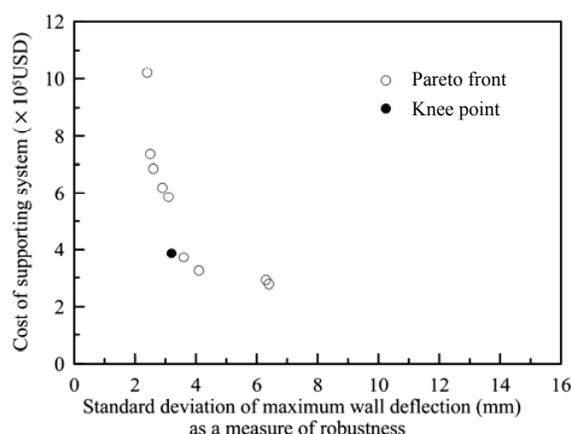
Fig. 8   Pareto front optimized to both cost and robustness based upon a stricter protection requirement, the maximum wall deflection not to exceed 0.3% of the final depth of excavation

Table 4   List of designs on the Pareto front based upon a stricter protection requirement*

No.	T (m)	L (m)	S (m)	EA	Robustness** (mm)	Cost ($\times 10^5$USD)
1	0.5	15.5	3	H400	6.4	2.78
2	0.5	15.5	2	H400	6.3	2.92
3	0.6	15.5	3	H400	4.1	3.25
4	0.7	15.5	3	H400	3.6	3.72
5	0.7	15.5	2	H400	3.2	3.86
6	1.1	15	3	2@H350	3.1	5.86
7	1.1	16	3	2@H350	2.9	6.19
8	1.2	16.5	3	2@H350	2.6	6.86
9	1.3	16.5	3	2@H350	2.5	7.36
10	1.3	23	2	2@H350	2.4	10.21

*   The maximum wall deflection (δ_{hm}) is not permissible to exceed 0.3% of the final depth of excavation (H_f).

**   Robustness is evaluated in terms of standard deviation of the maximum wall deflection. Higher standard deviation such as the one for Design 1 or Design 2 indicates lower design robustness.

signal-to-noise ratio (Phadke 1989) and feasibility robustness (Parkinson *et al*. 1993; Wang *et al*. 2013). Furthermore, other system response such as the factors of safety against various failure modes or the excavation-induced ground settlement may also be adopted as a basis for measuring the design robustness.

Third, in the previous analysis, TORSA is adopted as the deterministic model for stability and deformation analysis. TORSA is adopted for a couple of reasons: (1) it can handle complex soil conditions and its accuracy has been proven by years of practice, and (2) it can be implemented automatically and seamlessly in the RGD framework, as indicated in Fig. 3. Although TORSA is considered the best choice for the RGD framework at this point, it is possible to adopt the more sophisticated model such as AFENA (Kung *et al*. 2007b) and PLAXIS (Dang *et al*. 2013) as the deterministic model for stability and deformation analysis in the RGD framework.

In summary, the various components of the presented RGD framework may be further investigated and improved as warranted. At this point, the authors feel that the RGD framework and its current implementation for robust design of braced excavation systems represent a significant step toward practical application of this design methodology.

7. CONCLUDING REMARKS

In this paper, the authors presented a detailed formulation and implementation of robust design of braced excavations in multiple-layer strata. This robust geotechnical design (RGD) approach considers safety, robustness, and cost simultaneously in the design. The RGD methodology is demonstrated as an effective tool through an illustrative example using industrial-strength software TORSA for braced excavations. By enforcing design robustness in the face of uncertainties, which is not considered in any traditional design methods (either factor-of-safety-based or reliability-based), the variation of the system response caused by the input parameter uncertainties is controlled by the designer through a tradeoff consideration of cost efficiency and robustness, while safety is guaranteed.

It is interesting to note that robust design allows for reduction in the variation of the system response of concern without having to eliminate the sources of the uncertainties in the designed system. In the braced excavation design, such robustness is achieved by carefully adjusting the design parameters of both the diaphragm wall and the bracing system in a given set of design settings (*i.e.*, excavation geometry and excavation depth). As in many engineering problems, inevitably, higher cost is involved when the design robustness is sought. Thus, a tradeoff consideration based upon the Pareto front obtained through multi-objective optimization is required. In this regard, the knee point concept is shown as a valuable design aid for selection of the most preferred design in the design of braced excavation system.

Various components of the presented RGD methodology may be further investigated and improved as warranted. Some of the possible research topics to improve the RGD methodology are discussed. Further researches by the third parties are encouraged.

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